19

String Quartet Theory

Bei der nämlichen Gelegenheit fragte ich Haydn,
warum er nie ein Violinquintett geschrieben habe,
und erhielt die lakonische Antwort, er habe
mit vier Stimmen genug gehabt.[1]
Ferdinand Ries [116, p.287]

Why are we including this topic in a book on performance theory? Couldn't one include any other topic relating to a specific musical genre as well? Solo piano music, operas, what not? The reason for the special role of the string quartet is that this genre has a theory, a reflection about why these four instruments—the two violins, the viola and the violoncello—merge to such a perfect harmony or collaborative music, a theory that is not only relevant sociologically, but also with respect to the intrinsic messaging of musical contents.

Fig. 19.1. Ludwig Finscher.

But there is more: We have a string quartet theory in the sense of musicology or music theory as prominently described by Ludwig Finscher in his habilitation [32]. And we have a mathematical theory of the string quartet as developed by the author and presented in [75] and reproduced in [84]. This second theoretical approach perfectly confirms Finscher's findings about the singular role of the string quartet in the European history of music. But it is more prominently also a theory of the role of musical instrumental parameters in their function as expressive tools for the communication of musical contents. In this sense this theory is about performance: It deals with the question of how analytical musical contents are conveyed to the audience in the exquisite instrumentalization of the string quartet.

[1] On the said occasion I asked Haydn why he had never written a violin quintet, and I obtained the laconic answer that he had always had enough with four voices.

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The general topic behind this chapter is still another: It deals with the general question about the adequacy of instrumentation with respect to the musical contents of a composition. Why do we need a certain instrumentation to express a musical thought? Would it work if we played Beethoven's op.130 with four recorders? Or would it be advantageous to play *Happy Birthday to You* by a Wagner orchestra? The latter would be 'overdressed' while the former would flatten down the musical depth. It is interesting that this type of question has seldom been dealt with in classical orchestration and composition.

19.1 Historical and Theoretical Prerequisites

Finscher describes the historical root of the string quartet as follows (translation by the author in [84, p. 994]):

> *The prehistory of the string quartet is more complicated than that of any other instrumental art form of the eighteenth century. It cannot be causally deduced from any single one of the threads of tradition from where it comes.*
>
> *To a certain degree it is the creative act, the invention out of a moment of the delicate historic equilibrium, the kairos in the sense of ancient Greek thinking.*
>
> *The prehistory dates only from about 1720 to 1760 when Luigi Boccherini and Joseph Haydn independently invented the string quartet. In 1761, Boccherini wrote his first quartets in northern Italy, they were published 1767-68 in Paris under the name of "quatuor concertant." Probably Haydn had written quartet "divertimenti" already in the 1750s in Vienna, they were however only well known in 1760.*
>
> *The sparse regional, instrumental, and stylistic rootedness in the string quartet's prehistory, from which this new art form has quite spontaneously emerged, provokes the question whether beyond historical rationales a more systematic understanding could better enlighten the 'string quartet phenomenon.' The problem is to question this precise date (1760) of the rise of this precise instrumental art form (the string quartet) in the context of the European music from the systematic point of view.*

We should recall here the famous words of Carl Dahlhaus [20, vol. 10, p.104-105]:

> *Erst die systematische Konstruktion öffnet den Blick dafür, welche Tatsache einer Geschichte angehören, die zu erzählen lohnend erscheint.*

Only the systematic construction is capable of shedding light upon those facts of a story that are worth being narrated. The string quartet is a particularly challenging historical fact. Why did it emerge so unattendedly around 1750? And then grow instantly to a most profiled genre? Finscher:

> *Das Streichquartett ist die einzige Gattung der neueren Instrumentalmusik, die eine solche an einem einzigen künstlerischen Modell entwickelte, vergleichsweise genau und detailliert ausformulierte und als allgemeinverbindlich akzeptierte Theorie ausgebildet hat.*

The string quartet is the only genre of more recent instrumental music that has developed a comparably detailed and generally accepted theory built upon a single artistic model. Therefore, it is a good point of departure for our performance-oriented discussion of this genre that we have a good musicological theory. The model is based on

- the four-part texture,
- the topos of a conversation of four humanistically educated persons,
- the distinguished character of the family of violins.

