16

Analytical Weights

Weight and measure save a man's toil.

Analytical weights did not come up from empty space. In fact, our idea was taken from Hugo Riemann's definition of metrical weights: Meter relates to weights. So it was decided to generate an output in the form of numerical weights for any analytical engine. Here is the precise definition of a weight:

Definition 1 *An analytical weight is a continuous function*

$$w : PARA \to \mathbb{R}$$

defined on a space PARA of parameters such as $\mathbb{R}^E$, $\mathbb{R}^H$, $\mathbb{R}^{EHL}$, *etc., with non-negative real values.*

Such weights are calculated upon music analyses and correspond to associated semantics. For example, a metrical weight $w : \mathbb{R}^E \to \mathbb{R}$ might associate metrically important onsets with higher weights. We observe that weights are also defined where no notes are present. This is not a restriction, since any discrete functions defined only on notes, say, can be extended in a continuous way to the entire space. There are also deeper reasons for this setup. Since performance fields are defined on entire frames (see section 10.1), their shaping must be defined for any argument of those frames, also where there

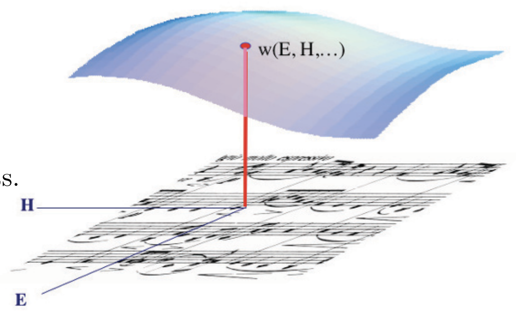

Fig. 16.1. An analytical weight on the space $\mathbb{R}^{EH}$.

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are no notes. Therefore, it is wise to use weights that are defined all over the place. There is also a more computer-driven argument for such an extension. If we are to apply a shaping to a given note, it might be that for certain computer-generated imprecisions, the position of the note cannot be identified with the position of a discrete weight, when applied to that note. Therefore, it is prudent to extend the discrete weight continuously to the neighborhood of each note point.

We have implemented the construction of continuous weight functions from discrete weights by use of cubic splines. Cubic splines are uniquely determined cubic polynomial functions $P(x) = a_3X^3 + a_2X^2 + a_1X^1 + a_0$, which connect two values f_0, f_1 at two arguments x_0, x_1, respectively, with the given slopes s'_0, s'_1. This means that $P(x_0) = f_0, P(x_1) = f_1, P'(x_0) = s_0, P(x_1) = s_1$. This construction can be extended by recursive procedures to functions on higher-dimensional spaces [84, section 32.3.2.1]. Our slopes are always set to zero, so that the local variation of the continuous extension is minimal, if the argument is slightly different from the required data.

In the following section, we present a bunch of analytical tools, which were implemented in the Rubato software to give prototypical examples of analytical procedures following the above weight philosophy. Although none of these was thought to be a particularly creative contribution to musical analysis, it turned out that they all quite ironically entailed successful scholarly careers of those specialists[1] who delved into these analytical topics without deeper connections to performance theory as such.

16.1 Metrical Weights

The metrical analysis that we developed in this context can be understood from its central concept: the local meter (figure 16.2). This is akin to the one proposed by Jackendoff and Lerdahl [53], but differs in essential points: A local meter is a finite sequence $M = (E_0, E_1, \ldots E_l)$ of regularly distributed symbolic onsets E_i with constant interval $d = E_i - E_{i-1}, i = 1, \ldots l$, the number $l = l(M)$ is called *the local meter's length*. Local meters are however always built from onsets that appear as attributes of objects, such as notes, pauses, etc. in a score. Onsets that are not related to concrete objects are not admitted, in contrast to the approach in [53], and also in accordance with Riemann's understanding of metrical structure being supported by existing events.

A maximal local meter in a score is a local meter, which cannot be embedded in a properly larger local meter. Figure 16.2 gives an example of a maximal and of a non-maximal local meter. In the metrical analysis of a piece, we then project all notes to their onsets and look at the covering of those onsets by maximal local meters. Figure 16.3 shows a simple music piece X and its covering $Max(X) = \{a, b, c, d, e\}$ by five maximal local meters.

