Stochastic Capturing Moving Intrusions by Mobile Sensors

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Abstract. In our previous work [1], we studied Intrusion/Detection Model 1 (IDM-1), in which intrusion events occur /arrive randomly at the points of the region of interest and then fade way after a random time. In this paper, we built a stochastic model analyzing the detection quality achieved by a single sensor moving along a certain track, based on velocity and mobility pattern. We consider the modes of intrusion events defined as follows: intruders occur/arrive at random points at the edge of the field of interest, and move directly to the center of the field of interest at a constant/maximum speed. We called this model as IDM-2. In order to compare the results, two detection scenarios are studied: the robot detection scenario and the radar detection scenario. In the robot detection scenario, a robot is set to move periodically along a certain route at a constant speed. In the radar detection scenario, radar is rotated at a constant speed in a clockwise/anti-clockwise direction. An intrusion is said to be captured if it is sensed by the moving robot or radar before it arrives at the center of the field of interest. For both scenarios, we derive general expression for intrusion loss probability and the expected time that it takes the robot or radar to make the first capture of the intruders.

Keywords: Sensor Networks, Mobile Sensors, robots, Intrusions.

1 Introduction

Recent advances in robotics and low power embedded systems have made dynamic detection [3, 4, 5, 6] an available choice for sensing applications. Due to their mobility, a small number of moving robots may be deployed to cover a large sensing field [7]. A properly designed routine of the moving robots makes the networking connection more reliable because the robots are capable of exchanging information with each other whenever one is within communication range of any of the others.

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However it is important to notice that the instantaneous areas covered by a moving robot and a stationery detection device of the same type are the same. Without properly designing the moving track, the moving robots may not have better detection quality than the stationary detection devices, especially when the sensing field is highly dynamic (either spatially or temporally) in nature.

In [2], the authors derived results for how the quality of coverage in mobile sensor networks is affected by the number of mobile sensors, their velocity, velocity pattern, and event dynamics. They analyzed the case where the PoI's (point of interest) are located on a simple closed curve and the sensors move along the curve. Their model is based on the assumption that the intrusions occur at one or more fixed points of the region that the sensors detect and that the intrusions do not occur randomly on the points of the region of interest.

In [1], we studied Intrusion/Detection Model 1 (IDM-1), in which intrusion events occur /arrive randomly at the points of the region of interest and then fade way after a random time. We investigated and drew conclusions on how the quality of the first intrusion capture in the moving robot detection scenario, as well as in the radar detection scenario, depends on parameters such as robot moving speed and event dynamics in both scenarios. Analysis and derivation of the first capture were also presented.

In this paper, we will study Intrusion/Detection Model 2 (IDM-2), in which intrusion events arrive at random points at the edge of the field of interest and move directly to the center of the field of interest at a constant (or the maximum) speed. There are many examples of applications in IDM-2 in the real world. For instance, thieves may arrive at the periphery of a house and try to get in the center of the house where there is jewelry as quickly as possible. We analyze how the quality of the first intrusion capture in moving robot detection scenario, as well as in the radar detection scenario, depends on parameters such as robot moving speed, event dynamics, and the speed of the intruder in both scenarios. We draw our conclusions for the problem above and for the first capture problem.

The rest of this paper is organized as follows. Section 2 presents problem definition including model, problems to be solved, and assumptions. Section 3 provides analysis of first capture of intrusions. Finally, we conclude our paper in Section 4.

2 IDM-2: Problem Definition

In this section, we define an intrusion/detection model, explain the problems to be solved, and give assumptions.

2.1 Intrusion/Detection Model 2 (IDM-2)

In this paper, we considered the following intrusion mode, called Intrusion/Detection Model 2 (IDM-2). The detection region studied here is a circle with radius R, shown in Fig. 1. An intrusion is a target/intruder that appears /occurs along the perimeter of the circle (i.e., the edge of the sensing field), and moves straightly towards the center

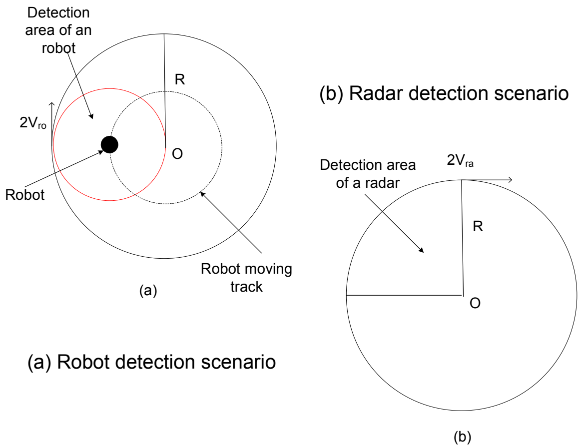

Fig. 1. Robot and radar detection scenarios

of the circle. An intrusion/intruder is considered to be detected if the target/intruder appears within the sensing range of a moving sensor (robot) or radar on its way to the center. The intruder is immediately captured by the robot or radar as soon as it is detected. If the intruder is captured, it will stop moving. Once the intruder arrives at the center, we regard it to be missed or not captured. Detection qualities for this intrusion mode are derived based on two different detection scenarios, the radar detection scenario and the robot detection scenario, respectively.

