Group Decisions in Interval AHP Based on Interval Regression Analysis

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Abstract. For encouraging communication in a group decision making, this paper proposes methods to aggregate individual preferences. The individual preferences are denoted as the interval priority weights of alternatives by Interval Analytic Hierarchy Process (Interval AHP). It is proposed to handle subjective judgments since the induced results are intervals reflecting uncertainty of given information. When each decision maker gives the judgments on alternatives, the priority weights of alternatives are obtained. In the sense of reducing communication barriers, such information helps group members to realize their own preferences and the others' opinions. Then, they are aggregated based on the concept of the interval regression analysis with interval output data, where two inclusion relations between the estimations and the observations are assumed. From the possibility view, the least upper approximation model is determined so as to include all observations. While, from the necessity view, the greatest lower approximation model is determined so as to be included in all observations. The former possible aggregations are acceptable for each group member and the latter necessary ones are useful for the supervisor at the upper level of decision making.

1 Introduction

The group decision support system is discussed from the scope of AHP (Analytic Hierarchy Process). AHP is a useful method in multi-criteria decision making problems [1]. It is structured hierarchically as criteria and alternatives. The priority or weight for each element of the hierarchy is obtained by eigenvector method, given

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the pairwise comparison matrix on the elements. They are summed up to reach a final decision. The advantages of AHP are the following two points. It helps decision maker structure complex problems hierarchically. In order to determine priority weights of elements, the decision makers only pairwisely compare elements at one time and give subjective judgments directly. This paper focuses on the latter advantage of AHP from the view of reducing communication barrier, since the decision makers often have some difficulties in representing and recognizing their own opinions.

The group decision making with AHP is discussed in [2, 3, 4, 5]. In the problem setting, more than two comparison matrices are given. The definitive purpose of the group is to reach a decision, that is, to choose one alternative which seems to be acceptable and agreeable for all members. However, it is sometimes difficult to reach a consensus among group members [6]. Especially when members do not have a face-to-face consultation, there exist some barriers to understand one another. It may happen that some members may exaggerate their preferences in order to influence the group decision. In this sense, it is important to support the interpersonal information exchange, as well as to find the agreeable alternative, in the group decision making. As a preparation for the consensus, it becomes necessary to remove communication barriers by representing individual opinions simply and clearly [6, 7]. In this paper, Interval AHP in [8, 9], which is suitable to handle uncertainty of given information, plays a significant role. Then, the individual opinions are aggregated based on the concept of Interval Regression analysis [10]. The group members can see the difference of the aggregated and their own opinions easily.

This paper consists as follows. As a preliminary, the definition and properties of the interval probability which are used for normalization of intervals are explained in Section 2. At first, Interval AHP as a tool to represent each group member's preference is shown briefly in Section 3. Then, in Section 4, the approaches to aggregate individual preferences which are obtained by Interval AHP are proposed. Finally, the proposed models are tested with a numerical example in the case of a group of four decision makers in Section 5.

2 Interval Probability as Preliminary

The interval probabilities are defined by a set of intervals as follows. This definition is originally proposed in [11] and also is used in [12, 13]. The conventional crisp probabilities are extended into interval ones.

Definition 2.1. Interval probability: The set of intervals denoted as $\{W_1,\dots,W_n\}$ where $W_i = [\underline{w}_i, \overline{w}_i]$ are called interval probabilities if and only if

$$\begin{array}{l} 1)\ 0 \leq \underline{w}_i \leq \overline{w}_i,\ \forall i \\ 2)\ \sum_{i\neq j} \overline{w}_i + \underline{w}_j \geq 1,\ \forall j \\ 3)\ \sum_{i\neq j} \underline{w}_i + \overline{w}_j \leq 1,\ \forall j. \end{array} \tag{1}$$

From (1), two inequalities, $\sum_i \underline{w}_i \le 1$ and $\sum_i \overline{w}_i \ge 1$, hold. Then, (1) is regarded as the normality condition of intervals corresponding to the conventional one $\sum_i w_i = 1$. It is noted that in interval probabilities there are many combinations of crisp values whose sum is one.

The combination of a pair of interval probability sets is denoted as follows.

