A Variable-Capacity-Based Fuzzy Random Facility Location Problem with VaR Objective

Shuming Wang, Junzo Watada, and Shamshul Bahar Yaakob

Abstract. In this paper, a Value-at-Risk (VaR) based fuzzy random facility location model (VaR-FRFLM) is built in which both the costs and demands are assumed to be fuzzy random variables, and the capacity of each facility is unfixed but a decision variable. A hybrid approach based on modified particle swarm optimization (MPSO) is proposed to solve the VaR-FRFLM. In this hybrid mechanism, an approximation algorithm is utilized to compute the fuzzy random VaR, a continuous Nbest-Gbest-based PSO and a genotype-phenotype-based binary PSO vehicles are designed to deal with the continuous capacity decisions and the binary location decisions, respectively, and two mutation operators are incorporated into the PSO to further enlarge the search space. A numerical experiment illustrates the application of the proposed hybrid MPSO algorithm and lays out its robustness to the parameter settings when dealing with the VaR-FRFLM.

1 Introduction

Facility location selection is a kind of optimization problems which aim to maximize the return or minimize the costs via determining the locations of facilities to open from a set of potential sites. Various kinds of facility location problems

Shuming Wang · Junzo Watada
Graduate School of IPS, Waseda University, 2-7 Hibikino, Wakamatsu,
Kitakyushu 808-0135, Fukuoka, Japan
e-mail: smwangips@gmail.com, junzow@osb.att.ne.jp

Shamshul Bahar Yaakob
Graduate School of IPS, Waseda University, 2-7 Hibikino, Wakamatsu,
Kitakyushu 808-0135, Fukuoka, Japan
and
School of Electrical Systems Engineering, Universiti Malaysia Perlis, 02600 Jejawi, Perlis, Malaysia
e-mail: shamshul@fuji.waseda.jp

V.-N. Huynh et al. (Eds.): Integrated Uncertainty Management and Applications, AISC 68, pp. 209–220.
springerlink.com

under uncertainty have been investigated in the literature. The first category of such uncertain location is the stochastic facility location problems which deal with the cases when the uncertain parameters, like customers' demands and operating costs of plants, are characterized by random variables. For the details on such facility location problems, one may refer to [1, 11]. Another category of facility location problems with uncertain parameters were developed based on fuzzy set theory [12] and possibility theory [8, 19], which aims at dealing with cases of imprecise or vague data. For this kind of location problems, one may refer to [4, 16].

In real-world applications, randomness and fuzziness may coexist in a facility location problem, there is a genuine need to deal with a hybrid uncertainty of randomness and fuzziness. Making use of the expected value operator of a fuzzy random variable (see [6, 9]) as the objective, Wang *et al.* [17] modeled a recourse-based facility location problem with fuzzy random uncertainty and discussed its binary particle swarm optimization (BPSO) approach. Wen and Iwamura [18] built an (α,β) -cost minimization model for random fuzzy facility location problems under the Hurewicz criterion in hybrid uncertain environment, and designed a genetic algorithm (GA) dealing with continuous decision variables for the location model.

In both studies of [17] and [18], the capacity of each facility is assumed to be fixed, however, in more practical situations, it should be variable and serve as a decision to be made. In this paper, we allow the capacity to be a decision variable and model fuzzy random facility location problems by using fuzzy random VaR (see [15]) as an objective. Herewith, in contrast with the model in [17] whose decisions are all binary variables, and the model in [18] where all the decisions are continuous variables, the fuzzy random location model built in this paper contains mixed decisions, that is the capacities are the continuous decisions while the location decisions are binary ones. As a consequence, we design a hybrid approach to the model which comprises an approximation algorithm to fuzzy random VaR and a mechanism of modified continuous-binary PSO.

2 Preliminaries

Let the triplet $(\Gamma,\mathscr{P}(\Gamma),\mathrm{Pos})$ be a possibility space, where $\mathscr{P}(\Gamma)$ is the power set of Γ, X be a fuzzy variable defined on $(\Gamma,\mathscr{P}(\Gamma),\mathrm{Pos})$ whose membership function is μ_X, and r be a real number. The possibility and credibility of an event $X\leq r$ are expressed as follows:

$$\begin{aligned}&\mathrm{Pos}\{X\leq r\}=\sup_{t\leq r}\mu_X(t),\text{and}\\&\mathrm{Cr}\{X\leq r\}=\tfrac{1}{2}\left(\sup_{t\leq r}\mu_X(t)+1-\sup_{t>r}\mu_X(t)\right).\end{aligned}\tag{1}$$

Suppose that $(\Omega,\mathscr{A},\Pr)$ is a probability space, $\mathscr{F}_v$ is a collection of fuzzy variables defined on possibility space $(\Gamma,\mathscr{P}(\Gamma),\mathrm{Pos})$. A fuzzy random variable is defined as a map $\xi : \Omega \to \mathscr{F}_v$ such that $\text{Pos}\{\xi(\omega) \in B\}$ is a measurable function of ω for any Borel subset B of $\Re$(see [9]).

