Single-Period Inventory Models with Fuzzy Shortage Costs Dependent on Random Demands

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Abstract. This paper considers single-period inventory models with fuzzy shortage costs dependent on discrete and continuous random demands considering the close relation between consumer's demands and shortage costs. Since these inventory models include randomness and fuzziness, they are formulated as fuzzy random programming problems. Then, in order to deal with the uncertainty and find the optimal order quantity analytically, the solution approach is proposed using Yager's ranking method with respect to the total expected future profit, and the strict solution is obtained. Furthermore, in order to compare with previous inventory models, basic random variables and fuzzy numbers are introduced, and differences between our proposed models and previous models are discussed.

1 Introduction

Inventory problems are generally common and important in production processes, maintenance services and business operations. Uncertainties such as randomness and fuzziness in inventory problems may be associated with demand, supply or various relevant costs. In the inventory models described in many previous literatures

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(e.g. [7, 13]), randomness has been the main subject of study. The single-period inventory model with randomness, called the newsboy problem, is one of these standard models, and is widely used in production management systems.

In a single-period inventory model, if we order too large a quantity and fail to sell it all, we pay the cost of storing the stocks, and if we order too little a quantity that some customers are not satisfied, we pay a penalty. Thus, we need to find an optimal order quantity for an item by considering the customer's demands, which are basically assumed to be random. However, in reality, due to the lack of reliable information regarding new commodities and the decision maker's subjectivity, some uncertainties may be treated as fuzziness, based on the fuzzy set theory introduced by Zadeh [17]. The fuzzy set theory has been applied to inventory problems in many studies [5, 10, 12, 14, 18]. In recent times, there has been a growing interest in the study of several single-period inventory models in fuzzy environments. Particularly, Ishii and Konno [4] applied fuzziness to shortage cost in the classical newsboy problem where the shortage cost is given by an L-shape fuzzy number while the demand is a discrete random distribution, and obtained the optimal order quantity using the fuzzy min order. Then, Li et al. [8] considered fuzzy models for the single-period inventory problem with a continuous random demand and analytically obtained the optimal order quantity using Yager's ranking method [16] revised by Liou and Wang [9]. This solution approach is one of the most analytical and strict approaches in fuzzy ranking methods. Furthermore, some researchers considered single-period inventory models in the mixed environment where randomness and fuzziness both appear simultaneously, called fuzzy random inventory problems [2, 6, 11, 15].

In previous fuzzy single-period inventory problems, each shortage cost is the same for all customers' demands. However, if a commodity is highly in demand and is sold out, it is definitely more disappointing for the customers than if it was not in demand, and vice versa. Therefore, considering these customer expectations, it is natural that the shortage cost of commodity be highly dependent on the customers' demands. Therefore, by extending studies [4] and [8], we consider single-period inventory models with the fuzzy shortage cost dependency on discrete and continuous random demands, and analytically obtain the optimal order quantity. Furthermore, using a basic random distribution such as the uniform distribution and a fuzzy number such as the triangle fuzzy number, we consider the relation between our proposed models, and previous standard models.

This paper is organized as follows. In Section 2, we focus on standard single-inventory problems based on the newsboy problem, and present each optimal purchasing quantity. In Section 3, we introduce a single-period inventory model with fuzzy shortage cost dependent on a discrete continuous random demand extending Ishii and Konno [4], and obtain the optimal order quantity. In Section 4, in a way similar to Section 3, we introduce a single-period inventory model with fuzzy shortage cost dependent on a continuous random demand extending Li et al. [8]. Finally, in Section 5, we conclude this paper.

2 Standard Single-Period Inventory Models with Discrete and Continuous Random Demands

In this section, we review the simple classical newsboy problem, which includes both discrete and continuous random demand. Generally, the single-period inventory model maximizing the total profit can be presented as the classical newsboy problem. We assume that a commodity can be procured either at the beginning of a period or after the end of the period. The notation of parameters in this paper is as follows:

b:unit purchasing price of an item
p:unit selling price of an item ($p > b$)
h:holding cost per each item after the end of the period ($h < p$)
s:unit shortage cost per item
Y:daily demand of the selling item
x:total purchasing quantity (decision variable)

First, we consider a standard inventory model with discrete random demand. In the classical newsboy problem, the daily demand of the selling item is assumed to be a random variable. We assume that he purchases x newspapers and the actual demand is y. If x is larger than y, he can sell y newspapers, but $x-y$ newspapers remain as the inventory. Therefore, he needs to pay the holding cost. On the other hand, if x is smaller than y, he can sell x newspapers but there is a shortage of $y-x$ newspapers. Therefore, he needs to pay the shortage cost for customers. Consequently, his total profit $e(x,y)$ is given as follows:

$$e(x,y) = \begin{cases} py - bx - h(x-y), & y \le x \\ px - bx - s(y-x), & y \ge x \end{cases} \tag{1}$$

