Chapter 8
Five Steps in Latent Curve and Latent Change Score Modeling with Longitudinal Data

John J. McArdle and Kevin J. Grimm

Abstract This paper describes a set of applications of one class of longitudinal growth analysis - latent curve (LCM) and latent change score (LCS) analysis using structural equation modeling (SEM) techniques. These techniques are organized in five sections based on Baltes & Nesselroade (1979). (1) Describing the observed and unobserved longitudinal data. (2) Characterizing the developmental shape of both individuals and groups. (3) Examining the predictors of individual and group differences in developmental shapes. (4) Studying dynamic determinants among variables over time. (5) Studying group differences in dynamic determinants among variables over time. To illustrate all steps, we present SEM analyses of a relatively large set of data from the National Longitudinal Survey of Youth (NLSY). The inclusion of all five aspects of latent curve modeling is not often used in longitudinal analyses, so we discuss why more efforts to include all five are needed in developmental research.

8.1 Introduction

Many debates in developmental research conclude with a suggestion that the collection of longitudinal data is a necessary ingredient for the study of developmental phenomena. Methodological researchers have defined these issues in extensive detail, but most rely on "the explanation of inter-individual differences (or similarities) in intra-individual change patterns" (e.g., Wohlwill, 1973; Baltes & Nesselroade, 1979). During the last two decades, many methodologists have contributed to the

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knowledge base, and the classic models for "growth curve analysis" seem to have been revived as an important research technique (e.g., see Rogosa & Willett, 1985; McArdle & Epstein, 1987; Meredith & Tisak, 1990). The term growth curve analysis denotes the processes of describing, testing hypotheses, and making scientific inferences regarding growth and change patterns in a wide range of time-related phenomena. Of course, these *curves* are not limited to the phases where the organism *grows*, but it can also be used to describe and analyze phases where the organism *declines, accelerates, decelerates, oscillates,* or even *remains stable*.

This paper describes a set of applications of one class of longitudinal growth models - *latent change score (LCS)* analysis using *structural equation modeling (SEM)* techniques. These techniques can be presented in many ways, but we organize this information in five sections, as steps of developmental data analysis, based on a sequential rationale inspired by Baltes & Nesselroade (1979):

Step 1 – *Describing the Observed and Unobserved Longitudinal Data* – We consider some useful ways to summarize longitudinal data, including statistical information from both the complete and incomplete cases.

Step 2 – *Characterizing the Developmental Shape of Individuals and Groups* – We try to describe both the group and individual characteristics of development and demonstrate the general ease and flexibility of the SEM approach.

Step 3 – *Examining the Predictors of Individual and Group Differences in Developmental Shapes* – We recognize individual differences in growth may be the result of combinations of other measured variables. We describe how SEM can be used in both multilevel and multiple-group forms to provide empirical evidence for hypotheses concerning the correlates of individual longitudinal patterns.

Step 4 – *Studying Dynamic Determinants among Variables over Time* – We show how the time-dependent nature of the latent variables can be represented in SEM and used to study lead-lag relations using simple dynamic expressions.

Step 5 – *Studying Group Differences in Dynamic Determinants Among Variables over Time* – We show how the multi-group and latent mixture dynamic models can be fit to examine heterogeneous lead-lag relationships for different groups of individuals.

As the reader will notice, we first use latent curve modeling (LCM) to begin the analyses, but we then emphasize the direct use of latent change scores (LCS) for more clarity in the model alternatives. This clarification may assist the researcher in considering the alternative change models available. This point is important because the LCS allows us to rather easily join seemingly different concepts about change from classical models based on time-series and auto-regression or latent growth curve analyses.

As an illustration for the five steps, we present SEM analyses of data from the well-known and publicly available *National Longitudinal Survey of Youth* (NLSY)

– Children and Young Adults. In this study, the children of female respondents were repeatedly measured biennially from 1986 through 2000. The longitudinal data of the NLSY includes measures of achievement (e.g., Peabody Individual Achievement Test; PIAT; Dunn & Markwardt, 1970) and behavior problems (e.g., Behavior Problems Index; BPI; Zill, 1990). These analytic illustrations are used to convey the main presumptions and techniques as well as the benefits and limitations of these approaches in developmental research.

Our main goal is to present an overview of the general developmental methodology, and to demonstrate the practical and flexible utility of these methods for developmental research. We do not provide extensive mathematical and statistical details, but the computer input and output scripts for each step of the SEM analyses are available from our website `http://kiptron.usc.edu/`as well as from `http://www.econ.upf.edu/˜satorra/longitudinallatent/readme.html`. Most importantly, the inclusion of all five aspects of latent curve modeling is often overlooked in longitudinal analyses, so we end by discussing why all five steps are needed in developmental investigations.

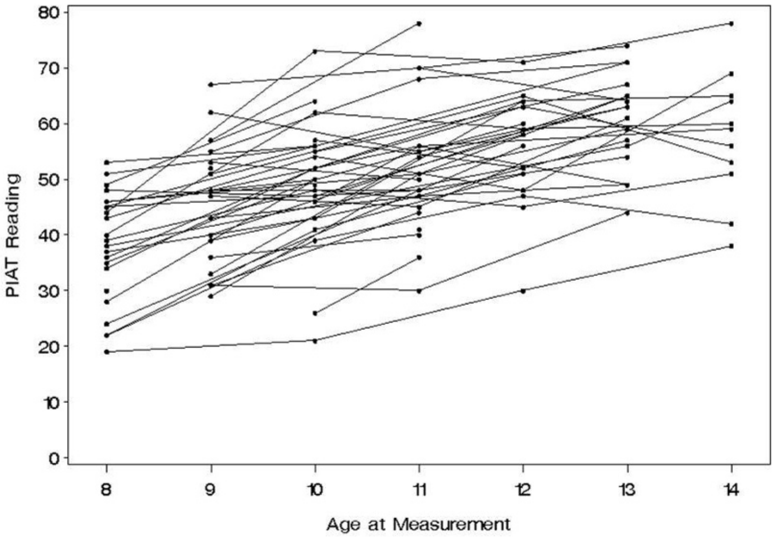
8.2 Step 1: Describing the Observed and Unobserved Longitudinal Data

The first step in any useful data analysis is an adequate description of the data. However, the collection and presentation of longitudinal data can be difficult, so the unique aspects of these data should be emphasized.

8.2.1 The National Longitudinal Survey of Youth – Children and Young Adults

The data examined here come from children who were measured at least once between age 8 and 14, so the overall $N = 6{,}790$. As previously mentioned, data collection occurred biennially with measurements occurring in every even year from 1986 through 2000. Figure 8.1 is a display of individual growth data for the (a) PIAT reading comprehension and (b) BPI antisocial behavior measure by age for a sub-sample of $n = 100$ randomly selected participants. The y-axis indexes the participants' scores and the x-axis is an index of the participants' age-at-testing. The connected lines in this figure are graphic descriptions of the change pattern for Reading Comprehension scores for each individual, so each line is termed a *growth curve* or *trajectory*. The plot allows us to see the overall trends for changes in achievement and antisocial behavior through childhood and adolescence as well as how the data are incomplete.

(A)



(B)

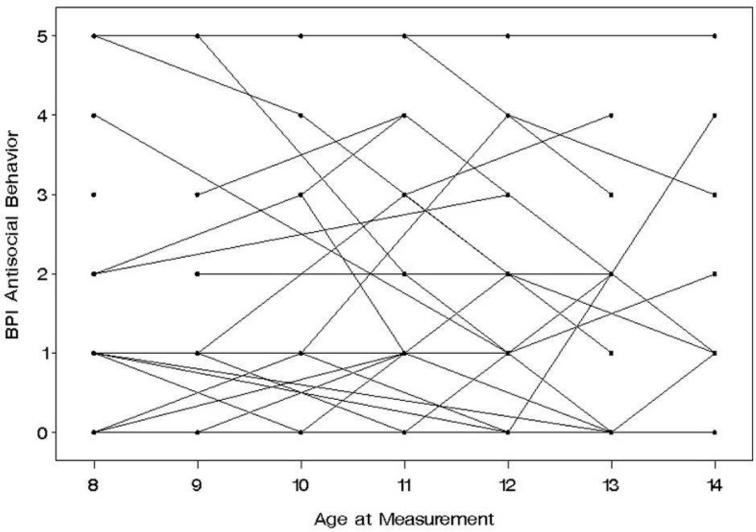


Fig. 8.1 Longitudinal plots of (A) Reading Comprehension from the Peabody Individual Achievement Test and (B) Antisocial Behavior from the Behavior Problems Index for a random sample of 100 participants.

8.2.2 Describing the Observed Data

The sample sizes, means, standard deviations, and correlations of these raw measures from age 8 to 14 are listed in Table 8.1. The means and standard deviations show a simple pattern from age 8 to 14 with increases in performance coupled with increases in variation. The correlations over time, the unique statistical information of longitudinal data, also present a relatively simple pattern of results with most correlations suggesting a relatively high level of the stability of individual differences (e.g., $r > .5$). We use SEM to test hypotheses about these longitudinal statistics.

8.2.3 Results from Dealing with Incomplete Information

The summary information presented in Table 8.1 is not limited to only those participants with complete data at all ages (no participants have complete data). To deal with this problem we present a description of the patterns of complete and incomplete data in Table 8.1c. The incomplete data patterns can be represented as the proportion of data or coverage for each covariance of these scores – at any time no more than 42% of the participants have data (age 8 and 9) and in some cases only 1% of the information is available (at ages 13 and 14).