Example 2.1. Let Y be a random variable defined on probability space $(\Omega, \mathscr{A}, \text{Pr})$. If we define that for every $\omega \in \Omega$, $\xi(\omega) = (Y(\omega), Y(\omega)+2, Y(\omega)+6)$ which is a triangular fuzzy variable defined on some possibility space $(\Gamma, \mathscr{P}(\Gamma), \text{Pos})$. Then, ξ is a (triangular) fuzzy random variable.

To measure an event $\xi \in B$ induced by a fuzzy random variable ξ, where B a Borel subset of $\Re$, the mean chance measure (see [10]) is given as

$$\text{Ch}\{\xi \in B\} = \int_{\Omega} \text{Cr}\{\xi(\omega) \in B\} \, \text{Pr}(\mathrm{d}\,\omega). \tag{2}$$

In the fuzzy random environment, let $\mathscr{L}$ be the loss variable with fuzzy random parameters of some investment. The fuzzy random Value-at-Risk of the investment with confidence $1-\beta$ is expressed in the following form (see [15]):

$$\text{VaR}_{1-\beta} = \sup\{\lambda \in \Re \mid \text{Ch}\{\mathscr{L} \geq \lambda\} \geq \beta\} \tag{3}$$

where $\beta \in (0,1)$, and Ch is the mean chance measure in (2).

3 VaR-Based Fuzzy Random Facility Location Model

In this section, we formulate a VaR-based fuzzy random facility location model (VaR-FRFLM) with variable capacity which is a task of two-stage mixed 0-1 integer fuzzy random programming. We introduce the following notation for this two-stage model:

Indices and constants

i	index of facilities, $1 \leq i \leq n$
j	index of clients, $1 \leq j \leq m$
r_j	unit price charged to client j
c_i	fixed cost for opening and operating facility i
W_i	maximum capacity of each facility i
t_{ij}	unit transportation cost from i to j
$1-\beta$	confidence level of the Value-at-Risk

Fuzzy random parameters

D_j	fuzzy random demand of client j
V_i	fuzzy random unit variable operating cost of facility i
ξ	fuzzy random demand-cost vector $\xi = (D_1, \cdots, D_m, V_1, \cdots, V_n)$

Decision variables

x_i	location decision which is a binary variable
x	location decision vector which is $x = (x_1, x_2, \cdots, x_n)$
s_i	capacity decision of facility i
s	capacity decision vector which is $s = (s_1, s_2, \cdots, s_n)$
$y_{ij}^{(\omega,\gamma)}$	quantity supplied to client j from facility i at scenario (ω, γ).

As usual, it is assumed that each customer's demand cannot be over served, but it is possible that not all demand is served. Furthermore, the total supply from one facility to all clients cannot exceed the capacity of the facility. Lastly, we assume that fuzzy random demand-cost vector $\xi = (D_1, \cdots, D_m, V_1, \cdots, V_n)$ is defined from a probability space $(\Omega, \mathscr{A}, \mathrm{Pr})$ to a collection of fuzzy vectors on possibility space $(\Gamma, \mathscr{P}(\Gamma), \mathrm{Pos})$.

Making use of the fuzzy random VaR in (3), a VaR-FRFLM at confidence level $1-\beta$ can be built as follows under the above notation and assumptions. The objective of this VaR-FRFLM is to minimize the VaR of the investment by determining the optimal locations as well as the capacities of the new facilities to open.

