

Pareto Optimal Based Evolutionary Approach for Solving Multi-Objective Facility Layout Problem

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Abstract. Over the years, various evolutionary approaches have been proposed in efforts to solve the facility layout problem (FLP). Unfortunately, most of these approaches are limited to a single objective only, and often fail to meet the requirements for real-world applications. To date, there are only a few multi-objective FLP approaches have been proposed. However, they are implemented using weighted sum method and inherit the customary problems of this method. In this paper, we propose an evolutionary approach for solving multi-objective FLP using multi-objective genetic algorithm that presents the layout as a set of Pareto optimal solutions optimizing both quantitative and qualitative objective simultaneously. Experimental results obtained with the proposed algorithm on test problems taken from the literature are promising.

Keywords: Multi-objective facility layout problem, Pareto optimal solutions, Quantitative objective, Qualitative objective.

1 Introduction

FLPs deal with arranging a given number of facilities (departments) on the factory floor of a manufacturing system to meet one or more objectives. These objectives may include minimizing the total cost of transporting materials (material handling costs) or maximizing adjacency requirement between the facilities. In essence, FLP can be considered as a searching or optimization problem, where the goal is to find the best possible layout. Traditionally FLP has been presented as a Quadratic Assignment problem (QAP) to find the best assignment of n facilities to m locations, where the number of facilities and locations must be equal. It is well known that QAP is NP-complete category due to the combinatorial function involved and cannot be solved for large layout problems. Despite its popularity, QAP is a difficult problem to solve using traditional optimal algorithms [1].

During the last decades many different methods have been developed to solve FLP with genetic algorithms (GAs). However, most of the researches in this field are generally concerned with a single objective, either qualitative or quantitative feature of the layout. By contrast, practical layout problems are multi-objective by nature and they require the decision makers to consider a number of criteria involving both quantitative and qualitative objectives before arriving at any conclusion. A solution that is optimal with respect to a given criterion might be a poor candidate for some other

criteria. Hence, the trade-offs involved in considering several different criteria provide useful insights for the decision makers. Surprisingly, the research in this important field has been scarce when compared to the research in single criterion.

Although dealing with multiple objectives has received attention over the last few years [2,3,4,5], to our knowledge these approaches are still considered limited, and mostly dominated by the unrealistic aggregation of preferences method, popularly known as weighted sum method. In this method, multiple objectives are combined into a single scalar objective using weighted coefficients. However, there are several disadvantages of this technique [6]. First, as the relative weights of the objectives are not exactly known in advance and cannot be pre-determined by the users, the objective function that has the largest variance value may dominate the multi-objective evaluation. As a result, inferior non-dominated solutions with poor diversity will be produced. Second, the user always has to specify the weight values for functions and sometimes this will not have any relationship with the importance of the objectives. Third, a single solution is obtained at one time. If we are interested in obtaining a set of feasible solutions, it has to be run several times. This also, is not a warranty that the solutions obtained in different runs are different. More importantly, since the selection of objective weights is critical in designing layout having multiple objectives, the objective weights therefore play an important role in the design process. In practice, it is selected randomly by the layout designer based on his/her past experience that restricts the designing process completely designer dependent and thus the layout varies from designer to designer. To overcome such difficulties, Pareto-based evolutionary optimization has become an alternative to classical weighted sum method. Goldberg [7] first proposed this approach and it explicitly uses Pareto dominance to determine the reproduction probability of each individual.

This paper presents a multi-objective evolutionary approach for FLP to find a set of Pareto optimal layout solutions optimizing both quantitative and qualitative objectives. In this work, we have used the Non-dominated Sorting Genetic Algorithm 2 (NSGA 2) proposed by Deb et. al. [8]. The goal of this proposed multi-objective FLP approach is to find as many different potential layouts as possible, each of which is near optimal and is not dominated by consideration of a particular objective. In an attempt to address multiple objectives simultaneously in this work, we apply material handling costs and closeness relationship among various departments as quantitative and qualitative objective respectively.

The paper is organized as follows. Section 2 explains the concept of multi-objective optimization and Pareto-optimal solution. Section 3 describes the related works on FLP. Section 4 justifies the necessities of multi-objective evolutionary FLP algorithm. The implementation of multi-objective FLP algorithm is presented in Section 5. Experimental results are presented and analyzed in Section 6, followed by the conclusions in the final section.