The notes are not all in the same position with respect to this covering. Some are contained in many maximal local meters, others in just one of them.

[1] This is Chantal Buteau for melodic analysis and Anja Volk-Fleischer for rhythmical analysis. And to a lesser degree Thomas Noll for harmonic analysis.

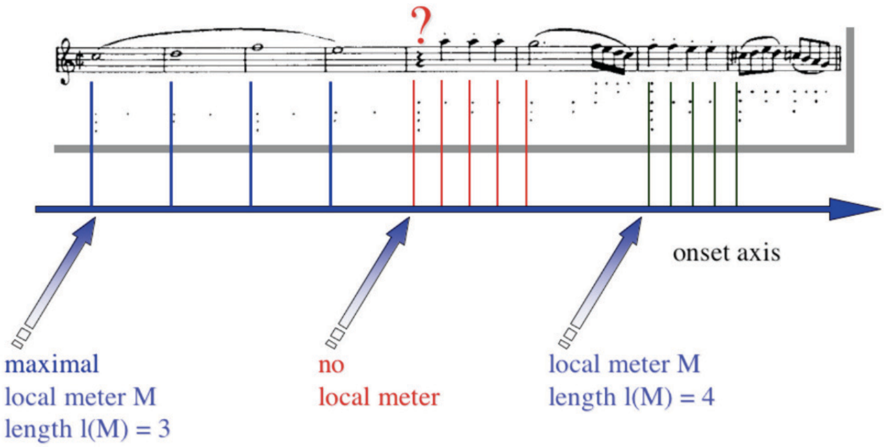

Fig. 16.2. Local meters: to the left a maximal one, in the middle a counterexample, to the right a non-maximal local meter. The metrical analysis is akin to the one proposed by Jackendoff and Lerdahl [53], from where we have taken the present score excerpt, the beginning of Mozart's *Jupiter Symphony*.

Some are contained in longer local meters, some in shorter. There are two views on this situation: a topological and a numerical. The topological one views notes of the composition X as being more or less dominant over others as a function of the maximal local meters which contain them. This is formally represented by the so-called *nerve* $\mathcal{N}(X)$ of the covering $Max(X)$. Figure 16.4 shows the situation.

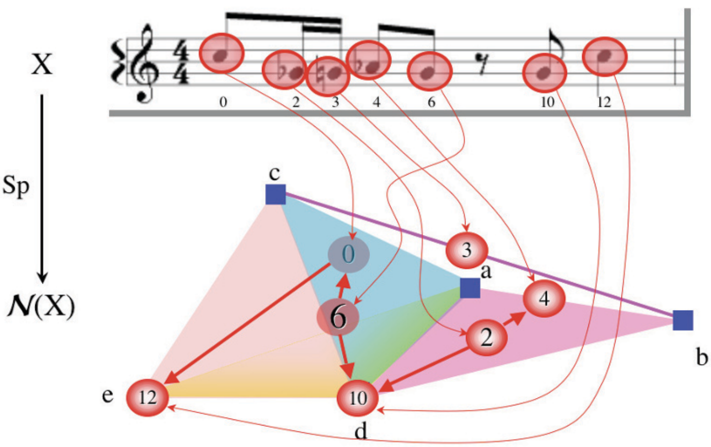

Fig. 16.4. The nerve of a composition X.

We have a map $Sp : X \to \mathcal{N}(X)$ that associates with each note $x \in X$ the set $Sp(x)$ of all maximal local meters containing x, this is called the *simplex of* x. We then see that certain notes are metrically dominant in the sense that they have larger simplexes than other notes. Musically speaking, this means that these notes participate in more local meters then others, so their metrical relevance is dominant. We see in our example that note 6 has a tetrahedron simplex—it is contained in maximal local meters a, c, d, e—whereas note 12 is only in the simplex that has a single maximal local meter e. So note 6 dominates note 12: That maximal local meter defines one vertex of the tetrahedron. Note 3 has a simplex built from two maximal local meters b, c, and we draw a line to visualize their common note 3. Note 2 has a triangle simplex: It is spanned by three vertexes, a, b and d.