Model

$$\left.\begin{array}{ll} \min & \mathrm{VaR}_{1-\beta}(x,s) \\ \text{subject to} & x_i \in \{0,1\}, i = 1,2,\cdots,n, \\ & 0 \le s_i \le W_i x_i, i = 1,2,\cdots,n, \end{array}\right\} \tag{4}$$

where

$$\mathrm{VaR}_{1-\beta}(x,s) = \sup\left\{\lambda \mid \mathrm{Ch}\left\{\sum_{i=1}^{n} c_i x_i - \mathscr{R}(x,s,\xi) \ge \lambda\right\} \ge \beta\right\}, \tag{5}$$

and the second-stage problem for each scenario (ω, γ) is

$$\left.\begin{array}{l} \mathscr{R}\Big(x,s,\xi(\omega,\gamma)\Big) = \max \sum_{i=1}^{n}\sum_{j=1}^{m}\big(r_j - V_i(\omega,\gamma) - t_{ij}\big) y_{ij}^{(\omega,\gamma)} \\ \qquad \text{subject to} \\ \qquad\qquad \sum_{i=1}^{n} y_{ij}^{(\omega,\gamma)} \le D_j(\omega,\gamma), j = 1,2,\cdots,m, \\ \qquad\qquad \sum_{j=1}^{m} y_{ij}^{(\omega,\gamma)} \le s_i x_i, i = 1,2,\cdots,n, \\ \qquad\qquad y_{ij}^{(\omega,\gamma)} \ge 0, i = 1,2,\cdots,n, j = 1,2,\cdots,m. \end{array}\right\} \tag{6}$$

In the VaR-FRFLM (4)-(6), the location-capacity decision (x,s) is called the first stage decision in the theory of two-stage fuzzy random programming with VaR criteria (see [15]), which should be made before the realizations $D_j(\omega,\gamma)$ and $V_i(\omega,\gamma)$ of the fuzzy random demand D_j and cost V_i, respectively, are observed, where the scenario $(\omega,\gamma) \in \Omega \times \Gamma$. Furthermore, we note that the objective function is

$$\mathrm{VaR}_{1-\beta}(x,s) = \sup\left\{\lambda \mid \int_{\Omega} \mathrm{Cr}\left\{\gamma \in \Gamma \mid \sum_{i=1}^{n} c_i x_i - \mathscr{R}\Big(x,s,\xi(\omega,\gamma)\Big) \geq \lambda \right\} \Pr(\mathrm{d}\omega) \geq \beta \right\}, \quad (7)$$

for each first stage decision (x,s), hence, in order to determine the value of objective $\mathrm{VaR}_{1-\beta}(x,s)$ we have to solve N second stage problems (6), where N is the number of all the scenarios $(\omega,\gamma) \in \Omega \times \Gamma$. Given (x,s), for each scenario (ω,γ), the quality distribution pattern $\left(y_{ij}^{(\omega,\gamma)}\right)_{n\times m}$ is determined by solving the second stage problem (6) at scenario (ω,γ). Here, the $y_{ij}^{(\omega,\gamma)}$ for $i = 1,2,\cdots,n; j = 1,2,\cdots,m$ are referred to as the second stage decisions (see [15]). From the model (4)-(6), we can see the second stage decision $y_{ij}^{(\omega,\gamma)}$ is up to the scenario $(\omega,\gamma) \in \Omega \times \Gamma$, it does not serve as the decision to all the scenarios but is determined for the calculation of the value of $\mathrm{VaR}_{1-\beta}(x,s)$. So the real decision in VaR-FRFLM is the first stage decision (x,s).

In general, the fuzzy random parameter V_i and D_j for $i = 1,2,\cdots,n; j = 1,2,\cdots,m$ are continuous fuzzy random variables which has infinite numbers of realizations, which follows from (7) that it requires to solve infinite second stage problems (6) to determine the objective value $\mathrm{VaR}_{1-\beta}(x,s)$. Hence, it cannot be calculated analytically. As a consequence, the VaR-FRFLM (4)-(6) within this nature cannot be solved analytically, we will design a hybrid metaheuristic approach to this two-stage mixed 0-1 fuzzy random programming problem.

4 Hybrid MPSO Approach

Recall that in this paper the proposed VaR-FRFLM (4)-(6) is a two-stage mixed 0-1 integer fuzzy random programming problems. We design a hybrid mechanism, which integrates the continuous PSO (see [13, 14, 15]), binary PSO (BPSO, see [5, 7, 16, 17]), and Approximation Algorithm to fuzzy random VaR (see [15]), to solve the model. Several modifications are made or implemented so as to enhance the performance of the hybrid approach:

i) We employ a phenotype-genotype mechanism (see [7, 16]) in the BPSO to further enhance the searching capability of the binary particles.

ii) To further improve the global search in the population-based optimization, it is desirable to consider the individual's neighborhood which is better than considering the individual itself. From this point of view, we introduce a Nbest-Gbest-based update rule (the 'Nbest' denotes the neighborhood-best particles) by adjusting the velocity in the direction of the personal best particles in the neighborhood and the global best particle.

iii) Two mutation operators are applied to the binary location particles and capacity particles, respectively, to further extend the search space of the hybrid algorithm so as to decrease the probability of its getting trapped in a local optimum.

