6

Tempo Curves

Time stays long enough for anyone who will use it.
Leonardo da Vinci

Tempo deals with the performance of time. There is a symbolic time, as used in the score notation, and the physical time of acoustical sounds, into which the symbolic time is transformed; for example, quarter note time units are transformed into seconds. We denote this transformation by $\wp_E$, E standing for symbolic onset time, whereas e stands for physical onset time (figure 6.1).

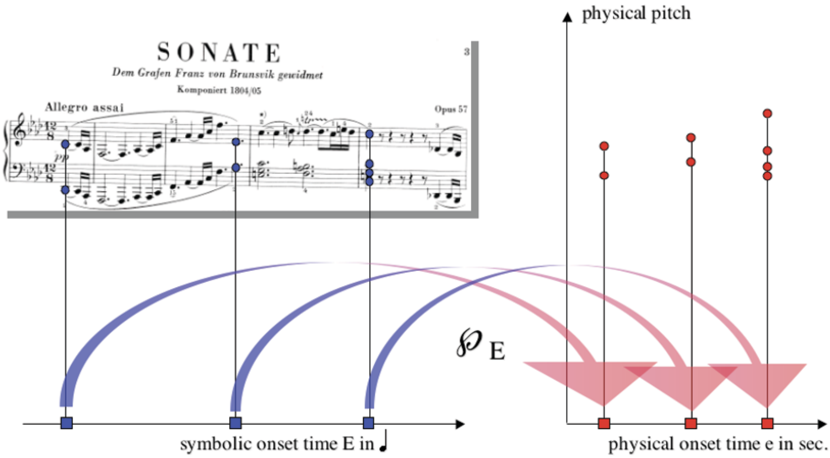

Fig. 6.1. Performance of time transforms symblic time units (typically quarters) into physical time units (typically seconds).

G. Mazzola, *Musical Performance*, Computational Music Science,
DOI 10.1007/978-3-642-11838-8_6, © Springer-Verlag Berlin Heidelberg 2011

What are these spaces of time? Carl Dahlhaus [21, p.53] distinguishes a “potential time” of the score versus the “real time” of performance. He states that “performances are contained in real time, while the score contains potential time.” We do not follow this distinction since the symbolic time of the score is as real as the performance time. The difference is not performative reality versus score’s non-reality—both are real, but pertain to different realities: the score to the mental reality and the performance to the physical reality in the ontological topography.

Peter Desain and Henkjan Honing [24] argue that symbolic time is discrete, whereas physical time, tempo and expressive timing are continuous. This is erroneous for several reasons:

- Metrical time is infinitely divisible in itself: No positive lower limit for mental durations has ever been envisaged. Mathematically speaking: Metrical time is a topologically dense, not discrete set in the field of real numbers. Hence, any reasonable (more precisely: uniformly continuous) time function from mental/symbolic time E to physical time e can uniquely be extended to a time function on the reals (see [59]). There is no conceptual reason to restrict metrical time to a discrete subdomain of the reals.
- Tempo does not deal with something more continuous than metrical time. It is another concept (see later in this chapter): the inverse differential quotient of a function $E \mapsto e(E)$ between two copies of the real number axis with irreducibly different ontological specifications, namely the musical mental status of the score and the physical status of performed music. Tempo is also constantly present, even within rests, fermatas, or glissandi.

So, summarizing, the space of symbolic and the space of physical time is the real line $\mathbb{R}$ of all real (= decimal) numbers, not just a discrete subset; it is the line of all real values, without holes, like in geometry. So performance of onset time is a map $\wp_E$ from the real time line of symbolic time to that of physical time.

6.1 What is Tempo?

To understand the concept of musical tempo, let us first look at the concept of speed in a physical environment, of a moving car, say (figure 6.2). As the car moves along the road, the driver may ask at any position on the road: What is the time? This is typically the case when the driver has an appointment, where he/she has to be at a certain place at a scheduled time.

