Modelling Reality: Motivate Your Students!

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Summary. Many universities have established modelling activities like special lectures on modelling, seminars or project work for math students; the ECMI Modelling Week is a nice example on a European level. In the past years there is a strong (and growing) interest in the integration of modelling activities at high school level: From the results of the PISA studies we know that students as well as their teachers need to enhance their modelling literacy.

In this paper we first motivate our strong focus on real world problems when conducting modelling activities and briefly summarise the general framework of our projects. Then we introduce three examples of modelling tasks we have presented to different groups of students at high school level as well as at university level. Finally, there is a short conclusion of our experiences made during the last 20 years.

1 Introduction – Motivation: The Hidden Component in Mathematical Modelling

Many modelling projects at university as well as at high school level have been conducted by the Department of Mathematics of the University of Kaiserslautern during the last 20 years. All of them have shown a big benefit for the participating students: they recognise that they can use the mathematical tools which are taught in school (or university, respectively) to understand and solve real world problems – and this insight gives a lot of motivation to learn mathematics! The logical consequence is to look for ways to incorporate mathematical modelling of real world problems (and there is a big emphasis on **real world**, for several reasons!) into standard mathematical education in high schools and universities.

Of course the concept of mathematical modelling is by no means a new one! Nor is it the idea of letting students deal with problems where they have to use at least some techniques of modelling in order to obtain a mathematical question to be solved. At the latest from publication of the results of the PISA studies [2] various activities to introduce mathematical modelling into

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standard math education have been started. There are many good ideas but from our experience quite a lot of them miss a very important point: the problem in focus has to be realistic and the modelling process should be complete![1] In order to save time many teachers simplify the problem. This can be done in several ways, among them:

- All information which is needed to solve the problem – and only that information! – is provided from the beginning.
- Some steps of the modelling process have been done before handing the problem to the students; as a consequence the students already start with a mathematical problem or obtain such a mathematical formulation of the original question from the pre-modelled parts without having a choice.
- Data is chosen in a way which simplifies the computations – with a possible loss of realism.

In certain situations there are reasons for introducing some simplifications either way, but from our point of view the experience of a complete modelling process without simplifying the original problem is worth all the effort and time. Especially those students who have a poor interest in mathematics as they know it from school gain a lot of motivation from such an experience. Very often it shows the interdisciplinary character of mathematical modelling and the usefulness of mathematical tools which are taught in school but rarely applied in a real context. From the feedback we have got from many participants of modelling projects in schools we learned that especially the 'soft features' like *correspondence to reality*, *applicability of mathematics*, *non-uniqueness of solutions* or *allowance to follow wrong ideas which are to be improved later* make the difference.

Hence our conclusion is to offer modelling projects which show a high degree of realism. Most of these projects are done in a compact form (1.5 up to 2.5 full days). They start with the presentation of the projects, then the participants choose a project they like to work on and form teams with 4–5 students; at the end of the whole project all teams present their results to the others and the important point is that they have to find explanations which can be understood by the problem poser – hence these presentations usually do not contain a lot of deep mathematics! In between the students try to really understand the original problem, obtain missing information/data, set up different mathematical models and try to solve them using the tools they have learnt before. Almost all projects have in common that a computer is necessary to solve the mathematical problems the students are facing. In an ideal setting there is a supervisor for each of the modelling groups who simulates the behaviour of the problem poser (since usually we do not have those people with us all the time). The supervisor is supposed to answer the

[1] These observations have been made in German schools and universities – and even here we do not claim them to be representative – but might be similar in other countries.

questions of the students in the way the problem poser (i.e. an engineer, a biologist, someone from administration) would do. In many situations the answer is again a question to the students – hence it is really them who determine the model and the solution process. In [1] we explain our framework for doing modelling projects with students in detail.