The proposed hybrid algorithm is referred to as a hybrid modified PSO (MPSO) algorithm, which is elaborated as follows.

4.1 Approximation to Fuzzy Random VaR

An approximation algorithm for fuzzy random VaR has been proposed in the VaR-based two-stage fuzzy stochastic programming (see [15]), and the convergence of the approximation algorithm is also proved in [15]. In this paper, we employ the approximation algorithm to estimate the objective value $\mathrm{VaR}_{1-\beta}(x,s)$ for each (x,s) in our VaR-FRFLM (4)-(6). The detailed Approximation Algorithm (Algorithms 2-3) to fuzzy random VaR can be found in [15].

4.2 Solution Representation

A real number vector $(x,s) \triangleq \Big(\langle x_1,s_1\rangle,\langle x_2,s_2\rangle,\cdots,\langle x_n,s_n\rangle\Big)$ is used as a particle pair to represent a solution (location-capacity) of the two-stage VaR-FRFLM (4)-(6), where $x_{p,i}\in\{0,1\}, 0\le s_{p,i}\le W_i x_i, i=1,2,\cdots,n$.

4.3 Initialization

First of all, we randomly generate the initial binary phenotype location particle $x_p = (x_{p,1},x_{p,2},\cdots,x_{p,n})$ as follows:

$$\begin{array}{l}\texttt{for}(i=1;i<=n;i++)\\ \quad\texttt{if}(\mathrm{rand}()>0.5)\ \texttt{then}\ x_{p,i}=1;\ \texttt{else}\ x_{p,i}=0;\end{array} \tag{8}$$

where rand() is a random number coming from the uniform distribution over the interval [0,1], and initialize the genotype location particle $x_g = x_p$. Then, we generate a capacity particle $s=(s_1,s_2,\cdots,s_n)$ by the following method:

$$\begin{array}{l}\texttt{for}(i=1;i<=n;i++)\\ \quad\texttt{if}(x_{p,i}=1)\ \texttt{then}\ s_i=\mathrm{rand}(0,W_i);\ \texttt{else}\ s_i=0;\end{array} \tag{9}$$

where rand(a,b) is a uniformly distributed random number over the interval [a,b]. Repeat the above process P_{size} times, we get P_{size} initial binary phenotype and genotype location particles $x_{p,1},x_{p,2},\cdots,x_{p,P_{size}};x_{g,1},x_{g,2},\cdots,x_{g,P_{size}}$, and P_{size} capacity particles $s_1,s_2,\cdots,s_{P_{size}}$, respectively.

4.4 Evaluation by Approximation Algorithm to VaR

Denote $\mathbf{Fit}(\cdot)$ the fitness function, and let the fitness of each decision (x,s) be the minus of the Value-at-Risk, i.e.,

$$\mathbf{Fit}(x,s) = -\mathrm{VaR}_{\mathbf{1}-\beta}(x,s).$$

Therefore, the particles of smaller objective values are evaluated with higher fitness. For each (x,s), the fitness value $\mathbf{Fit}(x,s)$ is calculated by the approximation algorithm mentioned in Subsection 4.1.

4.5 Update Process

4.5.1 Update of Genotype-Location and Capacity Particles

In the update process, we first need to determine the global best particle pair (x_{Gbest}, s_{Gbest}) (with the highest fitness), where the x_{Gbest} is the best phenotype location particle so far; and for each $(x_{p,k}, s_k)$, find the $(x_{Pbest,k}, s_{Pbest,k})$ with the highest previous fitness, where $k = 1, 2, \cdots, P_{size}$. Then, for each k, we determine the velocity vector pair $(v_{x,k}, v_{s,k})$ through the following *Nbest-Gbest-based* update formula:

$$v_{x,k} = \mathscr{W} * v_{x,k} + c_1 * d_N(x_{p,k}) + c_2 * \mathrm{rand}() * \left(x_{Gbest} - x_{p,k}\right), \tag{10}$$

$$v_{s,k} = \mathscr{W} * v_{s,k} + c_1 * d_N(s_k) + c_2 * \mathrm{rand}() * (s_{Gbest} - s_k). \tag{11}$$