Property 2.1. **Combination** Assuming a pair of interval probabilities on n elements as $\{W_1^A, \ldots, W_n^A\}$ and $\{W_1^B, \ldots, W_n^B\}$ which satisfy (1), their combination is denoted as $\{W_1^{AB}, \ldots, W_n^{AB}\}$. Each of elements is an interval $W_i^{AB} = [\underline{w}_i^{AB}, \overline{w}_i^{AB}]$ denoted as.

$$\underline{w}_i^{AB} = \min\{\underline{w}_i^A, \underline{w}_i^B\} \text{ and } \overline{w}_i^{AB} = \max\{\overline{w}_i^A, \overline{w}_i^B\}. \tag{2}$$

The set of combined intervals also satisfies (1) so that it is interval probability. As for the combination of more than two sets of intervals, it is also interval probability.

[Proof] Requirement 1) in (1) is apparent. Assuming $\overline{w}_j^A < \overline{w}_j^B$, Requirement 2) is verified as follows.

$$\begin{aligned}\textstyle\sum_{i\neq j} \overline{w}_i^{AB} + \underline{w}_j^{AB} &= \textstyle\sum_{i\neq j} \max\{\overline{w}_i^A, \overline{w}_i^B\} + \min\{\overline{w}_j^A, \overline{w}_j^B\} \\ &\ge \textstyle\sum_{i\neq j} \overline{w}_i^A + \overline{w}_j^A \ge 1.\end{aligned}$$

For $\overline{w}_j^A > \overline{w}_j^B$, Requirement 2) can be proved similarly. Requirement 3) can be shown in the same way. Therefore, all requirements in (1) are satisfied. (Q.E.D.)

The average of a pair of interval probability sets is denoted as follows.

Property 2.2. **Average** Assuming a pair of interval probabilities on n elements as $\{W_1^A, \ldots, W_n^A\}$ and $\{W_1^B, \ldots, W_n^B\}$ which satisfy (1), their average is denoted as $\{\overline{W_1^{AB}}, \ldots, \overline{W_n^{AB}}\}$. Each of elements is an interval $\overline{W_i^{AB}} = [\underline{\overline{w}}_i^{AB}, \overline{\overline{w}}_i^{AB}]$ denoted as

$$\underline{\overline{w}}_i^{AB} = (\underline{w}_i^A + \underline{w}_i^B)/2 \text{ and } \overline{\overline{w}}_i^{AB} = (\overline{w}_i^A + \overline{w}_i^B)/2. \tag{3}$$

The set of average intervals $\{\overline{W_1^{AB}}, \ldots, \overline{W_n^{AB}}\}$ also satisfies (1) so that it is interval probability. It is the same for more than two sets of interval probability.

[Proof] Requirement 1) in (1) is apparent. Requirement 2) is verified as follows.

$$\begin{aligned}\textstyle\sum_{i\neq j} \overline{\overline{w}}_i^{AB} + \underline{\overline{w}}_j^{AB} &= \textstyle\sum_{i\neq j} (\overline{w}_i^A + \overline{w}_i^B)/2 + (\overline{w}_j^A + \overline{w}_j^B)/2 \\ &= \{(\textstyle\sum_{i\le j} \overline{w}_i^A + \overline{w}_j^A) + (\sum_{i\le j} \overline{w}_i^B + \overline{w}_j^B)\}/2 \ge 1.\end{aligned}$$

Similarly, Requirement 3) is verified. Then, all requirements are satisfied. (Q.E.D.)

3 Interval AHP

AHP is an approach to multi-criteria decision making problems. The problem is decomposed into hierarchy by criteria and alternatives. The choice or preferences of alternatives are induced as a final decision from the decision maker's judgments given as pairwise comparison matrix. The decision maker compares all pairs of alternatives and gives the pairwise comparison matrix for n alternatives [1].

$$A = \begin{pmatrix} 1 & \cdots & a_{1n} \\ \vdots & a_{ij} & \vdots \\ a_{n1} & \cdots & 1 \end{pmatrix} \tag{4}$$

where a_{ij} shows the importance ratio of alternative i comparing to alternative j. The comparison matrix satisfies the following relations so that the number of given comparisons is $n(n-1)/2$.

$$a_{ii} = 1(\text{identical}) \text{ and } \; a_{ij} = 1/a_{ji} \; (\text{reciprocal}) \tag{5}$$

The decision maker can give his/her judgment intuitively without caring about the relative relations of comparisons. Although it is an advantage of AHP, the given comparisons are not always consistent each other. The consistent comparisons satisfy the following transitivity relations.

$$a_{ij} = a_{ik}a_{kj}, \; \forall i, j \tag{6}$$

In the following, inconsistency means that (6) is not satisfied. The proposed models in this paper deal with such inconsistency from the possibility view [10].

