Constructing Fuzzy Random Goal Constraints for Stochastic Fuzzy Goal Programming

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Abstract. This paper attempts to estimate the coefficient of the goal constraints through a fuzzy random regression model which plays a pivotal role in solving a stochastic fuzzy additive goal programming. We propose the two phase-based solutions; in the first phase, the goal constraints are constructed by fuzzy random-based regression model and, in the second phase, the multi-objective problem is solved with a stochastic fuzzy additive goal programming model. Further, we apply the model to a multi-objective decision-making scheme's use in palm oil production planning and give a numerical example to illustrate the model.

1 Introduction

In the formation of a classical goal programming model, goal constraints are formed for each associated objective under the consideration of the contained goal variables. The goal variables measure the deviation between goal levels and actual outcomes. The general formulation of goal constraints is written as $\sum_{j=1}^{n} a_{ij}X_j + d_i^- - d_i^+$ for $i = 1,...,m$ where, X_j denotes the decision variable, a_{ij} is the coefficient of the j^{th} decision variable, g_i is the target goal, and d_i^+ and d_i^- are the positive and the negative deviations with non-negative value, respectively.

Usually, it is assumed that decision makers are responsible for deciding the constant values of a system model; i.e., the relative weight, the actual values for the coefficients a_{ij} , and the goal target values of g_i . These, however, might contain errors, which should ruin the formulation of the model [15]. A number of studies have suggested methods in order to minimize these potential defects. For instance, analytic hierarchy process [18] and conjoint analysis [2] are used to determine the relative weights or priorities of the goal. Furthermore, logarithmic transformations

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of goal variables [14], input-output analysis [5] and regression analysis [7] have also been used to estimate the coefficients of goal constraints.

When the decision makers are responsible to decide the model's coefficient, it makes these decisions crucial and influential to the model's result. Nevertheless, the model coefficients are not exactly known, as relevant data is sometimes not given or is sometimes difficult to obtain or estimate. Therefore, the estimation of the goal constraint's coefficient in the fuzzy additive-goal programming model by using fuzzy random variable is proposed by Nureize and Watada to solve the multi-objective problem [12]. In addition, decision makers are assumed to provide the value of these coefficients from historical data or by the statistical inference, though this is sometimes uncertain [13]. Hence, it is more realistic to consider that estimated values of the coefficients are imprecise rather than precise value. Additionally, the given data may meanwhile include either stochastic information or fuzzy information.

Consequently, fuzzy set theory was introduced in the goal programming to deal with such imprecision. The use of fuzzy set theory in the goal programming was first formulated by [3, 4, 11]. Tiwari *et al.* have illustrated various aspects of decision problem using fuzzy goal programming [17]. Unlike conventional goal programming, which requires a decision maker to set definite aspiration values for each objective, fuzzy goal programming is treated in a flexible manner when specifying the aspiration values. Moreover, randomness in a goal programming problem results in a stochastic goal programming problem. A stochastic approach to the goal programming has put forward by Contini [1] to deal with the problem of attaining a set of targets (goals) with the sub-goals. The stochastic goal programming scheme with estimated parameters is proposed by Sengupta [16] and stochastic fuzzy goal programming in addressing the randomness in fuzzy goal programming problems has been addressed by Iskander [6].

Since it is sometimes difficult to estimate the coefficients of goal constraints in such situations, mathematical analysis is used to decide these coefficients by using statistical data. Hence, this paper attempts to estimate the coefficients a_{ij} of decision variables and further develop the fuzzy random model for the goal constraints. The objectives of this study are twofold. First, fuzzy random regression model is used to estimate the coefficients. Second, the stochastic fuzzy additive goal programming is then used to solve the multi-objective linear problem where the goal constraints are developed by mean of fuzzy random regression models.

The remainder of this paper is divided into five sections. Section 2 explains the fuzzy regression model based on fuzzy random variables. Section 3 describes the solution method for stochastic fuzzy goal programming where the goal constraints are developed by fuzzy random regression model. The numerical example to palm oil production planning is illustrated in Section 4. Section 5 covers conclusions.