Then, if $p(y)$ denotes the probability that $Y = y$, the total expected profit $E(x)$ becomes as follows:

$$\begin{aligned} E(x) &= \sum_{y=0}^{\infty} e(x,y)\, p(y) \\ &= \sum_{y=0}^{x} (py - bx - h(x-y))\, p(y) + \sum_{y=x+1}^{\infty} (px - bx - s(y-x))\, p(y) \end{aligned} \tag{2}$$

It is important for a seller to consider the maximization of the total profit $e(x,y)$, and so, taking into account the customer's random demand, we consider maximizing the expected total profit. In this discrete case, maximizing the expected total profit is equivalent to satisfying the following inequalities [4, 8]:

$$\begin{cases} E(x) - E(x-1) = (p-b+s) - (p+s+h) \sum_{y=0}^{x-1} p(y) \ge 0 \\ E(x+1) - E(x) = (p-b+s) - (p+s+h) \sum_{y=0}^{x} p(y) \le 0 \end{cases} \tag{3}$$

Therefore, an unique optimal purchasing quantity x^* is determined as follows:

$$\sum_{y=0}^{x^*-1} p(y) \leq \frac{p-b+s}{p+s+h} \leq \sum_{y=0}^{x^*} p(y) \tag{4}$$

In a way similar to the case of discrete random demand, we also consider the inventory model with a continuous random demand as follows:

$$\begin{aligned} E(x) &= \int_0^\infty e(x,y)\,\phi(y)\,dy \\ &= \int_0^x (py - bx - h(x-y))\,\phi(y)\,dy + \int_x^\infty (px - bx - s(y-x))\,\phi(y)\,dy \end{aligned} \tag{5}$$

where $\phi(y)$ is the probability density function of random variable y. With respect to the optimal order quantity for this continuous random demad, it is also determined as follows by using the first derivative of x and solving $\frac{\partial E(x)}{\partial x} = 0$ [7, 13]:

$$\Phi(x) = \frac{p-b+s}{p+s+h} \tag{6}$$

where $\Phi(x)$ is the value of probability distribution function at x.

3 Inventory Model with Fuzzy Shortage Cost for a Discrete Random Demand

In reality, by investigating production-goods markets and production processes, the decision maker obtains effective statistical data for the demand of the commodity and its selling price, and sets random distributions. However, with respect to invisible factors such as the shortage cost, for which it is difficult to obtain reliable numerical data, random distribution derived from the statistical analysis may have many errors and may lack reliability. Furthermore, predicted values of these factors collected from the experiences of veteran sellers are often more valuable than historical data. Here, each experience is his or her own, but affirmatively undetermined and flexible. Therefore, it is natural to set a fuzzy number rather than random distribution. Furthermore, the customer psychology is such that, they are more disappointed when a commodity highly in demand is sold out, than when a commodity lower in demand is, and vice versa. Therefore, considering these practical conditions and customer expectations, we consider single-period inventory models with fuzzy shortage costs dependent on random demands. In this section, we first consider the case of discrete random demand.

3.1 Fuzzy Numbers and Yager's Ranking Method

In this paper, we assume that the shortage cost includes fuzziness and that it depends on the customer demand of the commodity. Then, we introduce the following L-R fuzzy number for the shortage cost:

$$\mu_{\tilde{s}_y}(\omega) = \begin{cases} L\left(\frac{\bar{s}_y-\omega}{\alpha_y}\right) & (\bar{s}_y - \alpha_y \le \omega \le \bar{s}_y) \\ R\left(\frac{\omega-\bar{s}_y}{\beta_y}\right) & (\bar{s}_y \le \omega \le \bar{s}_y + \beta_y) \\ 0 & (otherwise) \end{cases} \tag{7}$$

where $\bar{s}_y$ is the center value of the fuzzy shortage cost dependent on each value of actual demands, and α_y and β_y are left and right spreads dependent on the demand, respectively. Then, $L(\omega)$ and $R(\omega)$ are reference functions and continuous strictly decreasing, and $L(0) = R(0) = 1$, and $L(1) = R(1) = 0$. To simplify, we denote this L-R fuzzy number $\mu_{\tilde{s}_y}(\omega)$ by $\mu_{\tilde{s}_y}(\omega) = (\bar{s}_y, \alpha_y, \beta_y)_{LR}$. Then, the discrete random demand y is introduced as follows:

$$y = \begin{cases} 0 & \Pr\{y=0\} = p(0) \\ 1 & \Pr\{y=1\} = p(1) \\ \vdots & \vdots \\ j & \Pr\{y=j\} = p(j) \\ \vdots & \vdots \end{cases} \tag{8}$$

$$\sum_{y=0}^{\infty} p(y) = 1$$

In this paper, considering the practical relation between shortage cost and demand of the commodity, we assume that $\bar{s}_y \le \bar{s}_{y+1}$.