This setup implies an interpretation of the car’s momentous speed at the spatial position S as follows (figure 6.3). The speed at time e is defined as derivative $\mathit{speed}(e) = dS/de(e)$ of the inverse function $e \mapsto S(e)$. Therefore, using the original function, speed at the position S is the inverse derivative

$$\mathit{speed}(S) = \frac{1}{de/dS(S)}$$

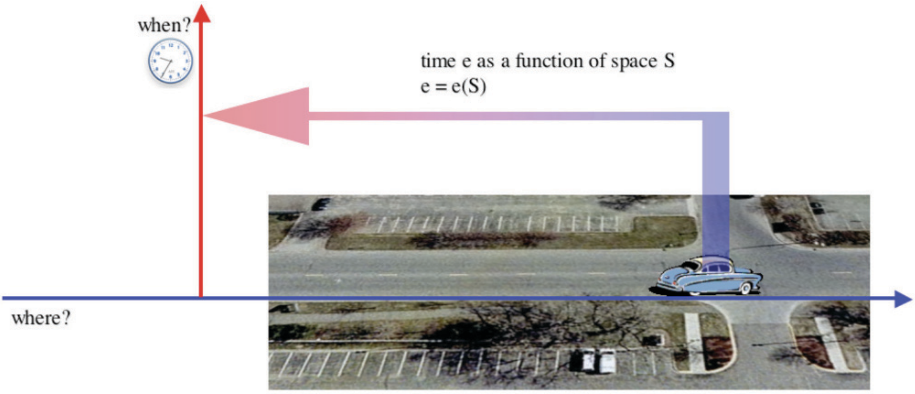

Fig. 6.2. When a car moves through a road, the time can be seen as a function of the position of the car.

as a function of S (!). This is a somehow different point of view since speed is viewed as a function of the position $S(e)$ at time e, and not of time e itself.

Now, the musical analog of this situation views the spatial dimension S being represented by the position of the performer when running though the score. The score takes over the role of the car, the car being replaced by the performer. This is shown in figure 6.4.

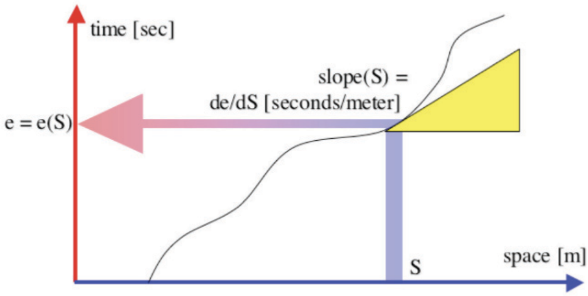

Fig. 6.3. Speed is viewed as the inverse slope of the curve defined by time being a function of space.

In this setup, S being replaced by E, speed takes the form

$$speed(E) = \frac{1}{de/dE(E)}$$

This is precisely what musicians mean when talking about *tempo* $T(E)$ *at a determined position* E on the score, i.e.

$$T(E) = \frac{1}{de/dE(E)} [♩/min]$$

where we have chosen the units quarter notes (♩) and minutes (min), as usual in Western music.

It is astonishing that the common understanding of tempo in music is still somewhat akin to what was the state of the art in physics in Galileo's

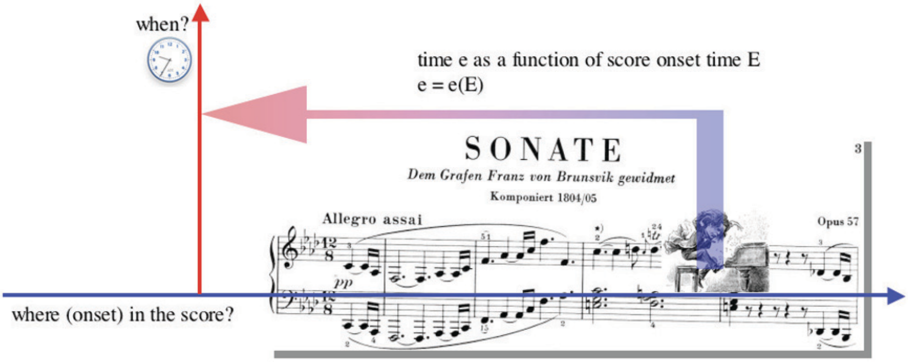

Fig. 6.4. When a performer moves through a score, physical time e can be seen as a function of the score's symbolic time E, the momentous position of the performing artist.

times. This means that tempo cannot be conceived as a momentous slope of a continuous curve, but must be thought as being locally constant. In other words, tempo is a step function that changes its values in discrete times. This view is made evident in Hermann Gottschewski's graphics [21] used to describe tempo variations for piano roll recordings (figure 6.5).

