Compensatory Fuzzy Ontology

Ariel Racet Valdés[1], Rafael A. Espin Andrade[1], and Jorge Marx Gómez[2]

[1] Management Studies Center, José Antonio Echeverría University
Calle 114 No. 11901, Marianao. Ciudad de La Habana, Cuba
{aracet,espin}@ind.cujae.edu.cu
[2] Dpt. Business Informatics I, Carl von Ossietzky University Oldenburg
Ammerländer Heerstr. 114-118, 26129 Oldenburg, Germany
marx-gomez@wi-ol.de

Abstract. Nowadays, to have relevant information is an important factor that contributes favorably to the decision making process. The usage of ontologies to improve the effectiveness in obtaining information has received special attention from researchers in recent years. However, the conceptual formalism supported by ontologies is not enough to represent the ambiguous information that is commonly founded in many domains of knowledge. An alternative is to incorporate the concepts of compensatory fuzzy logic in order to handle the uncertainty in the data, which take advantage of the benefits it provides for the formal representation of uncertainty. We present in this paper the formal definition of “Compensatory Fuzzy Ontologies" and attempt to bring to light the need for enhanced knowledge representation systems, using the advantages of this approach, which would increase the effectiveness of using knowledge in the field of decision making.

Keywords: Ontologies, Compensatory Fuzzy Logic, Decision Making Process, Compensatory Fuzzy Ontologies.

1 Introduction

Information is one of the strategic resources of any organization. To have a minimum of outstanding information could be an important factor to contribute favorably to the decision making process that takes place along the entire value chain of a company, as well on the training of the people involved in that process.

Making changes without information implies ignorance on the problem dimension, impossibility to monitor the advances and inability to evaluate the results [1]. Not all the information is vital for the decision making process, for this process is important to have the tangible and vague information relative to the environmental factors, organizational factors, decision – specific factors and the decision process characteristics. [2]

Human cognitive limitations make very difficult, questionable, subject to inconsistencies, to manage the multiple dimensions of the conflict when the number of attributes increases beyond a few [3]. This will influence significantly the decision making process, where the decision-maker must assess and manage several issues relevant to the decision.

It is necessary to have tools that are able to assist managers in the decision making process. The usage of ontologies to improve the effectiveness in obtaining information has received special attention from researchers in recent years. However, the conceptual formalism supported by ontologies is not enough to represent the ambiguous information that is commonly founded in many domains of knowledge [4], where decision making is not an exemption: while more strategic is the decision, greater is the amount of uncertainty and vagueness the decision maker has to deal with. It is well understood that relations among real life entities are always a matter of degree, and are, therefore, best modeled using fuzzy relations [5].

The incorporation of the concepts of compensatory fuzzy logic inside ontologies in order to take advantage of the use of vagueness in the knowledge domain, allows the enhancement of the formal representation and its employment in knowledge management.

2 Ontology and Fuzzy Ontology

Ontology is a conceptualization of a domain into a human understandable, machine readable format consisting of entities, attributes, relationships and axioms [6]. It is used as a standard knowledge representation for the Semantic Web.

There are several fuzzy concepts that we cannot conceive using conventional ontologies [7]. To handle uncertainty of information and knowledge, one possible solution is to incorporate fuzzy theory into ontology. Then we can generate fuzzy ontologies, which contain fuzzy concepts and fuzzy memberships. The fuzzy ontologies are capable of dealing with fuzzy knowledge [8].

A fuzzy ontology Fo consists of four elements $(C;A^C;R;X)$, where C represents a set of concepts, A^C represents a collection of attributes sets, one for each concept, and $R = (R_T;R_N)$ represents a set of relationships, which consists of two elements: R_N is a set of non-taxonomy relationships and R_T is a set of taxonomy relationships. X is a set of axioms. Each axiom in X is a constraint on the concepts and relationships attribute values or a constraint on the relationships between concept objects. [4]

Some new approach of multivalued fuzzy logic call compensatory fuzzy logic can improve some limitations that classical point of view of norm and co-norm has. Next sections introduce some of these criticisms and solutions, relevant to enhance the fuzzy ontologies role in Knowledge Management and Decision Making.