On the other hand, in previous literatures, a lot of ranking methods for fuzzy numbers have been proposed [1, 3]. Particularly, Yager's ranking method [16] is popular because (a) this method has the advantage of not requiring the knowledge of the explicit form of membership functions of the fuzzy numbers to be ranked and (b) its application is simple. Therefore, Yager's ranking method recently has been applied in some studies for inventory problems (e.g. [8]), and effective decision making has been proposed. This method calculates raking index $I\left(\tilde{C}\right)$ for the fuzzy number $\tilde{C}$ from its α-cut $C(\alpha) = \left[C_\alpha^L, C_\alpha^U\right]$ according to the following formula:

$$I\left(\tilde{C}\right) = \int_0^1 \frac{1}{2}\left(C_\alpha^L + C_\alpha^U\right) d\alpha \tag{9}$$

Using the Yager's ranking method, we discuss inventory models with the fuzzy shortage cost for the discrete random demand.

3.2 Formulation of Our Proposed Inventory Model and the Optimal Order Quantity

Using L-R fuzzy number (7), in the case that we set the total purchasing item to x, the total profit $e(x,y)$ including fuzzy numbers is as follows:

$$\tilde{e}(x,y) = \begin{cases} py - bx - h(x-y), & y \le x \\ (p-b)x - \tilde{s}(y-x), & y \ge x \end{cases} \tag{10}$$

Therefore, the expected total future profit becomes the following fuzzy numbers:

$$\tilde{E}(x) = \sum_{y=0}^{x} (py - bx - h(x-y))\, p(y) + \sum_{y=x+1}^{\infty} ((p-b)x - \tilde{s}(y-x))\, p(y) \tag{11}$$

Since fuzzy shortage cost $\tilde{s}$ is characterized by L-R fuzzy number (7), this fuzzy expected future profit $\tilde{E}(x)$ is also characterized by the following L-R fuzzy number:

$$\mu_{\tilde{E}(x)} = \begin{cases} R\left(\frac{\bar{s}(x)+c(x)-\omega}{\beta(x)}\right), & (\bar{s}(x)+c(x)-\beta(x) \le \omega \le \bar{s}(x)+c(x)) \\ L\left(\frac{\omega-(\bar{s}(x)+c(x))}{\alpha(x)}\right), & (\bar{s}(x)+c(x) \le \omega \le \bar{s}(x)+c(x)+\alpha(x)) \end{cases}$$

$$\begin{cases} \alpha(x) = \sum\limits_{y=x+1}^{\infty} (y-x)\, p(y)\, \alpha_y, \beta(x) = \sum\limits_{y=x+1}^{\infty} (y-x)\, p(y)\, \beta_y \\ \bar{s}(x) = -\sum\limits_{y=x+1}^{\infty} (y-x)\, p(y)\, \bar{s}_y \\ c(x) = \sum\limits_{y=0}^{x} (py-bx-h(x-y))\, p(y) + \sum\limits_{y=x+1}^{\infty} (p-b)xp(y) \\ \quad\;\; = (p-b)x + (p+h)\left(\sum\limits_{y=0}^{x} yp(y) - x\sum\limits_{y=0}^{x} p(y)\right) \end{cases} \tag{12}$$

Then, in order to deal with Yager's ranking method, we introduce the following t-cut of membership function $\mu_{\tilde{E}(x)}$:

$$\begin{aligned} \mu_{\tilde{E}(x)} &= \left[E_t^L(x), E_t^U(x)\right] \\ &= \left[\bar{s}(x) + c(x) - \beta(x) R^{-1}(t), \bar{s}(x) + c(x) + \alpha(x) L^{-1}(t)\right], 0 \le t \le 1 \end{aligned} \tag{13}$$

where $L^{-1}(t)$ and $R^{-1}(t)$ are inverse function of $L(\omega)$ and $R(\omega)$, respectively. Therefore, Yager's raking index $I\left(\tilde{E}(x)\right)$ is calculated as follows:

$$\begin{aligned} I\left(\tilde{E}(x)\right) &= \int_0^1 \tfrac{1}{2}\left(E_t^L(x) + E_t^U(x)\right) dt \\ &= \bar{s}(x) + c(x) + \tfrac{1}{2}\left(\alpha(x)\int_0^1 L^{-1}(t)\, dt - \beta(x)\int_0^1 R^{-1}(t)\, dt\right) \end{aligned} \tag{14}$$