The robot detection scenario is shown in Fig. 1a. The robot is moving clockwise or anti-clockwise along the dotted circle at a constant speed V_{ro}. At any time, the robot has a detection range of a circle with radius R/2, which can cover a quarter of the detecting region. In such a model, a single robot would have the ability to cover the whole circle field of interest in one time period.

The radar detection scenario is shown in Fig. 1b. It is very common in surveillance applications. At any given time, the radar has a detection range of a sector, which can cover a quarter of the detecting region. The radar will rotate in a clockwise/anti-clockwise direction, and the speed at the middle point of the radius is V_{ra}, which is also considered to be the speed of the radar . The whole detect region can be covered by the radar in one time period.

In the IDM-2, we assume that an intruder moves towards the center at its maximum speed. By intuition, once the intruder starts its intrusion, it always desires to reach the center as soon as possible. Therefore, the movement of the intruder can be considered to be a constant speed movement, where the speed is the maximum speed of the intruder, denoted as V_{in}.

2.2 Problems to Be Solved

We will address the following issues:

- *Problem 1*: What is the probability that an intruder is lost by a single moving sensor with a constant speed in the robot detection scenario?
- *Problem 2*: What is the probability that an intruder is lost by a single moving sensor with a constant speed in the radar detection scenario?
- *Problem 3*: If an intruder is captured by the robot, what is the expect time that the robot spends to capture it from the time when it occurs?
- *Problem 4*: If an intruder is captured by the robot, what is the expect time that the radar spends to capture it from the time when it occurs***?***

Note that an event that an intrusion is lost is critical, especially in some critical applications where even one intrusion would destroy the system.

2.3 Assumptions

We assume that an intrusion is a target/intruder moving from the edge of the sensing field straight towards the center. Let X_i denote the length of the time interval between the times when the *i*-th and *(i+1)*-th adjacent intruders appear. We assume X_i to be a stochastic variable that has an exponential probability distribution with a mean $1/\lambda_i$. Therefore, intruders appear at the edge of the sensing field one by one. In our math model, we further assume $\lambda_i = \lambda$ for any i .

Since what we are concerned with is the intrusion capture quality, we assume that no intrusion happens before the robot or radar starts to move to detect intrusions, and let zero to be the time when the robot begins to move to detect intrusions.

Let O(R) denote as the circular area with radius *R* that the robot or the radar will detect intrusions. Let O denote the center of the round region O(R). Any intrusion can appear randomly at any point on the perimeter of O(R). The point where the intrusion appears is a variable, which is subject to a uniform probability distribution along the perimeter. Without losing generality, we suppose that both the robot and the radar move a clockwise direction. If A and B are two points on the edge of O(R), an angle <AOB> represents the angle of line AO to line OB in a clockwise direction, and its value can vary from o to 2π .

3 IDM-2 Analysis of Capture of Intrusions

We consider only the capture of the first intrusion, called first capture. Let T denote the time that the first intruder/intrusion occurs. Let $\Pr_o[loss]$ and $\Pr_a[loss]$ denote the probabilities that the robot and the radar, respectively, misses (or does not capture) the first intruder. Let $\Pr_o[loss \mid T = t_0]$ and $\Pr_a[loss \mid T = t_0]$ denote the conditional probabilities that the robot and the radar, respectively, misses the first intruder under the condition that $T = t_0$.

3.1 Robot Detection Scenario

Let $\bar{r} \approx 4.6033$ .Then we can obtain that

$$\Pr{}_{o}[loss] = \Pr{}_{o}[loss \mid T = t_0]$$

$$= \begin{cases} 1-\dfrac{\theta}{2\pi} = 1-\dfrac{\pi}{2\pi} = \dfrac{1}{2} & \text{if } \dfrac{2V_{ro}}{V_{in}} \le 1 \\[2ex] 1-\dfrac{\theta}{2\pi} = 1-\dfrac{\dfrac{\pi}{2}+\arcsin(\dfrac{V_{in}}{2V_{ro}})+\dfrac{\sqrt{1-(\dfrac{V_{in}}{2V_{ro}})^2}}{\dfrac{V_{in}}{2V_{ro}}}}{2\pi} & \\[2ex] =\dfrac{3}{4}-\dfrac{\arcsin(\dfrac{V_{in}}{2V_{ro}})+\dfrac{\sqrt{1-(\dfrac{V_{in}}{2V_{ro}})^2}}{\dfrac{V_{in}}{2V_{ro}}}}{2\pi} & \text{if } 1 < \dfrac{2V_{ro}}{V_{in}} \le \bar{r} \\[2ex] 0 & \text{if } \dfrac{2V_{ro}}{V_{in}} > \bar{r} \end{cases}$$