In Table 8.1a and 8.1b we also use brackets to list an "incomplete data" estimate of the sample means, standard deviations, and correlations. These estimates are based on what is typically termed *full information maximum likelihood* (FIML; Little, 1995; McArdle, 1994; Cnaan, Laird, & Slasor, 1997). This approach allows us to examine the initial summary statistics "as if all persons were measured at all occasions" and, hopefully, deal with any selection bias in the longitudinal sampling strategy. These newly estimated statistics are not exactly the same as the pair-wise estimates, but they are not altered very much, indicating these data meet the minimal conditions of "missing at random" (MAR; Little, 1995). Most importantly, these estimated statistics do not suffer from some common statistical problems of pair-wise estimates based on different sample sizes, and use all available information from every person. As a result, we do not need to select a subset of persons because they have complete data, nor do we have to make our timing basis based on a data collection strategy (e.g., 1986, 1988, ..., 2000). Instead, we choose to examine age-at-testing (see equation (8.1) below) as opposed to wave-of-testing or year-of-testing, based on our developmental interest.

Let us be clear at the start that an age-based approach by itself does not guarantee that all model assumptions are met (e.g., see McArdle et al., 2002; cf., Sliwinski & Buschke, 1999; Miyazaki & Raudenbush, 2000). In fact, this age-based approach is not often used in standard developmental research, where it is much more likely to find time (e.g., the occasion of measurement) as the focal axis of development. However, in this specific case, the individuals were sampled from an ongoing developmental process that is likely to have strong age related components, and there was no common point of intervention except for the natural differences due to grade and maturity. The standard MAR assumptions that are needed do require an important belief on the part of the analyst – that the ways in which data are incomplete are somehow reflected in the data that are complete. While we think this is a reasonable

Table 8.1 Observed and unobserved summary statistics for the Peabody Individual Achievement Test Reading Comprehension (Read) and Behavior Problems Index Antisocial Behavior (Anti) scores from the National Longitudinal Survey of Youth data at eight time points (N = 6970; MLE-MAR estimates in brackets; Step 1, see Figures 8.1A and 8.1B

(a) Observed and unobserved means and standard deviations over age

Variable	N	Mean [MLE]	SD [MLE]	Skewness	Kurtosis	Min	Max
Read – Age 8	2847	31.1 [30.7]	9.8 [9.8]	.42	-.23	0	70
Read – Age 9	2833	36.7 [36.6]	10.3 [10.4]	.03	-.40	0	78
Read – Age 10	2660	41.5 [41.2]	10.6 [10.7]	-.09	.36	0	84
Read – Age 11	2566	44.7 [45.0]	11.4 [11.4]	-.13	.43	0	84
Read – Age 12	2226	48.1 [48.1]	11.4 [11.4]	-.15	.35	0	81
Read – Age 13	2047	50.3 [50.5]	12.1 [12.3]	-.19	.42	0	84
Read – Age 14	1734	52.1 [52.4]	12.0 [12.1]	-.24	.47	0	84
Anti – Age 8	3046	1.49 [1.53]	1.52 [1.53]	.97	.23	0	6
Anti – Age 9	2987	1.52 [1.52]	1.59 [1.59]	1.04	.36	0	6
Anti – Age 10	2722	1.52 [1.51]	1.61 [1.61]	.98	.17	0	6
Anti – Age 11	2644	1.54 [1.50]	1.63 [1.63]	.95	.02	0	6
Anti – Age 12	2287	1.60 [1.56]	1.62 [1.63]	.88	-.11	0	6
Anti – Age 13	2140	1.66 [1.60]	1.67 [1.68]	.81	-.35	0	6
Anti – Age 14	1798	1.70 [1.64]	1.72 [1.74]	.75	-.54	0	6

(b) Observed and unobserved correlations (each entry includes pairwise *r* and [MLE-MAR *r*])

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1. Read – Age 8	1.00													
2. Read – Age 9	.57 [.46]	1.00												
3. Read – Age 10	.63 [.64]	.64 [.58]	1.00											
4. Read – Age 11	.56 [.57]	.69 [.69]	.67 [.68]	1.00										
5. Read – Age 12	.57 [.58]	.56 [.60]	.68 [.70]	.72 [.65]	1.00									
6. Read – Age 13	.48 [.51]	.62 [.62]	.60 [.62]	.66 [.67]	.77 [.75]	1.00								
7. Read – Age 14	.53 [.54]	.57 [.59]	.62 [.63]	.69 [.69]	.67 [.68]	.63 [.71]	1.00							
8. Anti – Age 8	-.19 [-.20]	-.23 [-.20]	-.22 [-.22]	-.21 [-.24]	-.21 [-.22]	-.18 [-.17]	-.22 [-.24]	1.00						
9. Anti – Age 9	-.29 [-.23]	-.22 [-.21]	-.05 [-.16]	-.20 [-.21]	-.30 [-.23]	-.15 [-.17]	-.19 [-.22]	.58 [.53]	1.00					
10. Anti – Age 10	-.19 [-.17]	-.29 [-.25]	-.21 [-.20]	-.11 [-.19]	-.24 [-.22]	-.16 [-.21]	-.21 [-.21]	.59 [.60]	.59 [.64]	1.00				
11. Anti – Age 11	-.15 [-.22]	-.19 [-.19]	-.20 [-.23]	-.21 [-.22]	-.22 [-.23]	-.21 [-.22]	-.24 [-.28]	.52 [.50]	.64 [.64]	.57 [.57]	1.00			
12. Anti – Age 12	-.19 [-.18]	-.17 [-.13]	-.20 [-.20]	-.11 [-.15]	-.21 [-.20]	-.35 [-.21]	-.21 [-.22]	.50 [.50]	.54 [.54]	.59 [.60]	.54 [.58]	1.00		
13. Anti – Age 13	-.27 [-.29]	-.14 [-.16]	-.25 [-.22]	-.17 [-.19]	-.26 [-.20]	-.20 [-.21]	-.10 [-.20]	.45 [.45]	.54 [.53]	.49 [.52]	.58 [.59]	.48 [.51]	1.00	
14. Anti – Age 14	-.17 [-.19]	-.14 [-.15]	-.17 [-.19]	-.24 [-.23]	-.20 [-.21]	-.05 [-.20]	-.22 [.24]	.45 [.46]	.38 [.51]	.53 [.54]	.61 [.60]	.60 [.62]	.57 [.61]	1.00

Table 8.1 (Continued)

(c) Covariance coverage (proportion of participants with available data at each age and combination of ages)

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1. Read – Age 8	.42													
2. Read – Age 9	.02	.42												
3. Read – Age 10	.28	.02	.39											
4. Read – Age 11	.05	.28	.02	.38										
5. Read – Age 12	.22	.04	.25	.02	.33									
6. Read – Age 13	.04	.21	.04	.23	.02	.30								
7. Read – Age 14	.17	.03	.20	.03	.20	.01	.26							
8. Anti – Age 8	.38	.02	.28	.05	.22	.04	.17	.43						
9. Anti – Age 9	.02	.38	.02	.27	.04	.20	.03	.02	.42					
10. Anti – Age 10	.27	.02	.35	.02	.24	.04	.18	.28	.02	.38				
11. Anti – Age 11	.05	.27	.02	.34	.02	.22	.03	.05	.28	.02	.37			
12. Anti – Age 12	.21	.04	.24	.02	.29	.01	.19	.22	.04	.24	.02	.32		
13. Anti – Age 13	.04	.20	.04	.23	.02	.27	.01	.04	.21	.04	.23	.02	.30	
14. Anti – Age 14	.17	.03	.19	.03	.19	.01	.23	.17	.03	.19	.03	.20	.01	.25

set of assumptions, these will never be completely correct, and we try to point out critical junctures where a failure to meet MAR assumptions may be important.

8.3 Step 2: Characterizing Developmental Shapes for Groups and Individuals

The second step in a longitudinal data analysis is the attempt to highlight the key features of the data in terms of a *model*. In contemporary behavioral science research, one common approach to growth curve analysis is to write a *trajectory* equation for each group and individual. One such *trajectory* equation for *repeated measurements* of an observed variable, *Read*, at multiple times ($t = 1$ to T) for the same person ($n = 1$ to N), written in the mixed-model form of

$$Read[t]_n = g_{0n} + g_{1n} \cdot B[t] + e[t]_n \,. \tag{8.1}$$

This model includes three *unobserved or latent scores* representing the individual's (1) level (g_{0n}), (2) slope (g_{1n}) representing *linear change over time* and (3) independent errors of measurements ($e[t]_n$). To indicate the form of the systematic change, we use a set of group coefficients or *basis weights* (e.g., slope loadings) which define the timing or *shape of the trajectory over time* (e.g., $B[t] = t - 1$). It is typical to estimate the fixed group means for intercept and slopes (μ_0, μ_1) but also the implied random variance and covariance terms (σ_0^2, σ_1^2, σ_{01}) describing the distribution of individual deviations (d_{0n}, d_{1n}) around the group means. We also follow a traditional convention and assume there is a single random error variance within each time (σ_e^2), and the error terms are assumed to be normally distributed and uncorrelated with all other components. This final assumption about a single error variance mimics the assumptions of most other repeated measures models (e.g., Mixed-Effects ANOVA).

One important issue emerges when we recognize that there is nothing actually pre-defined about the basis of time ($B[t]$), and this allows us to investigate many alternative forms of the time axis (e.g., McArdle & Bell, 2000). For example, it may be more appropriate in this case to study multiple ages (e.g., age = 8 to 14) on the same person and write

$$Read[age]_n = g_{0n} + g_{1n} \cdot B[age] + e[age]_n \quad (8.2)$$

because using age as the basis of timing allows a more interpretable set of trajectories.