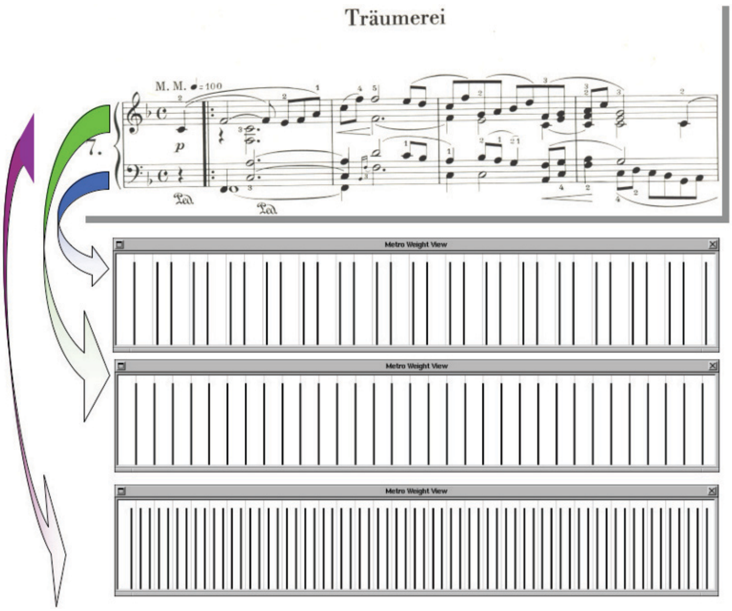

Fig. 16.5. The metrical analysis of Schumann's *Träumerei*, op.15/7, by Rubato's MetroRubette reveals a left hand weight of 3 +5 against a right hand of 4 beats, while the combined metric weight is an eight note offbeat metric.

The topological perspective is interesting, but far from what we expect: namely the weights associated with a given analysis. We would like to call the nerve the *global metric* of the piece, while the global rhythmic would be the following. We look at a given onset x. Then we look at all maximal meters containing that onset, and then we try to get a weight from that information. This information consists of two things: the maximal local meters containing that x, and then for each such object, a numerical value measuring this local meter's significance for that onset. We propose this formula for the metrical analysis we have implemented:

$$w(x) = \sum_{x \in M, m \leq l(M)} l(M)^p$$

where m is a minimal length of maximal local meters to be considered in this calculation, and where p (metrical profile) is a power that determines the relevance of lengths of local meters. The minimal admitted length m means that maximal local meters shorter than m are omitted. We only look at maximal local meters with sufficiently large length. We would call this function w the *global rhythmic* of X, the rhythmic being a function of the global metric structure but having a numerical expression, and this one being a weight function.

While this is a fairly satisfactory solution of the original problem, we are still left with some problems. In fact, in music scores, there are many different signs that relate to time: notes, pauses, notes from different instruments, bar lines, etc. How can we manage this? The approach is fairly simple. We take a number of such types of objects, like notes, pauses, etc. Let them be the types $t_1, t_2, \ldots t_k$. Then we have a weight function $w_i(x), i = 1, \ldots k$ for each of them according to the preceding theory. Further, we decide to give each of these weights a weight, i.e. a number $\nu_i \in \mathbb{R}$ measuring the strength of the weight w_i. Then we can define a combined weight by the function

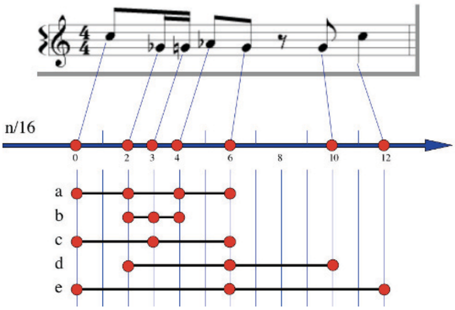

Fig. 16.3. The maximal local meters covering the piece X.

$$w(x) = \sum_{i=1,\ldots k} \nu_i w_i(x)$$

Let us look at an example to illustrate the general technique and its usage. In figure 16.5, the right hand shows a regular 4-beat weight, when we go to the longest possible minimal length m where there are maximal meters. In contrast, the left hand shows a 3+5 structure; this means a two-bar regularity, a marked opposition to the regular right hand. This creates a strong tension in performance; perhaps this is felt by many pianists performing the *Träumeri*?! The weighted sum of both of these weights (with $\nu_1 = \nu_2 = 0.5$) shows a half-measure offbeat metrical weight. The left hand sound with its 3+5 structure can be heard from a computer-generated performance in example ♫ 17.

