Trade-Off Analysis in Discrete Decision Making Problems Under Risk

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Abstract The paper considers a discrete stochastic multi-attribute decision making problem. This problem is defined by a finite set of alternatives A, a set of attributes X and a set E of evaluations of alternatives with respect to the attributes. In the stochastic case the evaluation of each alternative with respect to each attribute is characterized by a random variable. Thus, the comparison of two alternatives leads to the comparison of two vectors of probability distributions. In the paper a new interactive procedure for solving this problem is proposed. At each iteration a candidate alternative is proposed to the decision maker. If he/she is satisfied with the proposal, the procedure ends. Otherwise, the decision maker is asked to select the attribute to be improved and the attributes that can be decreased, ordered lexicographically starting with the one to be decreased first. The relations between distributions of trade-offs are used to generate a new proposal. An example is presented to illustrate the proposed technique.

1 Introduction

Interactive approach is probably the most often used method for solving multi-attribute decision making problems. It assumes that the decision maker (DM) is capable of defining attributes that influence his/her preferences and to provide preference information with respect to a given solution or a given set of solutions (local preference information). Two main advantages are usually mentioned for employing interactive techniques. First, such methods need much less a priori information about the DM's preferences. Second, as the DM is closely involved in all phases of the problem solving process, he/she puts much reliance in the generated solution, and as a result, the final solution has a better chance of being implemented.

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The kind of local preference information required varies for each interactive procedure. Two main paradigms are employed when the information about the DM's preferences is collected: direct and indirect. According to the first one, the DM expresses his/her preferences in relation to the values of attributes. Such approach is used in techniques proposed by Benayoun et al. (1971), Wierzbicki (1980), Steuer (1986), and Spronk (1981). Indirect collection of preferences means that the decision maker has to determine the trade-offs among attributes at each iteration, given the current candidate solution. The classical method by Geoffrion et al. (1972) is an example of such approach. These two classes are not disjoint. The methods proposed by Zionts and Wallenius (1976) and Kaliszewski and Michalowski (1999) combine both approaches.

This paper focuses on discrete multi-attribute decision making problems under risk. By "discrete" we mean that a finite number of alternatives are explicitly known. Evaluations of the alternatives with respect to attributes are characterized by random variables. Various approaches have been proposed for such problem. Keeney and Raiffa (1976) suggest multiattribute utility function approach for this problem. They show that if the additive independence condition is verified, then a multiattribute comparison of two alternatives can be decomposed into one-attribute comparisons. In practice, however, both the estimation of one-attribute utility functions and the assessment of the synthesis function are difficult. Saaty and Vargas (1987) proposed a version of the AHP that introduces uncertainty. Various techniques based on the outranking approach were also suggested: Dendrou et al. (1980), Martel et al. (1986), and D'Avignon and Vincke (1988).

In this paper stochastic dominance (SD) rules are used for comparing distributional evaluations. Huang et al. (1978) showed that if the additive independence condition is verified, then the necessary condition for multi-attribute stochastic dominance (MSD) is the verification of stochastic dominance with respect to each attribute. In practice the MSD rule is very rarely verified. Zaras and Martel (1994) suggested weakening the unanimity condition and accepting a majority attribute condition. They proposed MSD_r – multiattribute stochastic dominance for a reduced number of attributes. This approach is based on the observation that people tend to simplify the multiattribute problem by taking into account only the most important attributes. The procedure consists of two steps. First, the SD relations are verified for each pair of alternatives with respect to all attributes. Next, the multiattribute aggregation is realized – the ELECTRE I methodology is used to obtain the final ranking of alternatives.

Interactive procedures for discrete multi-attribute decision making problems based on stochastic dominance have been proposed in Nowak (2004, 2006). The first is an extension of the STEM method. In each step a candidate alternative, which has a minimal distance to the ideal solution, is generated. A min–max rule is used for measuring this distance. The decision maker examines the evaluations of the candidate alternative with respect to attributes and selects the one that satisfies him/her. Then the limit of concessions, which can be made on average evaluations with respect to this attribute, is defined. The procedure continues until a satisfactory solution is found. The INSDECM procedure proposed in Nowak (2006) combines the SD approach and mean-risk analysis. It is assumed that the decision maker is able to express his/her requirements defining the restrictions based both on average evaluations and on scalar risk measures.