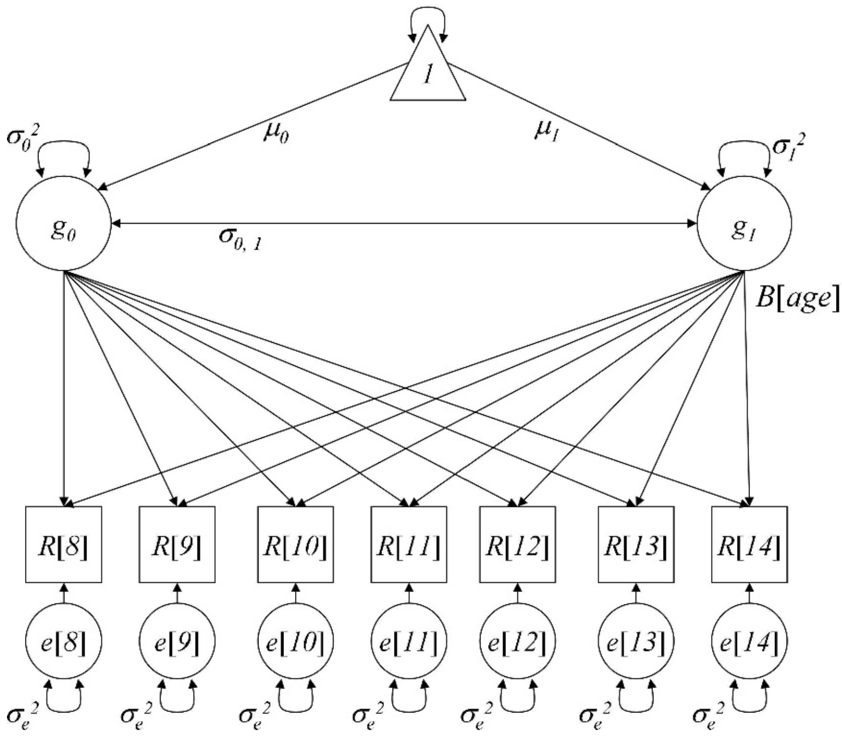


Fig. 8.2 Path diagram of a latent growth curve for Reading Comprehension.

A path diagram of this growth curve is presented in Figure 8.2 and is an exact translation of the necessary matrix algebra of these models (See Grimm & McArdle, 2005; McArdle, 2005; McArdle & McDonald, 1984). These diagrams can be conceptually useful devices for understanding the basic modeling concepts. In this path diagram the observed variables are drawn as squares, unobserved variables as circles, the required constant is included as a triangle, and parameters are labeled such that invariant parameters (e.g., residual variances) have the same label. Model parameters representing “fixed” or “group” coefficients are drawn as one headed arrows while “random” or “individual” features are drawn as two-headed arrows. In this case the level and slope are often assumed to be random variables with “fixed” means (μ_0, μ_1) but “random” variances (σ_0^2, σ_1^2) and covariance (σ_{01}). Of course, this is essentially a model based on means and covariances with MAR assumptions about the incomplete data.

8.3.1 Basic Linear Growth Models Results

Some initial growth curve modeling results for the NLSY Reading data are presented in Table 8.2. In these longitudinal models any change score (g_{1n}) is assumed to be constant *within* an individual but is not assumed to be the same *between* individuals. We do not estimate the unobserved scores but estimate several parameters that characterize the key features of the unobserved scores.

Table 8.2 Selected results from five latent growth models fitted to NLSY longitudinal data (N = 6970; Step 2).

Parameter	*2a*: No Growth	*2c*: Linear	*2d*: Latent
Fixed Effects			
Basis b[8]	=0	=0	=0
Basis b[9]	=0	=1	.28
Basis b[10]	=0	=2	.48
Basis b[11]	=0	=3	.66
Basis b[12]	=0	=4	.80
Basis b[13]	=0	=5	.92
Basis b[14]	=0	=6	=1
Level μ_0	42.1*	32.6*	30.7*
Slope μ_1	—	3.7*	21.7*
Random Effects			
Error σ_e^2	114.0*	42.7*	39.9*
Level σ_0^2	56.2*	61.1*	61.1*
Slope σ_1^2	—	1.0*	35.6*
Correlation ρ_{01}	—	.24*	.15*
Fit Indices			
Parameters	3	6	11
Degrees of Freedom	32	29	24
Log Likelihood	-66677	-61710	-61416
χ^2	10635	699	110
RMSEA	0.22	0.06	0.02

Note: The fit statistics for the time-based linear model (Model 2b) are not presented here because the data are considered to be different because of their organization.

The first model labeled 2a is the *no-growth* model fitted with only three parameters: a level mean ($\mu_0 = 42.1$), a level variance ($\sigma_0^2 = 56.1$), and an error variance ($\sigma_e^2 = 114.0$). The model yields a likelihood (L^2 = -66677) which shows the no-growth baseline is a poor fit compared to the totally unrestricted or saturated model (χ^2 = 10635, df = 32), in which means, variances, and covariances are estimated for all observed variables. This model is typically used as a baseline against which to judge the fit of more informative models. The second growth model (2b) uses a fixed set of basis coefficients that change linearly with the *number of occasions* representing the time passed since the participant was enrolled in the study. Therefore, $B[t] = t - 1$ where t is the occasion number. Based on the data collection paradigm of the NLSY, a one-unit change in t represents a two-year change (e.g., measurements occurred every two-years from 1986 to 2000). This model has three additional parameters compared to the no-growth model: a slope mean (μ_1), variance (σ_1^2), and a level-slope covariance (σ_{01}). This model yielded a new likelihood (L^2 = -61856), which was a distinct improvement over the no-growth model ($\Delta - 2LL = 9644$ for 3 additional parameters). The resulting estimates describe a function that begins at 35.2 at the first occasion and increased by 7.3 units every two years. The variance estimates of the level and slope parameters were significant (σ_0^2 = 76.4.1, $\sigma_1^2 = 3.8$) indicating inter-individual differences in reading ability upon entering the study and in the linear change over time. Additionally, the level-slope correlation was .20 indicating a small positive relationship between children's reading performance upon entering the study and their linear rate of change. The error variance was estimated to be 41.6.

The second *linear growth* model (2c) was fit uses a fixed set of basis coefficients or slope loadings that change linearly with *age* and formed by taking $B[age] = (age-8)$, or the fixed values of $B[age] = [0, 1, 2, 3, 4, 5, 6]$). This linear scaling is only one of many that could be used, but was chosen to permit a practical interpretation of the slope parameters in terms of a *per-year change* and centers the level to represent 8 years of age. Therefore, the parameters related to the level reflect parameters associated with age 8. This linear growth model has three additional parameters compared to the *no-growth* model: a slope mean (μ_1) and variance (σ_1^2), and a level-slope covariance (σ_{01}). This model yields a new likelihood (L^2 = -61710) that represents a relatively large distance from the unrestricted model (χ^2 = 699 on df = 29) but was an improvement over the no-growth model (*2c* vs *2a*: $\Delta\chi^2 = 9936$ on $\Delta df = 3$). The resulting means describe a function that started relatively low at age 8 ($\mu_0 = 32.6$) but increased by 3.7 units per year between ages 8 and 14 ($\mu_1 = 3.6$). The variance estimates of the level and slope parameters were significant ($\sigma_0^2 = 61.1$, $\sigma_1^2 = 1.0$) indicating inter-individual differences in the linear growth parameters. Additionally, the level-slope correlation was .24 indicating a small positive relationship between children's reading performance at age 8 and their linear rate of change. The error variance has been reduced ($\sigma_e^2 = 42.7$) compared to the *no-growth* model, which also indicates an improvement in fit.

The time-based and age-based models are not nested, which makes directly comparing their fit somewhat problematic. But this mimics a traditional problem that emerges when rescaling any X-variable in a traditional regression – if different transformations of X scores are used to predict Y scores the resulting parameters and fit can change. In the SEM framework the raw data always has the same likelihood and degrees of freedom, but different latent variable models based on $B[t]$ can have different likelihoods and degrees of freedom. One way to compare these models is based on likelihood comparison fit statistics, such as the AIC or BIC. In both cases here, the age-based model fit better. More importantly, the age-based model has important substantive interpretations. In this observational study we are observing the phenomena of changes in reading scores as they are unfolding. While the beginning of the study may be of paramount importance for the researchers, it is not likely that anything happened at this point to consider it important in the participants' lives (e.g., unlike a surgical procedure).

8.3.2 Nonlinearity Using Latent Basis Curves

An attractive nonlinear alternative of the linear growth model was proposed by Rao (1958) and Tucker (1958, 1966) in the form of summations of "latent curves" (see Meredith & Tisak, 1990). The use of this latent growth curve offers a simple way to investigate the shape of a growth curve - we allow the basis coefficients ($B[age]$) to take on a form based on the empirical data. In this approach we estimate the basis coefficients (e.g., $B[9-13]$) with the exception of two ($B[8]$ and $B[14]$) for identification purposes. In this latent basis model we end up with an optimal shape for the group curve and individual differences with one change component (see McArdle & Epstein, 1987; McArdle & Bell, 2000).

The fourth model fitted (2d) was this kind of latent basis growth model. For identification purposes, we fixed $B[8] = 0$ and $B[14] = 1$, but the remaining basis coefficients were estimated from the data. This results in a large improvement in the model likelihood ($L^2 = -61416$), which was much closer to the unrestricted model ($\chi^2 = 110$ on $df = 24$), and substantially better than the nested baseline ($\Delta\chi^2 = 10525$ on $\Delta df = 8$) and nested linear models ($\Delta\chi^2 = 589$ on $\Delta df = 5$). The error variance has also been reduced ($\sigma_e^2 = 39.9$). The estimated latent means were $\mu_0 = 30.7$ and $\mu_\iota = 21.7$, their variances were $\sigma_0^2 = 61.1$ and $\sigma_1^2 = 35.6$, and the intercept/slope correlation was $\rho_{01} = 0.15$. The estimated basis coefficients were .28, .48, .66, .80, and .92 for ages 9 – 13. The coefficients indicated a decelerating growth function. Additional nonlinear models, including multiphase (Cudeck & Klebe, 2002) and structured curves (Browne & du Toit, 1991) can be fit to these data, but are not described here. These additional nonlinear models may be able to adequately represent the data with fewer parameters. We refer the reader to Oud & Jansen (2000), Cudeck & Klebe (2002), Browne & du Toit (1991), and Ram & Grimm (2007) for further details regarding nonlinear models.