The four-part texture ([84, p. 995])

The four-part texture was the ideal type of structured polyphony that was oriented on the counterpoint with its long tradition. This is the formal—or better: formalized—element of string quartet theory. We have to take it in the full conceptual ambiguity, i.e., on the one hand, the texture is "note against note" in its linear temporal progression in the sense of classical counterpoint. On the other, it is a texture of vertical units as an expression of harmonic relations.

With Haydn, one could imagine that he could have added a fifth voice to "enrich the texture." But it is reported that he 'failed' on several occasions with this 'experiment.' Ries reports 1838 ([32, p.287]):

> *Bei der nämlichen Gelegenheit fragte ich Haydn, warum er nie ein Violinquintett geschrieben habe, und erhielt die lakonische Antwort, er habe immer mit vier Stimmen genug gehabt. Man hatte mir nämlich gesagt, es seien drei Quintette von Haydn begehrt worden, die er aber nie hätte komponieren können, weil er sich in den Quartettstil so hineingeschrieben habe, dass er die fünfte Stimme nicht finden könne; er habe angefangen, es sei aber aus einem Versuche am Ende ein Quartett, aus dem anderen eine Sonate geworden.*[2]

The topos of a conversation of four humanistically educated persons ([84, p. 996])

This topos must be cautiously distinguished from the well-known topos of a "sound speech," i.e. from the similarity of musical expression or semantics to the common language. In the case of the string quartet, what's more important

[2] On the said occasion I asked Haydn why he had never written a violin quintet, and I obtained the laconic answer that he had always had enough with four voices. They had told me that three quintets had been commissioned to Haydn, which he however was never able to compose because he had become so accustomed to the quartet style that he could not find a fifth voice; he had commenced, but from one attempt a quartet emerged, and from another a sonata.

than speaking is the dialogue, a fact that becomes more evident in the French expression "quatuor dialogué" for "string quartet" (in fact the invention of a publisher). The association of a discourse to the string quartet was initiated by the musician Johann Friedrich Reichhardt in 1777 ([32, p.287]):

> *Bei dem Quartett habe ich die Idee eines Gesprächs unter vier Personen gehabt.*[3]

Like Haydn, Reichhardt also views the number four as being the upper limit for a good dialogue. He (also) tries to add a fifth person to the quartet. But he fails:

> *Die fünfte Person ist hier ebensowenig zur Mannigfaltigkeit des Gesprächs nothwendig, als zur Vollstimmigkeit der Harmonie; und in jenem verwirrt sie nur und bringt Undeutlichkeiten in's Stück.*[4]

The same happens to Schumann ([32, p.289]) in a discussion about a viola that was added in a quintet:

> *Man sollte kaum glauben, wie die einzige hinzugekommene Bratsche die Wirkung der Saiteninstrumente auf einmal verändert, wie der Charakter des Quintetts ein ganz anderer ist, als der des Quartetts. Hat man im Quartett vier einzelne Menschen gehört, so glaubt man jetzt eine Versammlung vor sich zu haben.*[5]

The distinguished character of the family of violins ([84, p. 997])

The formation of the string quartet would not have been thinkable without the collaboration of the homogeneous sound of the instruments of the violin family.

Finscher describes ([32, p.124/125]) the characteristic of violins as compared to the gambas, which were the preferred solo instruments in the seventeenth century, as follows:

> *Die Violinen hatten gegenüber den Gamben jedoch noch eine weitere zukunftsträchtige Eigenart: Sie gliederten den Tonraum, der ehemals in Analogie zu den menschlichen Stimmengattungen gebildet, nun aber in der Tiefe wie in der Höhe längst kräftig erweitert worden war, klarer und sinnfälliger, mit deutlicherer Individualisierung ihrer jeweiligen Tonbereiche. (...) Für das klassische Streichquartett, das die Beweglichkeit, den Lagenwechsel, den Kontrast- und Farbreichtum des symphonischen Streichersatzes mit der grösstmöglichen Annäherung an eine*

[3] For the quartet I had the idea of a conversation among four persons.