2 Fuzzy Random Regression Model

Given a universe Γ, let *Pos* be a possibility measure defined on the power set $P(\Gamma)$ of Γ. Let $\Re$ be the set of real numbers. A function $Y : \Gamma \to \Re$ is said to be a fuzzy variable defined on Γ(see [10]). The possibility distribution μ_Y of Y is defined by $\mu_Y(t) = Pos\{Y = t\}$, $t \in \Re$, which is the possibility of event $\{Y = t\}$. For fuzzy variable Y with possibility distribution μ_Y, the possibility and necessity of event $\{Y \leqslant r\}$ are given, respectively, in the following forms:

$$\begin{aligned} \text{Pos}\{Y \leqslant r\} &= \sup_{t \leq r} \mu_Y(t), \\ \text{Nec}\{Y \leqslant r\} &= 1 - \sup_{t > r} \mu_Y(t). \end{aligned} \tag{1}$$

From [8], we can define the expectation based on an average of possibility and necessity. The motivation behind the introduction of the expectation is to develop a sound aggregate of the extreme cases such as the possibility (expressing a level of overlap) and necessity (articulating a degree of inclusion). The expected value of a fuzzy variable is presented as follows:

Definition 2.1. Let Y be a fuzzy variable. The expected value of Y is defined as

$$\boldsymbol{E}[Y] = \begin{array}{l} \int_0^\infty \left(\frac{1}{2} \left[1 + \sup_{t \geq r} \mu_Y(t) - \sup_{t < r} \mu_Y(t) \right] \right) dr \\ - \int_{-\infty}^0 \left(\frac{1}{2} \left[1 + \sup_{t \leq r} \mu_Y(t) - \sup_{t > r} \mu_Y(t) \right] \right) dr \end{array} \tag{2}$$

under assumption that the two integrals are finite. Making use of (2), we determine the expected value of Y to be $\boldsymbol{E}[Y] = \frac{a^l + 2c + a^r}{4}$ when Y is a triangular fuzzy number (c, a^l, a^r). What follows here is the definition of fuzzy random variables and their expected value operators.

Definition 2.2. Suppose that (Ω, Σ, Pr) is a probability space and F_v is a collection of fuzzy variables defined on possibility space $(\Gamma, P(\Gamma), Pos)$. A fuzzy random variable is a map $x : \Omega \to F_v$ such that for any Borel subset B of $\Re$, $Pos\{X(\omega) \in B\}$ is a measurable function of ω.

Let X be a fuzzy random variable on Ω . From the above definition, we know that, for each $\omega \in \Omega, X(\omega)$ is a fuzzy variable. Furthermore, a fuzzy random variable X is said to be positive if, for every ω, X is almost surely positive.

Let V be a random variable on probability space (Ω, Σ, Pr). Define that for every $\omega \in \Omega$, $X(\omega) = (V(\omega) - 2, V(\omega) + 2, V(\omega) + 6)_\Delta$ which is a triangular fuzzy variable on some possibility space $(\Gamma, P(\Gamma), Pos)$. Therefore, X is a (triangular) fuzzy random variable.

For any fuzzy random variable X on Ω, the expected value of the fuzzy variable $X(\omega)$ is denoted by $E[X(\omega)]$, which has been proved to be a measureable function of Ω; i.e., it is a random variable. *Theorem* 1 [9]. Given this, the expected value of the fuzzy random variable X is defined as the mathematical expectation of the random variable $E[X(\omega)]$.

Definition 2.3. Let X be a fuzzy random variable defined on a probability space (Ω, Σ, Pr). The expected value of X is defined as

$$\boldsymbol{E}[X] = \int_{\Omega} \begin{bmatrix} \int_{0}^{\infty} \left(\frac{1}{2} \left[1 + \sup_{t \geq r} \mu_{Z(\omega)^{(t)}} - \sup_{t < r} \mu_{Z(\omega)^{(t)}} \right] \right) dr \\ - \int_{-\infty}^{0} \left(\frac{1}{2} \left[1 + \sup_{t \leq r} \mu_{Z(\omega)^{(t)}} - \sup_{t > r} \mu_{Z(\omega)^{(t)}} \right] \right) dr \end{bmatrix} Pr(d\omega). \tag{3}$$

Definition 2.4. Let X be a fuzzy random variable defined on a probability space (Ω, Σ, Pr) with expected value e. The variance of X is defined as

$$Var[X] = E[(X-e)^2] \tag{4}$$

where $e = E[X]$ given by Definition 3.