3 Fuzzy Logic and Modeling of Decision

One way to implement the "principle of gradualness" - essential property of Fuzzy Logic - is the definition of logics where the predicates are functions of the universe X in the interval [0,1] and the conjunction, disjunction, negation and involvement operation are defined in such a way that when restricted to the domain {0,1} we obtain the Boolean Logic. The different ways of defining the operations and their properties determine different multivalued logics that are part of the Fuzzy Logic Paradigm [9].

The use of a set of different operators with properties that generalize the bivalued logic would seem to be the natural way to model decision problems from the language. In fact, applications in the field of decision making has been made basically from the operator concept, rather than multivalued logic [10]. However, this way to address the decisions does not provide the best base to exploit the capacity of Fuzzy Logic for knowledge transformation and decision maker preferences in logical formulas.

There are two main features that hinder the use of logic-based approaches in decision modeling:

- The associative property of conjunction and disjunction operators used
- No compensation among truth values of basic predicates when the compound predicates veracity is calculated using the operators.

The associativity property of a large part of the operators used for aggregation determines that objectives hierarchy trees, which represent different preferences, produce the same truth values of its compound predicates. Under the associativity property both trees in Fig. 1 represent the same preferences, something inappropriate in a decision making model. It is obvious, for example, that the target x has greater relevance in the tree on the right than in left.

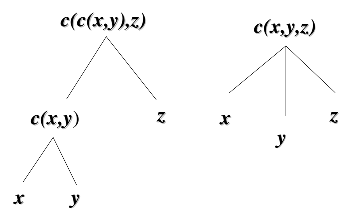

Fig. 1. Objectives hierarchy trees

The lack of compensation is an obstacle for a model that seeks to norm or to describe the reality of decision making; the classic approaches of decision theory, the base of normative thinking, includes models such as the additive ones, which accept the compensation without limits. Descriptive approaches accept partial compensation, which seems more akin to the reasoning of the actual agents. In this way the total lack of compensation and associativity are important limitations of operators frequently used for the addition of preferences.

The above suggests that is desirable the creation of non-associative multivalued logic systems, that facilitate the truth values compensation between basic predicates. Compensatory fuzzy Logic is a multivalued logic that meets these requirements. This is proposed as a decision logical approach, which joins decision modeling and reasoning.

4 Compensatory Fuzzy Logic

Let x = (x1, x2 ,..., xn) be any element of the Cartesian product $[0,1]^n$. A quartet of continuous operators (c, d, o, n), c and d $[0,1]^n$ in [0,1], or $[0,1]^2$ in [0,1] and n of [0,1] in [0,1] represents a compensatory logic, if it satisfies the following set of axioms:

Compensation Axiom, Symmetry or Commutativity Axiom, Strict Growth Axiom, Veto Axiom, Fuzzy Reciprocity Axiom, Fuzzy Transitivity Axiom and De Morgan's Laws.

Operators c and d are called conjunction and disjunction, respectively. Operator o is called fuzzy-strict ordering, and n the negation operator.

Among the multiple aggregation operators found in the literature the only operator which satisfies the axioms of compensation, symmetry, strict growth and veto is the geometric mean:

$$c(x_1, x_2, \ldots, x_n) = (x_1.x_2\ldots x_n)^{1/n} \quad (1)$$

and consequently with the De Morgan's laws, correspondent disjunction will be:

$$d(x_1, x_2, \ldots, x_n) = 1 - ((1-x_1)(1-x_2)\ldots(1-x_n))^{1/n} \quad (2)$$

From the above it follows that the quartet of operators formed by the geometric mean and its dual as conjunctive and disjunctive operators, together with the order (3) and negation n(x) = 1− x represent a compensatory logic.

The implication can be defined in general as (4) or (5).

$$o(x, y) = 0.5[C(x) - C(y)] + 0.5 \quad (3)$$

$$i_1(x, y) = d(n(x), y) \quad (4)$$

$$i_2(x, y) = d(n(x), c(x, y)) \quad (5)$$

thereby generalizes the truth tables of Boolean logic in two different ways.

