17

Shaping Operators

Those who are good at making shape don't usually fight.

Performance operators are those instances of our theory that shape a performance transformation. We have defined the relevant structures, namely performance cells, in chapter 10. A performance cell essentially includes the performance transformation $\wp$, the symbolic kernel K, and the sounding output data (the initial data are not central in this issue, so we neglect them here). Performace operators will also have to act on one of the three components of a performance cell: the symbolic kernel, the field defining the transformation $\wp$, or the physical sound output. According to one of these cases, we have called the operator *symbolic*, *field*, or *physical* (figure 15.10). Any performance operator will have to define such a performance cell.

This can be done in two ways: Either a completely new performance cell must be constructed or a given one is taken and then used to generate a new one. The first (uninteresting) case is known as *primavista performance*: One takes the score data and produces a primary performance with no artistically elaborate shaping. One could add the primavista operator as described in section 16.4, and that is all. That operator takes the weights and interprets them in a straightforward way. For example, it transforms the tempo weight function w_T as shown in figure 16.9 into a tempo curve without any change to the weight's shape. For example, if the default tempo is 100 [♩$/min$], then the primavista tempo is $T(E) = 100w_T(E)$. This produces a neutral first rendition.

As already pointed out in section 15.3, the second case is much more difficult and important since shaping operators must be applied in very different situations of performance with complicated conditions.

Before delving into delicate questions of this type, let us get off the ground with some easy operators. A very useful and easy symbolic operator is the split operator. It is used to split the given composition into parts that have to be treated separately, such as right and left hand for a piano composition, or pe-

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riods in order to shape such time slices independently. The operator takes certain parameters and creates cubes by defining intervals in those parameters. Then the composition (the symbolic kernel) is split into two portions: one being within the cube, and the complement. If we repeat this procedure, we may create quite sophisticated boxing configurations allowing for detailed processing. For example, it may be necessary to deal with a small motif or an ornament separately in its shaping. A trill, for example, might require a very special agogical treatment. It can also happen that we just need to redefine some symbolic objects for the sake of better symbolic representation. This might happen with regard to some time signatures or pitch shifting conditions, etc. This can be done with the symbolic operator. It allows for affine maps in any set of parameters. We omit the details here.

A second, relatively easy type of operator is the physical operator. It allows a weight to act upon any selection of parameters of the output of the given performance. This does not influence the transformation, nor does it change the symbolic kernel. It just takes the given output and then alters that data. For example, we may let a weight act upon loudness or duration or pitch, whichever.

We now want to give an instructive example of a tempo operator. Let us first deal with a straightforward idea to construct a tempo operator. We suppose that we are given a tempo field $T(E)$ in the performance cell that we want to modify by the tempo operator. The modification should be made using a weight function $w(E)$. The straightforward approach is to let this weight act as-is upon the tempo and to generate a new 'weighted' tempo $T_w(E) = w(E)T(E)$. This works supposing that the weight takes only positive values on the given time frame. Let us suppose this now. But how should we deal with the extension of this formula to articulation? If we are given a parallel field at the outset, we have $\partial T(E,D) = (T(E), 2T(E+D) - T(E))$. So we get the weighted parallel field

$$\partial T_w(E,D) = (T_w(E), 2T_w(E+D) - T_w(E)).$$

While this formula might work for a parallel field, if we apply it to a non-parallel field, it destroys the duration component of the given field and replaces it with the parallel component. This is precisely the delicate situation we alluded to above when saying that this is the straightforward approach: The given performance field might have a tempo component, but the articulation component (duration) is not a function of the tempo. Can such destructive action be avoided?

Yes, it can, and that works as follows. Restate the D component of the parallel field by

$$2T(E+D) = \partial T(E,D)_D + T(E)$$

and then get the formula for the weighted parallel field:

$$\begin{aligned}\partial T_w(E,D)_D &= w(E+D)(\partial T(E,D)_D + T(E)) - w(E)T(E)\\ &= w(E+D)\partial T(E,D)_D + (w(E+D) - w(E))T(E).\end{aligned}$$

So the total two-dimensional articulation field is as follows:

$$\partial T_w(E, D) = \begin{pmatrix} w(E) & 0 \\ w(E+D) - w(E) & w(E+D) \end{pmatrix} \partial T(E, D)$$