[4] Here the fifth person is as superfluous for the manifold of the conversation as it is for the full voicedness of harmony; and in the former it only creates confusion and makes the piece cloudy.

[5] It is incredible how a single added viola alters the effect of the string instruments, changing the character of the quintet radically as opposed to the quartet. If you heard four single persons in the quartet, you believe that now you are facing an assembly.

streng auskomponierte Vierstimmigkeit zu verbinden suchte, bot sich das vierstimmige Ensemble aus Gliedern der Violinfamilie als das ideale Instrument an.[6]

19.2 The General Plan

The general plan of our performance strategy of the string quartet is to generate optimal rhetorical expression of contents by use of the violin family's sound variety. This means that the sound configurations of the composition with respect to their constituting structures derived from counterpoint and harmony in the late eighteenth century must be played with maximal profile. What does "profile" mean? We must represent these configurations aurally such that these sound objects are optimally distributed in the sonic space-time. Despite this somewhat generic requirement, there is one condition that seems to be mandatory for optimal distribution: independent position, namely the condition that no three sound objects be aligned on a line, no four of them be lying on a plane instead of spanning a tetrahedron, and so on: any $n+1$ different points span an n-dimensional space. This is known as *general position*.

At first this looks more geometric than musical, but it was music theory that first introduced this seemingly geometric idea with the basic intervals of an octave, fifth, and major third in their just tuning. It is, in fact, well known that the pitches of all usual tunings (just, mixed, tempered) are (up to a basic reference pitch, such as the chamber pitch) rational linear combinations $p(o,f,t) = o\log(2) + f\log(3/2) + t\log(5/4), o,f,t \in \mathbb{Q}$. The three basic directions of these linear combinations, $\log(2), \log(3/2), \log(5/4)$, are linearly independent over the rationals; they are vectors of independent space directions. This was exactly how music teorists and composers conceived these basic intervals: They define musically independent harmonic "dimensions." That this is true in a very precise sense of modern linear algebra is a remarkable fact, and all the more since the mathematical machinery was invented several centuries after music's discovery.

In this spirit, general position will be the target to address when playing note assemblies with those rich instrumental colors. We therefore first have to make ourselves knowledgeable about the space of sounds that is opened by the violin family. When we know about that, we may ask questions about the

[6] Compared with the gambas, the violins had a further seminal peculiarity: They articulated the sound space, which originally was built in analogy to the human voices, now being however strongly extended in low and high pitches, with more clarity and evidence, with pronounced individualization of the respective regions of sound. (...) For the classical string quartet, which attempted to ideally connect flexibility, change of position, richness of contrast and colors of a symphonic string setting with the optimal approximation of a complete four-voice setting, the ensemble built from the four instruments of the violin family offered the ideal instrument.

configurations of sound objects in general positions, which are possible in such a sound space.

When we have those answers, we shall finally come back to the contents to be conveyed by these rhetorical sound representation tools. These contents must be analyzed with respect to general positions and then yield the number of instruments necessary to enable such rich-sounding representations. The result will be remarkable, but let us first get off the ground with the analysis of the violin family's sounds.

19.2.1 General Position for Performance: Four Examples

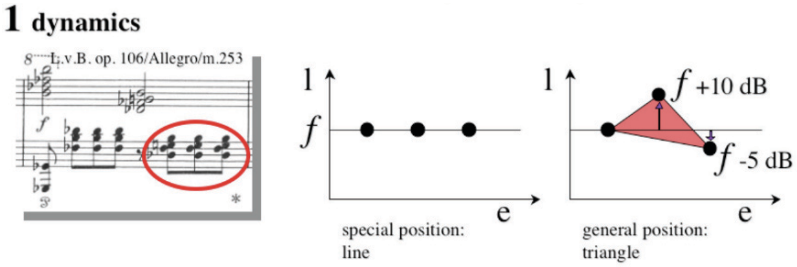

Fig. 19.2. Dynamical shaping of three chords in special position.