Sometimes the DM is not able to express his/her preferences directly – by defining the minimum or maximum values for one or more distribution characteristics. Often he/she is able to choose only that attribute which should be improved and the attributes that can be decreased without defining the limits of such concessions. In such a case trade-offs can be used for identifying a new proposal. The aim of this paper is to propose an interactive technique using dialog scenario of this type.

The paper is structured as follows. The problem is formulated in Sect. 2. Section 3 provides basic information about point-to-point trade-offs. In Sect. 4 an interactive procedure is presented. The next section gives a numerical example. The last section consists of conclusions.

2 Formulation of the Problem

The decision situation considered in this paper may be conceived as a problem (**A**, **X**, **E**) where **A** is a finite set of alternatives $a_i, i = 1, 2, \ldots, m$, **X** is a finite set of attributes $X^p, p = 1, 2, \ldots, n$, and **E** is a set of evaluations of projects with respect to attributes X_i^p, $i = 1, 2, \ldots, m, p = 1, 2, \ldots, n$. We assume that the attributes are defined in such a way that a larger value is preferred to a smaller one.

This work focuses on decision making problems under risk. Thus, we will assume that the evaluation of a_i with respect to X^p is a random variable with a cumulative probability distribution function $F_i^p(x)$ defined as follows:

$$F_i^p(x) = \Pr\left(X_i^p \le x\right)$$

The attributes are supposed to be probabilistically independent, and are also supposed to satisfy the preference independence condition. Thus, the overall comparison of two alternatives can be decomposed into one-attribute comparisons of probability distributions.

Two main approaches are usually used for such comparisons: mean-risk models and stochastic dominance. The former is based on two criteria: one measuring expected outcome and another one representing variability of outcomes. The latter is based on an axiomatic model of risk-averse preferences and leads to conclusions that are consistent with the axioms. In fact, mean-risk approaches are not capable of modeling even the entire gamut of risk-averse preferences. Moreover, for typical statistics used as risk measures, the mean-risk approach may lead to inferior conclusions (Ogryczak and Ruszczyński 1999).

In this paper we will use stochastic dominance rules for modeling preferences of the DM in relation to each attribute. First stochastic dominance and second stochastic dominance are defined as follows:

Definition 1. (FSD – First Degree Stochastic Dominance).
X_i^p dominates X_j^p by FSD rule ($X_i^p \succ_{\text{FSD}} X_j^p$) if and only if
$F_i^p(x) \neq F_j^p(x)$ and $F_i^p(x)$-$F_j^p(x) \leq 0$ for all $x \in \mathbf{R}$

Definition 2. (SSD – Second Degree Stochastic Dominance).
X_i^p dominates X_j^p by FSD rule ($X_i^p \succ_{\text{SSD}} X_j^p$) if and only if
$F_i^p(x) \neq F_j^p(x)$ and $\int_{-\infty}^{x} \left(F_i^p(y)\text{-}\ F_j^p(y)\right) dy \leq 0$ for all $x \in \mathbf{R}$

The FSD is the most general relation. If $X_i^p \succ_{\text{FSD}} X_j^p$, then X_i^p is preferred to X_j^p within all models preferring larger outcomes. The use of SSD requires more restrictive assumptions. If $X_i^p \succ_{\text{SSD}} X_j^p$, then X_i^p is preferred to X_j^p within all risk-averse preference models that prefer larger outcomes.

In this paper we assume that the DM is risk averse. Thus we will assume that X_i^p dominates X_j^p by stochastic dominance rule ($X_i^p \succ_{\text{SD}} X_j^p$) if $X_i^p \succ_{\text{FSD}} X_j^p$ or $X_i^p \succ_{\text{SSD}} X_j^p$. We will use this rule for comparing evaluations of alternatives with respect to attributes, and for analyzing relations between distributions of point-to-point trade-offs.

3 Point-to-Point Trade-Offs

A trade-off is defined for a particular solution and for a selected pair of the attributes. It specifies the amount by which the value of one attribute increases while that of the other one decreases when a particular solution is replaced by another given solution.