8.4 Step 3: Modeling Individual Differences in Developmental Scores and Patterns

The estimated means of the level and slope in the previous analyses allow us to plot the group trajectory over time. Similarly the estimated variance parameters allow us to consider the size of the between group differences at each age. However, no prior information obtained in model fitting tells us about the *sources of this variance*. To further explore the differences between persons we expand the basic latent growth model to include impacts on the latent parameters. There are several techniques to evaluate the sources of inter-individual differences and we consider three common methods including the growth model with an extension variable, the multiple group growth model, and the growth mixture model.

8.4.1 The Growth Model with an Extension Variable

Let us assume a variable termed X indicates some measurable difference between persons (e.g., sex, educational level). If we measure this variable at one occasion we might like to examine its influence in the context of a growth model for our outcome of interest (e.g., reading achievement). One popular model is based on the use of "adjusted" growth parameters as popularly represented in the *analysis of covariance*. In growth curve terms, this model is written with fixed (group) coefficients (γ) with some effect on the measured scores at each occasion ($Read[t]$), and the X is an independent observed (or assigned) predictor variable and written as

$$\begin{aligned} g_{0n} &= \nu_0 + \gamma_0 \cdot X_n + d_{0n}\,, \\ g_{1n} &= \nu_1 + \gamma_1 \cdot X_n + d_{1n}\,, \end{aligned} \tag{8.3}$$

where we have intercepts (ν) and regression slopes (γ) for the effect of X on the two latent components (g_0 and g_1) with residuals (d_0 and d_1). In this case the latent growth parameters ($\mu_{0:x}$, $\mu_{1:x}$, $\sigma_{0:x}$, $\sigma_{1:x}$, $\sigma_{0,1:x}$) are considered to be conditional on the expected values of the measured X variable. In the early factor analytic literature this relation between an observed X and a common factor score was termed an "extension analysis" (Horn, 1973). The apparent complexity of the covariance model leads to a simpler and increasingly popular way to add an external variable – we can write a *growth model with an extension variable* where the X variable has a direct effect on the parameters of the growth curve.

8.4.2 Results Growth Model with an Extension Variable

A variety of additional variables have been measured in the NLSY, including demographic (e.g., gender, mother's and father's education.), self reported health behaviors (e.g., smoking, drinking, physical exercise, etc.) and other problems (e.g., general health, illness, medical procedures, etc.). In the analyses presented here we consider two variables: gender (effect coded as -0.5 for males and $+0.5$ for females) and the mother's age at the child's birth (centered at 24 years of age).

We add gender and mother's age at child's birth as predictors of the level and slope. Table 8.3 is a list of results including the two variables as predictors of the level and slope. The model (3a) has a misfit ($\chi^2 = 125$ on $df = 34$) and this is an improvement when compared to the model in which the regression parameters were fixed at zero ($\chi^2 = 276$ on $df = 38$; $\Delta\chi^2 = 151$ on $\Delta df = 4$). The parameter estimates suggest the following interpretations. (0) The latent basis coefficients ($B[t]$) were unaffected by the inclusion of the predictors. (1) There were accurate (significant) differences between males and females on both the level and slope with females predicted to have a greater reading level at age 8 ($\gamma_0 = 1.9$), but a slightly slower rate of change from age 8 to 14 ($\gamma_1 = -1.3$). (2) The mother's age at the child's birth was also predictive of the level ($\gamma_0 = 0.19$) and slope ($\gamma_1 = 0.20$) of the growth model for reading comprehension. Older mothers at the child's birth were predicted to have children with a greater level of reading ability at age 8 as well as a faster rate of change from age 8 to 14.

Table 8.3 Results from latent growth models with extension variables fit to the NLSY longitudinal data (Step 3)

Parameters	*3a*: Level	*3a*: Slope
Fixed Effects		
Basis $B[t]$	= 0, .27*, .48*, .66*, .80*, .92*, = 1	
Intercept ν_0	30.8*	22.3*
Regression from gender γ_g	1.9*	-1.3*
Regression from mother's age at birth γ_a	.19*	.20*
Random Effects		
Residual δ_d^2	59.6*	34.6*
Error σ_e^2	39.9*	
Correlation $\rho_{d0,ds}$	.15	
Fit indices		
Parameters	20	
Degrees of Freedom	34	
Log Likelihood	-97446	
χ^2	125	
RMSEA	.02	

8.4.3 Group Differences from a Multiple Group Perspective

The initial representation of group differences uses a set of estimated parameters to summarize between group differences. This idea is clearly represented by coding a set of variables (X) to characterizing the group differences and then examining the effect of this set (X) on the model parameters. However, this method is limited in a number of important ways. For example, some reasonable forms of group differences in the growth processes (e.g., different developmental shapes) are not possible within the standard framework. For example, different groups of people could have different "amplitude" or be in different "phases" in their growth pattern. These group differences in the features of growth are not separated within the basic level and change parameters although they may be realistic features of development.

An SEM treatment of this kind of a model uses concepts derived from multiple-group factor analysis (e.g., Jöreskog & Sörbom, 1979; McArdle & Cattell, 1994). In these kinds of models, each group (g = 1 to G) is assumed to follow a latent growth model where the basis coefficients ($B[t]^{(g)}$) are allowed to vary across groups. Since the groups need to be independent (each person can only be in one group) this kind of grouping is most easily done for discrete categorical variables (i.e., sex, but not educational level or maternal age at birth). A multiple group growth model (see McArdle, 1989) with age as the time-basis can be written as

$$Read[age]_n^{(g)} = g_{0n}^{(g)} + g_{1n}^{(g)} \cdot B[age]^{(g)} + e[age]_n^{(g)} . \tag{8.4}$$

This multiple group growth model permits the examination of the presumed invariance of the latent basis functions (i.e., $B[age]^{(1)} = B[age]^{(2)} = \ldots\ B[age]^{(g)} = \ldots$ $B[age]^{(G)}$). The rejection of this model implies that each independent group has a different shape of growth. If invariance is found we can also examine the equality of the variances of the latent level and slope ($\sigma_0^{(g)} = \ldots\ \sigma_0^{(G)}$ and $\sigma_1^{(g)} = \ldots\ \sigma_1^{(G)}$) and their covariance ($\sigma_{01}^{(g)} = \ldots\ \sigma_{01}^{(G)}$). Further analyses could include the fixed effects (μ_0, μ_1), error deviations ($\sigma_e^{(g)}$), and functions of all the other parameters. These multiple group hypotheses represent additional types of group differences than was possible with the *growth modeling with an extension variable* approach.

8.4.4 Results for Group Differences in Growth of Reading for Males and Females

To illustrate this kind of analysis here, we fit multiple group growth models with gender as the grouping variable. Table 8.4 contains the parameter estimates and fit statistics for three models. In these cases the two groups were created, so the unrestricted likelihood for these data was based on two sets of mean and covariance matrices; one for males and one for females.

The first model (4a) allows both groups to have completely different latent growth curves. The model now includes 11 parameters for each group, and the 22 estimates are listed in the first two columns. This resulted in a reasonable fit to both data sets (χ^2 = 131 on df = 41). A few small differences in estimates can be seen between the two groups, but one key difference appears to be the smaller slope

Table 8.4 Numerical results from multiple group latent growth models fitted to male and female NLSY longitudinal data (Step 3)

Growth Model Parameters	*4a*: Latent Growth for Gender		*4b*: Loading Invariance over Both Groups		*4c*: Total Invariance over Both Groups
Fixed Effects	Males $n = 3448$	Females $n = 3342$	Males $n = 3448$	Females $n = 3342$	
Basis $b[8]$	=0	=0	=0		=0
Basis $b[9]$	.27*	.29*	.28*		.28*
Basis $b[10]$	.48*	.48*	.48*		.48*
Basis $b[11]$	.67*	.64*	.66*		.66*
Basis $b[12]$	.80*	.79*	.80*		.80*
Basis $b[13]$	.93*	.90*	.92*		.92*
Basis $b[14]$	=1	=1	=1		=1
Level μ_0	29.8*	31.5*	29.7*	31.6*	30.7*
Slope μ_1	22.1*	21.4*	22.4*	21.1*	21.7*
Random Effects					
Error σ_e^2	38.3*	41.4*	38.3*	41.5*	39.9*
Level σ_0^2	66.3*	54.2*	66.2*	54.1*	61.6*
Slope σ_1^2	41.7*	28.2*	42.6*	27.3*	35.6*
Correlation ρ_{01}	.21*	.11*	.20*	.11*	.15*
Fit Statistics					
Parameters	22		17		11
Degrees of Freedom	48		53		59
Log Likelihood	-61362		-61366		-61416
χ^2	131		138		237
RMSEA	.02		.02		.03

variance for the females. The second model (4b) adds the restriction that the latent basis coefficients, while free to vary, must be identical across males and females. This model was similar in fit to the free model (χ^2 = 138 on df = 53; $\Delta\chi^2$ = 7 on Δdf = 5), and this indicates the shapes of the curves may be considered the same across gender.

The third model (4c) adds the restriction that all parameters, while free to vary, must be identical across males and females. This model showed a loss in fit (χ^2 = 237 on df = 59) compared to the previous model (4b vs. 4c: $\Delta\chi^2$ = 99 on Δdf = 6), indicating some of the latent means and/or covariances are different. As previously seen in the model with gender as an extension variable, the growth factor means were somewhat different between males and females. Additionally, it appears that the slope variances were also somewhat different.