Equivalence is defined from operator *i* as

$$e(x, y) = i(x, y) \wedge i(y, x) \quad (6)$$

The universal and existential quantifiers must be introduced naturally from the conjunction and disjunction operators, for which introducing already the selected operators, we have:

$$\underset{x \in U}{\forall} p(x) = \underset{x \in U}{\wedge} p(x) = \sqrt[n]{\prod_{x \in U} p(x)} = \\ = \begin{cases} \exp\left(\frac{1}{n}\sum_{x\in U} \ln(p(x))\right) & if\ x\ p(x) \neq 0 \\ 0 & \text{in any other case} \end{cases} \quad (7)$$

$$\underset{x \in U}{\exists} p(x) = \underset{x \in U}{\vee} p(x) = 1 - \sqrt[n]{\prod_{x \in U}(1 - p(x))} = \\ = \begin{cases} 1 - \exp\left(\frac{1}{n}\sum_{x\in U} \ln(1 - p(x))\right) & if\ x\ p(x) \neq 0 \\ 0 & \text{in any other case} \end{cases} \quad (8)$$

For the case of limited sets over R^n, universal and existential quantifiers are defined naturally from the concepts of conjunction and disjunction respectively, moving on to the continuous case through integral calculus [12]:

$$\forall x\, p(x) = \begin{cases} e^{\frac{\int_x \ln(p(x))\, dx}{\int_x dx}} & if\ p(x) > 0 \text{ for any } x \in X \\ 0 & \text{in any other case} \end{cases} \tag{9}$$

$$\exists x\, p(x) = \begin{cases} 1 - e^{\frac{\int_x \ln(1-p(x))\, dx}{\int_x dx}} & if\ p(x) > 0 \text{ for any } x \in X \\ 0 & \text{in any other case} \end{cases} \tag{10}$$

4.1 Relation between Compensatory Fuzzy Logic and Boolean Logic

The formulas for the Propositional Calculus of Compensatory Logic (PCCL) are functions of operators c, d, n and i. Consistent with the definition given by expression (9), any function (11) from the PCCL is considered valid if f (x)> 0 for any element and (12).

$$.f:[0,1]^n \rightarrow [0,1]\ . \tag{11}$$

$$e^{\frac{\int_{[0,1]^n} \ln(1-p(x))\, dx}{\int_{[0,1]^n} dx}} > \frac{1}{2}\ . \tag{12}$$

According to the predicate calculus introduced through the definitions of the quantifiers, it's satisfied the following theorem of Compatibility with Boolean Logic:

Theorem 1: The valid formulas of the PCCL are exactly the ones from Boolean Propositional Calculus (BPC) for any of the two selections of the implication operator (i1 o i2), correspondent to formulas (4) and (5) respectively.

4.2 Compound Inference

The next theorem is the key for a new type of inference which join together the logical and statistical inference:

Theorem 2: Suppose M denote a random sample of the universe U. If p(x)>0 for all x in U, then the universal proposition (13) is normally distributed as (14) where σ^2 is the variance of ln(p(x)) in U and (15) is the mean of ln(p(x)) over U.

$$\underset{x \in M}{\forall} p(x) = \exp\left(\frac{1}{n}\sum_{x \in M} \ln(p(x))\right) \tag{13}$$

$$N(u, \frac{\sigma^2}{n}) \tag{14}$$

$$u = \frac{1}{n}\sum_{x \in U} \ln(p(x)) \tag{15}$$

This result is obtained from definition (7) using the central limit theorem, and allows the estimation of the truth of a universal proposition using a sample. It can be used to reason using a compound inference, which translates from the language for modeling scenarios using predicates of the CFL, and estimates the truth about the universe using the veracity of these predicate on the sample. This inference can also be employed using the same scheme of reasoning, applying Monte Carlo techniques to estimate the accuracy of a universal proposition.

5 Compensatory Fuzzy Ontology

A Compensatory Fuzzy Ontology is a conceptualization of a domain into a human understandable, machine-readable format consisting of fuzzy concepts and non-fuzzy concepts, fuzzy properties and non-fuzzy properties, fuzzy relationships and non-fuzzy relationships, axioms, instances, using compensatory fuzzy logic to obtain the truth values of fuzzy elements expressed through fuzzy predicates.