Therefore, Problem 1 is solved. To examine the expected time that the robot takes to capture the first intrusion under the condition that it can be captured, we first introduce a stochastic variable T_{ro}, which represents the time the robot takes to capture the first intrusion.

$$
\begin{aligned}
&E(T_{ro} \mid I[\text{the first intruder is captured}]=1) \\
&= \frac{E(T_{ro} I[\text{the first intruder is captured}] \mid T = t_0)}{1-\Pr_o[loss]} \\
&= \begin{cases}
4\int_0^{\frac{R}{V_{in}}} t\frac{1}{2\pi}\left(\frac{V_{in}}{R\sqrt{\frac{2V_{in}}{R}t-(\frac{V_{in}}{R}t)^2}}\right)dt & \text{, if } \frac{2V_{ro}}{V_{in}} \le 1 \\
\dfrac{\int_0^{\frac{1-\sqrt{1-(\frac{V_{in}}{2V_{ro}})^2}}{\frac{V_{in}}{R}}} t\frac{1}{2\pi}\left(\frac{V_{in}}{R\sqrt{\frac{2V_{in}}{R}t-(\frac{V_{in}}{R}t)^2}}-\frac{2V_{ro}}{R}\right)dt + \int_0^{\frac{R}{V_{in}}} t\frac{1}{2\pi}\left(\frac{2V_{ro}}{R}+\frac{V_{in}}{R\sqrt{\frac{2V_{in}}{R}t-(\frac{V_{in}}{R}t)^2}}\right)dt}{\frac{3}{4}-\dfrac{\arcsin(\frac{V_{in}}{2V_{ro}})+\dfrac{\sqrt{1-(\frac{V_{in}}{2V_{ro}})^2}}{\frac{V_{in}}{2V_{ro}}}}{2\pi}} & \text{, if } 1<\frac{2V_{ro}}{V_{in}} \le \bar{r} \\
\int_0^{\frac{1-\sqrt{1-(\frac{V_{in}}{2V_{ro}})^2}}{\frac{V_{in}}{R}}} t\frac{1}{2\pi}\left(\frac{V_{in}}{R\sqrt{\frac{2V_{in}}{R}t-(\frac{V_{in}}{R}t)^2}}-\frac{2V_{ro}}{R}\right)dt + \int_0^{\overline{T_{ro}}} t\frac{1}{2\pi}\left(\frac{2V_{ro}}{R}+\frac{V_{in}}{R\sqrt{\frac{2V_{in}}{R}t-(\frac{V_{in}}{R}t)^2}}\right)dt & \text{if } \frac{2V_{ro}}{V_{in}} > \bar{r}
\end{cases}
\end{aligned}
$$

Therefore, Problem 3 is solved.

3.2 The Radar Detection Scenario

We know that if $V_{ra}/V_{in} \ge 3\pi/4$, the first intruder is surely captured, namely,

$$\Pr_a[loss] = \Pr_a[loss \mid t_0] = 0;$$

if $\frac{V_{ra}}{V_{in}} < \frac{3\pi}{4}$,

$$\Pr a[loss] = \Pr a[loss \mid t_0] = \int_{\frac{2V_{ra}}{V_{in}}}^{\frac{3\pi}{2}} \frac{1}{2\pi} d\theta = \frac{\frac{3\pi}{2} - \frac{2V_{ra}}{V_{in}}}{2\pi} = \frac{3}{4} - \frac{\frac{V_{ra}}{V_{in}}}{\pi}.$$

Therefore, Problem 2 is solved. Let T_{ra} represent the time the radar talks to capture the first intrusion.

$$\begin{aligned} & E(T_{ra} \mid I[\text{the first intruder is captured}]) \\ & = \frac{E(T_{ra} I[\text{the first intruder is captured}] \mid T = t_0)}{1 - \Pr a[loss]} \\ & = \begin{cases} \dfrac{R \dfrac{V_{ra}}{V_{in}^{\ 2}}}{\dfrac{\pi}{2} + \dfrac{2V_{ra}}{V_{in}}} & \text{if } \dfrac{V_{ra}}{V_{in}} < \dfrac{3\pi}{4} \\ \dfrac{9\pi R}{32 V_{ra}} & \text{if } \dfrac{V_{ra}}{V_{in}} \geq \dfrac{3\pi}{4} \end{cases} \end{aligned}$$

Therefore, problem 4 is solved. Therefore, all of the proposed problems have been solved.

4 Conclusion

In this paper, under an intrusion mode, called IDM-2, we analyze intrusion capture performance under two different detection scenarios: the robot and radar detection scenarios. Intrusion loss probabilities for both detection scenarios are considered. We derive first capture probabilities and capture time durations under both the robot and the radar detection scenarios. For both detection scenarios, we consider the expected time (capture time) that the moving sensor or radar takes to capture the intruder from the time it occurs, in the case that the moving sensor or radar captures it. Our results (omitted due to limited space) show that the robot performs better than the radar in terms of the probability of capturing an intruder, and that the radar performs better than the robot in terms of capture time.

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