In the conventional AHP, crisp priority weights are obtained from the given comparison matrix by the eigenvector method as follows [1].

$$Aw = \lambda w \tag{7}$$

where λ and w are the eigenvalue and eigenvector, respectively. Solving (7), the eigenvector corresponding to the principal eigenvalue is obtained as the priority weight vector. The weights are extended to intervals in Interval AHP [8, 9]. The given comparisons are inconsistent each other, that is, they do not always satisfy (6). In order to reflect such inconsistency, the priority weight of alternative is denoted as the interval $W_i = [\underline{w}_i, \overline{w}_i]$, $\forall i$. For their normalization, they are represented as interval probabilities so that they satisfy (1) in Definition 2.1.

The pairwise comparison is an intuitive ratio of two alternatives so that they are approximated by the interval $\frac{W_i}{W_j} = \left[\frac{\underline{w}_i}{\overline{w}_j}, \frac{\overline{w}_i}{\underline{w}_j}\right]$ where $0 < \underline{w}_i, \forall i$ and the upper and lower bounds of the approximated comparison are defined as the maximum range with respect to the two intervals.

In the approximation model the probabilities are determined so as to include the given pairwise comparisons. Thus, from the possibility view, the obtained interval probabilities satisfy the following inclusion relation which leads to the inequalities.

$$a_{ij} \in \frac{W_i}{W_j} \Leftrightarrow \frac{\underline{w}_i}{\overline{w}_j} \leq a_{ij} \leq \frac{\overline{w}_i}{\underline{w}_j} \Leftrightarrow \underline{w}_i \leq a_{ij}\overline{w}_j \text{ and } \overline{w}_i \geq a_{ij}\underline{w}_j, \; \forall i, j \tag{8}$$

The approximations by the obtained interval priority weights include the given inconsistent comparisons.

For any inconsistent comparisons, assuming $[\underline{w}_i, \overline{w}_i] = [0, 1]$, $\forall i$, the above inclusion relation (8) is apparently satisfied. A decision maker does not need to revise his intuitive judgments so as to be consistent. When a decision maker gives completely inconsistent judgments, the obtained priority weights of all alternatives are equally $[0, 1]$. It represents complete ignorance. Inconsistency among the given comparisons is reflected in the uncertainty of interval probabilities.

The constraint conditions for determining the interval probabilities are (1) and (8). In order to obtain the least uncertain probabilities, the uncertainty of interval probabilities should be minimized. The uncertainty of interval probabilities can be measured by several indices, such as widths of intervals and entropy [14]. For simplicity, the sum of widths of intervals is used in this paper. The problem to determine the interval priority weights is formulated as follows.

$$\begin{array}{ll} \mathrm{I} = \min & \sum_i (\overline{w}_i - \underline{w}_i) \\ \text{s.t. Equation (1) and Equation (8)} & \end{array} \tag{9}$$

The greater optimal objective function value is, the more uncertain the given interval priority weight becomes.

4 Group of Decision Makers

Interval AHP is introduced to the group decision making by aggregating individual opinions. Each group member gives pairwise comparisons for alternatives based on his/her subjective judgments. The comparison matrix given by the member k, where $k = 1, ..., m$, is denoted as follows.

$$A_k = \begin{pmatrix} 1 & \cdots & a_{1nk} \\ \vdots & a_{ijk} & \vdots \\ a_{n1k} & \cdots & 1 \end{pmatrix}, \ \forall k \tag{10}$$

When pairwise comparison matrices are given, they can be aggregated at the beginning stage of group decision making process. The group members can see their differences on their giving comparisons. One of the comparison aggregation approaches is taking the geometric mean of comparisons $a_{ij} = \sqrt[n]{\prod_k a_{ijk}}, \ \forall i, j$ [4, 5, 15, 16]. Since the aggregated comparison matrix satisfies (5), the eigenvector method can be also applied to it and the priority weights are obtained.

