10

General Performance Fields

La musique mathématiquement discontinue
peut donner les sensations les plus continues.
Paul Valéry [142, I]

Let us now look at the general procedure for defining performance. We write $P.$ for a sequence $P_1 P_2 \ldots P_n$. We have seen in the previous analysis that a performance $\wp_{P.}$ on a parameter space $\mathbb{R}^{P.}$ gives rise to a performance field $\mathbf{Ts}_{P.}$, and that this field can be used to define performance. We do, however, have to make this precise in the general case.

In the classical case of tempo (the one-dimensional situation of onset performance), we have the formula

$$e_1 - e_0 = \int_{E_0}^{E_1} \frac{1}{T(E)} dE$$

that yields the physical time between the initial symbolic onset E_0 and the terminal onset E_1. It completes the necessary information once we know the initial physical onset time $e_0 = \wp_E(E_0)$. So we have everything once we have that one-dimensional field $T(E)$ and the initial performance $\wp_E(E_0)$, which we denote by $\wp_E^I(E_0)$ since it is given a priori and independently from the tempo field data.

10.1 General Performance Fields

The general case works similarly but is more demanding, mathematically and musically speaking. The easiest way to understand the process is to view the performance field $\mathbf{Ts}$ as being the inverse image under $\wp_{P.}$ of the constant diagonal field Δ. This means that integral curves of the Δ field are mapped to

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integral curves of $\mathbf{Ts}$. But what is an integral curve of a vector field? Suppose we take a point X of the underlying parameter space $\mathbb{R}^{P\cdot}$. Then there is a unique curve $\int_X \mathbf{Ts} : J \to \mathbb{R}^{P\cdot}$, defined on an open interval J of the real number line $\mathbb{R}$, such that $\int_X \mathbf{Ts}(0) = X$ and $d\int_X \mathbf{Ts}/dt(t) = \mathbf{Ts}(\int_X \mathbf{Ts}(t))$ for all $t \in J$, and such that the curve cannot be extended to a strictly larger domain of parameters t. So the curve's tangent at the curve point for curve parameter t is the given vector field at the curve point for that parameter. Intuitively speaking, if one imagines the vector field as being the velocity field of a water stream, the integral curve is the curve a small boat follows when floating on the water.

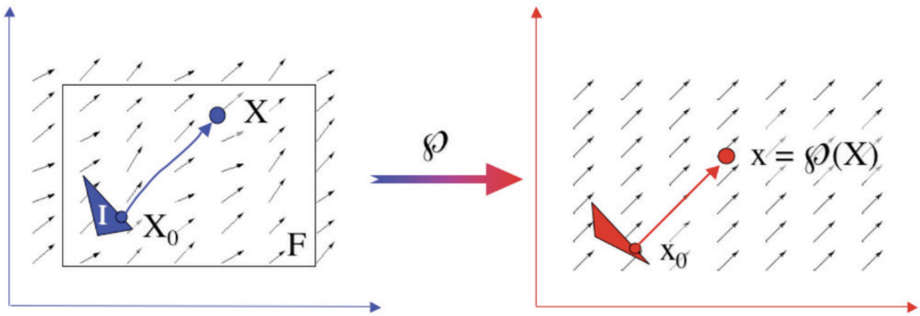

Fig. 10.1. Performance is defined by a field $\mathbf{Ts}$, defined on a frame F, and an initial performance map defined on an initial set I.

