# Chapter 12
# Evolutionary Synchronization of Chaotic Systems

Ivan Zelinka and Ales Raidl

**Abstract.** This chapter introduces a simple investigation on deterministic chaos synchronization by means of selected evolutionary techniques. Five evolutionary algorithms has been used for chaos synchronization here: differential evolution, self-organizing migrating algorithm, genetic algorithm, simulated annealing and evolutionary strategies in a total of 15 versions. Experiments in this chapter has been done with two different coupled systems (master - slave) - Rössler-Lorenz and Lorenz-Lorenz. The main aim of this chapter was to show that evolutionary algorithms, under certain conditions, are capable of synchronization of, at least, simple chaotic systems, when the cost function is properly defined as well as the parameters of selected evolutionary algorithm.This chapter consists of two different case studies. For all algorithms each simulation was 100 times repeated to show and check the robustness of proposed methods and experiment configurations. All data were processed to obtain summarized results and graphs.

## 12.1 Introduction

Synchronization is a dynamical process during which one system (synchronized, slaved) is remoted by another (synchronizing, master) so that the synchronized system is in a certain manner following the behavior of the master system. The

Ivan Zelinka
Tomas Bata University in Zlin, Faculty of Applied Informatics, Nad Stranemi 4511, Zlin 76001, Czech Republic
and
VSB-TUO, Faculty of Electrical Engineering and Computer Science, 17. listopadu 15, 708 33 Ostrava-Poruba, Czech Republic
e-mail: zelinka@fai.utb.cz

Ales Raidl
Charles University, Faculty of Mathematics and Physics,V Holesovickach 2, 180 00 Prague 8, Czech Republic
e-mail: ales.raidl@mff.cuni.cz

I. Zelinka et al. (Eds.): Evolutionary Algorithms and Chaotic Systems, SCI 267, pp. 383–407.
springerlink.com 

word “synchronization” come from the Greek word “synchronos” ($\sigma\upsilon\nu\chi\rho o\nu o\varsigma$) in which $\sigma\upsilon\nu$ (syn) means the same (common,...) and $\chi\rho o\nu o\varsigma$ (chronos) means the “time”. Synchronization can be divided into the following classes [9], [4], [12]:

- **Identical synchronization.** This synchronization may occur when two identical chaotic oscillators are mutually coupled (unidirectional or bidirectional coupling), or when one of them drives the other, which is the case of numerical study A (Lorenz-Lorenz), reported in this chapter. Basically, if $\{x_1, x_2, \ldots x_n\}$ is a set of state (dynamical) variables of the master system as well as $\{x'_1, x'_2, \ldots x'_n\}$ of the slave system, then both systems are synchronized if under certain initial conditions and $t \longrightarrow \infty$ is true that $|x_1 - x_1^{'}| \rightarrow 0$. This states that nothing more than large time is enough for dynamics of both systems in a good approximation. This kind of synchronization is usually called identical synchronization.
- **Generalized synchronization** differ from the previous case by the fact that coupled chaotic oscillators are different and that the dynamical state of one of the oscillators is completely determined by the state of the other. This is a case of numerical study B (Rössler-Lorenz), reported in this chapter.
- **Phase synchronization** is another case of synchronization which occurs when the oscillators coupled are not totally identical and the amplitudes of the oscillator remain unsynchronized, while only oscillator phases evolve in synchrony. There is a geometrical interpretation of this case of synchronization. It is possible to find a so called plane in phase space in which the projection of the trajectories of the oscillator follows a rotation around a well-defined center. The phase is defined by the angle $\varphi(t)$, described by the segment which is joining the center of rotation and the projection of the trajectory point onto the plane.
- **Anticipated and lag synchronization.** Lets say that we have a synchronizing system with state variables $\{x_1, x_2, \ldots x_n\}$ and a synchronized system with state variables $\{x'_1, x'_2, \ldots x'_n\}$. Anticipated and lag synchronization occurs when $x_1^{'}(t) = x_1(t + \tau)$ holds true. This relation, in fact, states that the dynamics of one of the systems follows, or anticipates, the dynamics of the other and whose dynamics is described by delay differential equations.
- **Amplitude envelope synchronization** is a kind of synchronization which may appear between two weakly coupled chaotic oscillators. Comparing with another cases of synchronization, there is no correlation between phases or amplitudes. One can observe a periodic envelope that has the same frequency in the two systems. Magnitude of that envelope has the same order of the difference between the average frequencies of oscillation of both systems. It is important to note that phase synchronization can develops from amplitude one, when the strength of the coupling force between two amplitude envelope synchronized oscillators increases in time.

A rich amount of literature of working with synchronization exist. We can recommend as a representative literature [9], [4] and [12]; all three books are well written and highly readable. Other research works are [13], [2] (synchronization based on time series analysis), [11] (robustness of synchronized systems), and many others. Avery good starting reference can be found in the above mentioned books [9], [4] and [12].

The main aim of this research is to show that evolutionary algorithms (EA) are capable of synchronizing simple chaotic systems, without the knowledge of internal system structure. The ability of EAs to successfully work with black box type of problems have been proven; see for example real-time control of plasma reactor [7], [8], [20] or CML non real-time control by evolutionary algorithms [17], [18], [21] and Chapter 6 in this book. This chapter is organized as follows. The first part outlines the motivation of EAs use on synchronization. This is followed by a very brief note about used evolutionary algorithms whose detailed description is presented in Chapter 6. Evolutionary synchronization is then studied, and finally experimental results are reported, followed by conclusion.

## 12.2 Motivation

Motivation of this research is quite simple. As mentioned in the introduction and also in the previous chapters, evolutionary algorithms are capable of hard problem solving. A lot of examples about evolutionary algorithms can be easily found like their use in control, artificial intelligence, electronic devices design and setting etc. For more, see for example mentioned references in Chapter 6, section Motivation. The main question in the case of this chapter was if EAs are able to synchronize simple chaotic systems. Main attention has been paid to continuous chaotic systems, i.e. to Rössler and Lorenz systems. All experiments here were designed to confirm or reject this idea and were designed to be as simple as possible, to show the methodology of evolutionary algorithms use.

## 12.3 Selected Evolutionary Algorithm – A Brief Introduction

For the numerical and symbolic experiments described here, stochastic optimization algorithms such as Differential Evolution (DE) [10], Self Organizing Migrating Algorithm (SOMA) [16], Genetic Algorithms (GA) [5], Simulated Annealing (SA) [6], [3] and Evolutionary Strategies (ES) [1] were selected. Description of all these algorithms can be found in mentioned references or in Chapter 6.

## 12.4 Evolutionary Synchronization

### *12.4.1 Used Hardware, Problem Selection and Case Studies*

Synchronization in this case study has been done on a special grid computer, comparing to simple PC as in [19]. This grid computer consist of two special Apple servers (for pictures, see Chapter 6). In total 78 CPUs are available. Emanuel has been used for calculations such that each CPU has been used like a single processor and thus a rich set of statistically repeated experiments was possible which was not time consuming. Typical parallel computing has been avoided in experiments described here.

Chaotic systems used in this chapter, has been selected from continuous domain, especially the Rössler and Lorenz systems. This selection has been done because in the remaining part of this book are used mostly discrete chaotic systems (1D as well as CML systems) prior to continuous systems. We would like to show that EAs are not restricted only to discrete domain, so this was the main reason of Rössler and Lorenz system use.

Two case studies (A and B) were defined and used. In the first a coupled system Lorenz-Lorenz (case A, eq. (12.1)) was used. Synchronization has been done via parameter $d$ and coupling "part" $d\,(x_1(t) - x_2(t))$ in eq. (12.1). The cost function in this case has been calculated according to eq. (12.3), i.e. difference in master-slave system in all three system variables.

In the second case study (B), Rössler-Lorenz system was used as described by eq. (12.2). Synchronization has been done via parameter $c$ and coupling "part" $c\,(y_1(t) - y_2(t))$ in eq. (12.2). Cost function in this case has been calculated according to eq. (12.4), i.e. only, comparing with eq. (12.3), for one system variable difference in master-slave system.