The other conceivable approach to aggregate the comparisons given by m decision makers is to take their minimum and maximum from the possibility view.

$$A_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}] = [\min_k a_{ijk}, \max_k a_{ijk}], \ \forall i, j \tag{11}$$

The interval priority weights are obtained from the aggregated interval comparison matrix by replacing the inclusion constraints (8) into

$$A_{ij} \in \frac{W_i}{W_j} \Leftrightarrow \frac{\underline{w}_i}{\overline{w}_j} \leq \underline{a}_{ij} \text{ and } \overline{a}_{ij} \leq \frac{\overline{w}_i}{\underline{w}_j}. \tag{12}$$

The inclusion relation (12) with interval comparisons is an extension of (8) with crisp ones. The aggregated comparisons are interval and included in the approximated ones by the interval priority weights.

In these methods, it is simple to aggregate the given comparisons directly. However, a decision maker might slip on in giving his/her judgment on a pair of alternatives and also he/she has no chance to check his own preferences on alternatives.

In the following section, each individual preference is induced beforehand and then they are aggregated. First, the priority weights of alternatives are obtained from the individually given pairwise comparison matrix. The interval priority weights based on the comparison matrix given by kth decision maker are denoted as $[\underline{w}_{ik}, \overline{w}_{ik}]$. Each decision maker can realize his/her priority weights on the alternatives, as well as others'. Then, in order to reach a consensus of the group, the obtained individual priority weights are aggregated.

If there is some information about the importance of each group member, it is reasonable to take it into consideration [17]. In the following, they are aggregated based on the concepts of interval regression analysis, which is so called the least upper and greatest lower approximations. The basic concept is that the estimations should be obtained so as to be the nearest to the individual preferences. By the former method, they include all the given intervals, which are the individual interval priority weights. While by the latter method, they are obtained so as to be included in each given interval. We do not need to be given nor calculate the importance weights of group members.

4.1 Least Upper Approximation Model

From the view of possibility, the aggregated interval priority weights W_i and the individually obtained ones W_{ik} should satisfy the following inclusion relation.

$$W_i = [\underline{w}_i, \overline{w}_i] \supseteq W_{ik} = [\underline{w}_{ik}, \overline{w}_{ik}] \Leftrightarrow \underline{w}_i \leq \underline{w}_{ik} \text{ and } \overline{w}_{ik} \leq \overline{w}_i, \ \forall i,k \tag{13}$$

The difference between two intervals W_i and W_{ik} can be measured as follows.

$$c_{ik} = \max(\underline{w}_{ik} - \underline{w}_i, \overline{w}_i - \overline{w}_{ik}) \tag{14}$$

One of the well-known definitions of difference is the sum of deviations of the upper and lower bounds; $c_{ik} = (\underline{w}_{ik} - \underline{w}_i) + (\overline{w}_i - \overline{w}_{ik})$. Let's assume the two cases shown in Fig.1, one is that each bound is overestimated as α, and the other is that only the upper bound is overestimated as 2α. Although the sums of deviations of both bounds of ${}^1W_i$ and ${}^2W_i$ are the same, for decision maker k they seem to be different. The former ${}^1W_i$ seems to fit his/her intuitive sense and reflect his/her preference more than the latter ${}^2W_i$. ${}^1W_i$ includes the decision maker's preference W_{ik} at its center, while ${}^2W_i$ includes it at its left. The maximum of deviations of ${}^1W_i$ is less than that of ${}^2W_i$. By using the maximum of deviations of the upper and lower bounds, the aggregated interval tends to include each group member's preferences at its center. Therefore, in our setting, it is more suitable to measure difference by the maximum of deviations than the sum of deviations.

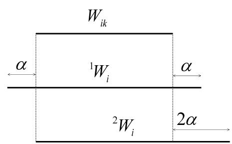

Fig. 1 Aggregated intervals including individual intervals

The problem to determine the aggregated interval priority weights is formulated.

$$\begin{array}{c} \min \sum_{ik} \max\{\underline{w}_{ik} - \underline{w}_i, \overline{w}_i - \overline{w}_{ik}\} \\ \text{s.t. Equation (1) and Equation (13)} \end{array} \tag{15}$$

The sum of deviations between all comparisons and priority weights is minimized by the objective function and the possible aggregations are obtained. Since the aggregated ones should be normalized, the conditions of interval probabilities (1) are added to the constraints. The aggregations W_i include the given intervals W_{ik} with minimum width so that they are called the least upper approximations.