Fuzzy random data, denoted as Y_i, X_{ik} for all $i = 1, \cdots, N$ and $k = 1, \cdots, K$ defined as

$$Y_i = \bigcup_{t=l}^{M_{Y_i}} \left\{ (Y_i^t, Y_i^{t,l}, Y_i^{t,r})_\triangle, p_i^t \right\} \tag{5}$$

$$X_{ik} = \bigcup_{t=l}^{M_{X_{ik}}} \left\{ (X_{ik}^t, X_{ik}^{t,l}, X_{ik}^{t,r})_\triangle, q_{ik}^t \right\} \tag{6}$$

respectively. That means all values are given fuzzy variables with probabilities, where fuzzy variables $(Y_i^t, Y_i^{t,l}, Y_i^{t,r})_\triangle$ and $(X_{ik}^t, X_{ik}^{t,l}, X_{ik}^{t,r})_\triangle$ are obtained with probability p_i^t and q_{ik}^t for $i = 1, \cdots, N$, $k = 1, \cdots, K$ and $t = 1, \cdots, M$ or $t = 1, \cdots, M_{X_{ik}}$ respectively.

Let us denote a fuzzy linear model with fuzzy coefficients $\overline{A}_i^*, \cdots, \overline{A}_K^*$ as follows:

$$\overline{Y}_i^* = \overline{A}_i^* X_{i1} + \cdots + \overline{A}_K^* X_{iK}, \tag{7}$$

where each $\overline{Y}_i^*$ denotes an estimate of the output and $\overline{A}_k^* = (\frac{[\overline{A}_k^{*l} + \overline{A}_k^{*r}]}{2}, \overline{A}_k^{*l}, \overline{A}_k^{*r})_\triangle$ are symmetric triangular fuzzy coefficients when triangular fuzzy random data X_{ik} are given for $i = 1, \cdots, N$, $k = 1, \cdots, K$.

The input data $X_{ik} = (x_{ik}, x_{ik}^l, x_{ik}^r)_\triangle$ and output data $Y_i = (y_i, y_i^l, y_i^r)_\triangle$ for all and $i = 1, \cdots, N$, $k = 1, \cdots, K$ are fuzzy random variables. Therefore, the following relation should hold:

$$\overline{Y}_i^* = \overline{A}_i^* X_{i1} + \cdots + \overline{A}_K^* X_{ik} \supset_{FR} Y_i, \ i = 1, \cdots, N \tag{8}$$

where $\supset_{FR}$ is a fuzzy random inclusion relation.

Let us use the one-sigma $(1 \times \sigma)$ confidence interval to express the confidence interval, which is induced by the expectation and variance of a fuzzy random variable as follows:

$$I\,[e_X, \sigma_X] \triangleq \left[E(X) - \sqrt{var(X)}, E(X) + \sqrt{var_(X)} \right] \tag{9}$$

Hence, the fuzzy random regression model with $\sigma-$confidence intervals is described as follows:

$$
\left.\begin{array}{l}
\min_{\overline{A}} \;\; J(\overline{A}) = \sum_{k=1}^{K} (\overline{A}_k^r - \overline{A}_k^l) \\
\qquad\qquad \overline{A}_k^r \geq \overline{A}_k^l, \\
\qquad\qquad \overline{Y}_i^* = \overline{A}_i^* I[e_{X_{i1}}, \sigma_{X_{i1}}] + \cdots + \overline{A}_K^* I[e_{X_{iK}}, \sigma_{X_{iK}}] \supset_{\tilde{h}} I[e_{Y_i}, \sigma_{Y_i}], \\
\qquad\qquad i = 1, \cdots, N;\; k = 1, \cdots, K.
\end{array}\right\} \quad (10)
$$