Before embarking on the systematic analysis of the sound space of the violin family, we want to discuss four examples of putting sounds in general position for performance purposes.

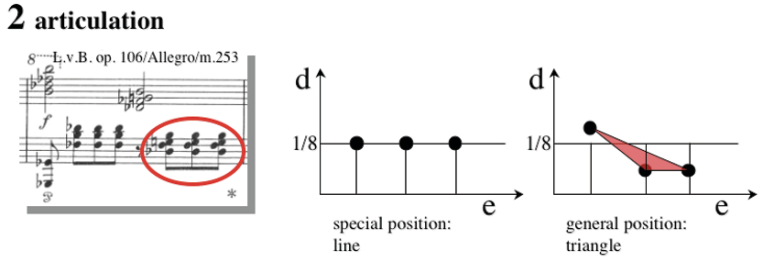

Fig. 19.3. Articulation shaping of three chords in special position.

The first example deals with dynamics (figure 19.2). We are given a piece where we want to express the rhythmical role of a sequence of chords in the left hand. They are all of identical loudness and occur an eighth note apart from each other. So they are in special position in the plane of onset and loudness. To change this, we increase loudness of the second by 10 dB and lower loudness

of the third chord by 5 dB. The result are three onset loudness points in general position—they define a proper triangle.

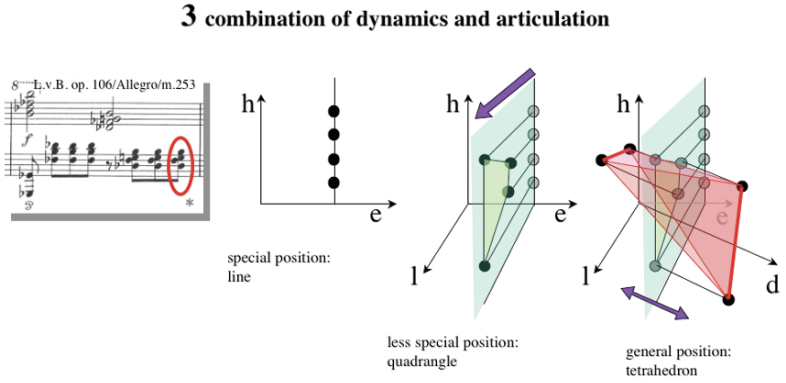

Fig. 19.4. Dynamics and articulation shaping of four chord notes in special position.

The second example is the same, but we now differentiate articulation instead of dynamics (figure 19.3). So we work in the plane of onset and duration. We are shortening the second and third notes, while the first is extended. The onset of the second is also shifted to enable the longer first duration. So the second starts while the first is still playing, and then we hear two staccati.

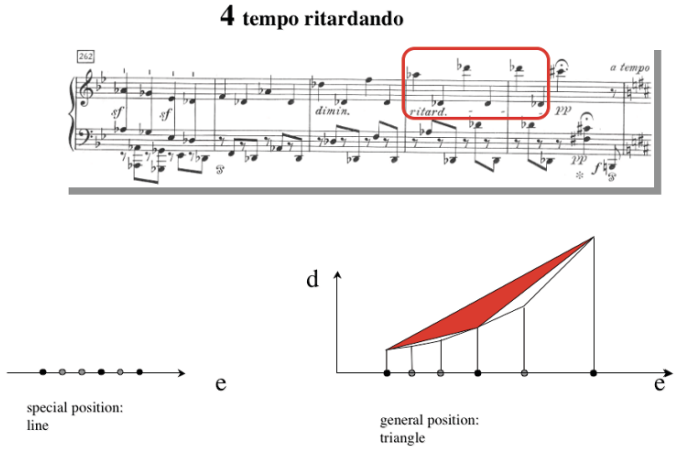

Fig. 19.5. Dynamics and articulation shaping of four chord notes in special position.