The optimal order quantity x^* is obtained by maximizing $I\left(\tilde{E}(x)\right)$, and so we consider the following conditions whose detail is shown in Appendix A:

$$\begin{cases} I\left(\tilde{E}(x)\right) - I\left(\tilde{E}(x-1)\right) = (p-b+E(I(\tilde{s}_y))) - \sum\limits_{y=0}^{x-1} (p+s+I(\tilde{s}_y))\, p(y) \\ I\left(\tilde{E}(x+1)\right) - I\left(\tilde{E}(x)\right) = (p-b+E(I(\tilde{s}_y))) - \sum\limits_{y=0}^{x} (p+h+I(\tilde{s}_y))\, p(y) \end{cases} \tag{15}$$

where $I(\tilde{s}_y) = \bar{s}_y + \frac{1}{2}\left(\beta_y \int_0^1 R^{-1}(\alpha)\, d\alpha - \alpha_y \int_0^1 L^{-1}(\alpha)\, d\alpha\right)$ is the Yager's ranking index for each fuzzy number $\tilde{s}_y$ and $E(I(\tilde{s})) = \sum\limits_{y=0}^{\infty} I(\tilde{s}_y)\, p(y)$ is the expected value of

$I(\tilde{s}_y)$. In the case that $I\left(\tilde{E}(x)\right) - I\left(\tilde{E}(x-1)\right) > 0$ and $I\left(\tilde{E}(x+1)\right) - I\left(\tilde{E}(x)\right) < 0$, $I\left(\tilde{E}(x)\right)$ is the maximum value of ranking index, and so we obtain the optimal order quantity x^*. That is, x^* satisfies the following condition:

$$\begin{cases} (p-b+E(I(\tilde{s}))) - \sum_{y=0}^{x^*-1}(p+h+I(\tilde{s}_y))\,p(y) \geq 0 \\ (p-b+E(I(\tilde{s}))) - \sum_{y=0}^{x^*}(p+h+I(\tilde{s}_y))\,p(y) \leq 0 \end{cases}$$
$$\Leftrightarrow \sum_{y=0}^{x^*-1}(p+h+I(\tilde{s}_y))\,p(y) \leq p-b+E(I(\tilde{s})) \leq \sum_{y=0}^{x^*}(p+h+I(\tilde{s}_y))\,p(y) \tag{16}$$

If each fuzzy shortage cost $\tilde{s}_y$ is independent of the demand, i.e., $\tilde{s}_y = \tilde{s}_c, \tilde{s}_c = (\bar{s}_c, \alpha, \beta)_{LR}$, this optimal condition is transformed into

$$\sum_{y=0}^{x^*-1}(p+h+I(\tilde{s}_c))\,p(y) \leq p-b+E(I(\tilde{s}_c)) \leq \sum_{y=0}^{x^*}(p+h+I(\tilde{s}_c))\,p(y)$$
$$\Leftrightarrow (p+h+I(\tilde{s}_c))\sum_{y=0}^{x^*-1}p(y) \leq p-b+I(\tilde{s}_c) \leq (p+h+I(\tilde{s}_c))\sum_{y=0}^{x^*}p(y) \tag{17}$$
$$\Leftrightarrow \sum_{y=0}^{x^*-1}p(y) \leq \frac{p-b+I(\tilde{s}_c)}{(p+h+I(\tilde{s}_c))} \leq \sum_{y=0}^{x^*}p(y)$$

and so it can be seen that it is almost the same as the standard inventory model. Therefore, our proposed model is the more versatile model among all the others, including standard classical inventory models with randomness.