The inclusion relation should be written as follows.

$$
\left.\begin{array}{l}
\overline{Y}_i^* + \{e_{X_{iK}} + \sigma_{X_{iK}}\} \leq (\overline{A}_K^r \cdot \{e_{X_{iK}} + \sigma_{X_{iK}}\})^T, \\
\overline{Y}_i^* - \{e_{X_{iK}} - \sigma_{X_{iK}}\} \geq (\overline{A}_K^l \cdot \{e_{X_{iK}} - \sigma_{X_{iK}}\})^T.
\end{array}\right\} \quad (11)
$$

Given this, finding the value of the solution of the problem may rely on some heuristics proposed in [19]. The solution of the fuzzy random regression model with confidence interval can be rewritten as a problem of samples with one output and input interval values [20, 21].

3 The Solution

In this section we introduce the stochastic based fuzzy additive goal programming (SFaGP) with fuzzy random goal constraints. In the first part, stochastic fuzzy goal programming with additive model was explained, and followed by the solution of SFaGP model with fuzzy random goal constraints.

3.1 Stochastic Based Fuzzy Additive Goal Programming

The additive modeling in stochastic fuzzy goal programming was explored by employing the usual addition as an operator to aggregate the fuzzy stochastic goals in the conventional fuzzy additive goal programming. The stochastic problem is reformulated based on the additive model [17] and defined as follows:

$$
\left.\begin{array}{ll}
\text{Find} & X, \\
\text{to satisfy} & G_i(X) \tilde{\geq}'' g_i; \;\; i = 1, \cdots, m \\
\text{subject to} & AX \leq b, \\
 & X \geq 0.
\end{array}\right\} \quad (12)
$$

where $\tilde{\geq}''$ refers to the stochastic and fuzzification of the aspiration level. A linear membership function for the fuzzy stochastic goal $G_i(X) \tilde{\geq}'' g_i$ is given according to [22]. The membership function is described as follows:

$$
\mu_i = \begin{cases} 1 & \text{if} \;\; \widetilde{G}_i(X) \leq d_{ij} \\ \frac{G_i(X) - d_{ij}}{g_{ij} - d_{ij}} & \text{if} \;\; g_{ij} \leq \widetilde{G}_i(X) \leq d_{ij} \\ 0 & \text{if} \;\; \widetilde{G}_i(X) \geq g_{ij} \end{cases} \quad (13)
$$

where $d_{ij} < g_{ij}$ and $\max_{j=1}^{m} d_{ij} < \min_{j=1}^{m} g_{ij}$.

The stochastic fuzzy additive goal programming (SFaGP) is given by adding the memberships function together as:

$$\left.\begin{array}{lll} \max & V(\mu)=\sum_{i=1}^{m} E[\mu_i(G_i(X))] \\ \text{subject to} & E[\mu_i(G_i(X))] \geq \alpha_i, \\ & AX \leq b, \quad \mu_i \leq 1, \\ & X,\mu \geq 0; \quad i=1,\cdots,m \end{array}\right\} \tag{14}$$

where $E[\mu_i(G_i(X))]$ denotes the expectation of $\mu_i(G_i(X))$ and α_i is a satisfactory threshold determined by the decision maker. Let us denote g_{ij} as the target value, d_{ij} as the tolerance or aspiration level, and p_{ij} as the probability decided for g_i. The value is decided by the decision maker with $d_{ij} \leq g_{ij}$. When the goal constraints $E[\mu_i(G_i(X))]$ are expressed by $\sum_{i=1}^{m} p_i^j \mu_i(G_i(X))$, model (14) is rewritten as:

$$\left.\begin{array}{ll} \max \;\; V(\mu)=\sum_{i=1}^{m} p_{ij}\mu_i(G_i(X)) \\ \text{subject to} \quad \sum_{i=1}^{m} p_{ij}\mu_i\left(\dfrac{G_i(X)-d_{ij}}{g_{ij}-d_{ij}}\right) \geq \alpha_i, \\ \qquad \max\{d_{ij}\} \leq \mu_i(G_i(X)) \leq \min\{g_{ij}\}, \\ \qquad AX \leq b; \quad x_i \geq 0; \quad i=1,\cdots,m. \end{array}\right\} \tag{15}$$

Considering the fuzzy stochastic decision function, μ_i and α_i are then maximized in the model (15).