Since an integral curve $\int_X \mathbf{Ts}$ is the inverse image of the corresponding integral curve $\int_x \Delta$, $x = \wp_{P\cdot}(X)$, the time that elapses when moving on integral curve $\int_X \mathbf{Ts}$ between two points is the same as the time that elapses on $\int_x \Delta$ between the corresponding points. Therefore, if X_0 is a point on $\int_X \mathbf{Ts}$ at time t, and if we know the "initial performance value" $x_0 = \wp^I_{P\cdot}(X_0)$ on $\int_x \Delta$, then the performance of X is given by

$$x = \wp_{P\cdot}(X) = x_0 - t\Delta.$$

We may therefore calculate the performance of a point X by taking the integral curve though that point, seeking a point X_0 on the curve whose initial performance is known, calculating the time that elapes from X to that point X_0 and then applying the above formula. We therefore need this data to calculate performance:

1. a performance field[1] $\mathbf{Ts}$,
2. defined on an n-dimensional cube $F = [a_1, b_1] \times [a_2, b_2] \times \ldots [a_n, b_n]$, Cartesian product of n closed intervals, $[a_i, b_i], a_i \leq b_i$, called the *frame of the performance*,

[1] $\mathbf{Ts}$ must be a Lipschitz field, see [84, Chapter 33.2.2] for such technicalities.

3. an *initial set* $I \subset F$, where
4. an *initial performance map* $\wp^I_{P.} : I \to \mathbb{R}^{p\cdot}$ is given.

Our performance is then defined for all points $X \in F$ that can be connected to points of I by integral curves (see also figure 10.1). The set of "notes" $K \subset F$ to be performed should consist of such connectable points, of course. It is called the *symbolic kernel of the performance*. The total information $\mathcal{C} = (\mathbf{Ts}, F, I, \wp^I_{P.}, K)$ is called a *performance cell*.

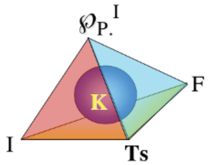

Fig. 10.2. Symbol for a performance cell.

10.2 Initial Sets

It is important to understand the deep meaning of initial performance. We have known performance as a transitional process from mental to physical reality. This is a transition from the score to the acoustical realization, to be archived on sound media such as a CD. Following the valid doctrine—as preconized by Paul Valéry and Theodor W. Adorno—performance is an integral part of the work of art, and this means that, in the sense of communication theory of art as described by Jean Molino, performance is part of the semiosis of the work, and its meaning is not complete except when it is performed.

Fig. 10.3. Sergiu Celibidache (1912-1996).

Put it the other way round: The mental score conveys a part but not the whole content, and only via performance can we complete the work's semiosis. Performance involves a kind of usage of the mental score sign by a performer. More specifically, those signs whose content are not only instantiated but also substantially depend on the user are the well-known shifters. Shifters (also called deictic morphemes) are signs that gain their full meaning only when used. In language, the most prominent shifters are "I," "here," and "now." The lexical meaning of these signs is incomplete and changes significantly when such a sign is used.

Every human individual using "I" creates the subjective individual portion of the pronoun's meaning. Same for the other two signs: Each usage changes their meaning—each time when I say "now," it means a different moment of presence. This contrasts with lexically determined signs, such as "cat," whose complete meaning can be looked up in a dictionary. Performance of a mental score is such a shift from lexicality to full-fledged meaning, since the pure score is essentially less than the work of art. In other words, performance is what semiotics calls a shifter characteristic of the score semantics.

Production of full-fledged meaning is only possible by means of performance, and this adds a semantic aspect to the sign that is a non-trivial function of the performer(s).

The shifter nature of performance is especially acute in initial performance, since this is where the fictitious reality of the score is anchored in reality, namely by the initial value of initial symbols. All the rest is defined by integration of performance fields, but these are only meaningful to physics via initial anchorage. This existential aspect was promimently stressed by the celebrated Romanian conductor Sergiu Celibidache.

10.3 Measuring Performance Fields

Although the formalism of performance cells is a perfect conceptual tool, whether and how it can be applied in practice is far from evident. There is a number of problems that are related to such far-out mysteries. First, the performance field of a normal parameter space with onset, pitch, loudness, and duration is four-dimensional and hard to visualize as such, so lower-dimensional images will be required if possible. Second, even if a representation has low dimension, it is difficult to visualize vector fields with strong intuitive expressivity. Third, if we are given a performance, how can we calculate and the visualize its performance field(s)?