8.4.5 Mixture Models for Latent Groups

Another fundamental problem is the discrimination between models of (a) multiple curves for one group of people from (b) *multiple groups of people with different curves*. It is possible for us to have, say, three clusters of people, each with a distinct growth curve, but when we aggregate information over all people we end up with a complex growth pattern with multiple growth factors for a single population as opposed to a simple growth pattern for three groupings of people. This is the essence of a latent grouping of people, and parallels the "person centered approach" to multivariate data analysis (e.g., Cattell, 1980; Magnussen, 2003).

The recent series of models termed *growth mixture models* have been developed for this purpose (Muthén & Muthén, 2000; Muthén & Shedden, 1999; Nagin, 1999; Wedel & DeSarbo, 1995). In these analyses the distribution of the latent parameters are assumed to come from a "mixture" of two or more overlapping distributions. Current techniques in mixture models have largely been developed under the assumption of a small number of discrete or probabilistic "classes of persons" based on mixtures of multivariate normals. More formally, we can write a model as a probability weighted sum of curves where the probability of class membership (π_{cn}) is defined for the person in c = 1 to C classes. With a age-based growth curve as the within-class model we can write the growth mixture model as

$$Read[age]_n = \sum_{c=1}^{C} \pi_{cn} \left(g_{0n}^{(c)} + g_{1n}^{(c)} \cdot B[age]^{(c)} + e[age]_n^{(c)} \right)$$
$$\text{where } \sum_{c=1}^{C} \pi_{cn} = 1 \text{ and } 0 \leq \pi_{cn} \leq 1\,. \tag{8.5}$$

In this kind of growth mixture analysis we estimate the threshold parameter for the latent distribution (τ_p, for the *pth* parameter) while simultaneously estimate separate model parameters for the resulting latent groups.

The growth mixture models may be seen as a *model-restricted fuzzy-set cluster analysis* – a multiple group model without exact knowledge of group membership for each individual. The concept of an unknown or latent grouping can be successively based on the logic of multiple group factorial invariance. The resulting estimates yield a likelihood which can be compared to the results obtained from a model with one less class, so the mixture model distribution can be treated as a hypothesis to be investigated. As in standard discriminant analysis, we can also estimate the probability of assignment of individuals to each class in the mixture. In growth mixture modeling, it is important to fully examine how the latent classes differ from one another. Building on the work of multiple group growth models, described above, we examine differences in the basis coefficients (i.e., $B[age]^{(1)} = \ldots B[age]^{(c)} = \ldots B[age]^{(C)}$). The rejection of this model implies that each latent class has a different shape of growth. If invariance is found we can also examine the equality of the variances of the latent level and slope ($\sigma_0^{(c)} = \ldots\ \sigma_0^{(C)}$ and $\sigma_1^{(c)} = \ldots\ \sigma_1^{(C)}$) and their covariance ($\sigma_{01}^{(c)} = \ldots\ \sigma_{01}^{(C)}$). Further analyses could include the fixed effects (μ_0, μ_1), error deviations ($\sigma_e^{(g)}$), and functions of all the other parameters.

8.4.6 Results from Latent Mixture Models

These latent growth mixture models were fit using the NLSY reading data and some of the results are described here. In a first latent mixture model (4d) we estimated a two-class model with free parameters for both groups. This model required 23 parameters and led to another likelihood ($L^2 = -61072$). We recognize the statistical basis of this comparison is still somewhat controversial, but if we consider the threshold as an implied parameter in some previous models, we can get some sense of the gain in fit. The threshold parameter is a point estimate of the position on the outcome distribution where the individuals would be separated in classification into one group or another. In this case, the threshold ($\tau = -1.29$) is a z-score that suggests the total group can be considered a mixture of two classes of different sizes, n_1 = 1,463 and n_2 = 5,327, with different growth patterns between groups but the same growth pattern within groups. By contrast to the one-class model ($L^2 = -61416$) this 2-class model appears to be an improvement; however, numerical instability (and convergence problems) was found (i.e.; for one of the classes as the level variance was near zero). In a second model the level variance and level/slope covariance was fixed at zero in the first class. The result was a model with most participants categorized into the second class ($n = 6121$; $\tau = -2.07$).

In second set of latent mixture model (4e) we allowed the possibility of two latent classes ($C = 2$) with different parameters for the latent means and variance but assumed the same growth basis. This model resulted in a model with convergence problems for the same reasons as the previous model (4d). Finally, the latent means were allowed to vary between latent classes, but the remaining parameters were forced to be equal across latent classes. This final mixture model resulted in convergence problems as the estimated within-class level-slope correlation was greater than 1. Therefore, the results from these growth mixture models did not provide any evidence of latent classes with divergent growth patterns. It's important to remember there was variability in the growth factors (Model 2c), but the results from these mixture models confirms that this variation was distributed normally.

8.5 Step 4: Studying Dynamic Determinants across Multiple Variables

In recent research we have considered some ways to improve the clarity of the basic dynamic change interpretations with conventional SEM analytic techniques. These dynamic change hypotheses have led to the development of a set of alternative models, based on classical principles of dynamic change, but represented in the form of *latent change scores* (e.g., McArdle, 2001; McArdle & Nesselroade, 1994). This alternative representation makes it relatively easy to represent a dynamic hypothesis about the change within a variable, and about the time-ordered determination of one variable upon another.

8.5.1 Modeling Latent Change Scores

The introduction of multiple variables at each longitudinal occasion of measurement leads naturally to questions about time-dependent relationships among growth. A classical SEM for multiple variables over time is based on a *latent variable cross-lagged regression model* (see Cook & Campbell, 1977; Rogosa, 1978). This model can be written for latent scores with over-time auto-regressions (φ_y, φ_x) and cross-regressions (δ_{yx}, δ_{xy}) for time-lagged predictors, but the standard applications of this model do not include systematic growth components (i.e., individual slopes). For this reason, recent SEM analyses have examined *parallel growth curves*, including the correlation of various components (McArdle, 1988, 1989; Willett & Sayer, 1994). A popular alternative used in multilevel and mixed effects modeling is based on the analysis of covariance with $X[t]$ as *time-varying covariates*. In this model the regression coefficient (e.g., $X[t] \rightarrow Y[t]$) is usually assumed to be the same at all occasions. These last two models are easy to implement using existing computer software (e.g., Sliwinski & Buschke, 1999; Sullivan et al., 2000; Verbeke et al., 2000), but the typical applications are limited to a few basic forms of dynamic hypotheses.

To expand our SEM for other dynamic concepts we now reconsider the trajectory equations from a different starting point. First, we assume we have a pair of observed scores ($Y[t]$ and $Y[t-1]$) measured over a defined interval of time ($\Delta t = 1$), and write a model with latent scores ($y[t]$ and $y[t-1]$), and corresponding errors of measurement ($e[t]$ and $e[t-1]$). We can now define a new latent variable that represents the change in the latent scores for y. The *latent change score* is defined as in equation (8.6a). This latent change score is not the same as an observed change score ($\Delta Y[t]_n$) because the latent score is considered separate from the model based error component. Now we can write the trajectory over time in the observed variables as with (8.6b).

$$\Delta y[t]_n = y[t]_n - y[t-1]_n\,, \tag{8.6a}$$

$$Y[t]_n = g_{0n} + \left(\sum_{t=2}^{T} \Delta y[t]_n\right) + e[t]_n\,. \tag{8.6b}$$

Of course, the main alteration in this approach is that in this LCS representation we do not directly define the basis coefficients ($B[t]$; as in equation (8.1)), but instead we directly define *change as an accumulation of the first differences among latent variables*. This deceptively simple algebraic device allows us to define the trajectory equation as an accumulation of the latent changes ($\Delta y[t]$) up to time t *based on any model of change*.

One benefit of this LCS approach is that all of the previous latent growth models can be re-conceptualized in terms of first differences, and some new models emerge (as in McArdle & Nesselroade, 1994; McArdle, 2001, 2009; McArdle & Hamagami, 2001). We first re-iterate traditional models and then present some new models. We can start with the simple baseline model of *no change* by stating (8.7a), so that this difference model represents a trajectory with (8.7b).

$$\Delta y[t]_n = 0\,, \tag{8.7a}$$
$$Y[t]_n = g_{0n} + e[t]_n\,. \tag{8.7b}$$

Thus, the baseline model allows systematic individual differences at all occasions, and random error at all occasions, but no systematic changes over time.

In contrast, we can write (8.8a), so that this change model represents a trajectory with (8.8b).

$$\Delta y[t]_n = g_{1n}\,, \tag{8.8a}$$
$$Y[t]_n = g_{0n} + \left(\sum_{t=2}^{T} g_{1n}\right) + e[t]_n\,. \tag{8.8b}$$

So,

$$Y[1]_n = g_{0n} + e[1]_n\,,$$
$$Y[2]_n = g_{0n} + g_{1n} + e[2]_n\,,$$
$$Y[3]_n = g_{0n} + g_{1n} + g_{1n} + e[3]_n\,,$$

or, in general,

$$Y[t]_n = g_{0n} + g_{1n}\,(t-1) + e[t]_n\,,$$

and so the trajectory is linear over time.

As another alternative, we can consider a model where the changes are directly proportional to the previous latent score by writing (8.9a) and this change model represents a trajectory with (8.9b).

$$\Delta y[t]_n = \beta \cdot y[t-1]_n \tag{8.9a}$$
$$Y[t]_n = g_{0n} + \left(\sum_{t=2}^{T} \beta \cdot y[t-1]_n\right) + e[t]_n \tag{8.9b}$$

So

$$Y[1]_n = g_{0n} + e[1]_n,$$
$$Y[2]_n = g_{0n} + (\beta \cdot y[1]) + e[2]_n,$$
$$Y[3]_n = g_{0n} + (\beta \cdot y[1] + \beta \cdot y[2]) + e[3]_n\,,$$

and so on.