The concepts, properties and relationships keep the exact same definitions and play the same roles as in a classical ontology. The concepts, properties and fuzzy relations are defined by compensatory fuzzy logic, used to represent elements of the fuzzy area modeled.

- A fuzzy concept $\tilde{C}$ is defined as a fuzzy set whose membership function over the universe U is the associated property c that is defined through a predicate c(x), $x \in U$; the correspondent crisp set is C=$\{x \in U: c(x) > 0.5\}$ that is a non fuzzy class.
- Axioms are compound predicates using defined concepts.

The definition of Compensatory Fuzzy Ontology allows taking into account Theorem 1 to ensure that the results of a bivalued reasoning, from the consideration that the classes are true when their value is greater than 0.5, are the same as if the compensatory fuzzy operators are applied to the calculation of the veracity of predicates and consequently the membership degree to a class. In this second case for each instance we get the truth values of each of the classes.

The knowledge representations obtained using Compensatory Fuzzy Ontology can be used for querying and knowledge discovering [3][11][13]; this means that it is possible from data and other properties to automate the process of ontology enrichment.

The fulfillment of Theorem 2 allows the use of the Compensatory Fuzzy Ontology to estimate real values of the universal proportions from samples, the compound inference associated with the theorem makes easy to work with a sample of instances to infer knowledge.

The knowledge represented trough fuzzy ontologies should be exploited by a reasoner using the definitions of all the elements in CFL. The reasoner can answer some requests trough simple evaluation of predicates and discover knowledge by the use of searching methods.

6 Case Study: Ontology for Competitive Enterprise

The following ontology represents the basic domain knowledge for competitive enterprise. The represented knowledge by the ontology is a consensual knowledge of BIOMUNDI consulting firm.

Following are presented the statements and their translation into the language of predicate calculus:

A firm is competitive in a product line at a given market if 1) the economy of the company is solid and 2) its technology is advanced and 3) it is very strong in the product line at the market

1. A company is financially sound if it has a good financial state and good sales. If the financial state is a not so good, it must be offset by very strong sales.
2. A company has an advanced technological position if your current technology is good and also owns patents, or it has products in research and development, or significant amounts of money devoted to this activity. If their technology is somewhat behind, then it must have many patents, many products in research and development, or spend very substantial amounts of resources to this effort.
3. A company is strong in a product line, if you have strength in the market, has a diversified product line and it is independent of the supplier.

The following notation associates concepts to names of predicates that will be defined using the "translation" of the statement.

- *c(x)*: Enterprise x is competitive
- *s(x)*: Enterprise x has solid economy
- *t(x)*: Enterprise x has advanced technological position
- *l(x)*: Enterprise x is strong in the product line
- *f(x)*: Enterprise x has good financial state
- *v(x)*: Enterprise x has good sales
- *g(x)*: Enterprise x has a good technology nowadays
- *p(x)*: Enterprise x is the owner of patents
- *i(x)*: Enterprise x many products in research and development
- d(x): Enterprise x or spend very substantial amounts of resources in research and development
- *m(x)*: Enterprise x has strength in the market
- *vl(x)*: Enterprise x has a diversified product line
- *ip(x)*: Enterprise x is independent of the supplier

The compound predicate c(x) obtained from the expressed definition of Competitive Enterprise is the property that defines the fuzzy class $\tilde{C}$ that represents competitive enterprise:

$$c(x) = s(x) \wedge t(x) \wedge l^2(x) \qquad (16)$$

where the use of the exponent 2 means "very" and is used as a modifier like is usual in Fuzzy Logic literature. The correspondent no fuzzy class is

$$C = S \cap T \cap L^2. \qquad (17)$$

where C,S,T and L^2 are the respective non fuzzy classes associated to the properties s(x), t(x) y l^2(x) respectively.