Let us start with a decision making problem under certainty. For a pair of alternatives a_i and a_j and a pair of attributes X^p and X^q, a point-to-point trade-off is the ratio of a relative value increase in one attribute (X^p) per unit of value decrease in the reference attribute (X^q) when the alternative a_i is replaced by the alternative a_j.

$$T_{ji}^{pq} = \frac{X_j^p - X_i^p}{X_i^q - X_j^q}$$

Let us assume that the DM analyzes the alternative a_i and decides that the evaluation with respect to X^p should be improved, while the evaluation with respect to X^q can be decreased. In this case we will look for alternatives a_j such that $X_j^p > X_i^p$ and $X_j^q \geq X_i^q$, and choose the one for which the increase of X^p is maximal. If such alternatives do not exist, then an alternative maximizing point-to-point trade-off will be proposed.

In the stochastic case the situation is much more complicated, as various situations have to be taken into account when a point-to-point trade-off for a pair of alternatives (a_j, a_i) and a pair of attributes (X^p, X^q) is computed. In fact, such a trade-off is a random variable whose distribution is a mixture of four distributions:

X_i^p, X_i^q, X_j^p, X_j^q. In this paper we will assume that the attributes are probabilistically independent and satisfy independence conditions allowing us to use an additive utility function. To generate probability distribution of point-to-point trade-off T_{ji}^{pq} we have to analyze the following cases:

1. $X_j^p > X_i^p$ and $X_i^q > X_j^q$,
2. $X_j^p > X_i^p$ and $X_i^q = X_j^q$,
3. $X_j^p > X_i^p$ and $X_i^q < X_j^q$,
4. $X_j^p = X_i^p$ and $X_i^q > X_j^q$,
5. $X_j^p = X_i^p$ and $X_i^q = X_j^q$,
6. $X_j^p = X_i^p$ and $X_i^q < X_j^q$,
7. $X_j^p < X_i^p$ and $X_i^q > X_j^q$,
8. $X_j^p < X_i^p$ and $X_i^q = X_j^q$,
9. $X_j^p < X_i^p$ and $X_i^q < X_j^q$.

Only the first case describes the classical trade-off situation. Cases (2) and (3) describe situations in which it is possible to improve the value of X^p without decreasing X^q. For such situations we will assume that $T_{ji}^{pq} = M$, where M is a "big number". If (4), (5), or (6) takes place, then we will assume that $T_{ji}^{pq} = 0$, as replacing a_i by a_j will not change the value of X^p. And finally for cases (7), (8), and (9) we will assume that $T_{ji}^{pq} = -M$, as replacing a_i by a_j will decrease the value of X^p. Thus for given $X_i^p, X_i^q, X_j^p, X_j^q$, the value of trade-off will be computed as follows:

$$T_{ji}^{pq} = \begin{cases} \frac{X_j^p - X_i^p}{X_i^q - X_j^q} & \text{if } X_j^p > X_i^p \text{ and } X_i^q > X_j^q \\ M & \text{if } X_j^p > X_i^p \text{ and } X_i^q \leq X_j^q \\ 0 & \text{if } X_j^p = X_i^p \\ -M & \text{if } X_j^p < X_i^p \end{cases}$$

Let us consider the following example. Alternatives a_1 and a_2 are evaluated with respect to attributes X^1 and X^2. Distributions of alternatives with respect to attributes are presented in Table 1. To generate the distribution of T_{21}^{12}, we have to consider all possible combinations of the values X_1^1, X_1^2, X_2^1, X_2^2 (Table 2).

Table 1 Example 1 – evaluations of alternatives

Distributions for X^1			Distributions for X^2		
	a_1	a_2		a_1	a_2
100			20		0.5
150	0.5	0.25	30	0.75	
200	0.5		40		0.5
250		0.75	50	0.25	

Table 2 Example 1 – generation of distribution of T_{21}^{12}

	X_1^1	X_2^1	X_1^2	X_2^2	Prob.	T_{21}^{12}
1	100	150	30	20	0.046875	5.00
2	100	150	30	40	0.046875	M
3	100	150	50	20	0.015625	1.67
4	100	150	50	40	0.015625	5.00
5	100	250	30	20	0.140625	15.00
6	100	250	30	40	0.140625	M
7	100	250	50	20	0.046875	5.00
8	100	250	50	40	0.046875	15.00
9	200	150	30	20	0.046875	–M
10	200	150	30	40	0.046875	–M
11	200	150	50	20	0.015625	–M
12	200	150	50	40	0.015625	–M
13	200	250	30	20	0.140625	5.00
14	200	250	30	40	0.140625	M
15	200	250	50	20	0.046875	1.67
16	200	250	50	40	0.046875	5.00

Let us again assume that the DM analyzes the alternative a_i and decides that the evaluation with respect to X^p should be improved, while the evaluation with respect to X^q can be decreased. Assuming that a set of potential new proposals has been generated, the following question arises: how can distributions of point-to-point trade-offs be compared to identify a new proposal? In this paper SD rules are employed for comparison of these distributions. We will assume that the decision-maker is risk-averse, and as a result, FSD and SSD rules can be used for the analysis of relations between distributions of point-to-point trade-offs.