It is clear that such questions are extremely important for performers, for their instructors, and for the empirical research on existing performances. The solution to this problem has been offered in collaboration with my computer science students Stefan Göller and Stefan Müller [86]. They have programmed a software component, called *Espresso Rubette*, of the music software environment RUBATO®. The component has a 3D interface that looks like an espresso machine (figure 10.4). It takes a MIDI file of a composition and the MIDI file of its performance, compares them and generates a corresponding performance field. The comparison is a highly non-trivial task since one has to match the score events with the performed events. These may be wrong by the musician's errors, or there might even be ghost events played by error but not corresponding to any written notes. The performed events are also, by the very action of performance, not in the same relative position as written on the score: The onset times of notes written in a chord may differ by slight arpeggi, and the duration will be articulated.

There are a number of rather good matching algorithms, but none of them is absolutely reliable, also because there is no a priori reason to have a perfect match in view of the mentioned errors and deformations. The program's second task, after a match has been found, is the construction of a performance filed. Here there are two subtasks: finding a finite number of "representative vectors" of the field from the finite number of events available from the given

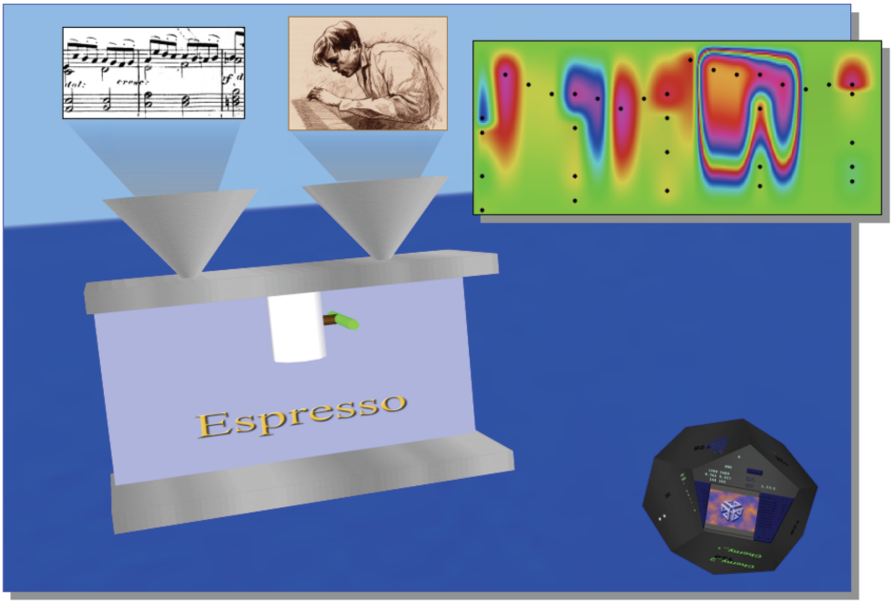

Fig. 10.4. The *Espresso Rubette* component of RUBATO® takes a MIDI file of a composition and the MIDI file of its performance, compares them and generates a corresponding performance field, which is visualized as a color field by use of the color circle.

composition and interpolating these vectors to a field that is defined everywhere on the composition's frame. The interpolation task is trivial. The vector field can also be represented as a color field matching the vectors' directions with positions on the color circle; the vctors' lengths determine the color intensity. Figure 10.4 shows such a visualzation of a part of the Czerny exercise shown as input to the *Espresso Rubette*. The black points on the color field to the right are the performed note events. The construction of the representative vectors is quite subtle, but it can be done on the basis of standard methods of linear algebra. The point here is to find good pairs of vectors that describe the Jacobian matrixes, where "good" means that the selection must cope with robust positions of the transformations' base vectors, see [86] for details. It is evident that such a software is the germ of a revolutionary tool for performance education because the student can play a piece on a MIDI piano, and—while the performance is ongoing—the color field on a big screen shows immediately the performance field as a common reference for the instructor and the student, a tool which enables a detailed, undelayd, and objective reference for the delicate pedagogical work of teaching refined performer artistry.