According with the statements the following predicates are the properties associated to fuzzy classes that represents Solid Economy Enterprises, Advanced Technology Enterprises, and very strong Line of Products Enterprises:

$$s(x) = f(x) \wedge v(x) \wedge (\neg(f(x))^{0.5} \rightarrow v^2(x)) \tag{18}$$

$$t(x) = g(x) \wedge (p(x) \vee i(x) \vee d(x)) \wedge \\ \wedge (\neg g^{0,5}(x) \rightarrow (p^2(x) \vee i^2(x) \vee d^2(x)) \tag{19}$$

$$l(x) = m(x) \wedge vl(x) \wedge ip(x) \tag{20}$$

The used predicates, like components in s, t and l are called basic predicates. All of them define fuzzy classes too.

Notice that 0.5 is used in the same way that 2 was used before, like a modifier expressing the word more or less.

Figure 2 illustrates the ontology through a logical tree. In the tree, conditional expressions are expressed, by placing on the arc the predicate corresponding to the premise; and as the end of the arc itself, the thesis.

The sigmoidal function of Fig. 3 illustrates the way to define the basic predicates and correspondent classes by membership functions.

Table 1 illustrates the answers that could be obtained requesting about if certain instances are part of each class defined in the ontology.

Compound predicates incorporated to the ontology using defined classes, defined connectives and modifiers are axioms. Axioms can be incorporated from human sources or by knowledge discovery.

For example the conditional s(x) → c(x), or the disjunction t(x) ∨ l(x) could be axioms. Equivalences of the form p(x)↔ a where p(x) is a predicate and "a" is a real number, could be very useful, using the possibilities of CFL to discover and to express knowledge from samples.

Table 1. Answer to fuzzy queries

Empresa x	f(x)	v(x)	g(x)	p(x)	i(x)	d(x)	ip(x)
A	0.5	0.47	0.3	0.93	0.81	0.61	0.6
B	0.6	0.63	0.5	0.41	1	0.95	0.8
C	0.9	0.75	0.7	0.62	0.55	0	1
D	0	0.99	0.8	0.81	0.79	0.7	0.5
	vl(x)	m(x)	s(x)	t(x)	l(x)	l2(x)	c(x)
A	0.23	0.1	0.5	0.516	0.234	0.058	0.246
B	0.77	0.4	0.611	0.682	0.627	0.393	0.545
C	0.92	0.8	0.812	0.584	0.903	0.815	0.728
D	0.39	1	0	0.763	0.58	0.336	0

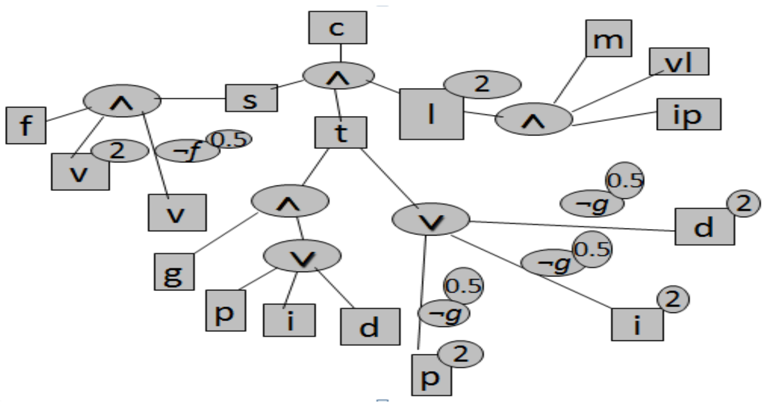

Fig. 2. Logical tree

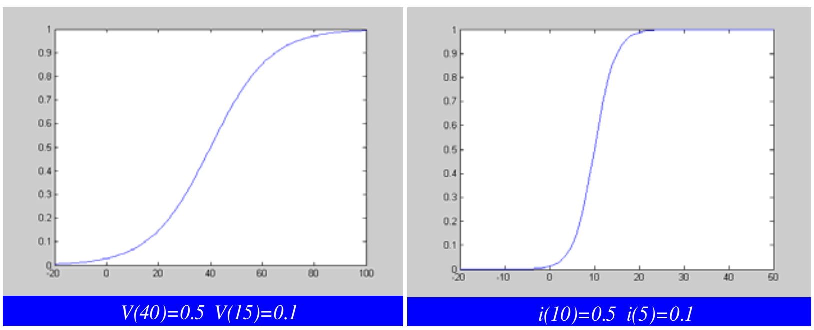

Fig. 3. Sigmoidal function

7 Conclusions

Properties of Compensatory Fuzzy Logic make possible a very useful representation of the knowledge: Compensatory Fuzzy Ontologies. They allow selection of relevant information, and useful knowledge discovery, very important for decision making.

