11

The Category of Performance Cells and Hierarchies

On trouve toujours l'homogène à un certain degré de division.
Paul Valéry [142, I, p.209]

This chapter completes our study of the structure of performance. Recall from chapter 5 that we had four local-global dichotomies for performance theory: instrumental, parts, dimensions, and evolution. We are not going to discuss the first two, but dimensions and evolution will be dealt with. Dimensions are what we want to discuss now, and evolution will be discussed in chapter 21.

We have defined the minimal units that are full-fledged data for the construction of performance, namely performance cells $\mathcal{C} = (\mathbf{Ts}, F, I, \wp^I_{P.}, K)$. But we have also seen that certain parameter spaces do not need other parameters in order to define performance of *these* parameters. The two examples we have dealt with are the default performance of onset and pitch on one side:

$$\begin{tikzcd} & \mathbb{R}^{EH} \arrow[dl, "pr_E"'] \arrow[dr, "pr_H"] & \\ \mathbb{R}^{E} & & \mathbb{R}^{H} \end{tikzcd}$$

and the performance of onset and duration on the other:

$$\begin{tikzcd} \mathbb{R}^{ED} \arrow[d, "pr_E"'] \\ \mathbb{R}^{E} \end{tikzcd}$$

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These arrows can now be made more precise when relating them to performance cells. This works as follows: Suppose in the first situation that we are given a performance cell $\mathcal{C} = (\mathbf{Ts}, F, I, \wp^I_{EH}, K)$ for onset and pitch. And also suppose that the field $\mathbf{Ts}$ is a Cartesian product $\mathbf{Ts} = \mathbf{Ts}_E \times \mathbf{Ts}_H$ of an onset and a pitch field, i.e., tempo and intonation. Then we can forget about pitch when performing onset: Tempo is independent of intonation, and vice versa. Take now the projection $K_E = pr_E(K)$, $K_H = pr_H(K)$ of the symbolic kernel to onset and pitch, respectively. Also suppose we have "reasonable" initial sets I_E, I_H and initial performances $\wp^I_E, \wp^I_H$ in onset and pitch (we are imprecise here, but this is a technical subtlety that would disturb our understanding), and take the projections $F_E = pr_E(F), F_H = pr_H(F)$ as onset or pitch frames, respectively. We therefore get two performance cells: $\mathcal{C}_E = (\mathbf{Ts}_E, F_E, I, \wp^I_{EH}, K)$ and $\mathcal{C}_H = (\mathbf{Ts}_H, F_H, I, \wp^I_{EH}, K)$. Then the projections p_E, p_H can be viewed as "morphisms" (a kind of generalized map) between these cells, yielding this diagram:

$$\begin{tikzcd} & \mathcal{C} \arrow[dl, "pr_E"'] \arrow[dr, "pr_H"] & \\ \mathcal{C}_E & & \mathcal{C}_H \end{tikzcd}$$

Similarly, we can generate an arrow of performance cells in the second case:

$$\begin{tikzcd} \mathcal{C} \arrow[d, "pr_E"'] \\ \mathcal{C}_E \end{tikzcd}$$

in which case there is no arrow to a cell in the space of durations, since with no performance fields here, no such cell is possible.

The general definition of such a morphism between performance cells is this: Take two cells $\mathcal{C}_1 = (\mathbf{Ts}_1, F_1, I_1, \wp^I_{P.}, K_1)$, $\mathcal{C}_2 = (\mathbf{Ts}_2, F_2, I_2, \wp^I_{Q.}, K_2)$, where $Q.$ is a subset of parameters of the set $P.$ and consider the projection $p : \mathbb{R}^{P.} \to \mathbb{R}^{Q.}$. Suppose these conditions are satisfied:

1. $p(F_1) \subset F_2$,
2. $p(K_1) \subset K_2$,
3. $p(\mathbf{Ts}_1) = \mathbf{Ts}_2$, which means that the components of $\mathbf{Ts}_1$ in $\mathbb{R}^{Q.}$ do not depend on parameters other than those in $\mathbb{R}^{Q.}$ and have the values of $\mathbf{Ts}_2$;

plus some technical conditions on the initial performance that we omit here (but see [84, Chapter 35.2]). These data define what we call a *morphism of performance cells*. It is denoted by

$$p : \mathcal{C}_1 \to \mathcal{C}_2.$$

To be clear, the underlying map of a performance cell morphism is always a projection of parameter spaces. The only serious point that turns it into a morphism is the set of the above three conditions (plus those technical conditions). With this conceptual architecture, we can now define a *performance hierarchy* $\mathcal{D}$ as being a diagram $\mathcal{D}$, whose vertexes are performance cells and whose arrows are morphisms of performance cells. So the above diagrams give rise to simple performance hierarchies.

The advantage of this hierarchical representation of performance is twofold. First, it eases calculations for parameter spaces that are projections of higher-dimensional ones. It is much easier to calculate, for example, three one-dimensional values than one three-dimensional one since the calculation of higher-dimensional cases requires the numerical integration of vector fields, which is equivalent to the numerical solution of ordinary differential equations (ODEs). Second, for the genealogical theory of performance, i.e. the theory that describes performance as an unfolding process, starting with the unshaped prima-vista rendition and ending with the artistically detailed shaping, one needs such hierarchical structures in order to derive more sophisticated performances from simpler ones.

In order to show concrete situations, we give in figure 11.1 an example of a performance hierarchy for the piano. It is the default hierarchy, i.e. a hierarchy defined for the most simple configuration of the piano. It is the starting points for more sophisticated performances.

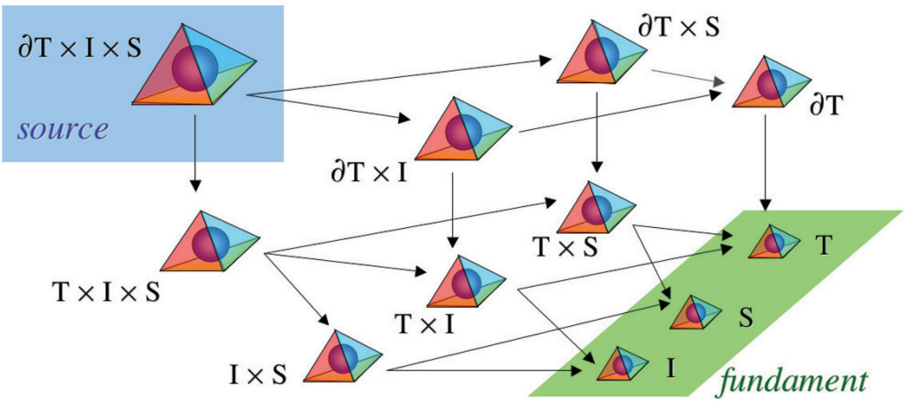

Fig. 11.1. The piano hierarchy spans between the source cell with all four piano parameters—onset, pitch, loudness, and duration—and the three fundamental cells for onset, pitch, and loudness, respectively.

It lives in the four-dimensional space $\mathbb{R}^{EHLD}$ and is realized in the source cell in figure 11.1. This cell has the Cartesian product of three fields: the one-dimensional fields I for intensity (dynamics) and S for intonation, and the two-dimensional parallel field ∂T of articulation. This cell projects onto three three-dimensional fields $T \times I \times S$, $\partial T \times I$, and $\partial T \times S$, where the first projections stem from the projection of the parallel field onto tempo. The triple Cartesian product cell for $T \times I \times S$ projects onto three two-dimensional cells $I \times S$, $T \times I$, $T \times S$, whereas both, $\partial T \times I$ and $\partial T \times S$ project onto the parallel field ∂T. All these two-dimensional cells project onto the one-dimensional cells I, S, and T of the fundament. When refining this default performance scheme, one will have to act on some of these cells and thereby define a new hierarchy that as a matter of fact will have fewer vertexes because the independence of parameters will be deranged. A more complex default hierarchy for the violin has been described in [84, Chapter 35.3.3].