### *12.4.2 Cost Function*

The fitness (cost function) has been calculated, as mentioned in the last paragraph, according to the distance between desired synchronizing system state and actual synchronized system state. The minimal value of this cost function, guarantees the best solution. The aim of all simulations was to find the best solution, i.e. a solution that returns the cost value as small as possible. The difference between eq. (12.3) and eq. (12.4) is in the number of used state variables. In the case of eq. (12.3) it is logical to expect that all three state variables will be synchronized perfectly, while in the case of the eq. (12.4) we expected that only synchronized variable, in this case $y_2(t)$, will be synchronized in an acceptable manner. Results depicted later in various figures has confirmed all these presumptions. The cost value was in fact the absolute value of summarization of grey areas between synchronizing and synchronized system output (time series) as demonstrated in Fig. 12.1.

$$
\begin{aligned}
&\textbf{Lorenz} - \textbf{Lorenz synchronization}\\
&\textbf{Lorenz system}\;(\textbf{master}):\\
&x_1'(t) = -a\,(x_1(t) - y_1(t))\\
&y_1'(t) = -x_1(t)z_1(t) + bx_1(t) - y_1(t)\\
&z_1'(t) = x_1(t)y_1(t) - cz_1(t)\\
&\\
&\textbf{Lorenz system}\;(\textbf{slave}):\\
&x_2'(t) = -\mathbf{a_2}\,(x_2(t) - y_2(t)) + \mathbf{d}\,(x_1(t) - x_2(t))\\
&y_2'(t) = -x_2(t)z_2(t) + bx_2(t) - y_1(t)\\
&z_2'(t) = x_2(t)y_2(t) - z_2(t)
\end{aligned}
\tag{12.1}
$$

$$
\begin{aligned}
&\textbf{Rössler} - \textbf{Lorenz synchronization}\\
&\textbf{Rössler system}\ (\textbf{master}):\\
&x_1'(t) = -y_1(t) - z_1(t)\\
&y_1'(t) = -x_1(t) - \tfrac{y_1(t)}{5}\\
&z_1'(t) = (x_1(t) - 5.7)\, z_1(t) + 0.2\\
\\
&\textbf{Lorenz system}\ (\textbf{slave}):\\
&x_2'(t) = -\mathbf{a}\,(x_2(t) - y_2(t))\\
&y_2'(t) = -x_2(t) z_2(t) + \mathbf{b} x_2(t) + \mathbf{c}\,(y_1(t) - y_2(t))\\
&z_2'(t) = x_2(t) y_2(t) - z_2(t)
\end{aligned}
\tag{12.2}
$$

$$CF_{LL}(a_2, d) = \int_0^{100} |x_1(t) - x_2(t)| + |y_1(t) - y_2(t)| + |z_1(t) - z_2(t)|\, dt \tag{12.3}$$

$$CF_{RL}(a, b, c) = \int_0^{200} |(y_1(t) - y_2(t)|\, dt \tag{12.4}$$

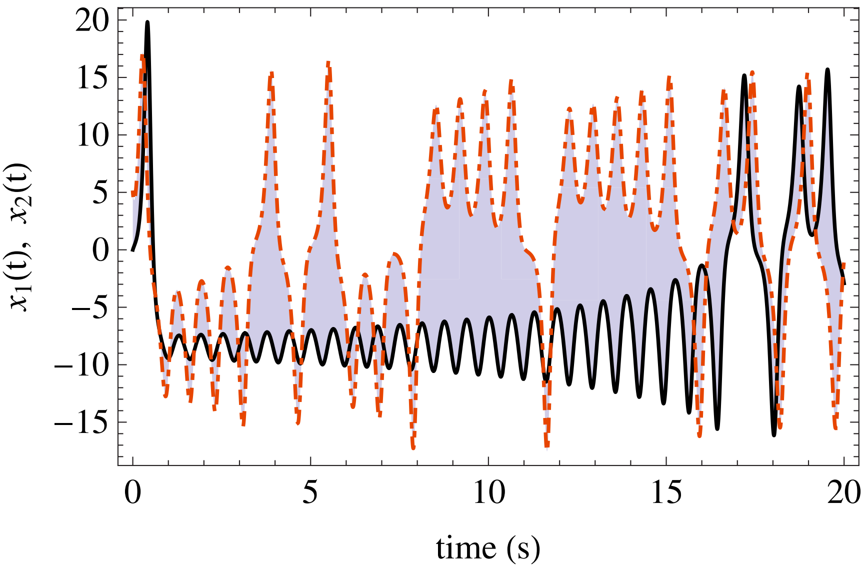


**Fig. 12.1** Principle of cost value calculation. For all three variables $x(t)$, $y(t)$ and $z(t)$ has been calculated difference between behavior of synchronizing and synchronized system (light grey area).

### 12.4.3 Parameter Setting

The control parameter settings have been found empirically and are given in Tables 12.1 - 12.6. The main criterion for this setting was to keep the same setting of parameters as much as possible alongside the same number of cost function evaluations as well as the population size. Individual length represents the number of optimized parameters (in this case coupling parameter $d$ (L-L system) or $c$ (R-L system)).

We would like to note that settings of all used algorithms has been based on our preliminary experiences and certainly can be improved. However this topic is quite numerically time consuming, so we let this topic open for future research.

All algorithms (SOMA, DE, SA, GA, ES) have been applied 100 times in order to find the optimum of both case studies. The primary aim of this comparative study is not to show which algorithm is better and worst, but to show that evolutionary synchronization can be really used for chaotic systems. Outputs of all simulations are depicted in Fig. 12.6 - 12.12, and Fig. 12.34 - 12.36, which shows results of all 100 simulations for each case study.

**Table 12.1** Algorithms abbreviation

| Algorithm | Version | Abbreviation |
|---|---|---|
| Differential Evolution | DEBest1JIter | D1 |
| | DEBest2Bin | D2 |
| | DELocalToBest | D3 |
| | DERand1Bin | D4 |
| | DERand1DIter | D5 |
| | DERand2Bin | D6 |
| Evolutionary strategies | $(\mu,\lambda)$ | ES1 |
| Evolutionary strategies | $(\mu+\lambda)$ | ES2 |
| Genetic Algorithm | | G |
| Simulated annealing with elitism | | SA1 |
| Simulated annealing without elitism | | SA2 |
| SOMA | AllToAllAdaptive | S1 |
| | AllToAll | S2 |
| | AllToOne | S3 |
| | AllToOneRandomly | S4 |

**Table 12.2** DE setting for case studies A and B

| Case Study | A | B |
|---|---|---|
| NP | 100 | 100 |
| F | 0.9 | 0.9 |
| CR | 0.3 | 0.3 |
| Generations | 500 | 500 |
| Individual Length | 2 | 3 |

**Table 12.3** ES setting for case studies A and B

| Case Study | A | B |
|---|---|---|
| $\mu$ ,$\lambda$ | 100 | 100 |
| $\sigma$ | 1 | 1 |
| Iterations | 100 | 100 |
| Individual Length | 2 | 3 |

**Table 12.4** GA setting for case studies A and A

| Case Study | A | B |
|---|---|---|
| Population size | 100 | 100 |
| Mutation | 0.4 | 0.4 |
| Generations | 500 | 500 |
| Individual Length | 2 | 3 |

**Table 12.5** SA setting for case studies A and B

| Case Study | A | B |
|---|---|---|
| No. of particles | 100 | 100 |
| $\sigma$ | 0.5 | 0.5 |
| $k_{\max}$ | 66 | 66 |
| $T_{min}$ | 0.0001 | 0.0001 |
| $T_{max}$ | 1000 | 1000 |
| $\alpha$ | 0.95 | 0.95 |
| Individual Length | 2 | 3 |

**Table 12.6** SOMA setting for case studies A and B

| Case Study | A | B |
|---|---|---|
| PathLength | 3 | 3 |
| Step | .11 | .11 |
| PRT | 0.1 | 0.1 |
| PopSize | 20 | 20 |
| Migrations | 10 | 10 |
| MinDiv | -0.1 | -0.1 |
| Individual Length | 2 | 3 |

### *12.4.4 Experimental Results*

Two main case studies has been done in this chapter. Case study A (synchronization of Lorenz-Lorenz system), and B (synchronization of Rössler-Lorenz system). In both cases attention was paid to parameter estimation, obtained cost value as well as to cost function evaluations needed to reach acceptable setting of given synchronization. All data has been processed in order to get viable statistics about the evolutionary dynamics behind these experiments. These statistics has been processed into figures, which shows the performance of evolutionary techniques from different point of views. Average values are depicted as horizontal line in each figure.

#### 12.4.4.1 Case Study A: Lorenz - Lorenz Synchronization

In the first case study, we have used for synchronization two identical systems Lorenz - Lorenz systems (eq. (12.1), Fig. 12.2). Synchronization has been done by coupling of variables $x_{1,2}(t)$ via parameter $d$. The difference between them has