2 Multi-Objective Optimization and Pareto Optimal Solution

In the world around us, there are few problems concerned with a single value or objective. Instead, most problems involve multiple objectives and constraints that often conflict with each other. For such multi-objective optimization problems (MOOP),

these conflicts have to be met or optimized before any adequate solution is reached. However, it is rarely the case that a single solution can simultaneously satisfy all the existing objectives. Therefore, when dealing with MOOPs, we normally look for the trade-offs rather than a single solution. In order to generate these trade-off solutions, an old notion of optimality called Pareto-optimum [9] is normally adopted. In a MOOP, a solution $x^{(1)}$ is said to dominate the other solution $x^{(2)}$, if both the following conditions are true: (1) The solution $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives, and (2) the solution $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective.

For a given finite set of solutions, we need to perform pair-wise comparisons to find out which solutions dominate and which are dominated by each other. From these comparisons, we can find a subset of the finite set of solutions such that, any two solutions of which do not dominate each other and all the other solutions of the finite set are dominated by one or more members of this subset. This subset is called the non-dominated set for the given set of solutions. Unfortunately the use of this concept almost always gives not a single solution but a set of solutions, which is called the Pareto-optimal set.

3 Related Works

The FLP has been extensively studied over the last few decades and a wide variety of approaches have been proposed. Traditionally, these approaches have been divided into two categories [10]: optimal and suboptimal. The optimal methods such as the branch-and-bound and cutting plane algorithm have been successfully applied to FLP when the number of facilities is less than 15. However, as the number of departments is larger than 15, QAP has been validated to be an NP-complete problem and its computational time is exponentially increased [1].

FLP is one of the truly difficult ill-structured, multi-criteria and combinatorial optimization problems. In recent years, a lot of sub-optimal and intelligent techniques have been developed to cope with this type of problems. Most of these approaches belong to heuristic ones such as simulated annealing [11], tabu search [12], and GAs [13, 14]. During the past three decades, numerous heuristic methods have been developed to obtain some good, rather than optimal, solutions for layout problems. However, Generally speaking, GAs outperform other heuristic methods [1]. GAs have been widely implemented to solve combinatorial optimization problems and are considered to be a robust approach by accompanying them with artificial intelligence [9]. Comprehensive surveys of the different approaches to FLPs are found in [10,15].

4 Importance of Multi-Objective FLP

Historically, FLPs have been solved only for one goal, either quantitative or qualitative aspect of the layout. Quantitative approaches involve primarily the minimization of material handling costs between various departments. More specifically, it tries to minimize the sum of distance among all facilities multiplied by the corresponding flows. Qualitative approaches used the closeness rating scores to indicate the desired relative "closeness" requirement for two departments to be next to each other. Here

the goal is to maximize the total closeness ratings among all departments. The closeness ratings are; A (absolutely necessary), E (essentially important), I (important), O (ordinary), U (un-important) and X (undesirable), to indicate the respective degrees of necessity that any two given departments be located close together.

In real-world FLPs, it is often necessary to optimize both quantitative and qualitative criteria simultaneously. In general, the minimization of the total material handling costs is often used as the optimization criterion in FLP. However, the closeness rating, hazardous movement or safety, and the like are also the important criteria in FLP. Many researchers have questioned the appropriateness of selecting a single-criterion objective to solve FLP because qualitative and quantitative approaches each have advantages and disadvantages [4]. The major limitations on quantitative approaches are that they consider only relationships that can be quantified and do not consider any qualitative factors. The shortcoming of qualitative approaches is their strong assumption that all qualitative factors can be aggregated into one criterion. In essence, FLPs fall into the category of a MOOP.

Accordingly, it is desirable to generate many near-optimal layouts considering multiple objectives according to the requirements of the production order or customer demand. Then, the production manager can selectively choose the most demanding one among all of the generated layouts for specific order or customer demand. On the other hand, if multiple objectives conflict with each other, then the production manager does not need to omit any required objective. Based on the principle of multi-objective optimization, obtaining an optimal solution that satisfies all of the objectives is almost impossible. However, it is desirable to obtain as many different Pareto-optimal layout solutions as possible, which should be non-dominated, converged to, and diverse along the Pareto-optimal front with respect to these multiple criteria.

5 The Proposed Pareto Optimal Based Approach

5.1 Chromosome Representation

In this study, a form of direct representation for strings is used. The solution is represented as a string of integers of length n , where n is the number of facilities. The integers denote the facilities and their positions in the string denote the positions of the facilities in the layout. For example, the following assignment of the 8/8 problem

$$\begin{array}{cccc} 7 & 6 & 5 & 3 \\ 8 & 4 & 1 & 2 \end{array}$$

will be represented by the solution string 7 6 5 3 8 4 1 2.