For a more detailed study of metrical weights, we refer to Anja Volk-Fleischer's work [33].

16.2 Melodic Weights

A second analysis that we have implemented was inspired by Rudolf Reti's work on thematic processes in music [113]. It turned out that here, much more than with the metrical/rhythmical analysis, there was no valid theory. The very concept of a melody or motif is not defined, and no precise theory about the body of motivic structures within a given composition is available. We do not discuss this dramatically underdeveloped theory here, but see [9] for a detailed account. Despite this deplorable state of the art of motivic analysis, we have initiated an analytical theory of motives in order to be able to implement such thoughts and to use them in the framework of the Rubato software.

To begin with, we suppose that the score is given as a set of events with onset and pitch and possibly some other coordinates. So they live in the space $\mathbb{R}^{EH\dots}$. Then

Definition 2 *A motif M in $\mathbb{R}^{EH\dots}$ is a finite set $M = \{n_1, n_2, \dots n_k\} \subset \mathbb{R}^{EH\dots}$ of k different notes having all different onsets, i.e. $E_{n_i} \neq E_{n_j}$ if $i \neq j$.*

With this definition, one may define different paradigms of similarity among motives, depending upon the information extracted from a motif's structure. For example, one may only look at the structure of increasing, equal, or decreasing successive notes. We omit the technical details here and refer to [84, Chapter 22]. Whatever is the similarity paradigm, we may then define precisely what it means when a motif is similar to, i.e. in a neighborhood of, another motif with the *same cardinality*. This is a concept of distance, so we may say that the distance $d(M_1, M_2)$ of motif M_1 to motif M_2 is less than 0.125.

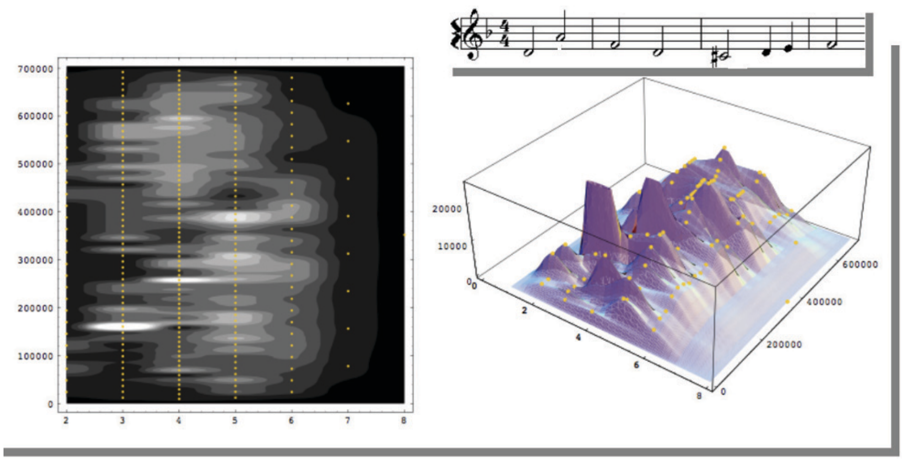

Fig. 16.6. The weights of motives in the main theme of Bach's *Kunst der Fuge*. The motives are grouped by their cardinality, and the weight to the left is encoded by brightness.

With these prerequisites, given a positive real number ϵ, we may define the melodic weight of motives (the definition is somewhat simplified here but gives the idea, see [84, Chapter 22] for a detailed account). Take a motif M in our composition and look at all motives N in the given composition such that there is a submotif $N^* \subset N$, with the same cardinality as M, such that $d(M, N^*) < \epsilon$. Call their number the *ϵ-presence* $pr_\epsilon(M)$ of M. Similarly, consider all motives L in the composition that are in the ϵ-neighborhood of a submotif of M, and call their number the *ϵ-content* $ct_\epsilon(M)$ of M. Then the *ϵ-weight* $w_\epsilon(M)$ of M is the product

$$w_\epsilon(M) = pr_\epsilon(M) \times ct_\epsilon(M).$$

So the weight of a motif 'counts' all motives that contain some motif similar to M or being similar to a submotif of M. This accounts for the motif's relation to other motives in the composition. See figure 16.6 for an example.