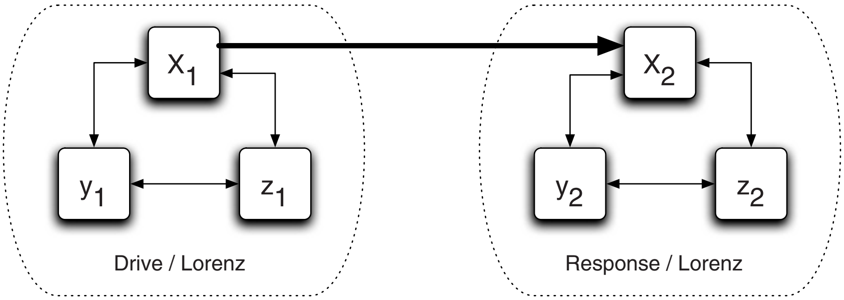


**Fig. 12.2** Schematic of Lorenz-Lorenz synchronization. Variable $x_2$ has been directly synchronized.

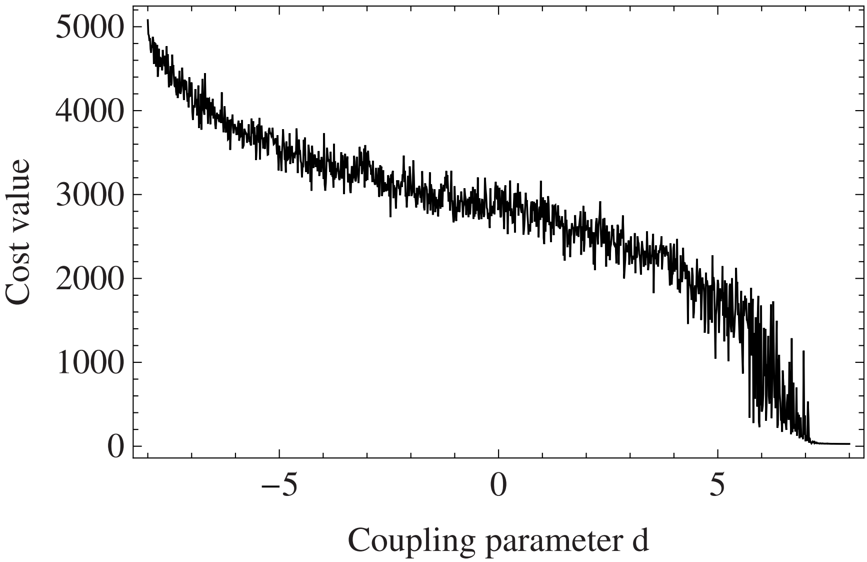


**Fig. 12.3** Dependance of cost function value on coupling parameter $d$.

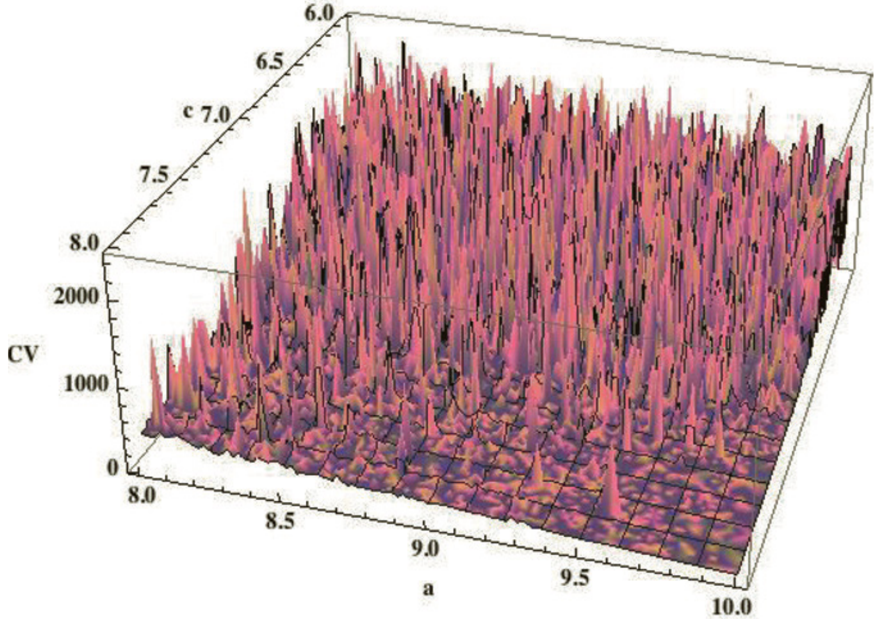


**Fig. 12.4** Dependance of cost function value on coupling parameter $d$ and parameter $a_2$.

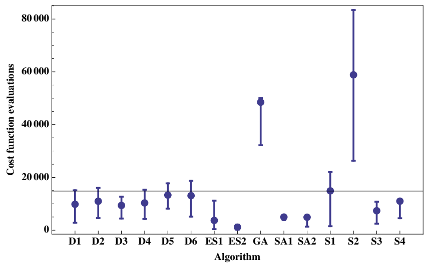


**Fig. 12.5** Cost function evaluations...

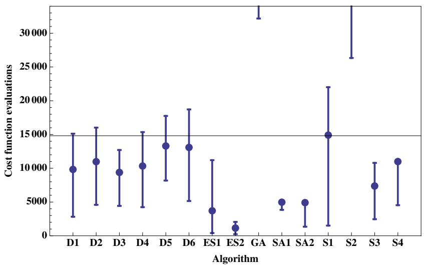


**Fig. 12.6** ... and detail view; horizontal line is an average value of all.

been calculated and multiplied by parameter $d$, see $(d\,(x_1(t) - x_2(t)))$ in eq. (12.1). In fact, the search for optimal parameter setting has been done with dependance on two parameters: on parameter $d$ and $a_2$ in eq. (12.1).

Complexity of cost function landscape, is depicted on Fig. 12.3 for dependance only on coupling parameter $d$ and Fig. 12.4 for dependance at both parameters. All parameters were varied around nominal values, as referred in the literature. Complexity is very high, as anyone can easily see. In the chaotic landscape a linear-like trend is visible. Thanks to this trend, it is visible that the minimum can be expected at position $\{d, a_2\} = \{8, 10\}$.

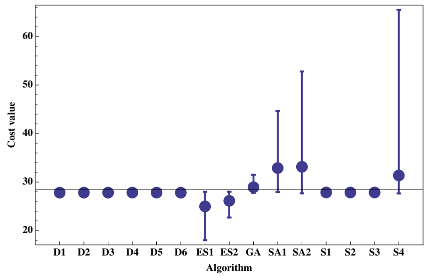


**Fig. 12.7** Cost value...

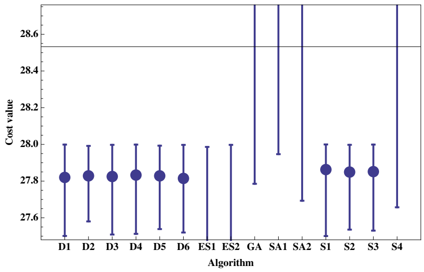


**Fig. 12.8** ... and detail view; horizontal line is an average value of all.

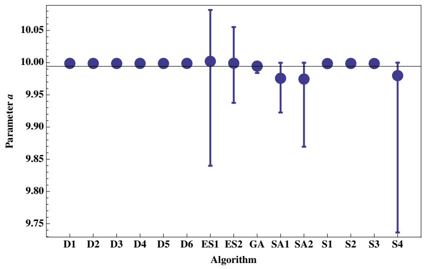


**Fig. 12.9** Estimation of parameter *a* ...

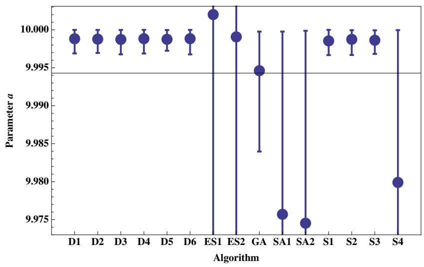


**Fig. 12.10** ... and detail view; horizontal line is an average value of all.

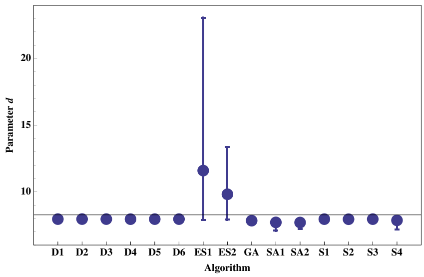


**Fig. 12.11** Estimation of parameter *d* ...

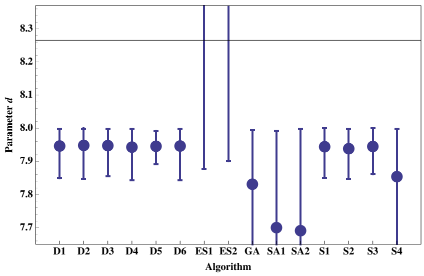


**Fig. 12.12** ... and detail view; horizontal line is an average value of all.

During this case study 5 algorithms has been used in 15 versions. Attention was focused on quality from the cost function evaluations point of view, further on cost value, parameter $a_2$ setting as well as setting of the coupling parameter $d$. Parameters $a_2$ and $d$ has been estimated simultaneously, i.e. individual dimension was 2. All these results are reported in Figs. 12.5 - 12.12. Together with minimal, maximal and average values, average vaue of all algorithms (horizontal line) is also depicted.

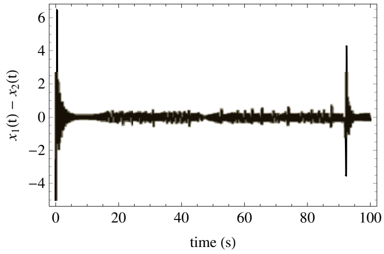


**Fig. 12.13** Difference between $x_1(t)$ and $x_2(t)$.

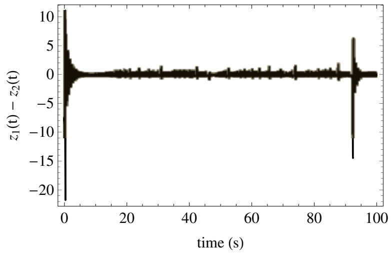


**Fig. 12.14** Difference between $z_1(t)$ and $z_2(t)$.

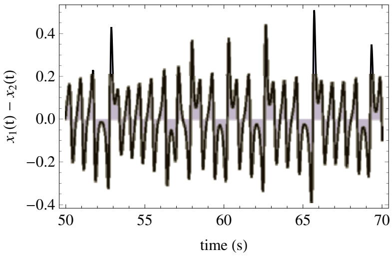


**Fig. 12.15** Difference between $x_1(t)$ and $x_2(t)$ - detail...

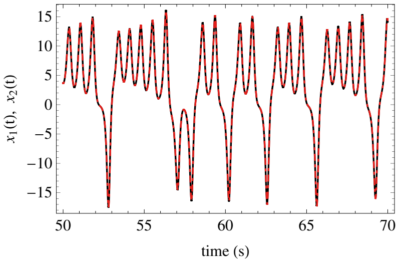


**Fig. 12.16** ...and total view on both variables $x_1(t)$ and $x_2(t)$. Both tme series are almost identical.

On Fig. 12.5 and Fig. 12.6 the performance (how many cost function evaluations was needed to find acceptable setting) of all algorithms is reported. Fig. 12.7 and Fig. 12.8 show the same for cost value related to used estimated setting (i.e. how much differs the behavior of both systems), Fig. 12.9 and Fig. 12.10 show how well the parameter $a_2$ has been estimated and Fig. 12.11 and Fig. 12.12 the give the same for parameter $d$. Difference of estimated parameters is still in the range of acceptable values (i.e. slaved system has been synchronized very well) ism shown in Fig. 12.14, as it is visible in Fig. 12.13 and 12.14. Difference between the worst and the best behavior of $x_{1,2}$ and $z_{1,2}$ is depicted there. The biggest impact on cost value come from interval [0, 7s], before systems are well synchronized. Another anomaly is at position 92s, which is just a sharp peak with little impact on cost value. Middle part is depicted in Fig. 12.15 and related time series of both variables $x_{1,2}$ is shown in Fig. 12.16.