Subsequently, in order to compare our proposed model with the previous models, we consider a basic case where each discrete random demand occurs with the same probability, and the relation between shortage cost and demand is linear and equally-spaced, i.e., $p(y) = \frac{1}{n}$, $(1 \leq y \leq n)$ and $\tilde{s}_y = (\bar{s}_y, a, a,)_{LR}, \bar{s}_y = ay$ where a is constant. In this case, $I(\tilde{s}_y)$ and $E(I(\tilde{s}))$ are calculated as follows:

$$I(\tilde{s}_y) = ay + \frac{a}{2}\left(\int_0^1 [a(y+1) - a\alpha]\,d\alpha - \int_0^1 [a\alpha + a(y-1)]\,d\alpha\right) = ay + \frac{a^2}{2}$$
$$E(I(\tilde{s})) = \frac{1}{n}\sum_{y=1}^{n} I(\tilde{s}_y) = \frac{1}{n}\left(\frac{a}{2}n(n+1) + \frac{a^2}{2}n\right) = \frac{a}{2}n + \frac{a(a+1)}{2} \tag{18}$$

Therefore, optimal condition (16) is the following condition:

$$\frac{1}{n}\sum_{y=1}^{x^*-1}\left(p+h+\frac{a^2}{2}+ay\right) \leq p-b+E(I(\tilde{s})) \leq \frac{1}{n}\sum_{y=1}^{x^*}\left(p+h+\frac{a^2}{2}+ay\right)$$
$$\Leftrightarrow \begin{cases} \left(p+h+\frac{a^2}{2}\right)(x^*-1) + \frac{a}{2}x^*(x^*-1) \leq n[p-b+E(I(\tilde{s}))] \\ n[p-b+E(I(\tilde{s}))] \leq \left(p+h+\frac{a^2}{2}\right)x^* + \frac{a}{2}x^*(x^*+1) \end{cases} \tag{19}$$
$$\Leftrightarrow \frac{-D_2+\sqrt{D_2^2+2anD_1}}{a} \leq x^* \leq 1 + \frac{-D_2+\sqrt{D_2^2+2anD_1}}{a}$$

where $D_1 = D_2 + \frac{a}{2}n - (b+h), D_2 = p+h+\frac{a(a+1)}{2}$. From optimal conditions (4) and (17), we also obtain the following conditions for optimal order quantities in cases of only random demand and fuzzy shortage cost independent of demand, respectively:

(Only random demand)

$$\begin{array}{l} \sum_{y=0}^{x^*-1} \frac{1}{n} \le \frac{p+s-b}{p+s+h} \le \sum_{y=0}^{x^*} \frac{1}{n} \\ \Leftrightarrow \frac{1}{n}(x^*-1) \le \frac{p+s-b}{p+s+h} \le \frac{1}{n}x^* \\ \Leftrightarrow \frac{n(p+s-b)}{p+s+h} \le x^* \le 1 + \frac{n(p+s-b)}{p+s+h} \end{array} \tag{20}$$

(Fuzzy shortage cost independent of demand)

$$\begin{array}{l} \sum_{y=0}^{x^*-1} \frac{1}{n} \le \frac{p-b+I(\tilde{s}_c)}{p+h+I(\tilde{s}_c)} \le \sum_{y=0}^{x^*} \frac{1}{n} \\ \Leftrightarrow \frac{n(p-b+I(\tilde{s}_c))}{p+h+I(\tilde{s}_c)} \le x^* \le 1 + \frac{n(p-b+I(\tilde{s}_c))}{p+h+I(\tilde{s}_c)} \end{array} \tag{21}$$

Comparing these optimal conditions, we find that each optimal order quantity is different from that derived in the other models by values of $\frac{n(p+s-b)}{p+s+h}$, $\frac{n(p-b+I(\tilde{s}_c))}{p+h+I(\tilde{s}_c)}$, and $\frac{-D_2+\sqrt{D_2^2+2anD_1}}{a}$. Consequently, even if the discrete random demand and the fuzzy number are general distributions such as L-R fuzzy numbers, we can construct an analytical solution approach to our proposed model in a manner similar to the discussion of previous inventory models with only random demand and independent fuzzy shortage cost.

4 Inventory Model with Fuzzy Shortage Cost for a Continuous Random Demand

In this section, in a way similar to the discrete random demand in Section 3, we focus on the case of continuous random demand. Using probability density function $\phi(y)$ in Section 2 and its probability distribution function $\Phi(y)$, the fuzzy total future profit is presented as follows:

$$\begin{array}{rl} \tilde{E}(x) &= \int_0^\infty \tilde{e}(x,y)\,\phi(y)\,dy \\ &= \int_0^x [py - bx - h(x-y)]\,\phi(y)\,dy + \int_x^\infty [(p-b)x - \tilde{s}_y(y-x)]\,\phi(y)\,dy \end{array} \tag{22}$$