5.2 Fitness Evaluation and Selection Scheme

As discussed early, there are two approaches in facility layout algorithms: minimize the distance-based objective function value (quantitative approach), and maximize the adjacency-based objective function (qualitative approach). In this work, we separately utilized both of these as objectives. The first fitness function, total material handling cost, is based on quantitative model. This function is subject to minimization, and measured as

$$Z_x = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n A_{ijkl} X_{ij} X_{kl} \tag{1}$$

subject to

$$\sum_{i=1}^n X_{ij} = 1; j = 1, \dots, n \quad , \quad \sum_{j=1}^n X_{ij} = 1; i = 1, \dots, n \quad , \quad \text{and} \quad X_{ij} = 0 \text{ or } 1$$

providing that

$$X_{ij} = \begin{cases} 1, & \text{if facility } i \text{ is assigned to location } j \\ 0, & \text{otherwise} \end{cases}$$

$$X_{kl} = \begin{cases} 1, & \text{if facility } k \text{ is assigned to location } l \\ 0, & \text{otherwise} \end{cases}$$

$$A_{ijkl} = \begin{cases} f_{ik} d_{jl} & \text{if } i \neq k \text{ or } j \neq l \\ f_{ii} d_{jj} + c_{ij} & \text{if } i = k \text{ and } j = l \end{cases}$$

where

c_{ij} = cost of assigning facility i to location j

d_{jl} = distance from location j to location l (rectilinear distance)

f_{ik} = material flow from facility i to facility k .

The second fitness function, the closeness rating score, is based on qualitative model. This function is subject to maximization, and measured as

$$Z_y = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n W_{ijkl} X_{ij} X_{kl} \tag{2}$$

subject to

$$\sum_{i=1}^n X_{ij} = 1; j = 1, \dots, n \quad , \quad \sum_{j=1}^n X_{ij} = 1; i = 1, \dots, n \quad , \quad \text{and} \quad X_{ij} = 0 \text{ or } 1$$

where

$$W_{ijkl} = \begin{cases} r_{ik}, & \text{if locations } j \text{ and } l \text{ are neighbors} \\ 0, & \text{otherwise} \end{cases}$$

r_{ik} = closeness ranking value when facility i and k are neighbors with common boundary.

The numerical values used for the closeness ranking values are: A=6, E=5, I=4, O=3, U=2, and X=1. After performing all these operations, a non-dominated sorting strategy employing the crowding-distance assignment [8] is performed to achieve the elitisms for the next generation.

5.3 Crossover and Mutation Operation

The chromosome representation in this multi-objective FLP is different from that of the conventional GA. As a result, the direct application of the traditional genetic operators may create an illegal solution. Hence, some problem-specific genetic operators are required. We follow the concept described by Suresh et. al [16] for crossover operation. It may be noted that this crossover maintains partial structure of the parents

to a large extent than by the existing crossover operations such as PMX, OX and CX [16]. Mutation provides and maintains diversity in a population. In our approach, two genes are picked up randomly and then they exchange their positions. As a result, the resultant chromosomes are legal and no repair is required.

6 Experimental Results and Discussion

Until now, almost all FLP algorithms try to optimize single criteria only (mainly minimizing the material handling cost). Though few multi-objective FLP approaches can be found in the literature, all of these existing approaches employed weighted sum approach. And to our knowledge, there are no published paper for the Pareto optimal solution. Therefore, to evaluate our proposed algorithm, first we compared the material handling costs obtained by our approach with the existing single objective approaches to justify its capability to optimize this cost. Then we demonstrated its performance as a multi-objective evolutionary FLP algorithm by optimizing material handling cost and closeness rating score. Note that, for both cases we have used the same results achieved by our approach.