This accumulated trajectory is an exponentially accelerating function over time. As yet another alternative, we can write a composite change expression model where we consider both a systematic constant change (g_{1n}) and a proportional change (β) over time. The change equation for this dual change score model can be written as (8.10a) and this change model represents a trajectory with (8.10b).

$$\Delta y[t]_n = g_{1n} + \beta \cdot y[t-1]_n\,, \tag{8.10a}$$

$$Y[t]_n = g_{0n} + \left(\sum_{t=2}^{T} g_{1n} + \beta \cdot y[t-1]_n \right) + e[t]_n\,. \tag{8.10b}$$

So

$$\begin{aligned} Y[1]_n &= g_{0n} + e[1]_n, \\ Y[2]_n &= g_{0n} + (g_{1n} + \beta \cdot y[1]) + e[2]_n, \\ Y[3]_n &= g_{0n} + (g_{1n} + \beta \cdot y[1] + g_{1n} + \beta \cdot y[2]) + e[3]_n\,, \end{aligned}$$

or, in general,

$$Y[t]_n = g_{0n} + g_{1n}\,(t-1) + \left(\sum_{t=2}^{T} \beta \cdot y[t-1] \right) + e[t]_n\,.$$

This accumulating of the composite change model (8.10a) leads to a potentially complex nonlinear growth trajectory (8.10b). Depending on the sign and size of the coefficients, this nonlinear growth trajectory follows an increasing or decreasing, accelerating or decelerating exponential form (e.g., $Y[t]_n = c_{0n} + c_{1n} \cdot (1 - e^{\pi \cdot t}) + e[t]_n$).

Of course, this use of latent change scores is a generic approach that can be extended to many other forms of change models. For example, McArdle (2001) examined the proportional change model with an independent residual (i.e., an autoregressive model) as well as a model of changes in the common factor scores. Hamagami & McArdle (2007) investigated the forms of changes based on second order difference operators. A key feature of this latent change score approach to defining trajectories over time is that we are not limited to the models discussed here. Instead, the latent change score approach opens up possibilities for other parametric analyses of repeated observations.

An immediate benefit of this approach is seen when we deal with multiple variables over time. In a simple case, we can first organize the model into a set of *bivariate dynamic change score* equations that relate the latent changes in each variable to the previous states of those variables and a constant change component. If we use the simple starting points of the models considered above, one set of dynamic equations can be written as

$$\begin{aligned} \Delta y[t]_n &= g_{1n} + \beta_y \cdot y[t-1] + \gamma_{yx} \cdot x[t-1] \\ \Delta x[t]_n &= h_{1n} + \beta_x \cdot x[t-1] + \gamma_{xy} \cdot y[t-1] \end{aligned} \tag{8.11}$$

Where g_{1n} and h_{1n} are the constant change components for y and x, β_y and β_x are the proportional change parameters describing how each variable influences itself over time, and γ_{yx} and γ_{xy} are the coupling parameters describing how each variable influences each other over time. It may be useful to note that all the desirable latent slope parameters are not jointly identifiable, so we typically estimate only the latent means (μ_{g1} and μ_{h1}; see Figure 8.3). Also, to simplify the expressions, we start with an explicit repetition of all model parameters across each time (i.e., β_x, β_y, γ_{yx}, and γ_{xy} do not depend on t), and we recognize this is not a necessary feature of real data. This simplified form of a bivariate trajectory model is depicted as a path diagram in Figure 8.3.

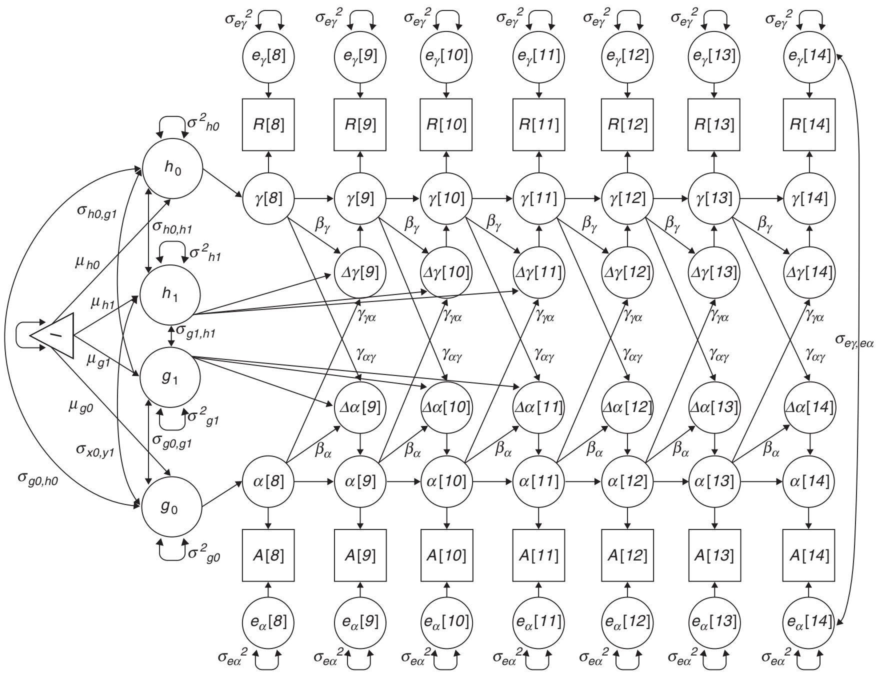


Fig. 8.3 Path diagram of a bivariate latent difference score model for Reading Comprehension and Antisocial Behavior.

In this simplified form of a bivariate dynamic system we assume a dual change score model within each variable but also permit *coupling* parameters (γ) across different variables. This model is used to estimate the time-dependent effect of latent $x[t]$ on $\Delta y[t+1]$ (γ_{yx}) as well as coupling parameter representing the time-dependent effect of latent $y[t]$ on $\Delta x[t+1]$ (γ_{xy}). This model subsumes all aspects of the previous cross-lagged, correlated growth, and time-varying covariate models as special cases. These latent change score models can lead to more complex nonlinear trajectory equations (e.g., non-homogeneous equations) and the use of latent change scores allow for the analysis of a variety of dynamic models using standard SEM (for more detailed explanations, see McArdle, 2001, 2009).

8.5.2 Results from Fitting Latent Change Score Models

The latent change score dynamic models were fitted to the reading comprehension variable and to the antisocial behavior scores. Several alternative Reading/Antisocial Behavior bivariate coupling models based on Figure 8.3 were fitted to the data. In the first model (5a), the coupling parameters (γ) were fixed to zero so the changes in reading and antisocial behaviors were not time-dependent. In the second model (5b), the coupling parameter from reading to changes in antisocial behavior was estimated whereas the coupling parameter from antisocial behavior to changes in reading was fixed to zero. In the third model (6c), the coupling parameter from reading to changes in antisocial behaviors was fixed to zero and the coupling parameter from antisocial behavior to changes in reading was estimated. These two models (5b and 5c) test whether reading was a leading indicator of changes in antisocial behavior (5b) and whether antisocial behavior was a leading indicator of changes in reading (5c). The final model (5d) was the bidirectional coupling model in which both coupling parameters were estimated.

The fitting of a sequence of alternative models was needed to interpret the replicability of the *coupling* across the reading and antisocial variables. Table 8.5 contains parameter estimates and fit statistics for the four bivariate dynamic models fit to reading and antisocial behaviors to determine whether one or more of the coupling parameters (γ) were different from zero. In the first model (5a), the coupling parameters were fixed at zero and led to a likelihood of $L^2 = -92162$. This model can be used as a baseline for comparison for the models in which coupling parameters were estimated. In the second model (5b) the parameter representing the effect of antisocial behavior on changes in reading was fixed to zero; however the effect of *reading* on changes in antisocial behaviors was estimated. This model resulted in a slight improvement in fit ($\Delta\chi^2 = 8$ on $df = 1$) compared to the *no coupling* model (5a). Similarly, the third model in which the coupling parameter from antisocial behavior to changes in reading was estimated and the parameter from reading to changes in antisocial behaviors was fixed to zero resulted in an improvement in fit ($\Delta\chi^2 = 8$ on $df = 1$) compared to the *no coupling* model. Finally, the bidirectional coupling model (5d) was fit and was an improvement over the *no coupling* model ($\Delta\chi^2 = 17$ on $df = 2$) and the two unidirectional coupling models (5b and 5c; $\Delta\chi^2 = 9$ on $df = 1$). Therefore, Model 5d, in which reading and antisocial behaviors were both dynamically related, was the most reasonable representation of the time-dependent relationships. The resulting interpretation is a dynamic process where scores on reading achievement have a tendency to impact changes in antisocial behavior in a positive manner and antisocial behavior has a tendency to effect subsequent change in reading achievement negatively. Therefore, children who have

Table 8.5 Results of bivariate latent change score dynamic models fitted to PIAT Reading Comprehension and BPI Antisocial Problem Behaviors (Step 4)

Model Parameters	*5a: No Coupling*		*5b: Read → ΔAnti*		*5c: Anti → ΔRead*		*5d: Bidirectional Coupling*	
	Read	Anti	Read	Anti	Read	Anti	Read	Anti
Fixed Effects								
Initial Mean μ_0	30.8*	1.5*	30.8*	1.5*	30.8*	1.5*	39.8*	1.5*
Slope Mean μ_1	11.7*	-.2*	11.8*	-.3*	13.6*	-.2*	13.7*	-.3*
Proportion β	-.19*	.16*	-.19*	.13*	-.19*	.15*	-.19*	.12*
Coupling γ	—	—	—	.004*	-1.32*	—	-1.32*	.004*
Random Effects								
Error Variance σ_e^2	39.9*	1.0*	39.9*	1.0*	39.7*	1.0*	39.7*	1.0*
Initial Variance σ_0^2	61.0*	1.5*	61.0*	1.5*	61.1*	1.5*	61.1*	1.5*
Slope Variance σ_1^2	5.5*	.06*	5.5*	.05*	5.6*	.05*	5.7*	.04*
Correlation ρ_{01}	.75*	-.90*	.75*	-.82*	.51*	-.88*	.52*	-.79*
Correlation ρ_{r0a0}	-.30*		-.30*		-.32*		-.31*	
Correlation ρ_{r1a1}	.22*		.05		-.31		-.40	
Correlation ρ_{r0a1}	.26*		.10		.26*		.09	
Correlation ρ_{r1a0}	-.30*		.29*		.38		.38	
Correlation ρ_{erea}	.01		-.01		-.00		-.00	
Fit Statistics								
Parameters	19		20		20		21	
Degrees of Freedom	100		99		99		98	
Log Likelihood	-92162		-92158		-92158		-92154	
χ^2	236		228		228		219	
RMSEA	.01		.01		.01		.01	

a greater reading comprehension scores tend to show slightly more positive changes in antisocial behaviors (negatively valenced) and children displaying more antisocial behaviors tend to show less positive changes in reading comprehension.