The combinations of the interval priority weights by all group members are $W_i^* = [\underline{w}_i, \overline{w}_i] = [\min_k \underline{w}_{ik}, \max_k \overline{w}_{ik}]$, $\forall i$. By Property 2.1, they are interval probability. Therefore, they are apparently the optimal solutions of (15) and there is no need to solve the above problem.

This model is based on possibility concept so that the aggregated preference includes all group members' preferences. Then, from the view of each group member, the aggregations are easy to accept. It works well on the assumption that all the group members give reasonable information. When the individual preferences are very different one another such as in a big group, the widths of the aggregated priority weights become large, that is, they are uncertain. Even if there is only one outlier, who gives apparently different preference from the others, the aggregated preferences highly depend on his/her preference. In order to reduce such an influence, the method to exclude outliers is proposed [17]. From the view of the supervisor who refers the results at the upper level of decision making process, such uncertain aggregated preference is not useful. In the next section, the model which induces the less uncertain aggregated preferences is proposed.

Remark 4.1. We obtain wide interval weights by the least upper approximation model. Indeed, we can show that the sum of widths of the obtained interval weights is usually larger than or equal to that obtained by a method based on (12).

Namely, from the constraints on inclusion relations (8) in (9) and (13) in (15), we have

$$\underline{w}_i \leq \underline{w}_{ik} \leq a_{ijk}\overline{w}_{jk} \leq a_{ijk}\overline{w}_j \text{ and } \overline{w}_i \geq \overline{w}_{ik} \geq a_{ijk}\underline{w}_{jk} \geq a_{ijk}\underline{w}_j,\ \forall i,j,k. \tag{16}$$

Therefore, we have

$$\underline{w}_i \leq \left(\min_k a_{ijk}\right)\overline{w}_j = \underline{a}_{ij}\overline{w}_j \text{ and } \overline{w}_i \geq \left(\max_k a_{ijk}\right)\underline{w}_j = \overline{a}_{ij}\underline{w}_j,\ \forall i,j. \tag{17}$$

This means that a feasible solution of (15) satisfies the constraints of the problem based on (12), i.e.,

$$\begin{array}{rl} \min & \sum_i(\overline{w}_i - \underline{w}_i) \\ \text{s.t.} & \underline{w}_i \leq \underline{a}_{ij}\overline{w}_j\ \forall(i,j) \\ & \overline{w}_i \geq \overline{a}_{ij}\underline{w}_j\ \forall(i,j) \\ & \text{Equation (1).} \end{array} \tag{18}$$

Because of the interval weights obtained by a method based on (12) are an optimal solution to (18), the sum of widths of the interval weights obtained from (15) is usually larger than or equal to that obtained by a method based on (12).

4.2 Greatest Lower Approximation Model

On the other hand, from the view of necessity, the aggregated interval priority weights are included in the individually given ones.

$$W_i = [\underline{w}_i, \overline{w}_i] \subseteq W_{ik} = [\underline{w}_{ik}, \overline{w}_{ik}] \Leftrightarrow \underline{w}_{ik} \leq \underline{w}_i \text{ and } \overline{w}_i \leq \overline{w}_{ik},\ \forall k \tag{19}$$

It is not always possible to find W_i included in W_{ik}. For instance, the individual priority weights are all crisp; $\underline{w}_{ik} = \overline{w}_{ik} = w_{ik}\forall k$ and they are different one another; $w_{ik} \neq w_{ik'}$. By relaxing each individual interval $[\underline{w}_{ik}, \overline{w}_{ik}]$ into $[\underline{w}_{ik} - \underline{d}_{ik}, \overline{w}_{ik} + \overline{d}_{ik}]$, the inclusion (19) can be satisfied. The width is enlarged and $\underline{d}_{ik}$ and $\overline{d}_{ik}$ are the positive variables and should be minimized.

Similarly to (14) the difference between two intervals W_i and W_{ik} is measured.

$$c_{ik} = \max\{\underline{w}_i - (\underline{w}_{ik} - \underline{d}_{ik}), (\overline{w}_{ik} + \overline{d}_{ik}) - \overline{w}_i\} \tag{20}$$

In Fig.2, two intervals included in the individual preference are shown and they are different by means of the maximum of deviations. The individual preference W_{ik} is underestimated to obtain the aggregated one ${}^1W_i$ or ${}^2W_i$. The difference is considered to represent the degree of compromise of each group member. From the view of compromise, ${}^1W_i$, which locates more centered, looks better than ${}^2W_i$. The advantage of using the maximum of deviations is that the aggregated preference can be located at more centered of the individual one.