Call

$$Q_w(E, D) = \begin{pmatrix} w(E) & 0 \\ w(E+D) - w(E) & w(E+D) \end{pmatrix}$$

this matrix. Then we can define this field equation

$$\mathbf{Ts}_w(E, D) = Q_w(E, D)\mathbf{Ts}(E, D)$$

for an arbitrary articulation field. The definition is independent of $\mathbf{Ts}(E, D)$ being parallel or not. This is what can be taken as a generic definition of the tempo operator! Whenever we have an articulation field $\mathbf{Ts}$ on $\mathbb{R}^{ED}$, the tempo operator is just the one defined by the matrix Q_w deduced from the weight w with the above formula. Of course, this specializes to the weighted parallel field if the original articulation field is parallel. But it works in complete generality.

This example shows where lie the difficulties and subtleties in the construction of clever performance operators: They act in maximal generality upon given performances, but specialize to what is expected for classical special cases. This has a deeper meaning than just a technical flexibility. The entire operator theme is about what it means to conceive expressive performance. This concept is about how we insert rhetorical architecture into the performance's unfolding. What is it that we want to influence by a given weight, and what is the essence of such an influence? For example, the tempo operator: Do we really understand this operator if we just apply it to the parallel situation? The above construction shows that we can interpret the tempo processing by a weight as being a deformation of any given articulation hierarchy $\mathbf{Ts}(E, D) \to T(E)$.

Behind this concern for flexible operators is also the deep question about the variety of operators as such. How many operators do we have? Are they all essentially different or are there some generic operators that specialize to more specific forms? Is there even a unique master operator, which can be specialized to any specific type?

Why is this musically speaking relevant? Because we would like to know about the unifying principles of expressive performance, at least in the analytical domain.

17.1 Are Lie Type Operators Universal?

There is a type of shaping operator that is both a well-known construction in mathematics as it is a quite general approach in the musical context. It uses a classical operator in differential geometry: the Lie operator. The Lie operator is defined for a given vector field X on a performance frame, say. It takes a differentiable function f on that frame and creates a new function $L_X(f)$, the Lie derivative of f. This operator $L_X : F \to F$ on the algebra F of functions on the given frame acts as a derivation: It is $\mathbb{R}$-linear and we have $L_X(fg) = fL_X(g) + gL_X(f)$. The relevance of this construction lies in the fact that the map $X \mapsto L_X$ is an isomorphism of the vector space of vector fields onto the vector space of derivations. A vector field is essentially the same as a derivation, which transforms functions into functions. Therefore:

Performance fields are essentially derivations on weights. Which means that performance fields are naturally associated with weights.

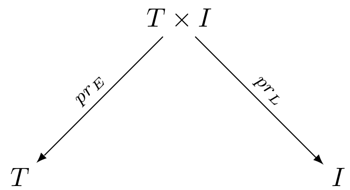
This is a strong argument for both, the performance field formalism and the usage of weights for the shaping of performance. Let us define a general operator using a weight and acting upon a given performance field. Take a performance field $\mathbf{Ts}$ on the source space $\mathbb{R}^{X.}$ of a performance hierarchy. Let Z, S be two subspaces of the hierarchy, Λ a weight on Z, and $Dir : S \to S$ an affine endomorphism. Let $i_S : S \to \mathbb{R}^{X.}$ be the embedding map of S, and $p_S : \mathbb{R}^{X.} \to S$ the projection onto S. Then we have this new performance field:

$$\mathbf{Ts}_{\Lambda,Dir} = \mathbf{Ts} - L_{\mathbf{Ts}_Z}(\Lambda) i_S \circ Dir \circ p_S$$

with $\mathbf{Ts}_Z$ being the Z-component of $\mathbf{Ts}$. This shaping operator type is called a *Lie operator*. So the operator acts trivially if the gradient of Λ is orthogonal to the given field $\mathbf{Ts}_Z$, i.e. the integral curves of the performance field move along constant weight hypersurfaces. Which is completely natural: When moving along an integral curve, the weight does not change, so it should not affect the given performance.

The point of this Lie type operator is that it cover quite a number of operators. Namely all those that create one of the following three deformations of hierarchies:

- The articulation hierarchy $\partial T \to T$ deforms to $Z_w \to T_w$ for a given weight w by the above matricial operator Q_w.
- The parallel articulation hierarchy $\partial T \to T$ deforms to a general hierarchy $Z \to T$.
- The hierarchy



deforms to the hierarchy $Z \to T$.