Properties expressed by theorems 1 and 2 are especially important. Theorem 1 establishes that Compensatory Fuzzy Logic offers a way of reasoning completely compatible with the Boolean Logic.

Compensatory Fuzzy Ontologies can be used for querying and knowledge discovery from data and other properties, getting a way of automated ontology enrichment.

The fulfillment of Theorem 2 allows the use of the Compensatory Fuzzy Ontology to estimate real values of the universal proportions from samples, the compound inference associated with the theorem makes easy to work with a sample of instances to infer knowledge.

References

1. Friedman, J.: Estrategias Administrativas para la Eficiencia Universitaria. Revista Chilena de Administración Pública (6), 197–209 (2004)
2. Rajagopalan, N., Rasheed, A.M.A., Datta, D.K.: Strategic Decision Processes: Critical Review and Future Directions. Journal of Management 19(2), 349–384 (1993)
3. Espin, R., Fernández, E.: La Lógica Difusa Compensatoria: Una Plataforma para el Razonamiento y la Representación del Conocimiento en un Ambiente de Decisión Multicriterio. In: Análisis Multicriterio para la Toma de Decisiones: Métodos y Aplicaciones. Coedición: editorial Plaza y Valdes/editorial Universidad de Occidente (2009)
4. Tho, Q.T., Hui, S.C.: Automatic Fuzzy Ontology Generation for Semantic Web. IEEE Transactions on Knowledge and Data Engineering 18(6), 842–856 (2006)
5. Wallace, M., Avrithis, Y.: Fuzzy relational knowledge representation and context in the service of semantic information retrieval. In: IEEE International Conference, Budapest, pp. 1397–1402 (2004)
6. Fensel, D., van Harmelen, F., Horrocks, I., McGuinness, D.L., Patel-Schneider, P.F.: OIL: an ontology infrastructure for the semantic web. IEEE Intelligent Systems 16(2), 38–45 (2001)
7. Ghorbel, H., Bahri, A., Bouaziz, R.: A Framework for Fuzzy Ontology Models. In: Proc. of journées Francophones sur les Ontologies JFO 2008, France, pp. 21–30 (2008)
8. Zhai, J., Li, Y., Wang, Q., Lv, M.: Knowledge Sharing for Supply Chain Management Based on Fuzzy Ontology on the Semantic Web. In: International Symposiums on Information Processing (ISIP), pp. 429–433 (2008)
9. Dubois, D., Prade, H.: Fuzzy Sets and Systems: Theory and Applications. Academic Press Inc., London (1980)
10. Dubois, D., Prade, H.: A review of fuzzy set aggregation connectives. Information Sciences 36, 85–121 (1985)
11. Espin, R., Mazcorro, G., Fenández, E.: Consideraciones sobre el carácter normativo de la lógica difusa compensatoria. In: Infraestructura de Datos Espaciales en Iberoamérica y el Caribe. IDICT, Cuba (2007)
12. Espin, R., Fernández, E., Mazcorro, G., Marx-Gómez, J., Lecich, M.I.: Compensatory Logic: A fuzzy normative model for decision making. Investigación Operativa. Universidad de la Habana 27(2), 188–197 (2006)
13. Delgado, T., Delgado, M.: Evaluación del Índice de Alistamiento de IDES en Iberoamérica y el Caribe a partir de un modelo de Logica Difusa Compensatoria. In: Delgado, T., Crompvoets, J. (eds.) Infraestructura de Datos Espaciales: Iberoamérica y el Caribe. IDICT-CYTED (2007)