The estimated model parameters were dependent on the scalings used, but the trajectory expectations allow us to interpret the results in a relatively "scale-free" form – Figure 8.4 gives a summary of this state-space plot as a *vector field* (for details, see Boker & McArdle, 1995; McArdle et al., 2001). Any pair of coordinates is a starting point (e.g., intercept for reading and antisocial behavior) and the directional arrow is a display of the expected pair of 1-year changes from this point. This figure shows an interesting dynamic property – *the change expectations of a dynamic model depend on the starting point*. From this perspective, we can also interpret the negative level-level correlation ($\rho_{r0,a0} = -.31$), which describes the placement of the individuals in the vector field, and the slope-slope correlation ($\rho_{r1,a1} = -.40$), which describes the location of the subsequent change scores for individuals in the vector field. The resulting "flow" shows a dynamic process where reading comprehension and antisocial behavior scores have a tendency to impact changes in each other from age 8 to 14.

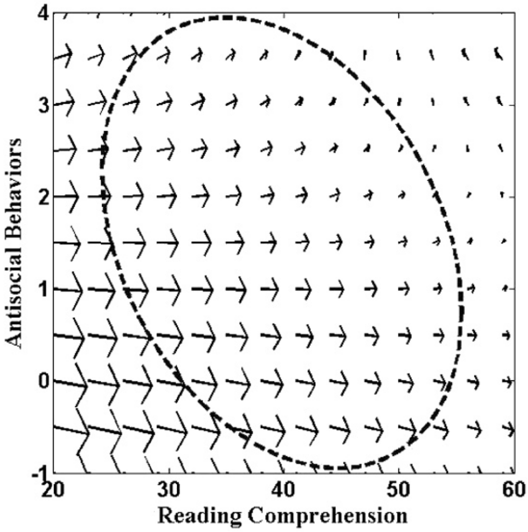


Fig. 8.4 Vector field for the pattern of results from the bivariate latent change score model for Reading Comprehension and Antisocial Behavior.

8.6 Step 5: Studying Group Differences in Dynamic Determinants across Multiple Variables

The final step was to examine whether there were group differences in the dynamic time-dependent associations. That is, we want to determine whether there were group differences in the lead-lag relationships (*Read* $\rightarrow$ $\Delta Anti$; *Anti* $\rightarrow$ $\Delta Read$). The models for this step are a combination of the previous two steps (Group Differences & Dynamics). That is, the bivariate dual change score model with bidirectional coupling is brought into a multiple group and latent mixture framework to study differences in lead-lag relationships for observed and unobserved groups.

8.6.1 Results for Multiple Group Dynamic Models with Gender

As in the univariate multiple group models for reading we begin with a model in which all of the parameters were separately estimated for males and females. This model yields a fit ($L^2 = -91876$) which is reasonable (χ^2 = 356 on *df* = 196) given the observed data for males and females. The estimated parameters were similar for males and females; however there were some small interesting differences. For example, the males tended to be more antisocial than females at age 8 and the effect of reading on changes in antisocial behavior appeared to be stronger for females than males. The dynamic parameters (β, γ) were then constrained to be equivalent for males and females and the resultant fit ($L^2 = -91878$; $\chi^2 = 360$ on $df = 200$) was similar ($\Delta\chi^2 = 4$, $\Delta df = 4$) to when the dynamic parameters were independently estimated for males and females indicating the lead-lag relationships that exist between reading and antisocial behavior were the same for males and females. Next, the variance/covariance parameters were set equal for males and females, which resulted in a substantial loss in fit ($\chi^2 = 615$ on $df = 213$). Therefore, there were variance/covariance parameters that were significantly different for males and females. From the previous model, it appeared the level and slope variances for reading and antisocial behaviors were greater for males than females. Additionally, males had greater level of antisocial behavior at age 8 and females tended to have higher levels of reading achievement at age 8.

8.6.2 Results for Dynamic Mixture Models

The first dynamic mixture model was a two-class bivariate dual change score model with bidirectional coupling. In this model all of the parameters were separately estimated for the two-classes. This model required 43 parameters and yielded a likelihood ($L^2 = -89639$) and likelihood based fit statistics (BIC = 179659). Comparing the likelihood and BIC from this two-class mixture model to the likelihood (-92154) and BIC (184493) from one-class model indicated an improvement. The threshold parameter (τ) was estimated to be 0.63 indicating the sample could be considered a mixture of two classes of different sizes, $n_1 = 4665$ and $n_2 = 2637$, with different dynamic relationships. The first class showed a dynamic pattern that was similar to the overall model with reading comprehension having a small positive (0.004) effect on changes in antisocial behaviors while antisocial behaviors had a large negative effect (-1.64) on changes in reading comprehension. The second class, on the other hand, had no significant coupling parameters indicating that reading comprehension and antisocial behaviors did not have a time-dependent relationship for this class of participants. This separation of individuals into people who did show a specific coupling from persons who seem uncoupled is an important theoretical issue that requires careful consideration and replication. Although an initial set of values can be estimated using this latent change mixture model approach, it also seems obvious that replicated results across multiple studies would give us a much stronger basis to form homogeneous groupings of people.

8.7 Discussion

This chapter serves to provide some methodological and analytical methods the examination of longitudinal data using the general rubric of growth curve modeling techniques in a structural equation modeling framework. We recognize that SEM is just one framework for longitudinal data analysis and represents a limited class of longitudinal data analytic techniques (e.g., Nesselroade & Baltes, 1979; Collins & Sayer, 2001). However, the analyses presented here include some of the most up-to-date combinations of longitudinal models dealing with the developmental-dynamic processes with unobserved heterogeneity. The five steps we outlined here represent one way to organize some of the inherent complexity of longitudinal data analysis, but these techniques are central to answering questions that are often posed and initiate the collection of longitudinal data.

These five steps form a sequence with increasing levels of practical and theoretical knowledge, so it is useful to consider them in the order presented here. The inclusion of all five aspects of latent curve modeling is often overlooked in longitudinal analyses. That is, latent curves models (#2) are often fit without first describing the basic data (#1). Group differences (#3) are presented without a full evaluation of various growth curves that may be appropriate for the data (#2). In many recent cases, inferences about latent curve dynamics across variables (#4 and #5) are offered using simpler models that are incapable of providing this information (e.g., #3). For these reasons, a longitudinal researcher should consider the issues within each step before moving on to the next step. Of course, it is easy to envision situations where it would be best to apply the steps in a different sequence, or even to elaborate on one step based on the research questions. Obviously, models of the complexity of Steps 4 and 5 may only be useful in the more advanced stages of research. Further steps beyond these are possible, and should deal with dynamic models from a time-series perspective (e.g., Nesselroade et al., 2002), models based on differential equations (e.g., Oud & Jansen, 2001), selection effects due to survival (e.g., McArdle et al., 2005), and deal with experimental group dynamics (e.g., McArdle, 2007).

The structural-dynamic models discussed here represent only a sample of the mathematical and statistical models appropriate for longitudinal data and the choice of longitudinal models should be based on the specific research question under investigation (see Grimm, 2007). Indeed, some of the most difficult problems for future work on latent curves will be focused on the rather elusive meaning of the latent model parameters themselves (Zeger & Harlow, 1987; McArdle & Nesselroade, 2003). The choice of an appropriate substantive-vs-methodological interface (see Wohlwill, 1991) creates problems that remain among the most difficult challenges for future work. In this sense, the five step sequence advocated here is mainly intended as a practical way to organize the otherwise daunting task of developmental analyses of multivariate multiple occasion data.

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References

Baltes, P. B., & Nesselroade, J. R. (1979). History and rationale of longitudinal research. In J. R. Nesselroade & P. B. Baltes (Eds.), *Longitudinal research in the study of behavior and development* (pp. 1-39). New York: Academic Press.

Boker, S. M., & McArdle, J. J. (1995). Statistical vector field analysis applied to mixed cross-sectional and longitudinal data. *Experimental Aging Research, 21*, 77-93.

Browne, M., & du Toit, S. H. C. (1991). Models for learning data. In L. Collins & J. L. Horn (Eds.), *Best methods for the analysis of change* (pp. 47-68). Washington, DC: APA Press.

Bryk, A. S., & Raudenbush, S. W. (1992). *Hierarchical linear models: Applications and data analysis methods*. Newbury Park, CA: Sage.

Cattell, R. B. (1980). The separation and evaluation of personal and environmental contributions to behavior by the person-centered model (PCER). *Multivariate Behavioral Research, 15*, 371-402.

Cnaan, A., Laird, N. M., & Slasor, P. (1997). Using the general linear mixed model to analyze unbalanced repeated measures and longitudinal data. *Statistics in Medicine, 16*, 2349-2380.

Cook, T. D., & Campbell, D. T. (1979). *Quasi-experimentation design and analysis issues for field settings*. Skokie, IL: Rand-McNally.

Cudeck, R., & Klebe, K. J. (2002). Multiphase mixed-effects models for repeated measures data. *Psychological Methods, 7*, 41-63.