From all figures it is visible that all EAs has demonstrated almost the same performance. Only a few differences has been recorded thanks to "outliers" (values "far" away from average), probably caused by non-optimal setting of selected algorithm. The problem of finding real optimal setting of used algorithms is quite complex and time consuming process and it was not an objective of this study.

**Table 12.7** Experiment summarization, Lorenz - Lorenz, part 1.

| Algorithm | D1 | D2 | D3 | D4 | D5 | D6 | ES1 | ES2 |
|---|---|---|---|---|---|---|---|---|
| **Cost function evaluations** see Fig. 12.5 | | | | | | | | |
| Minimum | 2820 | 4588 | 4428 | 4240 | 8180 | 5150 | 401 | 237 |
| Average | 9828.24 | 10977.46 | 9386.76 | 10330.74 | 13300.32 | 13088.38 | 3702.85 | 1145.04 |
| Maximum | 15120 | 16020 | 12704 | 15356 | 17748 | 18714 | 11201 | 2055 |
| Total for each algorithm | 982824 | 1097746 | 938676 | 1033074 | 252706 | 1308838 | 370285 | 114504 |
| **Cost Values** see. Fig 12.7 | | | | | | | | |
| Minimum | 27.50 | 27.58 | 27.50 | 27.51 | 27.53 | 27.51 | 18.03 | 22.69 |
| Average | 27.82 | 27.82 | 27.82 | 27.83 | 27.82 | 27.81 | 24.98 | 26.13 |
| Maximum | 27.99 | 27.99 | 27.99 | 27.99 | 27.99 | 27.99 | 27.98 | 27.99 |
| **Parameter *a* setting** see. Fig 12.9 | | | | | | | | |
| Minimum | 9.9969 | 9.9969 | 9.9967 | 9.9968 | 9.9972 | 9.9967 | 9.8399 | 9.9376 |
| Average | 9.9988 | 9.9987 | 9.9987 | 9.9988 | 9.9987 | 9.9988 | 10.002 | 9.999 |
| Maximum | 9.9999 | 9.9999 | 9.9999 | 9.9999 | 9.9999 | 9.9999 | 10.081 | 10.055 |
| **Parameter *d* setting** see. Fig 12.11 | | | | | | | | |
| Minimum | 7.8500 | 7.8476 | 7.8553 | 7.8434 | 7.8911 | 7.8424 | 7.8772 | 7.9012 |
| Average | 7.9464 | 7.9482 | 7.9476 | 7.9430 | 7.9456 | 7.9464 | 11.589 | 9.8122 |
| Maximum | 7.9987 | 7.9992 | 7.9990 | 7.9991 | 7.9917 | 7.9979 | 23.048 | 13.365 |

**Table 12.8** Experiment summarization, Lorenz - Lorenz, part 2.

| Algorithm | G | SA1 | SA2 | S1 | S2 | S3 | S4 |
|---|---|---|---|---|---|---|---|
| **Cost function evaluations** see Fig. 12.5 | | | | | | | |
| Minimum | 32180 | 3844 | 1346 | 1508 | 26338 | 2458 | 4534 |
| Average | 48490.67 | 4972.66 | 4909.23 | 14908.32 | 58855.69 | 7371.88 | 10993.90 |
| Maximum | 50100 | 4984 | 4984 | 22006 | 83430 | 10794 | 11220 |
| Total for each algorithm | 872832 | 497260 | 412375 | 1490832 | 3766764 | 737188 | 1099390 |
| **Cost Values** see. Fig 12.7 | | | | | | | |
| Minimum | 27.78 | 27.94 | 27.69 | 27.50 | 27.53 | 27.53 | 27.65 |
| Average | 28.93 | 32.89 | 33.15 | 27.86 | 27.84 | 27.85 | 31.36 |
| Maximum | 31.48 | 44.65 | 52.80 | 27.99 | 27.99 | 27.99 | 65.48 |
| **Parameter *a* setting** see. Fig 12.9 | | | | | | | |
| Minimum | 9.9839 | 9.9225 | 9.8695 | 9.9966 | 9.9966 | 9.9968 | 9.7364 |
| Average | 9.9946 | 9.9757 | 9.9745 | 9.9985 | 9.9987 | 9.9986 | 9.9799 |
| Maximum | 9.9997 | 9.9997 | 9.9998 | 9.9999 | 9.9999 | 9.9999 | 9.9999 |
| **Parameter *d* setting** see. Fig 12.11 | | | | | | | |
| Minimum | 7.5454 | 7.0813 | 7.2120 | 7.8497 | 7.8472 | 7.8618 | 7.1813 |
| Average | 7.8311 | 7.7000 | 7.6908 | 7.9442 | 7.9383 | 7.9450 | 7.8539 |
| Maximum | 7.9942 | 7.9929 | 7.9989 | 7.9995 | 7.9979 | 7.9997 | 7.9979 |

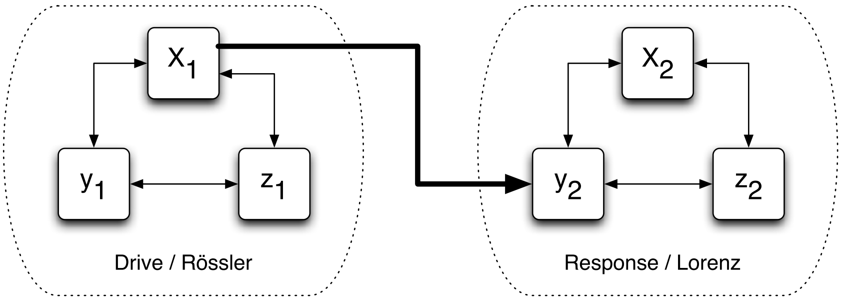


**Fig. 12.17** Schematic of Rössler-Lorenz synchronization. Variable $y_2$ has been directly synchronized.

#### 12.4.4.2 Case study B: Rössler - Lorenz Synchronization

In this case study we have used the selected evolutionary algorithms on synchronization of two different systems (Fig. 12.17), i.e. on synchronization of the Rössler - Lorenz systems (see Fig. 12.18 and Fig. 12.19). Synchronization has been done by the coupling of variables $y_{1,2}(t)$ so that the difference between them has been calculated and multiplied by parameter $c$, see $(c\,(y_1(t) - y_2(t)))$ in eq. (12.2). Typical difference between non-synchronized and synchronized behavior is depicted in Fig. 12.20 - Fig. 12.25. The effect of synchronization on synchronized system is depicted in Fig.12.26 and Fig. 12.27 (compare with Fig. 12.19).

To estimate the complexity of cost function landscape, a series of figures showing dependance on three parameters $a$, $b$ and $c$ in eq. (12.2), has been generated. All three parameters were varied around nominal values referred in the literature. For each parameter change, the behavior of master-slave system has been generated and the cost value calculated. In Fig. 12.28 and 12.29 dependance on various values of parameter $a$ is depicted, in Fig. 12.30 and 12.31 on parameter $b$ and in Fig. 12.32 and 12.33 on parameter $c$. It is clear that the cost function landscape (three dimensional surface (three variables $a$, $b$ and $c$) is in four dimensional space - fourth dimension is cost value) is very complex, nonlinear and almost erratic. Thus the use of evolutionary computation is again acceptable in this case.