Considering this assumption, the problem to determine the aggregated interval priority weights is as follows.

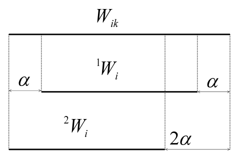

Fig. 2 Aggregated intervals being included in individual intervals

$$\begin{array}{ll} \min & \sum_{i,k}\{(\overline{d}_{ik}+\underline{d}_{ik})+\varepsilon\max\{\underline{w}_i-(\underline{w}_{ik}-\underline{d}_{ik}),(\overline{w}_{ik}+\overline{d}_{ik})-\overline{w}_i\}\} \\ \text{s.t.} & \underline{w}_{ik}-\underline{d}_{ik}\le\underline{w}_i\ \forall i,k \\ & \overline{w}_i\le\overline{w}_{ik}+\overline{d}_{ik}\ \forall i,k \\ & \underline{d}_{ik},\overline{d}_{ik}\ge 0 \\ & \text{Equation (1)} \end{array} \tag{21}$$

where variables are the bounds of the aggregated intervals, $\underline{w}_i$ and $\overline{w}_i$, and the added parts of individually given intervals, $\underline{d}_{ik}$ and $\overline{d}_{ik}$. By the objective function, primarily the added parts of the individual preference and secondary the difference of two intervals are minimized.

(21) is reduced to the following LP problem by adding new variable w_{ik}, which constrains the maximum deviation.

$$\begin{array}{ll} \min & \sum_{i,k}\{(\overline{d}_{ik}+\underline{d}_{ik})+\varepsilon w_{ik}\} \\ \text{s.t.} & \underline{w}_i-(\underline{w}_{ik}-\underline{d}_{ik})\le w_{ik}\ \forall i,k \\ & (\overline{w}_{ik}+\overline{d}_{ik})-\overline{w}_i\le w_{ik}\ \forall i,k \\ & \text{constraints of (21)} \end{array} \tag{22}$$

The necessary aggregations are obtained and they are included in the enlarged given intervals with maximum width so that they are called the greatest lower approximations. The aggregated preferences do not depend on the outlier too much. Although each group member has to compromise to some extent, from the view of the supervisor at the upper level of decision making process, such less uncertain information based on the necessity concept is useful.

4.3 Least Squares Model

In the former two sections, the inclusion relation between the aggregated and individual preferences is assumed. In this section, such an assumption is excluded so that the difference of two intervals can be defined as follows.

$$c_{ik}=\sum\{(\underline{w}_i-\underline{w}_{ik})^2+(\overline{w}_i-\overline{w}_{ik})^2\} \tag{23}$$

By minimizing the sum of difference for all alternatives and all group members, the problem to determine the aggregated interval priority weights is formulated.

$$\begin{array}{l} \min \sum_{i,k}\{(\underline{w}_i - \underline{w}_{ik})^2 + (\overline{w}_i - \overline{w}_{ik})^2\} \\ \text{s.t. Equation (1)} \end{array} \quad (24)$$

Using squared deviations avoids any of bounds of the aggregation to be extremely far from each of the individual preference. It does not matter whether the aggregations locate inside or outside of the individual preferences. The object is simple, that is, to determine the upper and lower bounds of the aggregated interval so as to be close to those of the individual intervals as possible. Comparing the upper or lower bounds of the individual preferences and the aggregation, some members' preferences are underestimated and the others' are overestimated.

The interval whose bounds consist of the average of the individually given interval priority weights are $W_i^* = [\sum_k \underline{w}_{ik}/n, \sum_k \overline{w}_{ik}/n]\ \forall i$. By Property 2.2, they are interval probability. Their sum of squared deviations from the individual intervals is the minimum. Therefore, they are the optimal solutions of (24) so that there is no need to solve the above problem.