In the above formula, $d_N(x_{p,k}), k = 1, 2, \cdots, P_{size}$ are the average distance from $x_{p,k}$ to the best positions in its neighborhood, which are defined as

$$d_N(x_{p,1}) = \sum_{j=1}^{2} \mathrm{rand}() * \left(\frac{x_{Pbest,j} - x_{p,1}}{2}\right), \tag{12}$$

$$d_N(x_{p,k}) = \sum_{j=k-1}^{k+1} \mathrm{rand}() * \left(\frac{x_{Pbest,j} - x_{p,k}}{3}\right), k = 2, 3, \cdots, P_{size} - 1, \tag{13}$$

$$d_N\left(x_{p,P_{size}}\right) = \sum_{j=P_{size}-1}^{P_{size}} \mathrm{rand}() * \left(\frac{x_{Pbest,j} - x_{p,P_{size}}}{2}\right), \tag{14}$$

respectively, and the $d_N(s_k), k = 1, 2, \cdots, P_{size}$ can be given similarly. Here, c_1 and c_2 are learning rates, to well adjust the convergence of the particles, we employ the time-varying learning rates (see [14]) as follows:

$$c_1 = 2 * \frac{G_{max} - G_n}{G_{max}} + 1, \quad \text{and} \quad c_2 = 2 * \frac{G_n}{G_{max}} + 1, \tag{15}$$

where G_{max} and G_n are the indexes of the maximum and current generations, respectively. $\mathscr{W}$ is the inertia weight which is set by the following expression [3]:

$$\mathscr{W} = \frac{2}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|},$$

where $\phi = c_1 + c_2$.

Next, each genotype location particle $x_{g,k}$ and capacity particle s_k are updated by the following operations

$$x_{g,k} = x_{g,k} + v_{x,k} \quad (16)$$

$$s_k = s_k + v_{s,k} \quad (17)$$

respectively.

4.5.2 Update of Phenotype-Location Particles and Re-update of Capacity Particles

All the phenotype location particles $x_{p,k}, k = 1,2,\cdots,P_{size}$ are updated according to the following rule [7]:

$$\begin{array}{l}\texttt{for}(i=1; i<=n; i++)\\ \quad \texttt{if}(\text{rand}() < S(x_{g,ki}))\ \texttt{then}\ x_{p,ki} = 1; \texttt{else}\ x_{p,ki} = 0;\end{array} \quad (18)$$

where $x_{g,ki}$ and $x_{p,ki}$ are the components of the vectors $x_{g,k}$ and $x_{p,k}$, respectively, and $S(\cdot)$ is a sigmoid function with $S(x) = 1/1+e^{-x}$. Furthermore, we re-update the capacity particles s_k with the following constraint:

$$\begin{array}{l}\texttt{for}(i=1; i<=n; i++)\\ \{ \quad \texttt{if}(x_{p,ki} = 0)\ \texttt{then}\ s_{ki} = 0;\\ \quad\quad \texttt{else}\\ \quad\quad \texttt{if}(s_{ki} = 0)\ \texttt{then}\ s_{ki} = \text{rand}(0, W_i); \}\end{array} \quad (19)$$

where s_{ki} is a component of capacity particle s_k, for $k = 1,2,\cdots,P_{size}$.

Making use of formulas (10)-(19), we yield a new generation of phenotype-location and capacity particle pairs $\left(x'_{p,1}, s'_1\right), \left(x'_{p,2}, s'_2\right), \cdots, \left(x'_{p,P_{size}}, s'_{P_{size}}\right)$.

4.5.3 Mutation

We predetermine 2 parameters $P_{m,L}, P_{m,C} \in (0,1)$ representing the probability of mutation for the location and capacity particles, respectively. The following mutation operation is applied to all velocity vectors of location particles after the update (16) of the genotype location particles:

$$\begin{array}{l}\texttt{for}(k=1; k<=P_{size}; k++)\\ \quad \texttt{if}(\text{rand}() < P_{m,L})\ \texttt{then}\ v_{x,k} = -v_{x,k};\end{array} \quad (20)$$

On the other hand, the mutation of capacity particles is implemented following the update operation (19). For each capacity particle $s_k = (s_{k1}, s_{k2}, \cdots, s_{kn}), k = 1,2,\cdots,P_{size}$, if rand() $< P_{m,C}$, then we generate a number N_m between 1 & n, and mutate the capacity particle as follows

$$\begin{array}{l}\texttt{for}(i=1; i<=N_m; i++)\\ \quad \texttt{if}(s_{ki} > 0)\ \texttt{then}\ s_{ki} = \text{rand}(0, W_i).\end{array} \quad (21)$$

4.6 Hybrid Algorithm Procedure

The hybrid MPSO algorithm to VaR-FRFLM (4)-(6) can be summarized as follows.