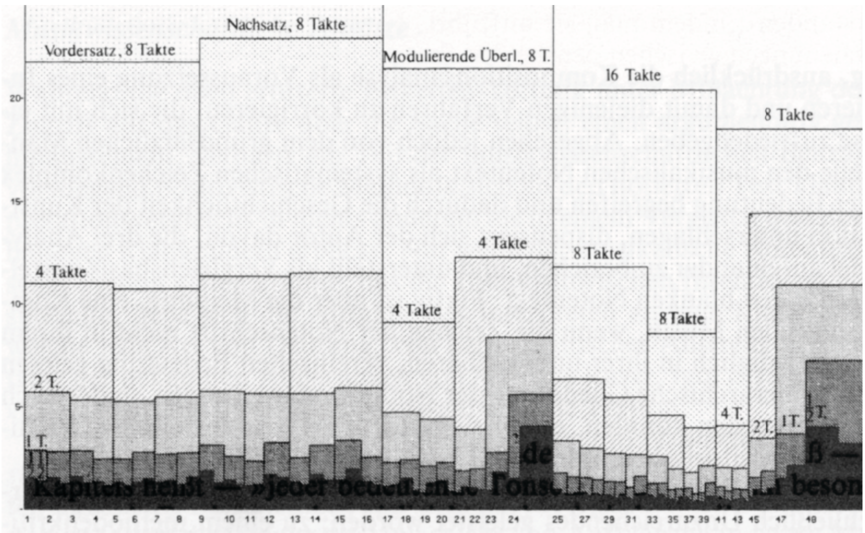

Fig. 6.5. Hermann Gottschewski's locally constant tempo curves.

In the ongoing experimental research on tempo performance, as investigated by Alf Gabrielsson [41], for example, tempo representation has already moved from a step-wise representation to continuous—more precisely to the polygonal representation as a concatenation of linear curves of inverse tempo (figure 6.6). Here the average tempo is shown as level 0 percent for a performance of Mozart's piano sonata KV 331, A major. The local deviation of tempo (every onset of a melody note being measured) is indicated by percentage in terms of duration, i.e., the inverse of tempo. The linearly interpolated curve therefore relates to $1/T$, what means that the tempo is interpolated by $\frac{1}{aE+b}$ between the measured points. The result of this measurements shows two curves, one solid, the other dotted. Both relate to performances of that piece by one and the same performer. They show that tempo is shaped in a systematic way, since the curves are very similar to each other, resulting in a typical rallentando (i.e., increasing note durations) at the end of the first and second phrase of the period.

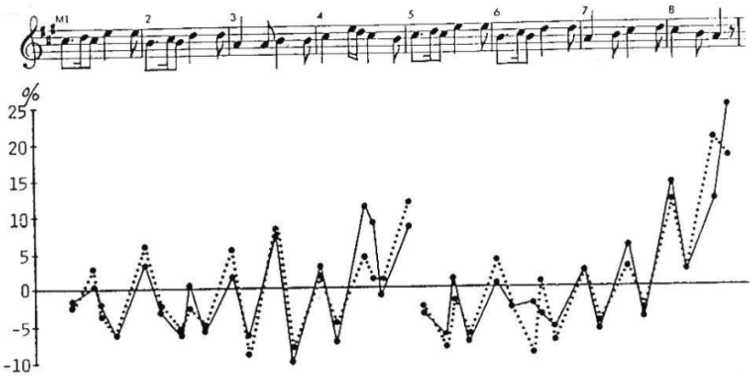

Fig. 6.6. Two $1/T$ curves from performances of Mozart's piano sonata KV 331, A major, by the same performer.