With respect to this fuzzy expected total profit derived from the continuous random demad, the membership funtion is as follows:

$$\begin{array}{l} \mu_{\tilde{E}(x)} = \begin{cases} R\left(\frac{\bar{s}(x)+c(x)-\omega}{\beta(x)}\right), & (\bar{s}(x)+c(x)-\beta(x) \le \omega \le \bar{s}(x)+c(x)) \\ L\left(\frac{\omega-(\bar{s}(x)+c(x))}{\alpha(x)}\right), & (\bar{s}(x)+c(x) \le \omega \le \bar{s}(x)+c(x)+\alpha(x)) \end{cases} \\ \begin{cases} \alpha(x) = \int_x^\infty (y-x)\,\alpha_y\phi(y)\,dy, \beta(x) = \int_x^\infty (y-x)\,\beta_y\phi(y)\,dy \\ \bar{s}(x) = -\int_x^\infty (y-x)\,\bar{s}_y\phi(y)\,dy \\ c(x) = \int_0^x (py-bx-h(x-y))\,\phi(y)\,dy + \int_x^\infty (p-b)x\phi(y)\,dy \end{cases} \end{array} \tag{23}$$

Therefore, by considering α-cut (13), we calculate Yager's ranking index $I\left(\tilde{E}\left(x\right)\right)$ for the expected total profit as follows (The detail is shown in Appendix B.):

$$I\left(\tilde{E}\left(x\right)\right) = \int_0^x \left[py - bx - h\left(x-y\right)\right]\phi\left(y\right)dy + \int_x^\infty \left[\left(p-b\right)x - I\left(\tilde{s}_y\right)\left(y-x\right)\right]\phi\left(y\right)dy \tag{24}$$

In order to obtain the optimal order quantity x^*, we need to consider the first derivative $\frac{\partial I\left(\tilde{E}(x)\right)}{\partial x}$ and solve equation $\frac{\partial I\left(\tilde{E}(x)\right)}{\partial x} = 0$. Therefore, we calculate $\frac{\partial I\left(\tilde{E}(x)\right)}{\partial x}$ as the following form whose detail is shown in Appendix B:

$$\frac{\partial I\left(\tilde{E}\left(x\right)\right)}{\partial x} = \left[p - b + E\left(I\left(\tilde{s}_y\right)\right)\right] - \int_0^x \left[p + h + I\left(\tilde{s}_y\right)\right]\phi\left(y\right)dy \tag{25}$$

In the case that we introduce functions $f\left(y\right) = \left[p+h+I\left(\tilde{s}_y\right)\right]\phi\left(y\right)$ and $F\left(y\right) = \int f\left(y\right)dy$, we obtain the following optimal order quantity derived from $\frac{\partial I\left(\tilde{E}(x)\right)}{\partial x} = 0$:

$$\begin{aligned} &\left[p - b + E\left(I\left(\tilde{s}_y\right)\right)\right] - \textstyle\int_0^x \left[\left(p+h\right) + I\left(\tilde{s}_y\right)\right]\phi\left(y\right)dy = 0 \\ &\Leftrightarrow F\left(x\right) - F\left(0\right) = p - b + E\left(I\left(\tilde{s}_y\right)\right) \\ &\Leftrightarrow x^* = F^{-1}\left(F\left(0\right) + \left[p - b + E\left(I\left(\tilde{s}_y\right)\right)\right]\right) \end{aligned} \tag{26}$$

If each fuzzy shortage cost $\tilde{s}_y$ is independent of the demand, i.e., $\tilde{s}_y = \tilde{s}_c, \tilde{s}_c = \left(\bar{s}_c, \alpha, \beta\right)_{LR}$, this optimal condition is transformed into

$$\begin{aligned} &\left[p - b + E\left(I\left(\tilde{s}_c\right)\right)\right] - \textstyle\int_0^x \left(p + h + I\left(\tilde{s}_c\right)\right)\phi\left(y\right)dy = 0 \\ &\Leftrightarrow \left(p + h + I\left(\tilde{s}_c\right)\right)\textstyle\int_0^x \phi\left(y\right)dy = p - b + I\left(\tilde{s}_c\right) \\ &\Leftrightarrow \Phi\left(x^*\right) = \tfrac{p-b+I\left(\tilde{s}_c\right)}{p+h+I\left(\tilde{s}_c\right)} \end{aligned} \tag{27}$$

and so it is almost the same as the standard inventory model with continuous random demand. Therefore, we find that our proposed model is versatile due to including some previous inventory models with fuzzy shortage cost such as Li et al. [8].