Dunn, L. M., & Markwardt, F. C. (1970). *Peabody Individual Achievement Test manual*. Circle Pines, MN: American Guidance Service.

Grimm, K. J. (2007). Multivariate longitudinal methods for studying developmental relationships between depression and academic achievement. *International Journal of Behavioral Development, 31*, 328-339.

Grimm, K. J., & McArdle, J. J. (2005). A note on the computer generation of structural expectations. In F. Dansereau & F. Yammarino (Eds.), *Multi-level issues in strategy and research methods* (Volume 4 of Research in multi-level issues) (pp. 335-372). Amsterdam: JAI Press/Elsevier.

Hamagami, F., & McArdle, J. J. (2007). Dynamic extensions of latent difference score models. In S. M. Boker & M. J. Wenger, *Data analytic techniques for dynamical systems. Notre Dame series on quantitative methodology* (pp. 47-85). Mahwah, NJ: Lawrence Erlbaum Associates.

Horn, J. L. (1973). On extension analysis and its relation to correlations between variables and factor scores. *Multivariate Behavioral Research, 8*, 477-489.

Jöreskog, K. G., & Sörbom, D. (1979). *Advances in factor analysis and structural equation models*. Cambridge, MA: Abt Books.

Laird, N. M., & Ware, J. H. (1982). Random effects models for longitudinal data. *Biometrics, 38*, 963-974.

Littell, R. C., Miliken, G. A., Stoup, W. W., & Wolfinger, R. D. (1996). *SAS system for mixed models*. Cary, NC: SAS institute.

Little, R. J. A. (1995). Modeling the dropout mechanism in repeated-measures studies. *Journal of the American Statistical Association, 90,* 1112-1121.

Magnussen, D, (2003). Person-Centered methodology. In S. C. Peck & R. W. Roeser (Eds.), *Person-centered approaches to studying development in context*. San Francisco: Jossey-Bass.

McArdle, J. J. (1988). Dynamic but structural equation modeling of repeated measures data. In J. R. Nesselroade & R. B. Cattell (Eds.), *Handbook of multivariate experimental psychology: Vol. 2* (pp. 561-614). New York: Plenum.

McArdle, J. J. (1989). Structural modeling experiments using multiple growth functions. In P. Ackerman, R. Kanfer & R. Cudeck (Eds.), *Learning and individual differences: Abilities, motivation, and methodology* (pp. 71-117). Hillsdale, NJ: Lawrence Erlbaum Associates.

McArdle, J. J. (1994). Structural factor analysis experiments with incomplete data. *Multivariate Behavioral Research, 29,* 409-454.

McArdle, J. J. (2001). A latent difference score approach to longitudinal dynamic structural analyses. In R. Cudeck, S. du Toit & D. Sörbom (Eds.), *Structural equation modeling: Present and future* (pp. 342-380). Lincolnwood, IL: Scientific Software International.

McArdle, J. J. (2005). The development of the RAM rules for latent variable structural equation modeling. In J. J. McArdle & A. Maydeu-Olivares (Eds.), *Contemporary psychometrics: A festschrift for Roderick P. McDonald* (pp. 225-273). Mahwah, NJ: Lawrence Erlbaum Associates.

McArdle, J. J. (2007). Dynamic structural equation modeling in longitudinal experimental studies. In K. van Montfort, J. Oud & A. Satorra (Eds.), *Longitudinal models in the behavioral and related sciences* (pp. 159-188). Mahwah, NJ: Lawrence Erlbaum Associates.

McArdle, J. J. (2009). Latent variable modeling of differences and changes. *Annual Review of Psychology, 60*, 577-605.

McArdle, J. J., & Bell, R. Q. (2000). An introduction to latent growth models for developmental data analysis. In T. D. Little, K. U. Schnabel & J. Baumert (Eds.), *Modeling longitudinal and multilevel data: Practical issues, applied approaches, and specific examples* (pp. 69-107). Mahwah, NJ: Lawrence Erlbaum Associates.

McArdle, J. J., Caja-Ferrer, E., Hamagami, F., & Woodcock, R. W. (2002). Comparative longitudinal structural analyses of the growth and decline of multiple intellectual abilities over the life span. *Developmental Psychology, 38,* 115-142.

McArdle, J. J., & Cattell, R. B. (1994). Structural equation models of factorial invariance in parallel proportional profiles and oblique confactor. *Multivariate Behavioral Research, 20*, 63-113.

McArdle, J. J., & Epstein, D. B. (1987). Latent growth curves within developmental structural equation models. *Child Development, 58*, 110-133.

McArdle, J. J., & Hamagami, F. (1992). Modeling incomplete longitudinal and cross-sectional data using latent growth structural models. *Experimental Aging Research, 18*, 145-166.

McArdle, J. J., & Hamagami, F. (2001). Linear dynamic analyses of incomplete longitudinal data. In L. Collins & A. Sayer (Eds.), *New methods for the analysis of change.* (pp. 137-176). Washington, DC: APA Press.

McArdle, J. J., Hamagami, F., Meredith, W., & Bradway, K. P. (2001). Modeling the dynamic hypotheses of Gf-Gc theory using longitudinal life-span data. *Learning and Individual Differences, 12*, 53-79.

McArdle, J. J., & McDonald, R. P. (1984). Some algebraic properties of the Reticular Action Model for moment structures. *The British Journal of Mathematical and Statistical Psychology, 37*, 234-251.

McArdle, J. J., & Nesselroade, J. R. (1994). Using multivariate data to structure developmental change. In S. H. Cohen & H. W. Reese (Eds.), *Life-span developmental psychology: Methodological innovations* (pp. 223-267). Hillsdale, NJ: Lawrence Erlbaum Associates.

McArdle, J. J., & Nesselroade, J. R. (2003). Growth curve analyses in contemporary psychological research. In J. Schinka & W. Velicer (Eds.), *Comprehensive handbook of psychology: Vol. 2. Research methods in psychology* (pp. 447-480). New York: Pergamon Press.

McArdle, J. J., Small, B. J., Backman, L., & Fratiglioni, L. (2005). Longitudinal models of growth and survival applied to the early detection of Alzheimer's Disease. *Journal of Geriatric Psychiatry and Neurology, 18* , 234-241.

Meredith, W., & Tisak, J. (1990). Latent curve analysis. *Psychometrika, 55*, 107-122.

Muthén, B., & Muthén, L. (2000). Integrating person-centered and variable-centered analysis: Growth mixture modeling with latent trajectory classes. *Alcoholism: Clinical and Experimental Research, 24*, 882-891.

Muthén, B., & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. *Biometrics, 55*, 463-469.

Miyazaki, Y., & Raudenbush, S. W. (2000). Tests for linkage of multiple cohorts in an accelerated longitudinal design. *Psychological Methods, 5*, 24-63.

Nagin, D. (1999). Analyzing developmental trajectories: Semi-parametric, group-based approach. *Psychological Methods, 4*, 139-177.

Nesselroade, J. R., & Baltes, P. B. (Eds.) (1979). *Longitudinal research in the study of behavior and development*. New York: Academic Press.

Nesselroade, J. R., McArdle, J. J., Aggen, S. H., & Meyers, J. M. (2002). Dynamic factor analysis models for representing process in multivariate time-series. In D. S. Moskowitz & S. L. Hershberger (Eds.), *Modeling intraindividual variability with repeated measures data: Methods and applications* (pp. 235-265). Mahwah, NJ: Lawrence Erlbaum Associates.

Oud, J. H. L., & Jansen, R. A. R. G. (2000). Continuous time state space modeling of panel data by means of SEM. *Psychometrika, 65*, 199-215.

Ram, N., & Grimm, K. J. (2007). Using simple and complex growth models to articulate developmental change: Matching method to theory. *International Journal of Behavioral Development, 31*, 303-316.

Rao, C. R. (1958). Some statistical methods for the comparison of growth curves. *Biometrics, 14*, 1-17.

Rogosa, D. R. (1978). Some results for the Johnson-Neyman technique. *Dissertation Abstracts International, 38(9-A).*

Rogosa, D., & Willett, J. B. (1985). Understanding correlates of change by modeling individual differences in growth. *Psychometrika, 50*, 203-228.

Singer, J. D. (1998). Using SAS PROC MIXED to fit multilevel models, hierarchical models, and individual growth models. *Journal of Educational and Behavioral Statistics, 24*, 323-355.

Sliwinski, M., & Buschke, H. (1999). Cross-sectional and longitudinal relationships among age, cognition, and processing speed. *Psychology and Aging, 14,* 18-33.

Sullivan, E. V., Rosenbloom, M. J., Lim, K. O., & Pfefferman, A. (2000). Longitudinal changes in cognition, gait, balance in abstinent and relapsed alcoholic men: Relationships to changes in brain structure. *Neuropsychology, 14*, 178-188.

Verbeke, G., Molenberghs, G., Krickeberg, K., & Fienberg, S. (Eds.) (2000). *Linear mixed models for longitudinal data*. New York: Springer Verlag.

Wedel, M., & DeSarbo, W. S. (1995). A mixture likelihood approach for generalized linear models. *Journal of Classification, 12*, 21-55.

Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin, 116*, 363-381.

Wohlwill, J. F. (1973). *The study of behavioral development.* Oxford, England: Academic Press.

Wohlwill, J. F. (1991). The merger of developmental theory and method. In P. Van Geert & L. P. Mos (Eds.), *Annals of theoretical psychology: Vol. VII* (pp. 129-138).

Zeger, S. L., & Harlow, S. D. (1987). Mathematical models from laws of growth to tools for biologic analysis: Fifty years of growth. *Growth, 51*, 1-21.

Zill. N. (1990). Behavior problem index based on parent report. In *National Health Interview Survey. Child Health Supplement*. Washington, DC: National Center for Health Statistics.