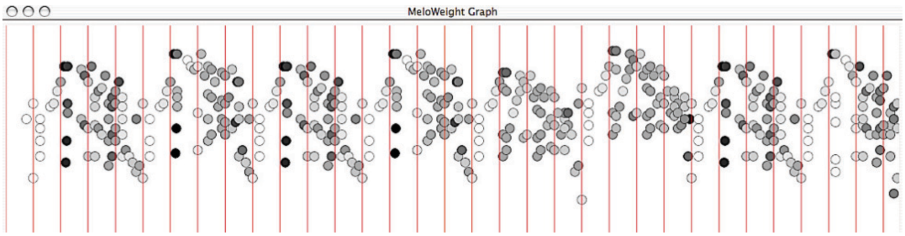

Fig. 16.7. The weights of notes in Schumann's *Träumerei* in Rubato's MeloRubette. The vertical lines are the barlines.

Given these numbers, we can define the *melodic ϵ-weight of a note x* of our composition to be the number

$$w_\epsilon(x) = \sum_{M, x \in M} w_\epsilon(M)$$

Its music-theoretical meaning is the account of all motives' weights, where x is a member. See figure 16.7 for an example, where the gray value of disks encodes the weight of the notes that are represented by these disks.

16.3 Harmonic Weights

The harmonic analysis that we have implemented is quite involved Riemann theory. Riemann harmony is designed to attribute to chords three types of harmonic values: tonic, dominant, or subdominant. Such a value is always related to the tonality where the given chord is situated. This valuation of chords generates a syntax of harmonic values, which reflects the harmonic semantics of tonal music. Harmony then makes statements about the harmonic meaning of given sequences of chords. Despite this fundamental role of Riemann harmony, Rubato's HarmoRubette for harmony is the rist to make Riemannian function theory fully explicit. The reason is that Riemann's harmony has never been completed because complex chords have never been given harmonic values by a reliable system of rules, but see [84, Chapter 25] for details. Our idea is this: We start with the sequence $(Ch_1, Ch_2, \dots Ch_n)$ of all chords in a given piece X. We first calculate the Riemann function values for each chord. This means that for every Riemannian value $riem = T, D, S, t, d, s$ of major tonic T, dominant D, subdominant S, and minor tonic t, dominant d, subdominant s, and every tonic $ton = C, C_{\#}, D, D_{\#}, E, F, F_{\#}, G, G_{\#}, A, A_{\#}, B$, we calculate a fuzzy value $val_{ton,riem}(Ch_i) \in \mathbb{R}$ of chord Ch_i. This defines the *Riemann matrix* $val_{.,.}(Ch_i)$ *of* Ch_i. The fact that we do this in a fuzzy way means that we do not oversimplify the situation: It might happen that a chord is 'more or less' dominant in D major; this is the meaning of fuzzy values here.

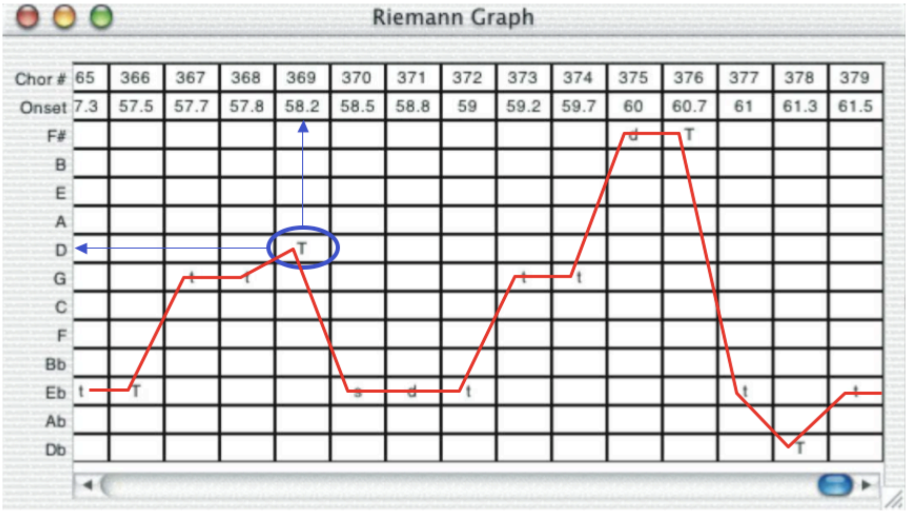

Fig. 16.8. The Riemann graph of a composition in Rubato's HarmoRubette. The sequence of chords is given Riemann values as a function of least transition weights.