3.2 Stochastic Fuzzy Additive Goal Programming with Fuzzy Random Goal Constraints

In this study we propose the fuzzy random regression model to estimate the coefficient of the goal constraints in the fuzzy goal programming definition. Using the fuzzy random variables based regression model, we estimate the coefficient $\widetilde{c}_{ij}$ for decision parameters. The solution for fuzzy random variables results in interval numbers $[a_j^l, a_j^r]$ where a^l and a^r are the lower and upper boundaries, respectively. Considering the center coefficient value $\widetilde{c}=0.5(a^l+a^r)$, the fuzzy stochastic goal constraints are turned into $\widetilde{G}_j(X)=\xi_{ij}(x_i)$ where the ξ_{ij} expresses the fuzzy random coefficient for decision variables, x_i. The SFaGP with fuzzy random goal constraints are then described as follows:

$$\left.\begin{array}{ll} \max & V(\mu)=\sum_{i=1}^{m} p_i^j \mu_i(\xi_{ij}(X)) \\ \text{subject to} & \sum_{i=1}^{m} p_i^j \mu_i\left(\dfrac{\xi_{ij}(X)-d_{ij}}{g_{ij}-d_{ij}}\right) \geq \alpha_i, \\ & \max\{d_{ij}\} \leq \mu_i(\xi_{ij}(X)) \leq \min\{g_{ij}\}, \\ & AX \leq b; \quad x_i \geq 0; \quad i=1,\cdots,m \end{array}\right\} \tag{16}$$

where $V(\mu)$ is the fuzzy achievement function or fuzzy decision function, and ξ_{ij} is fuzzy random coefficient. The proposed two-phase programming solution of the model can be illustrated as in Fig. 27.1.

Hence, the two phases of solving process are explained from the solution's steps. First, we use fuzzy random variables based regression model to build the fuzzy random goal constraint. Second we model the SFaGP consisting of fuzzy random goal constraints to solve fuzzy multi-objective problem. The subsequent chapter will provide the numerical example to illustrate the proposed models.

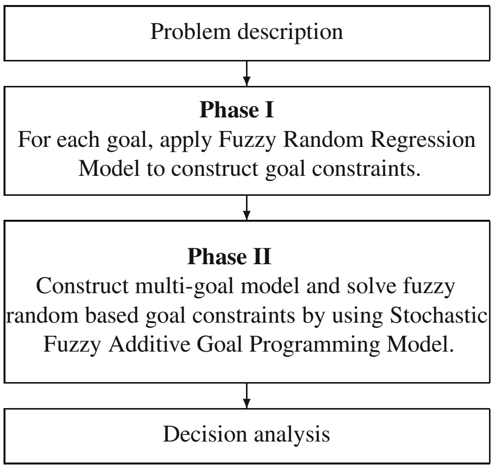

Fig. 1 A Flowchart for Solution Model

Table 1 Palm Oil Production and Profit Target

Oil Palm Product	Production		Profit	
	2009	2010	2009	2010
Crude Palm Oil	17.62	18.56	38.76	40.83
Crude Palm Kernel	2.09	2.19	4.70	5.04

4 A Numerical Example: Palm Oil Production Planning

The first step involves the problem description such as determine the decision parameters, the objectives, the constraints and the constant value for each parameters. We consider a national planning of palm oil production in Malaysia. The national target for the year of 2009 and 2010 are as in Table 27.1. Two objectives are considered; which are maximizing the total production and the profit returns. Let us assume two functional objectives are investigated under four system constraints. Two main products represents the decision variables are crude palm oil (CPO) and crude palm kernel oil (CPKO). The main resources that govern the production target are such as the fresh fruit bunch, the land area that cultivated the fruit, the mills capacity and the oil extraction rate.