2 Examples: Three Real World Problems of Different Type

In this section we introduce three different problems which all have a real world character. The idea of this section is not to discuss various models and solutions of these problems (there is by far not enough space to do this even for one of the projects) but to present them as we usually do for the students: using the language of the problem poser, i.e. a darts player, an alpinist or a biologist, we describe a problem to be solved. There is no mathematics in these descriptions, some terms and notions might be new for the reader and a lot of information is missing for sure. Let's start!

2.1 How to Play an Optimal Darts Game?

In some parts of the world – and many European countries can be considered to belong to them – the game of *darts* is known and many people have even played the game. In Fig. 1 a dartboard is shown. The line behind which the throwing player must stand is generally 2.37 m from the face of the dartboard measured horizontally; the centre of the board should be at a distance of 1.73 m from the floor. The dartboard is divided into 20 numbered sectors scoring from 1 to 20 points. Moreover, there are several rings with a different meaning for the scores: The outermost ring (*double* ring) doubles the score of the corresponding sector, hitting the next ring (*triple* ring) gives you three times the number of points of the corresponding sector and the centre (*bull*) which is again divided into two areas gives 25 and 50 (for the innermost circle) points.

The rules of the game – in the standard variant – are quite simple (cf. *wikipedia*): The sport of darts is usually contested between two players who take turns in throwing up to three darts. Starting from a set score, usually 501 or 301, a player wins by reducing his score to zero. The last dart in the leg must hit either a double or the inner portion of the bullseye, which is the double of the outer bull, and must reduce the score to exactly 0.

Now the question sounds simple but nevertheless it is hard to grasp for some people: How to play an optimal darts game, i.e. where should a player aim in order to win the game? For sure this depends on his or her abilities to hit certain points on the board, the corresponding abilities of the opponent and the actual score... but how does this dependence look like?

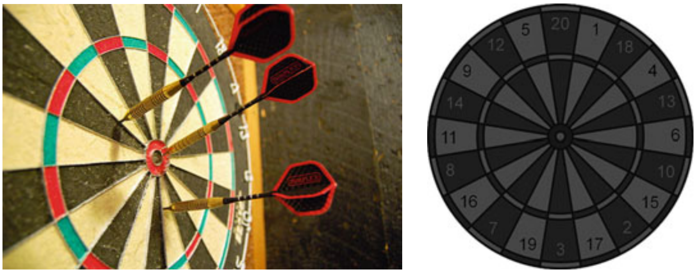

Fig. 1. Darts in a dart board (*left*) and different sectors of a dartboard (*right*)

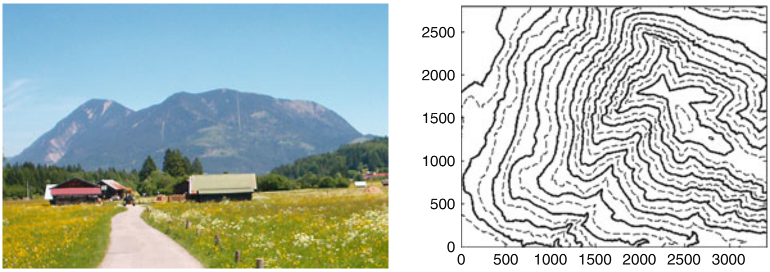

Fig. 2. *Wank* (1780m absolute altitude) near Garmisch-Partenkirchen, Germany

Most professional players aim at the triple 20, since three times 60 points would give the optimal result of 180 points per leg. But clearly, most amateur players are not able even to come close to 180! And this rises the question if the triple 20 is the optimal point to aim at – why not triple 19 or triple 14 or even the bull's eye (the central circle)?

2.2 The Optimal Way to the Top of a Hill

In Fig. 2 you can see a photo of mountain *Wank* near Garmisch-Partenkirchen together with an altitude profile (digital data based on a 1 m × 1 m grid is available). The simple question is: What is the best way in the sense of energy consumption to reach the top of the Wank?

Besides the high resolution contour data of the mountain there is some additional information regarding the relation between energy consumption when climbing a mountain and inclination of the path which was published by Italian physiologists (see Fig. 3) – but that's all the information given to the students!