(Hybrid MPSO Algorithm)

Step 1. Initialize a population of phenotype-genotype location particles $x_{p,k}, x_{g,k}$, and capacity particles s_k, for $k = 1,2,\cdots,P_{size}$, by using (8)-(9).
Step 2. Calculate the fitness $\mathbf{Fit}(x_p, s)$ for all particles through the Approximation Algorithm to VaR, and evaluate each particle pair according to the fitness;
Step 3. Determine the $d_N(x_p)$ and $d_N(s)$ for each phenotype location particle x_p and capacity particle s, and find the global best particles x_{Gbest} and $s_{Gbest}$for the population;
Step 4. Update all the genotype location and capacity particles by formulas (10)-(17);
Step 5. Run mutation operator (20) to each location velocity with probability $P_{m,L}$.
Step 6. Update each phenotype location particle by (18), and re-update each capacity particle with (19).
Step 7. Run mutation operator (21) to each capacity particle with probability $P_{m,C}$.
Step 8. Repeat Step 2 to Step 7 for a given number of generations;
Step 9. Return the particle pair (x_{Gbest}, s_{Gbest}) as the optimal solution to the VaR-FRFLM (4)-(6), and $\text{VaR}_{1-\beta}(x_{Gbest}, s_{Gbest}) = -\mathbf{Fit}(x_{Gbest}, s_{Gbest})$ the corresponding optimal value.

---

5 Numerical Experiments and Comparison

We consider a firm which plans to open new facilities in 10 potential sites, the capacity limits W_i, fixed costs c_i and fuzzy random operating costs V_i of the sites $i, i = 1,2,\cdots,10$ are given in Table 1. We suppose that there are 5 customers whose fuzzy random demands $D_j, j = 1,2,\cdots,5$ are given in Table 2, where $\mathscr{U}(a,b)$

Table 1 Capacity limits, fixed and variable costs

Facility site i	Capacity limit W_i	Fixed cost c_i	Variable cost V_i	Parameter Y_i
1	250	8	$(7+Y_1, 9+Y_1, 10+Y_1)$	$\mathscr{U}(1,2)$
2	220	15	$(6+Y_2, 8+Y_2, 10+Y_2)$	$\mathscr{U}(2,3)$
3	300	16	$(8+Y_3, 10+Y_3, 11+Y_3)$	$\mathscr{U}(1,2)$
4	290	12	$(12+Y_4, 13+Y_4, 15+Y_4)$	$\mathscr{U}(0,1)$
5	260	6	$(13+Y_5, 15+Y_5, 16+Y_5)$	$\mathscr{U}(1,2)$
6	250	12	$(8+Y_6, 9+Y_6, 10+Y_6)$	$\mathscr{U}(0,2)$
7	320	17	$(6+Y_7, 7+Y_7, 8+Y_7)$	$\mathscr{U}(2,4)$
8	330	8	$(8+Y_8, 10+Y_8, 12+Y_8)$	$\mathscr{U}(2,3)$
9	280	9	$(13+Y_9, 15+Y_9, 16+Y_9)$	$\mathscr{U}(3,4)$
10	370	12	$(10+Y_{10}, 11+Y_{10}, 12+Y_{10})$	$\mathscr{U}(1,2)$

Table 2 Fuzzy random demands

Customer j	t_j	Demand D_j	Parameter Z_j
1	24	$(20+Z_1, 22+Z_1, 23+Z_1)$	$\mathscr{U}(1,2)$
2	22	$(18+Z_2, 20+Z_2, 21+Z_2)$	$\mathscr{U}(1,3)$
3	28	$(16+Z_3, 18+Z_3, 19+Z_3)$	$\mathscr{U}(2,4)$
4	26	$(22+Z_4, 23+Z_4, 24+Z_4)$	$\mathscr{U}(2,3)$
5	19	$(20+Z_5, 22+Z_5, 23+Z_5)$	$\mathscr{U}(3,4)$