Table 1. Comparison with existing algorithms

Problem (naug n)	Lower Bound	Best Known	H63	H63-66	CRAFT	Biased Sampling	FLAC	DISCON	FATE	TAA	GESA	HU & Wang	Proposed Approach
6	41	43	43 (44.2)	43 (44.2)	43 (44.2)	43 (44.2)	43 (43)	43 (47.5)	50.6 (50.6)	43 (43)	43 (43)	43 (43)	43 (43)
8	91	107	109 (114.4)	107 (110.2)	107 (113.4)	107 (107)	107 (107)	107 (118.8)	126.7 (126.7)	116 (116)	107 (107)	107 (107.8)	107 (107.75)
12	243	289	301 (317.4)	304 (310.2)	289 (296.2)	289 (293)	289 (289)	295 (322.2)	326.2 (326.2)	314 (314)	289 (289.36)	289 (290.6)	287 (292.57)
15	479	575	617 (632.6)	578 (600.2)	583 (600)	575 (580.2)	585 (585)	597 (630.8)	660.8 (660.8)	596 (596)	575 (575.18)	575 (576.4)	575 (677.01)
20	1014	1285	1384 (1400.4)	1319 (1345)	1324 (1339)	1304 (1313)	1303 (1303)	1376 (1416.4)	1436.3 (1436.3)	1414 (1414)	1285 (1287.38)	1285 (1290.5)	1286 (1290.6)
30	2238	3062	3244 (3267.2)	3161 (3206.8)	3148 (3189.6)	3093 (3189.6)	3079 (3079)	3330 (3436.4)	3390.6 (3390.6)	3326 (33269)	3062 (3079.32)	3064 (3075.1)	3062 (3081.02)

To evaluate the proposed multi-objective FLP approach, we run the algorithm on various benchmark data. The 6, 8, 12, 15, 20 and 30 facility problem (*naug6*, *naug8*, *naug12*, *naug15*, *naug20*, *naug30*) formulated by Naugent et al. [17], the 6 and 8 facility problem proposed by Dutta and Sahu (*ds6*, *ds8*) [18], the 9 facility problem (*ct9*) used in [19], the 6 and 8 facility problem (*singh6*, *singh8*) proposed in [2], and 42, 72 and 100 facility problem (*ska42*, *ska72*, *wil100*) available in the web [20] are used here. As discussed above, very few benchmark problems are available for multi-objective FLP, particularly, in the case of closeness rating score. Thus, the authors have themselves created their own data sets for closeness rating score. The experiments are conducted using 50 chromosomes and 40 generations for up to 15 facility

problem, where as 100 chromosomes and 80 generations for more than 15 facility problem. The probabilities of crossover and mutation are 0.9 and 0.3 respectively. Each benchmark problem is tested for thirty times with different seeds. Then each of the final generation is combined and a non-dominated sorting is performed to constitute the final non-dominated solutions.

Table 2. Comparison of the material handling cost obtained by various evolutionary methods

Problem	<i>n</i>	SA	TS	GAs	Proposed Approach
naug30	30	6128	6150	6202	3062
sko42	42	NA	15866	15982	15796
sko72	72	NA	66920	67160	66034
will100	100	274022	NA	275290	273988

6.1 Single Objective Context

In Table 1, the performance of the proposed approach is compared with some existing algorithm in term of material handling cost. This table is partially cited from [1] and value in the parenthesis represents the average objective function value. From this table, it can be easily found that the proposed FLP algorithm is capable in producing near-optimal values for the test problems. As indicated in the table, the performance of the proposed approach is superior or equivalent to other approaches and match up with most of the best-found solutions; although some other algorithms may achieve this value. In case of *naug12*, it achieves the new best-found material handling cost. Only exception is in the case of *naug20* problem, where it fails to achieve the best-known result by a slight margin. Also, in some cases, the average results obtained by the proposed algorithm are a little higher than that of some existing algorithms. Despite that it should be mentioned that all the compared algorithms are designed for single objective only, and the main goal of our proposed algorithm is to find trade-off solutions for multi-objective FLP, which is very rare in literature. Also, according to the Pareto optimal theory, the final and average value may be influenced by the presence of other objective (closeness rating score). While considering this, the overall performance of the proposed approach is very promising for all the problems.

Table 3. Results of Test Problems

Problem	Material Handling Cost		Closeness rating Score	
	Best	Average	Best	Average
ds6	96	96.8	48	43.40
ds8	179	209.84	82	70.3
ct9	4818	4822.9	90	74.79
singh6	94	98.28	48	40.48
Singh8	179	199.84	82	73.1
naug30	3062	3081.02	292	254.05

We also perform experiments to compare our proposed evolutionary FLP algorithm with some existing evolutionary FLP approaches to justify its efficiency as an evolutionary approach. Table 2 summarizes the material handling cost in comparison with these methods for *naug30*, *sko42*, *sko72*, *will100* problems. The column

headings, SA, TS and GAs indicate simulated annealing, tabu search and genetic algorithm based approaches respectively. This table is partially cited from [21]. From the table, it can be found that the proposed approach clearly outperforms the other evolutionary approaches by a significant margin.