In musical practice and notation, tempo has been denoted in three different ways:

- by absolute tempo signs, such as Maelzel's metronomic notation—e.g. "M.M. ♩ = 120", meaning that tempo is 120 ♩-s per minute—or else verbal descriptions such as *andante*, *adagio*, etc.
- relative punctual tempo signs such as *fermata*, *general pause* (G.P.), *caesura*, *breath*, *mark*, etc.
- relative local tempo signs such as (1) coarse indications like *ritardando*, *rallentando*, *accelerando*, *stringendo*, etc., (2) notation of correspondence between adjacent tempi, such as, for example, 𝅗𝅥 = ♩., which means that M.M. ♩ = 93 is replaced by M.M.* ♩ $= 93\times(1/2)/(3/8) = 93\times4/3 = 124$, (3) signs of type *a tempo* or *istesso tempo*, re-establishing the original tempo.

6.1.1 Some Remarks on the History and Ethnography of Tempo

In the Western musical tradition, the practice of notating tempo has evolved slowly since approximately 1600 [122]. As pitch and rhythmic notation developed and became more precise, attempts were made to do the same with tempo indications. However, as musicians and conductors honed their skills to interpret the symbols of notated rhythm and pitch as accurately as possible, any attempt by a composer to exercise the same control over the tempo of a composition was (and is, to this day) met with much resistance. This may be (in part) due to the fact that given all of the factors of a piece that can be manipulated by a performer, tempo is the most malleable.

The first attempt to codify tempo markings happened between 1600 and 1750 during the Renaissance in Italy [31]. These descriptive words (*presto*, *andante*, etc.) are still used today as general indicators of tempo. However, the definitions of these words are highly subjective; attempts to clarify exactly what they mean (i.e. with a metronome marking) seem to go unnoticed by those who are performing the music to which they are attached. This confusion is partly due to the fact that the range of these terms implies a 1:8 ratio. That is, the fastest tempo (*prestissimo*) will be eight times faster than the slowest (*molto-adagio*) [119] . Curt Sachs in [119] also points out that the physical tempo of the piece (the tempo at which a conductor will beat quarter notes in 4/4) is a much smaller ratio than 1:8; it is more like 1:4. This is not to say that a piece cannot be thought of as having a tempo outside that range, and it does stand to reason that on the extreme ends of the range (i.e. less than M.M. 50 b.p.m. (beats per minute) and more than 140 b.p.m.) both performer and conductor alike will either “half” or “double up” their physical beat depending on which direction the tempo changes.

One would think that the metronome (invented by Johann Nepomuk Maelzel and made commercially available in 1815 [99]), would alleviate the uncertainty that previously existed in regard to “correct” tempi. Now that composers had a tool to mark exactly how fast or slow they wanted a particular composition, there could be no more of the ambiguity that allowed for such wide interpretation by performers. This has proven not to be the case, however. Some composers, such as Brahms, refused to use Maelzel markings because they were too rigid. And others, such as Beethoven, put different markings on different copies of the same score [122]. This inconsistency from composers, combined with doubts of the early metronomes’ accuracy and the tendency of publishers to make adjustments to Maelzel markings according to their personal taste, have made it difficult to trust (or provide a justification to ignore) the metronome markings on scores from the early days of the metronome.

Even in modern scores, Maelzel markings are still understood to be a guideline more than a rule when it comes to tempo. Other factors, such as the performer’s ability, artistic intuition, and personal taste, are seen as more important in tempo determination than the composer’s notion of the ideal tempo. One cannot help but question this attitude that is prevalent among music practitioners. There is evidence that listeners are able to detect a slowing of tempo more readily than a raise in tempo and that the natural preference of both listeners and performers alike is toward faster tempi [147]. From this we can gather that we are naturally more comfortable and more in tune with fast tempi. This phenomenon seems to also be illustrated in the activities of students who are learning a new exercise or piece of music. Their tendency is to go faster and faster. For some reason they equate speed with success.