Similarly, like in the previous case study, 5 algorithms in 15 versions have been used. Attention, like in the previous case, was focused also on cost value, parameter $a$, $b$ setting as well as the setting of the coupling parameter $c$. All three parameters has been estimated simultaneously, i.e. individual dimension was 3. All these results are reported in Figs. 12.34 - 12.39. Together with minimal, maximal and average values, the average vaue of all algorithms (horizontal line) is also depicted. Fig. 12.37 show the same for the cost value related to used estimated setting (i.e. how much differs behavior of both systems), Fig. 12.34 show how well parameter $a$ has been estimated and the same is done in Fig. 12.35 for parameter $b$ and Fig. 12.36 for parameter $c$. The number of cost function evaluations needed for each algorithm is reported in Fig. 12.38 and 12.39. From all figures, it is visible that all EAs have

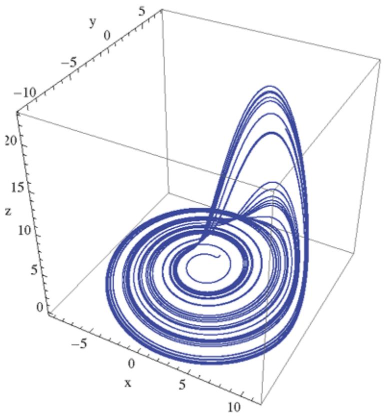


**Fig. 12.18** Rössler attractor (master), see. eq. (12.2)

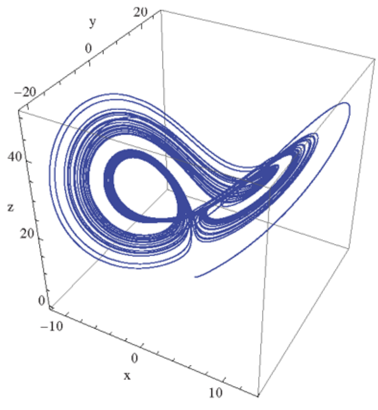


**Fig. 12.19** Lorenz attractor (slave), see. eq. (12.2)

demonstrated almost the same performance and generally works as well as in the previous case.

After evolution, it was discovered that EAs had found different setting than is reported in literature, i.e. $\{a,b,c\} = \{0.13089, 3.35025, 69.9999\}$ (cost value was 7.69753) instead of $\{a,b,c\} = \{3, 26.5, 69\}$ as used in Fig. 12.20 - Fig. 12.25. The behavior of evolutionary synchronized R-L system is depicted in Fig. 12.41 - Fig. 12.44. From figures is clearly visible that variable $y_2(t)$ has been synchronized very well while another state variables were almost supressed. Fig. 12.44 shows the difference between $y_1(t)$ and $y_2(t)$. When comparing evolutionary setting of parameter $a$ with Fig. 12.28, then it is clear that EAs have found different settings (extreme on function) than is visible in this figure (around a = 1). Value $a = 0.13089$ signalize, that there is more deeper extreme, which is depicted in Fig. 12.40. It is located almost on the left side. Despite the fact that its location is on the border of the searched space and there are another extremes (including many of chaotic) EAs has successfully found this setting repeatedly.

## 12.5 Conclusion

In this chapter we have studied the possibility on synchronization with evolutionary estimation of coupling parameters as well as internal parameters of selected chaotic systems. Two kind of synchronized systems were used: Lorenz - Lorenz (L-L) and Rössler - Lorenz (R-L) systems. Attention has been paid on synchronization of variable $x_2(t)$ for L-L synchronization and state variable $y_2(t)$ in the case of R-L synchronization. For the comparative study optimization algorithms such as Differential Evolution (DE) [10], Self Organizing Migrating Algorithm (SOMA) [16], Genetic Algorithms (GA) [5], Simulated Annealing (SA) [6], [3] and

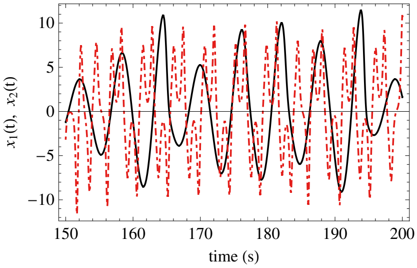


**Fig. 12.20** Rössler-Lorenz system for variables $x_1(t)$ and $x_2(t)$ (eq. (12.2)) with c = 0 (not synchronized)...

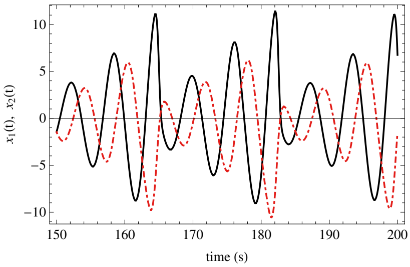


**Fig. 12.21** ... and under synchronization (c = 69.4458). Lorenz system is dotted light red curve.

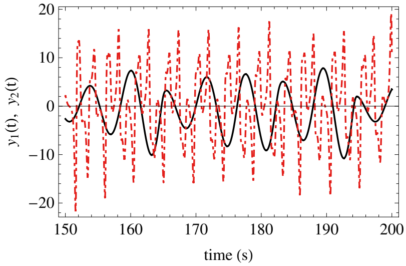


**Fig. 12.22** Rössler-Lorenz system for variables $y_1(t)$ and $y_2(t)$ (eq. (12.2)) with c = 0 (not synchronized)...

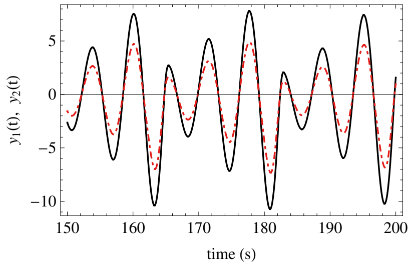


**Fig. 12.23** ... and under synchronization (c = 69.4458). Lorenz system is dotted light red curve.

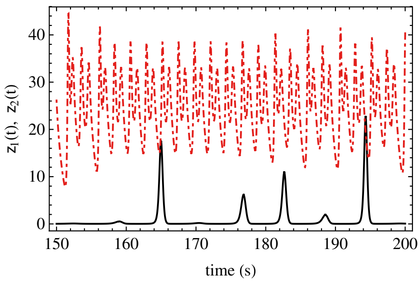


**Fig. 12.24** Rössler-Lorenz system for variables $z_1(t)$ and $z_2(t)$ (eq. (12.2)) with c = 0 (not synchronized)...

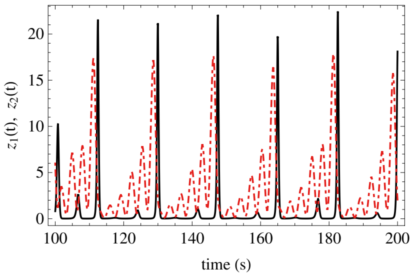


**Fig. 12.25** ... and under synchronization (c = 69.4458). Lorenz system is dotted light red curve.

Evolutionary Strategies (ES) [1] were selected. As a conclusion the following statements are presented:

- **Algorithm performance.** Based on all informations and figures reported in this chapter, it can be stated, that all EAs showed good performance. In both

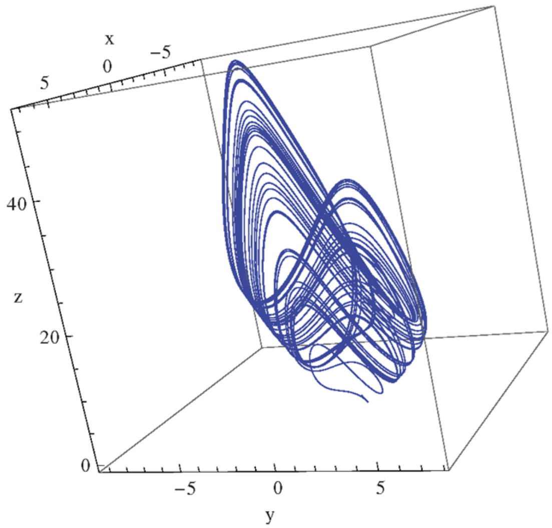


**Fig. 12.26** Synchronized Lorenz attractor

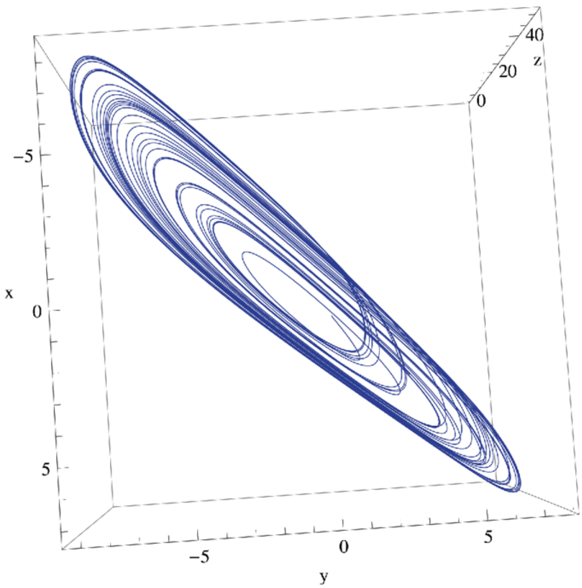


**Fig. 12.27** Synchronized Lorenz attractor, another view.

case studies the averages of each algorithm were "almost" on the same value (average of all) which is depicted by a horizontal line. For L-L synchronization see Fig. 12.5 - 12.16; for R-L synchronization see Fig. 12.34 - 12.39. Values far from average can be explained by the fact that **a)** algorithms are