Subsequently, in a way similar to Section 3, we consider the special case where the continuous random distribution becomes a uniform distribution and the relation between shortage cost and demand is linear and equally-spaced, i.e.,

$$\phi\left(y\right) = U\left(0,n\right) = \begin{cases} \frac{1}{n} & \left(0 \le y \le n\right) \\ 0 & \left(otherwise\right) \end{cases}, \ \tilde{s}_y = \left(\bar{s}_y, a, a,\right)_{LR}, \ \bar{s}_y = ay \tag{28}$$

In this case, since the value of $I\left(\tilde{s}_y\right)$ and $E\left(I\left(\tilde{s}\right)\right)$ are same as equation (18) in Section 3, we obtain the following strict optimal order quantity:

$$\begin{aligned} &\left[p - b + E\left(I\left(\tilde{s}_y\right)\right)\right] - \textstyle\int_0^x \left(p + h + \frac{a^2}{2} + ay\right)\left(\frac{1}{n}\right)dy = 0 \\ &\Leftrightarrow ax^2 + 2\left(p + h + \frac{a^2}{2}\right)x - 2n\left[p - b + E\left(I\left(\tilde{s}_y\right)\right)\right] = 0 \\ &\Leftrightarrow x^* = \frac{-\left(D_2 - \frac{a}{2}\right) + \sqrt{\left(D_2 - \frac{a}{2}\right)^2 + 2anD_1}}{a} \end{aligned} \tag{29}$$

Consequently, we can also construct the analytical solution approach to our proposed model in a manner similar to the discussion of previous inventory models with only continuous random demand and independent fuzzy shortage cost.

5 Numerical Example

We provide a toy numerical example for a discrete random distribution in Section 3. Let $b = 200$, $p = 500$, $h = 60$, and let the shortage cost not including fuzziness be $s = 120$. Furthermore, we assume that fuzzy shortage costs independent of or dependent on the random demand in Section 3 are triangle fuzzy numbers $(120, 10, 10)_{LR}$ and $(10y, 10, 10)_{LR}$, respectively. In the case that the random demand is presented as $p(y) = \frac{1}{24}, (1 \leq y \leq 24)$, we obtain $\frac{n(p+s-b)}{p+s+h} = \frac{420\times 24}{680}$, $\frac{n(p-b+I(\tilde{s}_c))}{p+h+I(\tilde{s}_c)} = \frac{470\times 24}{730}$, and $\frac{-D_2+\sqrt{D_2^2+2anD_1}}{a} = \frac{-635+\sqrt{635+480\times 495}}{10}$. Therefore, optimal order quantities are $x_r^* = 15$ (not fuzzy), $x_i^* = 16$ (fuzzy, but independent of the demand), and $x_d^* = 17$ (fuzzy, and dependent on the demand), respectively. This result shows that we should order more quantities when there exist ambiguity and dependency on the demand.

6 Conclusion

In this paper, we have considered single-period inventory problems with fuzzy shortage cost dependent on discrete and continuous random demands, respectively. In order to obtain these optimal order quantities analytically, we have introduced the Yager's ranking method for fuzzy expected future profits, and developed optimal conditions including optimal order quantities. Furthermore, we have introduced standard practical cases such that same probabilities for each occurrence discrete demand, uniformed distributions as a continuous random demand, and linear relation between each center value of fuzzy shortage cost and the demand. Then, we obtain the analytical optimal order quantity. Our proposed model includes some previous inventory models with randomness and fuzziness, and so it becomes one of the wider and more practical inventory models.

As future works, we will consider the more general cases of multi-commodity inventory problems, general relation between fuzzy shortage cost and consumer's demand, and the other uncertain environments. Then, this solution approach is analytical, but a little complex and not efficient. Therefore, we will develop not only analytical but also more efficient solution approaches using approximate methods and heuristics.

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Appendix A: Calculation of (15)

$$\begin{aligned}
&I\left(\tilde{E}(x)\right) - I\left(\tilde{E}(x-1)\right) \\
&= \sum_{y=x}^{\infty} p(y)\bar{s}_y + (p-b) - (p+h)\sum_{y=0}^{x-1} p(y) \\
&\quad + \tfrac{1}{2}\left(\sum_{y=x}^{\infty} p(y)\beta_y \int_0^1 R^{-1}(t)\,dt - \sum_{y=x}^{\infty} p(y)\alpha_y \int_0^1 L^{-1}(t)\,dt\right) \\
&= (p-b) - (p+h)\sum_{y=0}^{x-1} p(y) + \sum_{y=x}^{\infty} I(\tilde{s}_y)\,p(y) \\
&= (p-b+E(I(\tilde{s}))) - \sum_{y=0}^{x-1} (p+s+I(\tilde{s}_y))\,p(y)
\end{aligned} \tag{30}$$