In the case of figure 19.4, four notes are all in one line on the pitch axis and have identical loudness, duration, and onset. To put them into general position, we first alter their loudness: The lowest and the highest notes are made louder

than the two internal notes of the chord. The notes are now somehow more generally positioned: They build a quadrangle. The second step increases the duration of the lowest and the second to highest notes, whereas the other two notes are given shorter duration. The result is a tetrahedral configuration. Now the four notes are in general position.

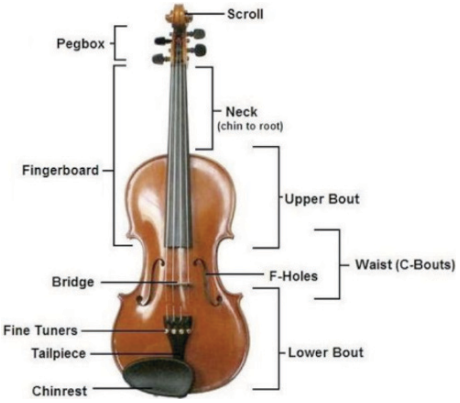

Fig. 19.6. The violin's components.

The fourth example, figure 19.5, shows the general position effect induced by a non-linear ritardando. We see that in the plane of onset and duration, the six points are such that some of the three-element subsets are in general position—for example the first, fourth, and last. To put the entire series of six notes into general position, we need a five-dimensional space. This is possible if we consider the parameters of onset, pitch, duration, loudness, and sound color (supposing that this one can be shaped here).

19.3 The Sound Space of the Violin Family

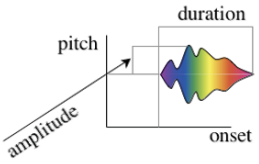

Fig. 19.7. A sound with its four geometric parameters.

The sound space of the violin family is a complex one, so let us start with the simplest information about a violin's anatomy (figure 19.6). The violin has a number of loci where it can be played, and this can be done in different ways, so, for example, the contact point of the string, or the angle of the bow on the string can vary. Let us give the complete list of parameters and then discuss them:

1 *Geometric parameters*
   a) onset, duration, and pitch define a three-dimensional space G
   (loudness will be added via amplitude to the technical parameters)
2 *Sound color parameters*
   a) instrumental parameters:
   violin type, choice of strings, performance conditions
   b) global technical parameters:
   vibrato: delay/pitch modulation (range of finger movement)/ modulation frequency/
   amplitude modulation (contact point of finger tip)

bow angle, bow contact point
they are a strong function of the individual player and remain relatively stable in time;

c) local technical parameters:
bow pressure, bow velocity = two-dimensional space H
they can be steered quickly and independently of each other, while bow pressure relates to amplitude.

The characteristic feature of the violin family is that the instrumental and the global technical parameters enable a much larger variety of sound color vectors than other instrument families. For example, if we compare two pitches, g and $g_{\#}$, being played by a Guarneri and by an F-horn, we get the image shown in figure 19.8. The Fourier spectra of the two instruments for the two pitches are markedly more independent from each other with the Guarneri violin as compared with the F-horn.