The following step involves data preparation for estimating decision coefficients by using Fuzzy Random-based Regression Model. Two data sets are collected from Malaysian Palm Oil Board for CPO and CPKO production and price for 5 years from 2003 to 2007. Since two objectives were defined for this problem, two fuzzy random regression models were used to estimate the weights of decision parameters x_1 and x_2. From Equation (10), the fuzzy regression model corresponding to the input-output data was obtained as follows. The linear program of Equation (17)and Equation (18)are for oil palm production and for profit of oil palm products respectively. *Production Model:*

$$\left.\begin{array}{ll}\min_{\overline{A}} & J(\overline{A}) = \sum_{k=1}^{2} (\overline{A}_k^r - \overline{A}_k^l) \\ \text{subject to} & \overline{A}_1^r \geq \overline{A}_1^l \geq 0; \quad \overline{A}_2^r \geq \overline{A}_2^l \geq 0; \\ & (1.3221 \times 10^7) \times \overline{A}_1^l + (3.8000 \times 10^7) \times \overline{A}_2^l \leq 1.6472 \times 10^7, \\ & (1.3916 \times 10^7) \times \overline{A}_1^l + (4.3841 \times 10^7) \times \overline{A}_2^l \leq 1.7602 \times 10^7, \\ & (1.4333 \times 10^7) \times \overline{A}_1^l + (2.4976 \times 10^7) \times \overline{A}_2^l \leq 1.8111 \times 10^7, \\ & (1.5404 \times 10^7) \times \overline{A}_1^l + (3.9312 \times 10^7) \times \overline{A}_2^l \leq 1.9065 \times 10^7, \\ & (1.5600 \times 10^7) \times \overline{A}_1^l + (3.5612 \times 10^7) \times \overline{A}_2^l \leq 1.9940 \times 10^7, \\ & (1.3755 \times 10^7) \times \overline{A}_1^r + (4.1073 \times 10^7) \times \overline{A}_2^r \geq 1.7152 \times 10^7, \\ & (1.4455 \times 10^7) \times \overline{A}_1^r + (6.3622 \times 10^7) \times \overline{A}_2^r \geq 1.8290 \times 10^7, \\ & (1.4692 \times 10^7) \times \overline{A}_1^r + (3.2105 \times 10^7) \times \overline{A}_2^r \geq 1.8604 \times 10^7, \\ & (1.6039 \times 10^7) \times \overline{A}_1^r + (4.0302 \times 10^7) \times \overline{A}_2^r \geq 1.9545 \times 10^7, \\ & (1.6126 \times 10^7) \times \overline{A}_1^r + (3.9362 \times 10^7) \times \overline{A}_2^r \geq 2.0498 \times 10^7. \end{array}\right\} \quad (17)$$

Return Model:

$$\left.\begin{array}{ll}\min_{\overline{A}} & J(\overline{A}) = \sum_{k=1}^{2} (\overline{A}_k^r - \overline{A}_k^l) \\ \text{subject to} & \overline{A}_1^r \geq \overline{A}_1^l \geq 0; \quad \overline{A}_2^r \geq \overline{A}_2^l \geq 0; \\ & (1.5106 \times 10^3) \times \overline{A}_1^l + (1.5067 \times 10^3) \times \overline{A}_2^l \leq 2.9749 \times 10^3, \\ & (1.5334 \times 10^3) \times \overline{A}_1^l + (2.4746 \times 10^3) \times \overline{A}_2^l \leq 3.8034 \times 10^3, \\ & (1.3434 \times 10^3) \times \overline{A}_1^l + (2.0622 \times 10^3) \times \overline{A}_2^l \leq 3.6000 \times 10^3, \\ & (1.4498 \times 10^3) \times \overline{A}_1^l + (1.5762 \times 10^3) \times \overline{A}_2^l \leq 3.2675 \times 10^3, \\ & (2.5816 \times 10^3) \times \overline{A}_1^l + (2.9733 \times 10^3) \times \overline{A}_2^l \leq 5.2029 \times 10^3, \\ & (1.5695 \times 10^3) \times \overline{A}_1^r + (1.5285 \times 10^3) \times \overline{A}_2^r \geq 3.0170 \times 10^3, \\ & (1.5658 \times 10^3) \times \overline{A}_1^r + (3.0251 \times 10^3) \times \overline{A}_2^r \geq 3.9698 \times 10^3, \\ & (1.3818 \times 10^3) \times \overline{A}_1^r + (2.1727 \times 10^3) \times \overline{A}_2^r \geq 3.6970 \times 10^3, \\ & (1.4956 \times 10^3) \times \overline{A}_1^r + (1.7427 \times 10^3) \times \overline{A}_2^r \geq 3.3633 \times 10^3, \\ & (2.6437 \times 10^3) \times \overline{A}_1^r + (3.4558 \times 10^3) \times \overline{A}_2^r \geq 5.4196 \times 10^3. \end{array}\right\} \quad (18)$$