Table 3 Results of hybrid MPSO algorithm with Different Parameters

	System Parameters			Results		
No.	$1-\beta$	$P_{m,L}$	$P_{m,C}$	Optimal solution	Objective	Error(%)
1	0.90	0.2	0.4	$(\langle 0,0\rangle,\langle 1,220.0\rangle,\langle 0,0\rangle,\langle 1,67.4\rangle,$ $\langle 1,185.0\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,200.3\rangle,\langle 0,0\rangle)$	-285.8	0.14
2	0.90	0.3	0.3	$(\langle 0,0\rangle,\langle 1,97.7\rangle,\langle 0,0\rangle,\langle 1,119.1\rangle,$ $\langle 1,142.3\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,51.3\rangle,\langle 0,0\rangle)$	-284.0	0.77
3	0.90	0.4	0.2	$(\langle 0,0\rangle,\langle 1,127.0\rangle,\langle 0,0\rangle,\langle 1,204.0\rangle,$ $\langle 1,182.4\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,234.3\rangle,\langle 0,0\rangle)$	-283.0	1.10
4	0.90	0.2	0.3	$(\langle 0,0\rangle,\langle 1,148.8\rangle,\langle 0,0\rangle,\langle 1,95.6\rangle,$ $\langle 1,121.2\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,49.5\rangle,\langle 0,0\rangle)$	-286.2	0.00
6	0.85	0.2	0.4	$(\langle 0,0\rangle,\langle 1,92.8\rangle,\langle 0,0\rangle,\langle 1,100.9\rangle,$ $\langle 1,260.0\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,247.8\rangle,\langle 0,0\rangle)$	-294.6	0.44
7	0.85	0.3	0.3	$(\langle 0,0\rangle,\langle 1,91.7\rangle,\langle 0,0\rangle,\langle 1,46.7\rangle,$ $\langle 1,180.2\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,147.4\rangle,\langle 0,0\rangle)$	-292.5	1.15
8	0.85	0.4	0.2	$(\langle 0,0\rangle,\langle 1,111.7\rangle,\langle 0,0\rangle,\langle 1,208.8\rangle,$ $\langle 1,239.7\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,177.6\rangle,\langle 0,0\rangle)$	-295.9	0.00
9	0.85	0.2	0.3	$(\langle 0,0\rangle,\langle 1,138.4\rangle,\langle 0,0\rangle,\langle 1,175.3\rangle,$ $\langle 1,54.6\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,91.8\rangle,\langle 0,0\rangle)$	-291.7	1.43
11	0.80	0.2	0.4	$(\langle 0,0\rangle,\langle 1,172.7\rangle,\langle 0,0\rangle,\langle 1,144.0\rangle,$ $\langle 1,199.3\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,105.4\rangle,\langle 0,0\rangle)$	-310.2	0.00
12	0.80	0.3	0.3	$(\langle 0,0\rangle,\langle 1,215.7\rangle,\langle 0,0\rangle,\langle 1,167.1\rangle,$ $\langle 1,173.9\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,54.6\rangle,\langle 0,0\rangle)$	-308.8	0.45
14	0.80	0.4	0.2	$(\langle 0,0\rangle,\langle 1,49.3\rangle,\langle 0,0\rangle,\langle 1,266.0\rangle,$ $\langle 1,250.7\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,241.5\rangle,\langle 0,0\rangle)$	-304.0	1.90
14	0.80	0.2	0.3	$(\langle 0,0\rangle,\langle 1,160.3\rangle,\langle 0,0\rangle,\langle 1,183.1\rangle,$ $\langle 1,260.2\rangle, \langle 0,0\rangle,\langle 0,0\rangle,\langle 0,0\rangle,\langle 1,260.0\rangle,\langle 0,0\rangle)$	-309.0	0.39

represents a random variable with uniform distribution on $[a,b]$, also the unit price r_j charged to each customer is also listed there. In addition, the unit transportation costs $t_{ij}, i = 1,2,\cdots,10; j = 1,2,\cdots,5$ are given by a matrix T as follows:

$$T = \left(t_{ij}\right)_{5\times 10} = \left(\begin{array}{r|cccccccccc} & i=1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \hline j=1 & 16 & 21 & 19 & 18 & 14 & 18 & 16 & 20 & 18 & 20 \\ 2 & 17 & 15 & 17 & 14 & 18 & 16 & 17 & 18 & 17 & 14 \\ 3 & 24 & 20 & 25 & 22 & 23 & 22 & 24 & 22 & 20 & 22 \\ 4 & 19 & 22 & 18 & 15 & 21 & 17 & 22 & 21 & 22 & 16 \\ 5 & 13 & 10 & 16 & 13 & 14 & 11 & 13 & 15 & 14 & 13 \end{array}\right).$$

The hybrid MPSO algorithm (Algorithm 1) which integrates the Approximation Algorithm is run to solve this VaR-based fuzzy random facility location problem with above settings. In the hybrid MPSO, we set the population size $P_{size} = 20$, and run the hybrid algorithm (Algorithm 1) with 200 generations for different confidence levels of 0.9, 0.85, and 0.8, respectively. The optimal location solutions with different parameters is listed in Table 3, where the $RelativeError = (optimal\ value - objective\ value)/(optimal\ value)$ is given in the last column. It follows from Table 3 that the relative error does not exceed $1.10\%, 1.43\%$ and 1.90% for the different confidence levels $1-\beta = 0.9, 1-\beta = 0.85$, and $1-\beta = 0.8$, respectively, when different parameters are selected. The performance implies the hybrid MPSO algorithm is robust to the parameter settings when solving the VaR-FRFLM.