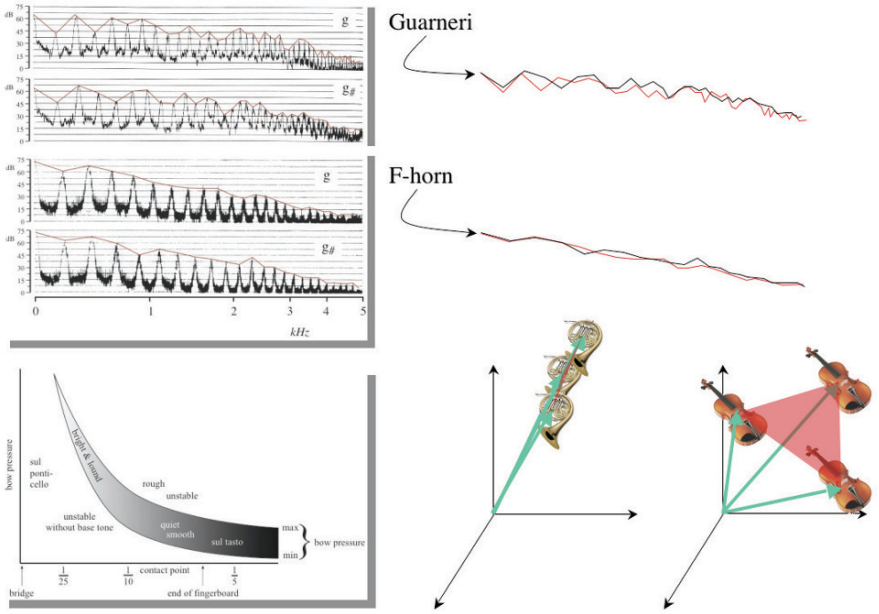

Fig. 19.8. The Fourier spectra of a Guarneri violin for two pitches g and $g_{\#}$, as compared with that of a F-horn. In their representation as vectors, these spectra look much more independent for Guarneri. The lower left graphic shows the variety of ways to play a violin.

19.4 Notes in General Position

The above sound space for n instruments of the violin family can be decomposed as follows:

1 Given n instruments of the violin family, we have a sequence $v_1, v_2, \ldots v_n$ of linearly independent vectors defined by the instrumental and the global technical parameters;
2 the two-dimensional space $H7\text{index}H$ of local technical parameters: bow pressure and bow velocity
3 the three-dimensional space G of geometric parameters: onset, duration, pitch.

This configuration enables the placement of sound events as played by the n instruments in a big space, see figure 19.9.

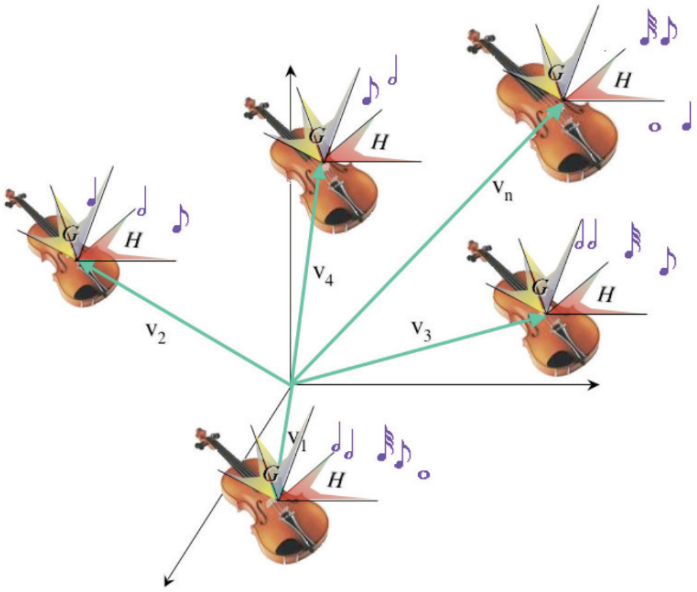

Fig. 19.9. The sound space of the violin family enables a distribution of the sound events played by n such instruments in a big sound parameter space.

As we are interested in general position questions, we need the answer to the question about how many sound events can be maximally placed in the space spanned by the instrumental vectors at which the common spaces G, H are attached, i.e. in the union

$$U = \bigcup_{i=1,\ldots n} v_i + G + H.$$

The answer is this mathematical theorem:

Theorem 1 *With the hypotheses and notations made on the parameter spaces available for the instrumental variety of the violin family, an ensemble of n such instruments can play at most $n+5$ points in general position on the space U.*

This means that when we look at all notes being played within a given string quintet, i.e. $n = 5$, then it is not possible to select more than 10 notes in general position.