We obtained the optimal solution of $\overline{A}^l$ and $\overline{A}^r$ for production and profit; by solving the linear program problem of Equations (17) and (18). The interval of $[\overline{A}^l, \overline{A}^r]$ shows the estimated weight for decision parameters. Using the central values of fuzzy intervals, the fuzzy random regression models are written with confidence interval as follows:

$$\left.\begin{aligned}\overline{Y}_{production} &= \left(\frac{\overline{A}^l+\overline{A}^r}{2}\right)_T I\left[e_{X_{iK}},\sigma_{X_{iK}}\right]\\ &= (1.254)_T I\left[e_{X_1},\sigma_{X_1}\right]+(0.000)_T I\left[e_{X_2},\sigma_{X_2}\right],\end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned}\overline{Y}_{return} &= \left(\frac{\overline{A}^l+\overline{A}^r}{2}\right)_T I\left[e_{X_{iK}},\sigma_{X_{iK}}\right]\\ &= (0.856)_T I\left[e_{X_1},\sigma_{X_1}\right]+(1.100)_T I\left[e_{X_2},\sigma_{X_2}\right].\end{aligned}\right\} \tag{20}$$

The two models of Equations (19) and (20) represent the goal constraints in fuzzy goal programming. Therefore the stochastic fuzzy goal program model is then written as follows:

$$\left.\begin{array}{ll} & (1.250x_1+0.000x_2)\tilde{\geq}'' g_1\\ & (1.250x_1+0.000x_2)\tilde{\geq}'' g_2\\ \text{subject to} & \\ & (3.83x_1+0.99x_2)\leq 87.75,\\ & (x_1+x_2)\leq 4.49,\\ & (17.73x_1+2.13x_2)\leq 96.55,\\ & (x_1+x_2)\leq 20.21,\\ & \mu_i\leq 1; \quad x_i,\mu_i\geq 0.;\end{array}\right\} \tag{21}$$

In this example, two decision makers are assumed to decide the target value for each goal with the probabilities, tolerance value and the threshold value. The parameters values used in the model are as shown in Table 27.3, which including the results from fuzzy random regression derived from the first stage.

Now the stochastic fuzzy goal are converted into crisp ones by using membership function as defined in (13). Thus the problem in Model (21) reduces to