6 Conclusions

This paper built a Value-at-Risk-based facility location model with variable capacity and fuzzy random demands and costs. The proposed model is inherently a two-stage mixed 0-1 integer fuzzy random programming problem. To solve the model, a hybrid MPSO algorithm is proposed, in which an Approximation Algorithm is utilized to compute the fuzzy random VaR, a continuous Nbest-Gbest-based PSO and a genotype-phenotype-based binary PSO vehicles are employed to deal with the continuous capacity decisions and the binary location decisions, respectively, and two mutation operators are implemented to enlarge the search space. The numerical experiments show that the hybrid MPSO is robust to the parameter settings.

Acknowledgment

The first author would like to thank the Research Fellowships of the Japan Society for the Promotion of Science (JSPS) for Young Scientists.

References

1. Berman, O., Drezner, Z.: A probalistic one-centre location problem on a network. Journal of the Operational Research Society 54(8), 871–877 (2003)
2. Bongartz, I., Calamai, P.H., Conn, A.R.: A projection method for lp norm location-allocation problems. Mathematical Programming 66(1-3), 283–312 (1994)
3. Clerc, M., Kennedy, J.: The particle swarm — Explosion, stability, and convergence in a multidimensional complex space. IEEE Transactions on Evolutionary Computaion 6(1), 58–73 (2002)
4. Ishii, H., Lee, Y.L., Yeh, K.Y.: Fuzzy facility location problem with preference of candidate sites. Fuzzy Sets and Systems 158(17), 1922–1930 (2007)
5. Kennedy, J., Eberhart, R.C.: A discrete binary version of the particle swarm algorithm. In: Proceedings of the 1997 IEEE International Conference on Systems, Man, and Cybernetics, Orlando, pp. 4104–4108 (1997)
6. Kwakernaak, H.: Fuzzy random variables–I. Definitions and theorems, Information Sciences 15(1), 1–29 (1978)
7. Lee, S., Soak, S., Oh, S., Pedrycz, W., Jeon, M.: Modified binary particle swarm optimization. Progress in Natural Science 18(9), 1161–1166 (2008)
8. Liu, B., Liu, Y.K.: Expected value of fuzzy variable and fuzzy expected value models. IEEE Transaction on Fuzzy Systems 10(4), 445–450 (2002)
9. Liu, Y.K., Liu, B.: Fuzzy random variable: A scalar expected value operator. Fuzzy Optimization and Decision Making 2(2), 143–160 (2003)
10. Liu, Y.K., Liu, B.: On minimum-risk problems in fuzzy random decision systems. Computers & Operations Research 32(2), 257–283 (2005)
11. Louveaux, F.V., Peeters, D.: A dual-based procedure for stochastic facility location. Operations Research 40(3), 564–573 (1992)
12. Pedrycz, W., Gomide, F.: An Introduction to Fuzzy Sets: Analysis and Design. MIT Press, Cambridge (1998)
13. Pedrycz, W., Park, B.J., Pizzi, N.J.: Identifying core sets of discriminatory features using particle swarm optimization. Expert Systems with Applications 36(3), 4610–4616 (2009)
14. Ratnweera, A., Halgamuge, S.K., Watson, H.C.: Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coeffients. IEEE Transactions on Evolutionary Computation 8(3), 240–255 (2004)
15. Wang, S., Watada, J.: Value-at-Risk-based fuzzy stochastic optimization problems. In: Proceedings of the 2009 IEEE International Conference on Fuzzy Systems, Jeju Island, Korea (2009)
16. Wang, S., Watada, J., Pedrycz, W.: Value-at-Risk-based two-stage fuzzy facility location problems. IEEE Transactions on Industrial Informatics 5(4) (2009) (in press)
17. Wang, S., Watada, J., Pedrycz, W.: Fuzzy random facility location problems with recourse. In: Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics, San Antonio, Texas, USA (2009)
18. Wen, M., Iwamura, K.: Facility location–allocation problem in random fuzzy environment: Using (α,β) -cost minimization model under the Hurewicz criterion. Computers & Mathematics with Applications 55(4), 704–713 (2008)
19. Zadeh, L.A.: Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems 1(1), 3–28 (1978)