4 The Procedure

The main ideas of the procedure are as follows:

- A candidate for most preferred solution is presented to the DM at each iteration
- If the DM is satisfied with the proposal – the procedure ends
- Otherwise – the DM is asked to select the attribute to be improved and the attributes that can be decreased, ordered lexicographically starting with the one to be decreased first
- Information about relations between trade-offs distributions is used to generate a new candidate

To start the procedure we have to identify the first proposal. In the approach presented here, SD rules and min–max criterion are employed in this phase. The first proposal is identified in the following steps:

1. Identify SD relations between distributional evaluations for each pair of alternatives and for each attribute.
2. For each alternative compute:

$$\overline{d}_i = \max_{p \in \{1,\ldots,n\}} \{d_i^p\}$$

where:

$$d_i^p = \text{card} \quad D_i^p$$
$$D_i^p = \{a_j : X_j^p \succ_{\text{SD}} \ X_i^p\}$$

3. Choose the alternative for which $\bar{d}_i$ is minimal.

In our problem the evaluations of alternatives with respect to attributes are expressed by probability distributions. In such a case it is not easy for the DM to compare alternatives. On the one hand, the DM is usually interested in maximizing the expected outcomes, on the other hand, however, he/she finds the variability of outcomes very important as well. In the approach presented here, as in the INSDECM procedure (Nowak 2006), it is assumed that the decision maker is able to specify the method of data presentation. For each attribute he or she may choose one or more scalar measures to be presented to him or her. Both expected outcome measures (mean, median, mode) and variability measures (standard deviation, semideviation, probability of getting outcomes not greater or not less than target value) can be chosen. Moreover, the DM may change his/her mind while the procedure is in progress, and specify other sets of measures at successive iterations. For example, while initially the DM may be interested mainly in the expected outcomes, in subsequent phases of the procedure he/she may focus on risk measures.

Let us denote:

$\mathbf{A}^{(l)}$ – the set of alternatives considered at iteration l, $\mathbf{A}^{(1)} = \mathbf{A}$
B – the set of potential new proposals
a_s – the candidate alternative

At each iteration the following steps are executed:

1. Ask the DM to specify the data he/she is interested in – the parameters of distributional evaluations such as mean, standard deviation, probability of getting a value not less (not greater) than ξ, etc.
2. Compute values of parameters for each alternative under consideration, identify the best value of each parameter.
3. Present the data to the DM:
   - The values of parameters for the candidate alternative a_s
   - Best values of parameters attainable within the set of alternatives
4. Ask the DM whether he/she is satisfied with the proposal. If the answer is YES – the procedure ends – the proposal is assumed to be the final solution of the problem.

5. If the DM is not satisfied with the proposal, ask him/her to specify the attribute be improved first and to set the order of the remaining attributes, starting from the one that can be decreased first. Let p be the number of the attribute that the DM would like to improve, while $\{q_1, q_2, \ldots, q_{n-1}\}$ is the order of the attributes that can be decreased.
6. Identify the set of alternatives satisfying the requirements expressed by the DM:

$$\mathbf{A}^{(l+1)} = \left\{a_i : a_i \in \mathbf{A}^{(l)}, a_i \neq a_s, \neg X_s^p \succ_{\mathrm{SD}} X_i^p\right\}$$

If the set $\mathbf{A}^{(l+1)}$ is empty, notify the DM that it is not possible to find an alternative satisfying his/her requirements, unless previous restrictions are relaxed. Then ask the DM whether he/she would like to relax the previous requirements. If the answer is NO, return to 5. Otherwise, generate the set of alternatives to be considered in the next phases of the procedure:

$$\mathbf{A}^{(l+1)} = \left\{a_i : a_i \in \mathbf{A}^{(1)}, a_i \neq a_s, \neg X_s^p \succ_{\mathrm{SD}} X_i^p\right\}$$