$$\left.\begin{array}{lll}\max & V(\mu) = & 0.8\frac{1.250x_1+0.000x_2-0.6}{2.075-0.6}+0.2\frac{1.250x_1+0.000x_2-0.2}{2.075-0.2}\\ & & +0.3\frac{1.126x_1+0.867x_2-0.4}{4.705-0.4}+0.7\frac{1.126x_1+0.867x_2-0.1}{4.705-0.1}\\ \text{subject to} & & 0.8\frac{1.250x_1+0.000x_2-0.6}{2.075-0.6}+0.2\frac{1.250x_1+0.000x_2-0.2}{2.075-0.2}\geq 0.25,\\ & & 0.3\frac{1.126x_1+0.867x_2-0.4}{4.705-0.4}+0.7\frac{1.126x_1+0.867x_2-0.1}{4.705-0.1}\geq 0.15,\\ & & \max_{i=1}^{2} k_1\leq 1.250x_1+0.000x_2\leq \min_{i=1}^{2} g_1,\\ & & \max_{i=1}^{2} k_2\leq 1.126x_1+0.867x_2\leq \min_{i=1}^{2} g_2,\\ & & 3.83x_1+0.99x_2\leq 87.75,\\ & & x_1+x_2\leq 4.49,\\ & & 17.73x_1+2.13x_2\leq 96.55,\\ & & x_1+x_2\leq 20.21,\\ & & x_i\geq 0; \quad i=1,2.\end{array}\right\} \tag{22}$$

Computer software LINGO was used to run the equivalent ordinary linear programming model (22).

The first stage uses the fuzzy random regression to estimate the coefficient of the goal constraints. The regression models (17) and (18) were applied to the data set of palm oil production and returns, respectively. The fuzzy random coefficients are obtained as shown in Table 27.3, where the center value is then used for solving the second stage. This result illustrates the coefficients for each attribute and shows the range of the evaluation. The result depicts that the production of CPO oil has significant contribution with weights of (1.237, 1.271) compared to the CPKO. This is related to the expert's judgment where about 90% CPO will be extracted from FFB. For the profit returns, the CPO and CPKO have the weights (0.56) and (1.006, 1.194), respectively. The interval form of the coefficient shows the flexibility which reflects the fuzzy judgment in the evaluation.

The second stage concerns solving the multi-objective problem with stochastic problem. The conventional FAGP is used by adding the stochastic properties and the goal constraints consist of fuzzy random model which was developed in the first stage. The model (9) is solved by using the simplex method. The results obtained by the proposed method are $x_1 = 1.66, x_1 = 2.83$ with achieved goal values $G_1 = 2.07$, $G_2 = 4.32$ and membership values $\mu_1 = 0.98$ and $\mu_2 = 0.93$. Hence, the first goal (production) is 98% achieved whereas the latter (revenue) is achieved by 93% .

Table 2 The Result from Fuzzy Random Regression

Goal	Fuzzy Random Coefficient ξ_{ij}	
	x_1	x_2
Production	$[1.237, 1.271]$	$[0.000, 0.000]$
Return	$[0.856, 0.856]$	$[1.006, 1.194]$

Table 3 Parameters Value for Stochastic-FaGP

Goal	Fuzzy Random Coefficient		Target	Probability	Tolerance	Threshold
	x_1	x_2	g_i	p_i	d_i	α_i
Production	1.254	0.000	$[2.075, 2.310]$	$[0.8, 0.2]$	$[0.6, 0.2]$	0.25
Return	0.856	1.100	$[4.705, 4.587]$	$[0.3, 0.7]$	$[0.4, 0.1]$	0.15

5 Conclusions

In this paper, we proposed the stochastic fuzzy additive goal programming method with fuzzy random goal constraints which consists of two phases of solution in order to solve the fuzzy stochastic multi-objective problem. The fuzzy random based regression model was introduced in the first phase to build the goal constraints, and is used to estimate the coefficient of goal constraints using historical data. The property of fuzzy random regression model is utilized to resolve the co-existence of fuzziness and randomness in the data, in which many real situations might occur. In the second phase, fuzzy stochastic multi-objective problem was solved using stochastic fuzzy additive goal and goal constraints model were derived from the first part of the solution. In this study, apart from solving the multi-objective problem, the proposed method suggests to use historic data to approximate the coefficient due to the difficulty of determining such value. The analytical results demonstrate that the proposed method using the central values of fuzzy intervals can achieve the stochastic based fuzzy goal programming. This shows that the fuzzy random regression used in the first stage of solution enables us to reduce the difficulty of determining the coefficient value for the multi-objective model as produced by the conventional fuzzy goal programming. In this study fuzzy random regression model effectively determines the coefficients value from historical data.

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