19.5 Performance of the String Quartet

We now have the general geometric conditions for the display of notes in general position. But we have to add the music-theoretical situation given from the historical moment of the ending of the eighteenth century in order to understand the requirements for general position of notes as an expression of those theories.

For the contrapuntal theory, the essential structure is the first species. It is the basis for all other species, and we want therefore to look at the basic configuration of notes in this theory.

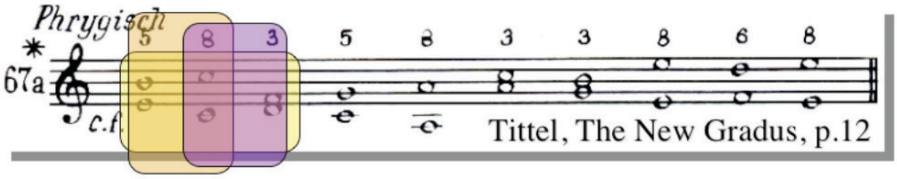

Fig. 19.10. Sequences of contrapuntal intervals must comply with the rules, such as forbidden parallels of fifths. But we also have to look at sequences of three successive intervals to grasp hidden parallels of fifths.

Figure 19.10 shows a sequence of two intervals that must comply with the rules, such as forbidden parallels of fifths. But we also have to look at sequences of three successive intervals to grasp hidden parallels of fifths. This requires a set of six notes. So we need "charts" of maximally six notes to be positioned in general position to see them all at once as a structural unit.

The other basic theory is harmony. Here, we have the most complex basic situation which is tonal modulation. Let us follow this theory in the lines exposed by Schönberg in his Harmony treatise [123], and take an example from page 197 (figure 19.11).

The example shows a covering of the modulatory phase by two charts of three triads each. The first one contains the modulation process $Cmajor \rightarrow$

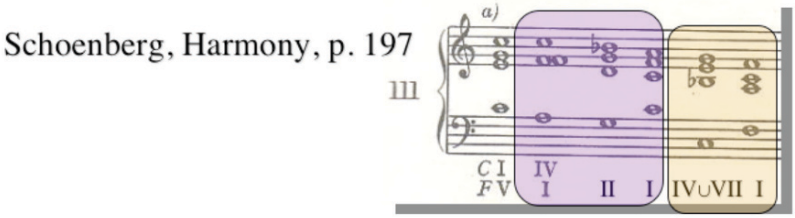

Fig. 19.11. A modulation from $Cmajor$ to $Fmajor$, showing the relevant charts for the tonal transition, followed by a cadence.

$Fmajor$ as described by Schönberg: neutral degree IV in $Cmajor$, pivotal chord II in $Fmajor$, tonic if $Fmajor$. And the second chart shows the cadence of the new tonality with the two chords $IV_F^7 = IV_F \cup VII_F, I_F$. So the maximal chart here has nine tones.

This means that we have an overall maximum of nine tones to be put in general position. This is, of course, a very rough requirement, since we do not represent the precise contents of these note assemblies, but only the geometric framework where to position them. But we believe that this geometric action is crucial for the best representation of theoretically important note groups.

With this in mind, we now have the following corollary of the above theorem:

Corollary 1 *We need at least four instruments of the violin group to position nine notes in general position in order to represent the essentials of contrapuntal and harmonic thinking in the late eighteenth century.*

This follows at once from the inequality $9 \leq n + 5$ from Theorem 1 of section 19.4.

Therefore, four instruments from the violin family are sufficient; of course, more can be used, but they are not necessary. *I argue that this is the deeper reason why Haydn, Schumann, and others did not need a fifth voice in these string instrumentations.*

String quartet theory therefore turns out to give a beautiful account on the question of expressing musical contents with a specific instrumental ensemble. It does so not only on a theoretical level of musical symbols, but also on the level of performance as a subtle shaping of sophisticated instrumental parameters of instruments that cannot be understood and played except by highly trained and cultivated performing musicians.