5 Numerical Example

Assuming four decision makers, $k = 1,2,3,4$, each of them gives the pairwise comparison matrix on four alternatives, $i = 1,2,3,4$ and the obtained interval priority weights W_k by (15) are shown in Table 1. All decision makers roughly think that alternative 1 and 4 are the most and least preferable. The priority weights of the alternative 1 by all group members are crisp. For comparison, at the right column of Table 1, the crisp priority weights by (7) and C.I. which represents consistency of the pairwise comparison matrix in the sense of eigenvector method are shown. If C.I.=0, then the obtained priority weights by (7) and (15) are crisp and the same.

The individual preferences are aggregated by the proposed three methods and the aggregations are shown in Table 2. Based on the least upper approximation model (15), all the individual preferences are considered to be possible. Since the obtained interval priority weight of each alternative by each decision maker is included in the aggregated intervals, the aggregated preferences are acceptable from the group members' viewpoints. However, the widths of the aggregated intervals tend to be too large, in case that the individual priority weights diverse.

When it comes to be referred the aggregated preferences at the upper level of decision making process by a supervisor, such uncertain information is not useful. Based on the greatest lower approximation model (22), the necessity parts of the individual preferences are focused. The aggregated interval priority weight is determined so as to be included in the obtained interval priority weights by all group members as much as possible. Instead of being included, the aggregated priority weights of alternatives 1 and 2 are between the four individually given crisp priority weights. Each group member compromises to some extent, which is measured by the maximum of deviations of the upper and lower bounds. From the view of the supervisor, such less uncertain information from the necessity view is preferable.

By the third model (24), the inclusion relations of the individual and aggregated preferences are not assumed. The difference is measured by the sum of squared deviations of both bounds by all group members. The interval priority weights of alternative 3 by A_1 and A_4 are included in and that by A_2 includes the aggregated one. As for A_3, its upper bound is underestimated, while its lower bound is overestimated. The aggregated intervals by the least squares model are between those by the least upper and greatest lower models.

Table 1 Comparison matrices by four decision makers

A_1	W_1 I=0.083	w_1C.I.=0.010	A_2	W_2 I=0.350	w_2 C.I.=0.081
1 2 3 4	0.500	0.402	1 1 4 6	0.390	0.470
1 2 3	0.250	0.337	1 3 4	[0.244,0.390]	0.255
1 2	[0.125,0.167]	0.164	1 4	[0.098,0.244]	0.192
1	[0.083,0.125]	0.097	1	[0.065,0.122]	0.083

A_3	W_3 I=0.283	w_3 C.I.=0.102	A_4	W_4 I=0.375	w_4 C.I.=0.150
1 3 3 4	0.571	0.487	1 1 2 2	0.375	0.260
1 3 3	[0.190,0.214]	0.269	1 3 1	[0.219,0.375]	0.335
1 4	[0.071,0.190]	0.168	1 3	[0.125,0.188]	0.246
1	[0.048,0.143]	0.076	1	[0.063,0.219]	0.159

Table 2 Aggregated interval priority weights by three methods

	Least upper	Greatest lower	Least suquare
A_1	[0.375,0.571]	0.5	0.459
A_2	[0.190,0.390]	0.250	[0.226,0.307]
A_3	[0.071,0.244]	[0.128,0.167]	[0.105,0.197]
A_4	[0.048,0.219]	[0.083,0.122]	[0.065,0.152]

6 Conclusion

The group decision support system based on Interval AHP has been discussed focusing on the aggregation of individual preferences. By Interval AHP the priority weights of elements are obtained as interval from the pairwise comparison matrix given by a decision maker based on his/her intuitive judgments. The obtained interval priority weights reflect all the possibilities in the given information. Interval AHP is one of the useful tools for them to realize their preferences as well as others. At first, the individual preferences are obtained and then they are aggregated based on the concept of interval regression analysis. From the possibility view by the least upper approximation model, the aggregations are determined so as to include the individual preferences. The obtained possible aggregations are easily acceptable for each group member. They are the same as the combinations of all group members' interval priority weights. From the necessity view by the greatest lower approximation model, the aggregations are determined so as to be roughly included in the individual preferences. Since the necessary aggregations are less uncertain, they are useful for the supervisor. Without assuming the inclusion relations, the aggregations are also obtained by the least squares model, where the sum of squared deviations is minimized. The obtained aggregations are the same as the average of all group members' interval priority weights. Although the suitable approach depends on the situations, the proposed three methods to aggregate group members' preferences help them understand one another and reach consensus.

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