We must also take into account historical aspects of tempo that do not fall under the purview of "European Art Music." For example, in jazz there exists a similar ambiguity as to the notation of tempo. There are some conventions similar to those of the Italian descriptors used in the European tradition. However, the terms are less codified and equally vague. For instance, "Medium Swing" could refer to anything played from M.M. 100 - 188. Other terms such as "Medium Up," "Up," and "Ballad Tempo" are also used. They too are approximate in their meaning. In addition, some composers take it upon themselves to create new tempo designations like, "quasi-walking-latin-ballad," or "tempo di A-train." The latter refers to a song (*Take the A-Train* by Duke Ellington) that has been recorded hundreds of times at hundreds of different tempi, and the former seems to be an attempt to define tempo in the spirit of Beethoven's *C Major Mass* op.86, which was inscribed "Andante con moto assai vivace quasi Allegretto ma non troppo." In both cases, the descriptive markings seem to do more to cloud the issue than to clarify it.

Another non-European terrain that deserves exploration is that of the music of Africa. As Westerners, we cannot purport to have a clear understanding of the subject based solely on musicological readings. And research on the integration of African concepts of tempo and time with those of the Western tradition is limited, to say the least. Most of the writing on tempo to this point has been in regard to its mechanical function in Classical music (i.e. what bearing it has on the musical experience as a whole) or semantic matters (i.e. how to determine the "proper" tempo). And most of the writing on African concepts of time and tempo regards how it functions within itself. The African concept of tempo and time, in general, is much different—and as we understand it—is much closer in spirit to something that resembles David Epstein's [29] definition of tempo[1], in that it is inextricably linked to the everyday lives of all of the music's participants. It is fully integrated into the existence of the

[1] "Tempo is yet more complex as a phenomenon, for it embodies more than pacing per se. Tempo has generally been acknowledged as a consequence of the sum of all factors within a piece—the overall sense of a work's themes, rhythms, articulations, 'breathing,' motion, harmonic progressions, tonal movement, [and] contrapuntal activity. In this respect tempo is a product of the music—its Gestalt, so to speak, as well as an element of that Gestalt. Yet tempo, in its capacity as pacing, is another product still—a reduction of this complex Gestalt to the element of speed per se, a speed that allows the overall, integrated bundle of musical elements to flow with a rightful sense."

participants and is an inextricable part of the whole of their being. In effect, it has taken Epstein's supposition to a higher level and applied those axioms to a grander concept of "everything" as opposed to keeping it localized to just the performance of music. The subject of adapting this philosophy and attitude to the Western tradition bears further investigation.

The history of tempo is a massive subject on which many writers have expounded with great skill and academic rigor. We have no illusions that this history is complete in any sense of the word. It is our hope, however, that we have provided an adequate cross section of some of the work that has been done on this subject to date. And that if the reader is interested in expanding his/her knowledge of the subject, we have provided an interesting starting point as well as planted seeds for new research and philosophical inquiry.

6.2 Calcuating Time from Tempo

Let us now terminate the tempo discussion with the question of how time relates to tempo. Intuitively: If I drive my car, starting at position S_0 on the street, and if I observe the speed on my speedometer until I arrive at position S_1, how much time has then elapsed? Musically, this means the following: If I play a piece, starting at onset time E_0 on the score, and if I observe the music's tempo on my tempo curve until I arrive at position E_1, how much time (in seconds, say) has then elapsed?

Let us look at figure 6.7 for this calculation. To the left, we see the physical time as a function of symbolic time, the onset position on the score. The physical time that elapses between the initial physical onset $e_0 = e(E_0)$ and the final onset $e_1 = e(E_1)$ is $\int_{E_0}^{E_1} \frac{de}{dE}(E)dE$. But $\frac{de}{dE} = \frac{1}{T}$, whence

$$e_1 - e_0 = \int_{E_0}^{E_1} \frac{1}{T(E)} dE.$$

Let us give two representative examples:

1. Constant tempo $T(E) = T = \text{const}$. Then we have

$$e_1 - e_0 = \int_{E_0}^{E_1} \frac{1}{T(E)} dE = \frac{E_1 - E_0}{T}.$$

2. Linear tempo $T(E) = T_0 + S \cdot (E - E_0)$, $S \neq 0$, where $S = \frac{T_1 - T_0}{E_1 - E_0}$ with $T_0 =$ tempo at E_0 and $T_1 =$ tempo at E_1. Then

$$e_1 - e_0 = \int_{E_0}^{E_1} \frac{1}{T_0 + S \cdot (E - E_0)} dE = \frac{1}{S} \cdot \ln(\frac{T_1}{T_0}).$$

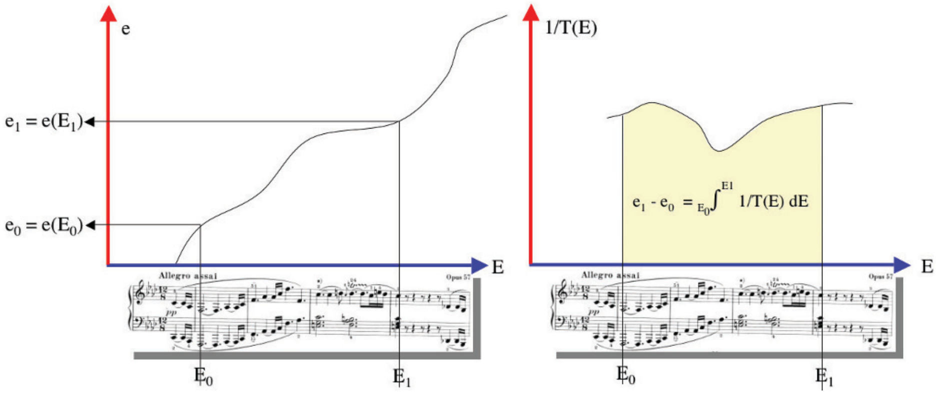

Fig. 6.7. Calculating time differences from inverse tempo curves.

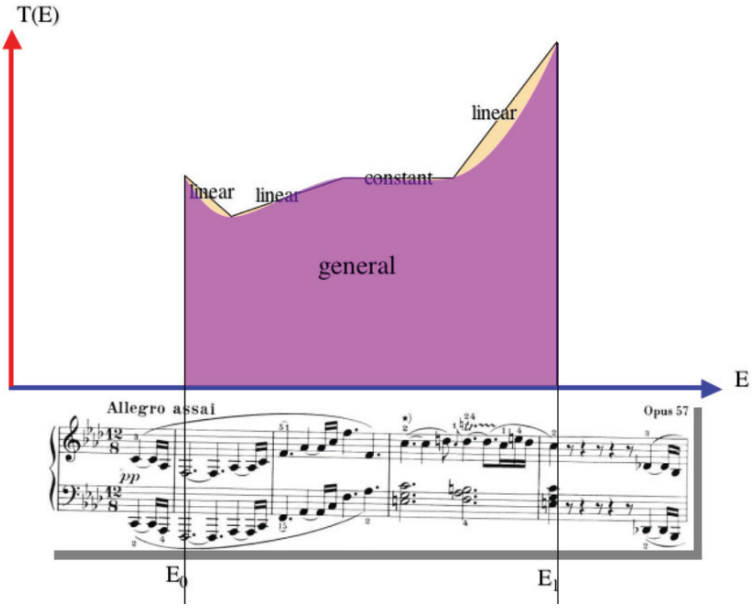

Fig. 6.8. Polygonal approximation of general tempo curves by linear and constant tempi.

With these results we may calculate the general case by sufficiently fine polygonal approximation of the tempo curve, as in figure 6.8.

We have not said the last word about the concept of tempo—we shall come back to this later—but want to emphasize that there are a number of open questions, such as:

- Does every piece have a tempo?
- Could it happen that a piece has several simultaneous tempi?

- Could it be that different parameters have a specific relations to their unfolding in time, and thereby enforcing a multplicity of tempi?
- Are all tempi, if there are several such things, equivalent, or may it be that we have herarchies of tempi, such as the conductor's tempo versus the musicians' or the soloists' tempi?