7. Assume: $\mathbf{B} = \mathbf{A}^{(l+1)}, k = 1$.
8. Generate probability distributions of trade-offs $T_{i\,s}^{p\,q_k}$ for each i such that $a_i \in \mathbf{B}$.
9. Compare distributions of trade-offs with respect to SD rules and identify the set of non-dominated distributions. If the number of non-dominated distributions is equal to 1, assume the corresponding alternative to be the new proposal and go to 13.
10. Identify the alternatives with dominated trade-offs and exclude them from the set $\mathbf{B}$.
11. If $k < n - 1$, assume $k := k + 1$ and go to 8.
12. The trade-offs for each pair of attributes have been compared, and the set of potential new proposals $\mathbf{B}$ still consists of more than one alternative. As the analysis of trade-offs has not provided a clear recommendation for the new proposal, analyze the relations between alternatives with respect to attributes. Start from attribute X^p and identify the set of alternatives with non-dominated evaluations according to SD rules. If the number of such alternatives is equal to 1, assume the corresponding alternative to be a new proposal and go to 13. Otherwise exclude from $\mathbf{B}$ the alternatives that are dominated according to SD rules with respect to attribute X^p. Next, analyze relations with respect to other attributes. In this phase of the procedure use a reversed lexicographic order of attributes: $q_{n-1}, q_{n-2}, \ldots, q_1$. For each attribute identify the dominated alternatives using SD rules and exclude them from $\mathbf{B}$. Continue until $\mathbf{B}$ consists of one alternative. If all attributes have been considered and $\mathbf{B}$ still consists of more than one alternative, assume any of them to be a new proposal a_s.
13. Assume $l := l + 1$ and go to 1.

5 Numerical Example

To illustrate our procedure let us consider the project selection problem. Ten proposals are evaluated with respect to four attributes. The evaluations of alternatives with respect to attributes are presented in Table 3. We assume that the DM is risk-averse. To identify the first proposal, stochastic dominance relations were identified (Table 4).

The first proposal is the alternative a_6, as $d_{61} = 5$, $d_{62} = 5$, $d_{63} = 3$, $d_{64} = 5$, and $\overline{d}_6 = 5$. We assume: $l = 1$,

$$\mathbf{A}^{(0)} = \mathbf{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}.$$

Iteration 1:

1. The DM decides that for each attribute means should be presented in the dialog phase of the procedure.
2. The data are presented to the decision maker (Table 5).
3. The DM is not satisfied with the proposal.
4. The DM would like to improve the evaluation with respect to attribute X_2.
5. The DM sets the order of other attributes starting from the one that can be decreased first: X_3, X_4, X_1.
6. The alternatives with evaluations that are not dominated by the evaluation of alternative a_6 with respect to attribute X_2 are identified:

$$\mathbf{A}^{(1)} = \{a_1, a_4, a_5, a_7, a_9, a_{10}\}$$

7. To identify a new proposal we analyze the relations between point-to-point trade-offs for pairs of attributes: (X_2, X_3), (X_2, X_4), (X_2, X_1). The set of potential new proposals is:

$$\mathbf{B} = \mathbf{A}^{(1)} = \{a_1, a_4, a_5, a_7, a_9, a_{10}\}.$$

8. We start to analyze point-to-point trade-offs with the pair of attributes (X_2, X_3). We generate distributions of trade-offs for each pair (a_i, a_6) such that $a_i \in \mathbf{A}^{(1)}$ and analyze SD relations (Table 6).
9. As distributions of trade-offs for the pairs (a_4, a_6), (a_5, a_6), (a_9, a_6), (a_{10}, a_6) are dominated, the alternatives a_4, a_5, a_9 and a_{10} are excluded form the set of potential new proposals:

$$\mathbf{B} = \mathbf{B} \backslash \{a_4, a_5, a_9, a_{10}\} = \{a_1, a_7\}.$$

10. As the set **B** consists of more than one alternative, we analyze relations between trade-offs distributions for the next pair of attributes (X_2, X_4). Unfortunately no SD relations can be identified for this pair of alternatives. The same situation is for attributes (X_2, X_1).