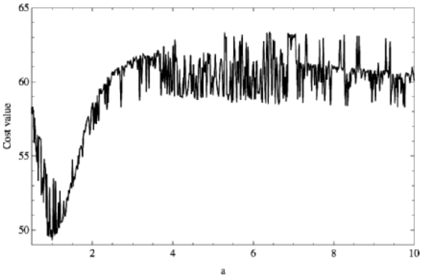


**Fig. 12.28** Dependance of parameter *a* in Rössler-Lorenz system (eq. (12.2))...

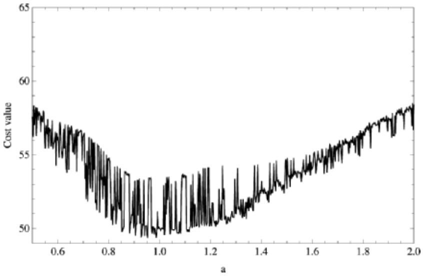


**Fig. 12.29** ... and its detail.

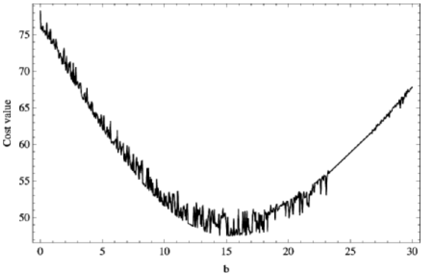


**Fig. 12.30** Dependance of parameter *b* in Rössler-Lorenz system (eq. (12.2))...

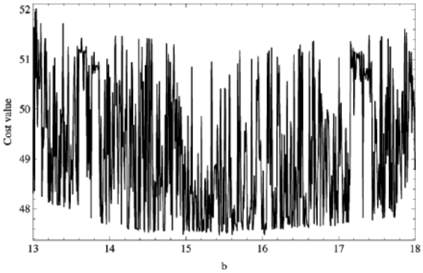


**Fig. 12.31** ... and its detail.

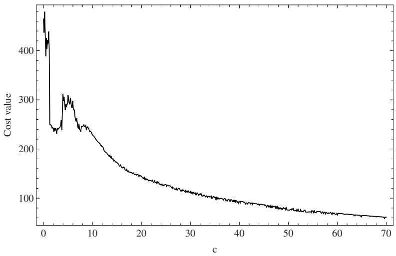


**Fig. 12.32** Dependance of parameter *c* in Rössler-Lorenz system (eq. (12.2))...

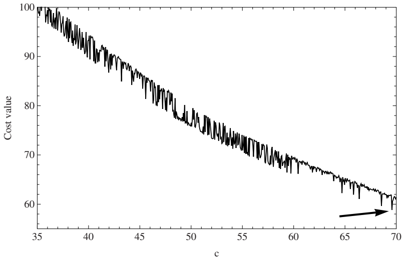


**Fig. 12.33** ... and its detail.

of evolutionary (pseudorandom, etc...) nature; **b)** settings of algorithm control parameters is not optimal; **c)** algorithms need longer time to get better values. However the most simplest explanation is that all those extreme values are just "outliers", i.e. only a few values estimated not so optimally, due to randomness of evolutionary algorithms.Two randomly selected histograms showing outliers are depicted in Fig. 12.45 and 12.46. From Fig. 12.45 it is clearly visible that cost function evaluation (approx. 30000, see Fig. 12.38, algorithm S4)

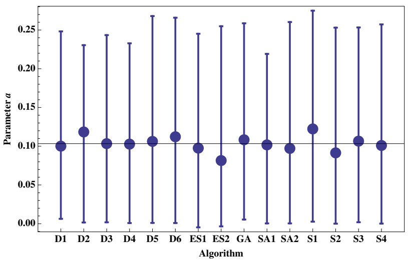


**Fig. 12.34** Estimation of parameter *a* ...

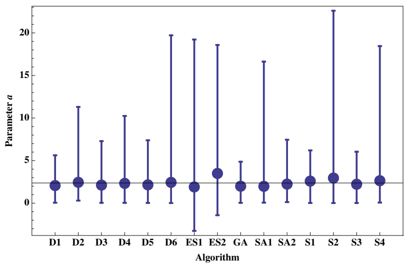


**Fig. 12.35** ... and of parameter *b* .

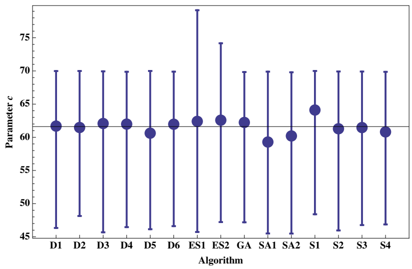


**Fig. 12.36** Estimation of parameter *c* ...

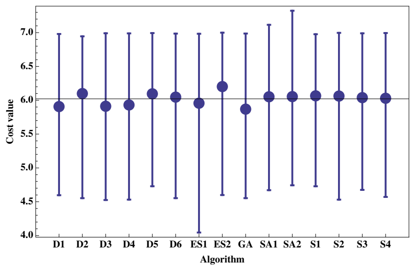


**Fig. 12.37** ... and summarization of cost values.

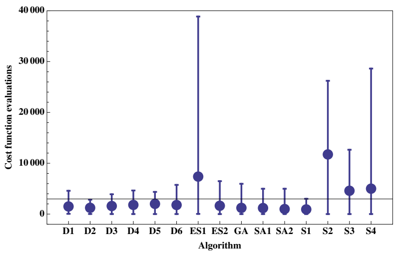


**Fig. 12.38** Cost function evaluations...

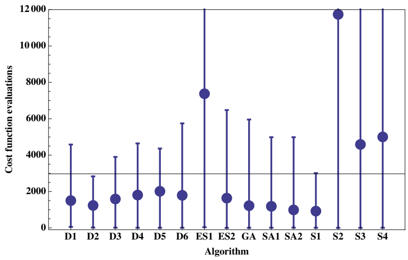


**Fig. 12.39** ... and detail view.

is really not a rule, but an outlier. The same is visible in Fig. 12.46, compared with Fig. 12.11, algorithm ES1.

- **Statistical robustness.** In the frame of this case study 30 ($2\times15$) simulations has been done, and each has been 100 $\times$ repeated. Thus the total number of simulations was 3000 simulations equal to 14975294 / 4387854 (L-L / R-L) cost function evaluations, see for more detailed description see Table 12.7 and 12.8 (L-L system) and Tables 12.9 - 12.10 (R-L system). All those calculations led to positive results, i.e. systems have been synchronised succesfully. All calculations has been done on a grid computer which consist of two special Apple servers: 16 XServers, each 2x2 GHz Intel Xeon, 1 GB RAM, 80 GB

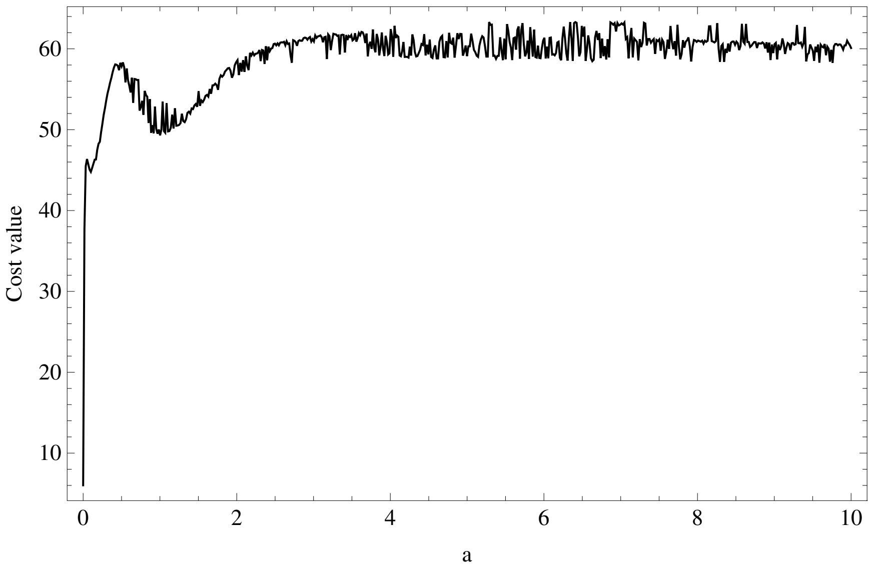


**Fig. 12.40** Total view of cost value dependance on parameter *a*

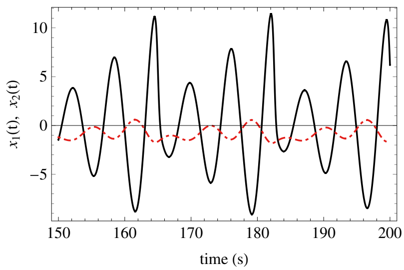


**Fig. 12.41** Synchronization of $x_{1,2}(t)$ according to the best estimated setting.

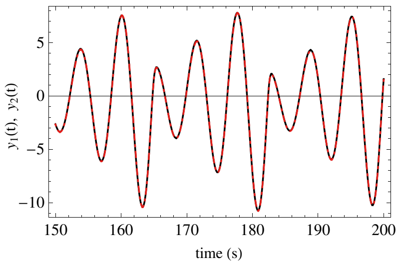


**Fig. 12.42** Synchronization of $y_{1,2}(t)$ according to the best estimated setting.

HD i.e. 64 CPUs) and 7 Apple Minimacs CoreDuo i.e. number of accessible CPUs is 14. In total 78 CPUs there was available for computation.

- **Results divergence.** As mentioned before, results in both case studies are slightly different. Obtained averages are mostly on the same level, however their divergence is for algorithms like ES and SA is different. As mentioned before, probably better setting should be applied. On the other side, there is so called "No Free Lunch" theorem, see [14], according to which universal algorithm does not exist, i.e. some of selected evolutionary algorithm is not much suitable for this task. But this is probably not a case of the algorithms used here.