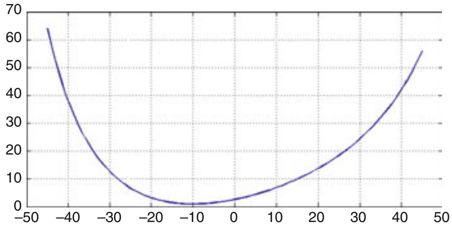

Fig. 3. Possible relation between inclination of a path (horiz. axis, angle in degree) and energy consumption of an alpinist (vert. axis, kJoule per meter height difference)

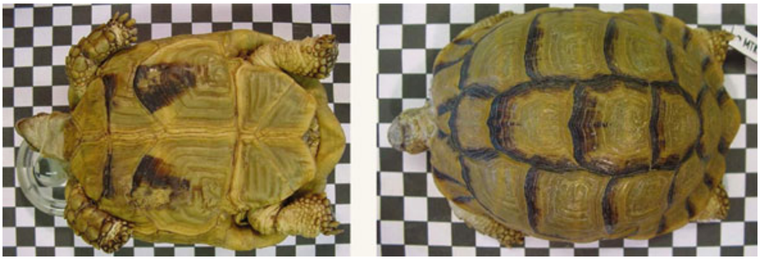

Fig. 4. Plastron & carapax of testudo kleinmanni

2.3 Design of an Identity Card for Turtles

How to identify a turtle using non-invasive methods (i.e. no transponder or other electronic devices being used)?[2] This question is motivated by CITES which is accepted by 173 countries (by October 2008). The aim of CITES is to ensure that international trade in specimens of wild animals and plants does not threaten their survival and hence it is necessary to be able to check the identity of animals! Fig. 4 shows photos of the plastron and carapax of an individual of testudo kleinmanni.

At first glance, there seem to be enough features in the images which allow for an identification of an individual. On the other hand, some animals clearly differ a lot while there are also some which are quite similar... But if you want to distinguish between 1,000,000 animals you clearly have to identify some appropriate features and determination of quantitative values has to be somewhat robust against various kinds of perturbations. Different models and solutions of student teams are discussed in [1].

2.4 It is Your Choice!

Having presented three different problems it is now your choice: which one would you like to think about or even to work on with your students? For the

[2]Original research project by biologist Dr. Carolin Bender (Kaiserslautern).

students it is very important to be able to choose a project. At the beginning of our activities we built the modelling teams based on various information about the abilities and interests of the participants. But over the years we have learnt that 'having the choice' has quite a big influence on the motivation of the students to work on a project for a longer period of time.

For all three problems various approaches exist – which will of course influence the solution and the final answer to our original question. But the very nice thing about modelling is that *the* solution does not exist (in the sense that there is only one model resulting in a certain answer)! The time you can spend for the whole process as well as the mathematical tools are going to have a very strong influence on the results.

3 Conclusions and Outlook

From our some twenty years experience of modelling with students, we can formulate two main consequences:

- The most important aspect concerning organization and implementation of modelling projects is learning by doing. Nobody would expect to become a good driver or even a pilot just by reading some books at this, quality is more or less proportional to the amount of practice (at least at the beginning. . .).
- Mathematical modelling should be integrated into teacher training including the learning by doing component, training of the supervisor role and learning how to find problems to the same extent. To achieve this one idea is to include student teachers in organization and implementation of modelling events in schools; we started to test this concept at TU Kaiserslautern some time ago and results are quite promising. A similar approach can be followed in advanced teacher trainings.

It is not our claim that the above consequences are completely new or particularly original but we have met many people thinking that having a collection of modelling problems at hand together with a good book on mathematical modelling is all they need to successfully do modelling projects with their students – and this is definitely not the case!

To conclude this paper we would like to appeal to all teachers interested in mathematical modelling of real world projects: just start modelling with your students – it is a lot of fun and it is worth one's while!

References

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2. OECD: *First Results from PISA 2000/2003/2006*, (OECD Paris, 2001/2004/ 2007)