Next, preferences allow us to set the transition weights for any pair of successive chords and Riemann parameters $(ton, riem, Ch_i), (ton', riem', Ch_{i+1})$, using also the Riemann matrix values $val_{ton,riem}(Ch_i), val_{ton',riem'}(Ch_{i+1})$. Harmonically difficult transitions will get larger weights than easier transitions. With this information, one then looks at all paths of Riemann parameters of chords

$$(ton_1, riem_1, Ch_1), (ton_2, riem_2, Ch_2), \dots (ton_n, riem_n, Ch_n)$$

and calculates the weight of such a path as a function of the pairwise transition weights. The lightest path is then chosen as being a solution of the Riemann function attribution for all chords. Figure 16.8 shows an example of a harmonic path.

The calculation of harmonic weights of single notes is now easy (although very intense in terms of computer calculation work). We select a note x, living within a chord Ch_{i_0}. Then we calculate the weights for the chord $Ch_{i_0} - \{x\}$ and look at the ratio of the full path as compared with the weight of the path with the chord after omitting x. The weight of x increases if the ratio is large, and so we get weights of single notes. The technical details are described in [84, section 41.3], we omit them here.

Whereas the rhythmical weight is essentially a function on the onset space $\mathbb{R}^E$, both the melodic and the harmonic weights are functions on $\mathbb{R}^{EH}$.

16.4 Primavista Weights

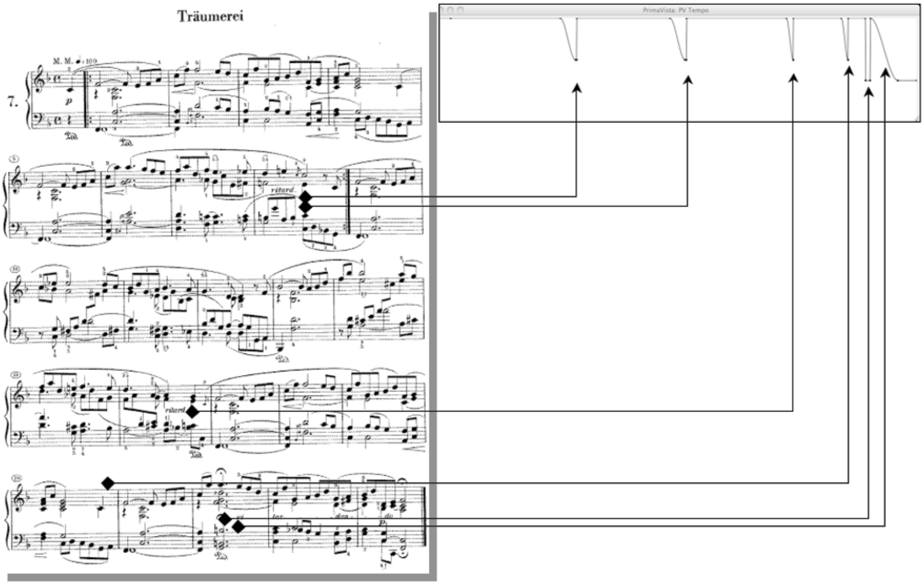

Fig. 16.9. The PrimavistaRubette deals with performance commands given from the score's structure. Here, we are giving the primavista agogics defined by the score's tempo indications.

The primavista weights are a special case, but one can interpret them in terms of weights. It deals with performance signs that are written on the score and need a representation by means of weights. We can do this for all primavista signs. Let us show how it is done just for agogics. In the example shown in figure 16.9, we have a set of tempo indications: several ritardandi and one fermata. This can be encoded as a weight function that shows a tempo curve that reflects these signs. The precise shape and quantity of these commands can be defined on the preferences of the PrimavistaRubette, so it is up to the user to make precise the meaning of a ritardando or a fermata. But the resulting curve is understood as a weight function, which, when applied to tempo shaping, yields the quantitative and qualitative forms of these agogical signs.