6.2 Multiple-Objective Context

MOEAs do not try to find one optimal solution but all the trade-off solutions, and deal with two goals - finding a set of solutions as close as possible to the Pareto-optimal front and as diverse as possible. Table 3 shows the performance statistics of the evolutionary multi-objective FLP in the context of material handling cost and closeness rating score. All the results for these problems available in the literature used weighted sum approach for handling multiple objects and present only a single final value. However, by extracting the material handling cost from these results, it can be shown that our proposed algorithm outperforms the best-known results for *ds6* and *ds8* problems. The available best-known results are 98 and 190 respectively. In others cases, the results are the same. On the other hand, the closeness rating score may vary in different cases. This is due to the fact that different authors use different rating scores for the test problems.

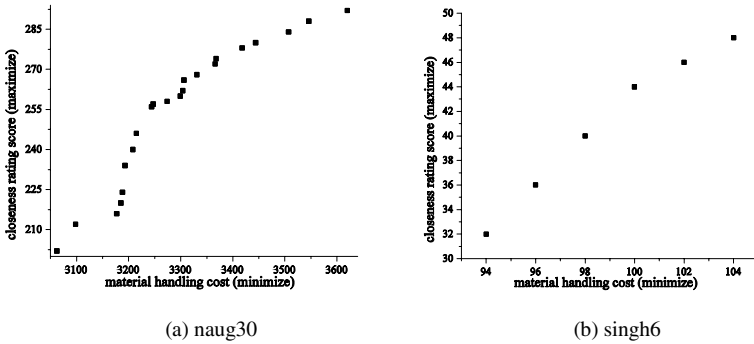


Fig. 1. Final Pareto-optimal front

To illustrate the convergence and diversity of the solutions, non-dominated solutions of the final generation produced by the proposed algorithm for the test problems *naug30* and *singh6* are presented in Fig. 1. From these figures, it can be observed that the final solutions are well spread and converged. And for this reason it is capable of finding extreme solutions. It is worthwhile to mention that in all cases, most of the solutions of the final population are Pareto optimal. In the figures, the occurrences of the same non-dominated solutions are plotted only once.

Fig. 2 demonstrates the convergence behavior of the proposed methods over generations for *ds8*. These figures also justify that our proposed approach clearly optimizes both of the objectives with generations. From the figures, it can be found that from first generations to last generations, the proposed method is able to optimize both of the material handling cost (minimize) and closeness rating score (maximize).

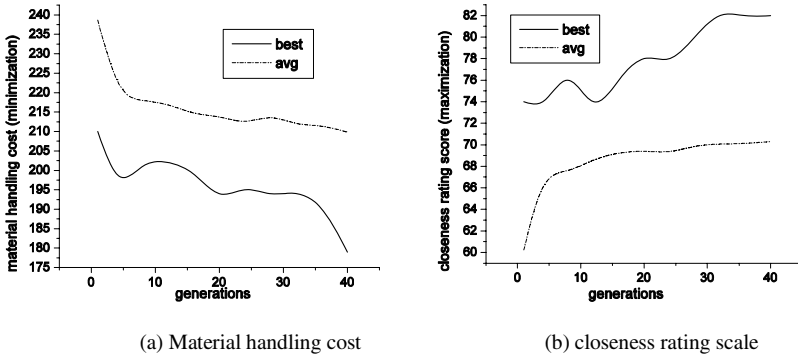


Fig. 2. Two objectives over generations of *ds8* problem

To summarize the result, the proposed approach is capable of producing near-optimal and non-dominated (Pareto optimal) solutions, which are also the best-known results in many cases. The simulation results clearly show that our proposed approach is able to find a set of diverse Pareto optimal solutions, which fulfills the two main goals of the multi-objective FLP algorithm.

7 Conclusion

FLP has attracted the attention of many researchers because of its practicality and interdisciplinary importance. Although several schemes for solving FLP are available in the literature, very few of them deal with multi-objective approach to optimize both qualitative and quantitative objectives. This is in contrast to real life, where FLP must consider both types of objectives. At present, several multi-objective FLP techniques have been proposed. However, most of them are limited to the weighted sum approach and suffer from a number of problems. In this work, we have presented an evolutionary approach for solving the multi-objective FLP that searches for the Pareto-optimal solutions. The experimental results demonstrate that the proposed approach can produce an overall strong performance for all of the applied benchmark problems related to the material handling cost in the context of single-objective optimization. For the multi-objective optimization, the results show that it is capable in finding a set of Pareto optimal solutions that optimizes both material handling cost and closeness rating score simultaneously throughout the evolutionary process, considering both diversity and convergence of the non dominated solutions.

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