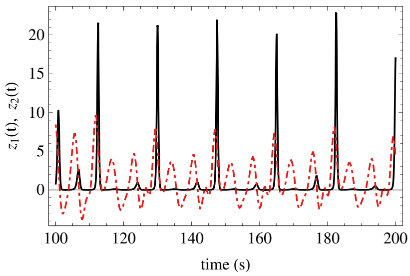


Fig. 12.43 Synchronization of $z_{1,2}(t)$ according to the best estimated setting.

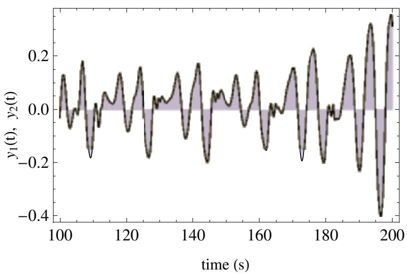


Fig. 12.44 Synchronization: difference between $y_1(t) - y_2(t)$.

**Table 12.9** Experiment summarization, Rössler - Lorenz, part 1.

| Algorithm | D1 | D2 | D3 | D4 | D5 | D6 | ES1 | ES2 |
|---|---|---|---|---|---|---|---|---|
| **Cost function evaluations** | | | | | | | | |
| see Fig. 12.5 | | | | | | | | |
| Minimum | 54 | 19 | 11 | 7 | 6 | 8 | 36 | 1 |
| Average | 1500 | 1234 | 1589 | 1806 | 2013 | 1791 | 7376 | 1637 |
| Maximum | 4582 | 2832 | 3900 | 4644 | 4364 | 5744 | 38861 | 6477 |
| Total for each algorithm | 149963 | 56767 | 158851 | 180646 | 201257 | 179055 | 737582 | 158797 |
| **Cost Values** | | | | | | | | |
| see. Fig 12.37 | | | | | | | | |
| Minimum | 4.5966 | 4.5531 | 4.5254 | 4.5314 | 4.7290 | 4.5533 | 4.0446 | 4.5991 |
| Average | 5.9067 | 6.0994 | 5.9115 | 5.9307 | 6.0956 | 6.0467 | 5.9568 | 6.2024 |
| Maximum | 6.9800 | 6.9453 | 6.9906 | 6.9888 | 6.9933 | 6.9855 | 6.9829 | 6.9992 |
| **Parameter *a* setting** | | | | | | | | |
| see. Fig 12.34 | | | | | | | | |
| Minimum | 0.0063 | 0.0015 | 0.0018 | 0.0008 | 0.001 | 0.0008 | -0.004 | -0.003 |
| Average | 0.1001 | 0.1184 | 0.1033 | 0.1027 | 0.1062 | 0.1122 | 0.0975 | 0.0815 |
| Maximum | 0.2482 | 0.2304 | 0.2433 | 0.2328 | 0.2679 | 0.2658 | 0.2451 | 0.2548 |
| **Parameter *b* setting** | | | | | | | | |
| see. Fig 12.35 | | | | | | | | |
| Minimum | 0.0534 | 0.3000 | 0.0416 | 0.0391 | 0.0313 | 0.0104 | -3.257 | -1.413 |
| Average | 2.0676 | 2.4464 | 2.1227 | 2.3213 | 2.1520 | 2.4390 | 1.9063 | 3.4911 |
| Maximum | 5.6232 | 11.309 | 7.2874 | 10.248 | 7.3880 | 19.717 | 19.221 | 18.584 |
| **Parameter *c* setting** | | | | | | | | |
| see. Fig 12.36 | | | | | | | | |
| Minimum | 46.333 | 48.138 | 45.681 | 46.458 | 46.144 | 46.607 | 45.726 | 47.220 |
| Average | 61.686 | 61.474 | 62.077 | 61.985 | 60.619 | 61.969 | 62.397 | 62.576 |
| Maximum | 69.984 | 69.990 | 69.941 | 69.873 | 69.995 | 69.890 | 79.150 | 74.171 |

**Table 12.10** Experiment summarization, Rössler - Lorenz, part 2.

| Algorithm | G | SA1 | SA2 | S1 | S2 | S3 | S4 |
|---|---|---|---|---|---|---|---|
| **Cost function evaluations** | | | | | | | |
| see Fig. 12.5 | | | | | | | |
| Minimum | 5 | 8 | 12 | 9 | 3 | 3 | 7 |
| Average | 1222 | 1188 | 989 | 923 | 11738 | 4588 | 5002 |
| Maximum | 5957 | 4984 | 4984 | 3008 | 26220 | 12656 | 28628 |
| Total for each algorithm | 122169 | 118765 | 98879 | 92258 | 1173843 | 458846 | 500176 |
| **Cost Values** | | | | | | | |
| see. Fig 12.37 | | | | | | | |
| Minimum | 4.5533 | 4.6710 | 4.7431 | 4.7287 | 4.5311 | 4.6754 | 4.5709 |
| Average | 5.8689 | 6.0519 | 6.0542 | 6.0657 | 6.0624 | 6.0390 | 6.0297 |
| Maximum | 6.9866 | 7.1153 | 7.3234 | 6.9764 | 6.9965 | 6.9898 | 6.9938 |
| **Parameter *a* setting** | | | | | | | |
| see. Fig 12.34 | | | | | | | |
| Minimum | 0.0053 | 0.0004 | 0.0004 | 0.0026 | 0. | 0.0019 | 0.0001 |
| Average | 0.1083 | 0.1017 | 0.0971 | 0.1223 | 0.0914 | 0.1065 | 0.1009 |
| Maximum | 0.2586 | 0.2191 | 0.2602 | 0.2749 | 0.253 | 0.2533 | 0.2572 |
| **Parameter *b* setting** | | | | | | | |
| see. Fig 12.35 | | | | | | | |
| Minimum | 0.0403 | 0.0617 | 0.1234 | 0.0261 | 0.0157 | 0.0167 | 0.0674 |
| Average | 1.9918 | 1.9770 | 2.2398 | 2.5879 | 2.9517 | 2.2215 | 2.637 |
| Maximum | 4.8671 | 16.629 | 7.4493 | 6.2005 | 22.604 | 6.0407 | 18.449 |
| **Parameter *c* setting** | | | | | | | |
| see. Fig 12.36 | | | | | | | |
| Minimum | 47.188 | 45.505 | 45.489 | 48.398 | 45.961 | 46.766 | 46.868 |
| Average | 62.236 | 59.287 | 60.202 | 64.109 | 61.287 | 61.465 | 60.811 |
| Maximum | 69.834 | 69.892 | 69.797 | 69.996 | 69.920 | 69.911 | 69.864 |

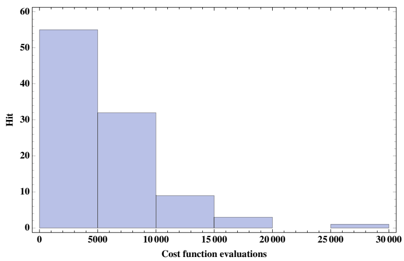


**Fig. 12.45** Histogram of cost function evaluations; R-L system.

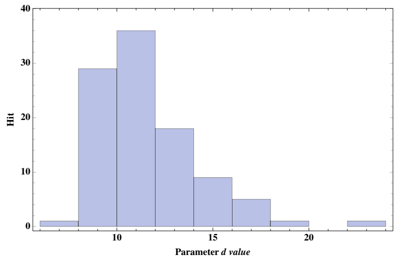


**Fig. 12.46** Histogram for parameter $d$; L-L system.

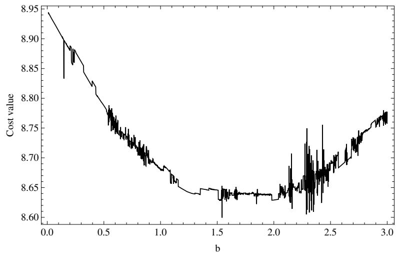


**Fig. 12.47** Dependance of cost value on the parameter $b$ for $a = 0.1$ and $c = 69.4458$ ...

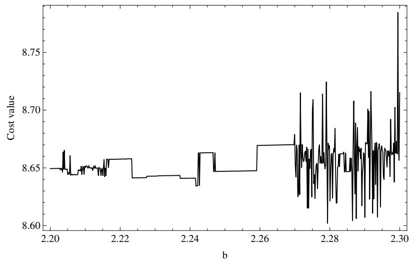


**Fig. 12.48** ... and detail view.

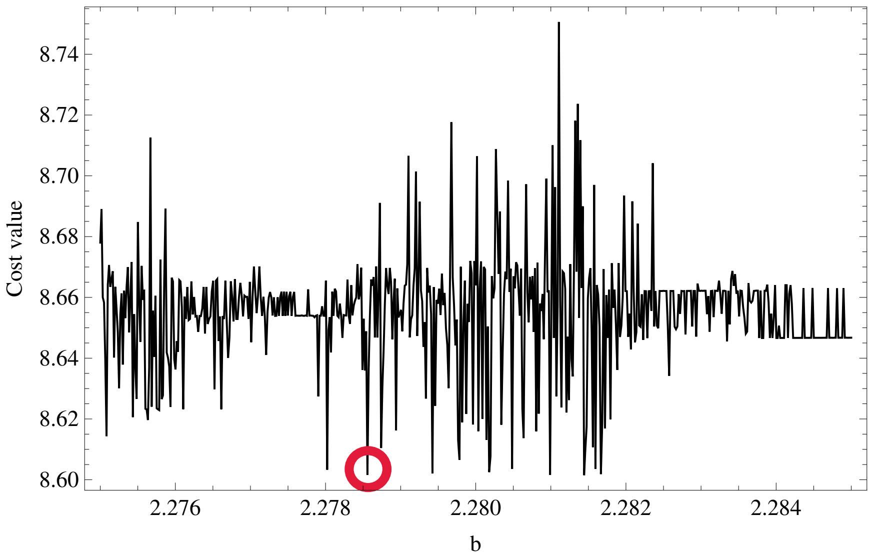


**Fig. 12.49** Dependance of cost value on the parameter $b$ in very tiny region. Global extreme is marked by red (light grey) circle. Its estimation is approximate, because this picture (and dataset used for) has been calculated with certain, limiting accuracy.