and similarly,

$$\begin{aligned}
& I\left(\tilde{E}\left(x+1\right)\right)-I\left(\tilde{E}\left(x\right)\right) \\
& =\sum_{y=x+1}^{\infty} p\left(y\right)\bar{s}_y+\left(p-b\right)-\left(p+h\right)\sum_{y=0}^{x} p\left(y\right) \\
& \quad +\frac{1}{2}\left(\sum_{y=x+1}^{\infty} p\left(y\right)\beta_y\int_0^1 R^{-1}\left(t\right)dt-\sum_{y=x+1}^{\infty} p\left(y\right)\alpha_y\int_0^1 L^{-1}\left(t\right)dt\right) \\
& =\left(p-b\right)-\left(p+h\right)\sum_{y=0}^{x} p\left(y\right)+\sum_{y=x+1}^{\infty} I\left(\tilde{s}_y\right)p\left(y\right) \\
& =\left(p-b+E\left(I\left(\tilde{s}\right)\right)\right)-\sum_{y=0}^{x}\left(p+h+I\left(\tilde{s}_y\right)\right)p\left(y\right)
\end{aligned} \tag{31}$$

Appendix B: Calculation of (24) and (25)

Yager's ranking index $I\left(\tilde{E}\left(x\right)\right)$ for fuzzy expected total profit (24) derived from the continuous random demand is given as follows:

$$\begin{aligned}
& I\left(\tilde{E}\left(x\right)\right) \\
& =\frac{1}{2}\int_0^1\left\{\left(\bar{s}\left(x\right)+c\left(x\right)-\beta\left(x\right)R^{-1}\left(t\right)\right)+\left(\bar{s}\left(x\right)+c\left(x\right)+\alpha\left(x\right)L^{-1}\left(t\right)\right)\right\}dt \\
& =\int_0^x\left(py-bx-h\left(x-y\right)\right)\phi\left(y\right)dy+\int_x^{\infty}\left(p-b\right)x\phi\left(y\right)dy-\int_x^{\infty}\left(y-x\right)\bar{s}_y\phi\left(y\right)dy \\
& \quad +\frac{1}{2}\left\{\left(\int_x^{\infty}\left(y-x\right)\alpha_y\phi\left(y\right)dy\right)\int_0^1 L^{-1}\left(t\right)dt-\left(\int_x^{\infty}\left(y-x\right)\beta_y\phi\left(y\right)dy\right)\int_0^1 R^{-1}\left(t\right)dt\right\} \\
& =\int_0^x\left[py-bx-h\left(x-y\right)\right]\phi\left(y\right)dy+\int_x^{\infty}\left[\left(p-b\right)x-I\left(\tilde{s}_y\right)\left(y-x\right)\right]\phi\left(y\right)dy
\end{aligned} \tag{32}$$

Using $I\left(\tilde{E}\left(x\right)\right)$, the first deviation $\frac{\partial I\left(\tilde{E}\left(x\right)\right)}{\partial x}$ is calculated as follows:

$$\begin{aligned}
\frac{\partial I\left(\tilde{E}\left(x\right)\right)}{\partial x} & =-\left(b+h\right)\int_0^x\phi\left(y\right)dy-\left(b+h\right)x\phi\left(x\right)+\left(p+h\right)x\phi\left(x\right) \\
& \quad +\left(p-b\right)\int_x^{\infty}\phi\left(y\right)dy-\left(p-b\right)x\phi\left(x\right) \\
& \quad +I\left(\tilde{s}_x\right)x\phi\left(x\right)+\int_x^{\infty}I\left(\tilde{s}_y\right)\phi\left(y\right)dy-xI\left(\tilde{s}_x\right)\phi\left(x\right) \\
& =\left(p-b\right)-\left(p+h\right)\int_0^x\phi\left(y\right)dy+\int_x^{\infty}I\left(\tilde{s}_y\right)\phi\left(y\right)dy \\
& =\left(p-b\right)-\left(p+h\right)\int_0^x\phi\left(y\right)dy+\left(E\left(I\left(\tilde{s}_y\right)\right)-\left(\int_0^x I\left(\tilde{s}_y\right)\phi\left(y\right)dy\right)\right) \\
& =\left[p-b+E\left(I\left(\tilde{s}_y\right)\right)\right]-\int_0^x\left[p+h+I\left(\tilde{s}_y\right)\right]\phi\left(y\right)dy
\end{aligned} \tag{33}$$