- **Algorithm settings.** Algorithm setting has been established according to heuristically known setting for each algorithm as well as on our own experiences. We would like to remind that it does not mean that there is no better settings for any of used algorithms. Main aim was not focused on speed of used algorithms but on successful synchronization.
- **Synchronization settings.** During all simulations different setting for estimated parameters has been found, comparing with literature and our heuristically obtained setting ($a = 3$, for $b = 26.5$ and $c = 70$ for R-L system). Compare Fig. 12.21 with Fig. 12.41, Fig. 12.23 with Fig. 12.42 and Fig. 12.25 with Fig. 12.43. Thus EAs have found better setting. Differences of

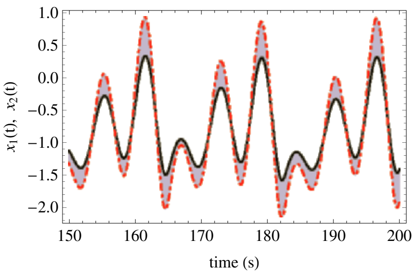


**Fig. 12.50** Difference between $x_{2-best}(t)$ (solid line) and $x_{2-worst}(t)$ (dotted line).

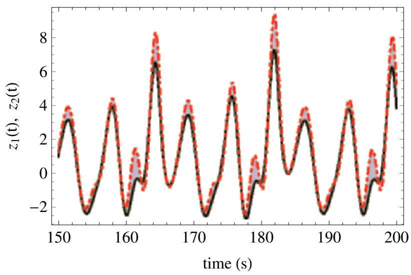


**Fig. 12.51** Difference between $z_{2-best}(t)$ (solid line) and $z_{2-worst}(t)$ (dotted line).

behavior between synchronizing and synchronized variables are also depicted in Fig. 12.13, 12.14, 12.15 and Fig. 12.16.

- **Problem complexity.** Problem complexity, represented by the cost function landscape, is depicted in Fig. 12.3, Fig. 12.4, Fig. 12.28 - 12.33 and Fig. 12.40. It is clearly visible that cost function landscape is very erratic, nonlinear and multimodal. Another view on its complexity is given in Fig. 12.47 - 12.49. Compare Fig. 12.47 with Fig. 12.30. The extreme of parameter $b$ dependance has moved for different $a$ and $c$ from positions (approx.) 15 to 2.2785. In other words, optimal setting cannot be found simply by visual checking of each parameter dependance, due to their mutual influence. Thus naturally, problem of synchronization, is suitable for evolutionary algorithms.
- **Different results.** When comparing R-L and L-L systems, then one can see, that there is visible difference in the range of the parameter value estimation, as well as in cost function evaluations and cost values. It is obvious because **a)** both systems are different, **b)** in both systems is estimated by different number of parameters, **c)** cost values are differently calculated, see eq. (12.3) and (12.4). For L-L system the cost value (eq. (12.3)) is calculated in interval $t \in [0,100]$ like difference between all three variables, while in R-L system (eq. (12.4)) for only $y_{1,2}(t)$ variable and $t \in [0,200]$. Another important point is, that because in R-L system $y_1(t) - y_2(t)-$ is minimized only, then the variables $x_2(t)$ and $z_2(t)$ were not pressed by evolution into exact values. Variables $x_2(t)$ and $z_2(t)$ has thus a "freedom" to reach different values. It is visible in Fig. 12.50 and 12.51. A difference between the best and the worst estimated synchronization is depicted there for R-L system and variables $x_2(t)$ and $z_2(t)$. Gray area represent "space" for all possible values of both variables.
- **Estimated parameters.** In the synchronization experiments coupling parameters are usually estimated. In this numerical study, the internal parameters of chaotic systems has been selected for optimization are also estimated. It can reflect, for certain systems, situation that some of physical parameters (pressure, current, ...) can be remoted by an external observer. Because the parameters has some certain physical meaning (they are not abstract

numbers), one has to carefully work with them and sometimes this is simply not allowed. We did not follow this idea in numerical studies here.

- **Negative values.** In the reported results it can easily be found that for some values, negative values are returned, especially for evolutionary strategies. It was caused by the fact that in evolutionary strategies procedure "watching" has not been applied whether evolutionary process overstepped the allowable search space or not. Thus, evolutionary strategies, has also searched a little bit behind of searchable space borders.

According to the author's opinion, this is a promising area of evolutionary algorithms use. Experiments designed, numerically simulated and reported here were one of the most simplest. Based on results obtained here and also in other chapters, it is possible to say that EAs are viable and should also work on more complicated cases of synchronization, for example the CML systems.

**Acknowledgements.** This work was supported by grant No. MSM 7088352101 of the Ministry of Education of the Czech Republic and by grants of the Grant Agency of the Czech Republic GACR 102/09/1680.

## References

1. Beyer, H.: Theory of Evolution Strategies. Springer, New York (2001)
2. Brown, R., Rulkov, N., Tracy, E.: Modeling and synchronization chaotic system from time-series data. Phys. Rev. E 49, 3784 (1994)
3. Cerny, V.: Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm. J. Opt. Theory Appl. 45(1), 41–51 (1985)
4. Gonzalez-Miranda, J.: Synchronization and Control of Chaos. An introduction for scientists and engineers. Imperial College Press (2004)
5. Holland, J.: Adaptation in Natural and Artificial Systems. Univ. Michigan Press, Ann Arbor (1975)
6. Kirkpatrick, S., Gelatt, C., Vecchi, M.: Optimization by simulated annealing. Science 220(4598), 671–680 (1983)
7. Nolle, L., Goodyear, A., Hopgood, A., Picton, P., Braithwaite, N.StJ.: On Step Width Adaptation in Simulated Annealing for Continuous Parameter Optimisation. In: Reusch, B. (ed.) Fuzzy Days 2001. LNCS, vol. 2206, pp. 589–598. Springer, Heidelberg (2001)
8. Nolle, L., Zelinka, I., Hopgood, A., Goodyear, A.: Comparison of an self organizing migration algorithm with simulated annealing and differential evolution for automated waveform tuning. Adv. Eng. Software 36(10), 645–653 (2005)
9. Pikovsky, A., Rosemblum, M., Kurths, J.: Synchronization: A Universal Concept in Nonlinear Sciences. Cambridge University Press, Cambridge (2001)
10. Price, K.: An Introduction to Differential Evolution. In: Corne, D., Dorigo, M., Glover, F. (eds.) New Ideas in Optimization, pp. 79–108. McGraw-Hill, New York (1999)
11. Rulkov, N., Sushchik, M.: Robustness of synchronized chaotic oscillations. Int. J. Bifurcat Chaos Appl. Sci. Eng. 7, 625 (1997)
12. Schuster, H. (ed.): Handbook of Chaos Control. Wiley-VCH, New York (2007)
13. Sushchik, M., Rulkov, N., Tsimring, L., Abarbanel, H.: Generalized synchronization of chaos in directionally coupled chaotic systems. In: Proceedings of Intl. Symp. on Nonlinear Theory and Appl., vol. 2, pp. 949–952. IEEE, Los Alamitos (1995)

14. Wolpert, D., Macready, W.: No Free Lunch Theorems for Search, Technical Report SFI-TR-95-02-010, Santa Fe Institute (1995)
15. Wolpert, D., Macready, W.: No Free Lunch Theorems for Optimization. IEEE Trans. Evol. Comput. 1(67) (1997)
16. Zelinka, I.: SOMA – Self Organizing Migrating Algorithm. In: Babu, B., Onwubolu, G. (eds.) New Optimization Techniques in Engineering, pp. 167–218. Springer, New York (2004)
17. Zelinka, I.: Investigation on Evolutionary Deterministic Chaos Control. In: IFAC, Prague (2005a)
18. Zelinka, I.: Investigation on Evolutionary Deterministic Chaos Control – Extended Study. In: 19th International Conference on Simulation and Modeling (ECMS 2005), Riga, Latvia, June 1-4 (2005b)
19. Zelinka, I.: Real-time deterministic chaos control by means of selected evolutionary algorithms. Eng. Appl. Artif. Intell (2008), doi:10.1016/j.engappai.2008.07.008
20. Zelinka, I., Nolle, L.: Plasma reactor optimizing using differential evolution. In: Price, K., Lampinen, J., Storn, R. (eds.) Differential Evolution: A Practical Approach to Global Optimization, pp. 499–512. Springer, New York (2006)
21. Zelinka, I., Senkerik, R., Navratil, E.: Investigation on Evolutionary Optimitazion of Chaos Control. Chaos, Solitons & Fractals (2007), doi:10.1016/j.chaos.2007.07.045