Table 3 Evaluations of alternatives with respect to attributes

X_1	Projects									
	1	2	3	4	5	6	7	8	9	10
1						1/7		1/7		
2	3/7						1/7	2/7		1/7
3	1/7				1/7			2/7		2/7
4							2/7	1/7		2/7
5	2/7		3/7	1/7		3/7	1/7	1/7	2/7	1/7
6		1/7	1/7		2/7	1/7	2/7		1/7	
7	1/7	2/7	1/7	1/7	2/7				3/7	1/7
8		2/7	2/7	1/7		1/7	1/7		1/7	
9				3/7	2/7					
10		2/7		1/7		1/7				
X_2	Projects									
	1	2	3	4	5	6	7	8	9	10
1		1/7						3/7		
2		3/7	2/7					3/7		1/7
3		1/7	1/7	1/7		4/7			1/7	
4				1/7				1/7	1/7	
5		1/7	2/7		1/7					
6	1/7	1/7		1/7	2/7		1/7		1/7	
7					1/7	1/7	1/7		4/7	2/7
8	2/7		1/7	3/7	2/7	2/7	3/7			3/7
9	3/7		1/7	1/7	1/7		2/7			
10	1/7									1/7
X_3	Projects									
	1	2	3	4	5	6	7	8	9	10
1		2/7								1/7
2		1/7			3/7					2/7
3	1/7	4/7		1/7	1/7				1/7	
4	3/7				1/7	1/7			2/7	
5					2/7	1/7		1/7	2/7	
6	1/7									2/7
7				1/7			2/7	1/7	2/7	2/7
8	1/7		4/7	2/7		2/7	3/7	2/7		
9	1/7		3/7	1/7		1/7	1/7	3/7		
10				2/7		2/7	1/7			
X_4	Projects									
	1	2	3	4	5	6	7	8	9	10
1				2/7						
2									1/7	
3	3/7						1/7			
4				1/7					1/7	
5	2/7			2/7		1/7	1/7			
6				1/7	1/7	1/7			3/7	3/7
7			1/7		1/7	1/7				1/7
8	1/7	4/7	4/7	1/7	3/7	2/7	3/7	3/7	1/7	1/7
9		2/7			1/7	1/7	1/7	1/7		1/7
10	1/7	1/7	2/7		1/7	1/7	1/7	3/7	1/7	1/7

Table 4 Stochastic dominance relations between distributional evaluations

X_1	Projects									
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1								FSD		
a_2	FSD		FSD		FSD	FSD	FSD	FSD	FSD	FSD
a_3	FSD					SSD	FSD	FSD		FSD
a_4	FSD		FSD		FSD	FSD	FSD	FSD	FSD	FSD
a_5	FSD					SSD	FSD	FSD		FSD
a_6								FSD		
a_7	FSD							FSD		FSD
a_8										
a_9	FSD		SSD			SSD	FSD	FSD		FSD
a_{10}	SSD							FSD		
X_2	Projects									
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1		FSD	FSD	FSD	FSD	FSD	FSD	FSD	FSD	FSD
a_2								FSD		
a_3		FSD						FSD		
a_4		FSD	FSD			FSD		FSD	FSD	
a_5		FSD	FSD	SSD		FSD		FSD	FSD	
a_6		FSD	SSD					FSD		
a_7		FSD	FSD	FSD	FSD	FSD		FSD	FSD	SSD
a_8										
a_9		FSD	SSD			SSD		FSD		
a_{10}		FSD	FSD					FSD		
X_3	Projects									
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1		FSD			FSD					SSD
a_2										
a_3	FSD	FSD		SSD	FSD	SSD	SSD	FSD	FSD	FSD
a_4	FSD	FSD			FSD				FSD	FSD
a_5		FSD								
a_6	FSD	FSD			FSD				FSD	FSD
a_7	FSD	FSD		SSD	FSD	SSD		SSD	FSD	FSD
a_8	FSD	FSD		SSD	FSD	SSD			FSD	FSD
a_9		FSD			FSD					SSD
a_{10}		FSD								
X_4	Projects									
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1				SSD						
a_2	FSD		SSD	FSD	FSD	FSD	FSD		FSD	FSD
a_3	FSD			FSD	FSD	FSD	FSD		FSD	FSD
a_4										
a_5	FSD			FSD		FSD	FSD		FSD	FSD
a_6	FSD			FSD			SSD		FSD	
a_7	FSD			FSD					FSD	
a_8	FSD	FSD	FSD	FSD	FSD	FSD	FSD		FSD	FSD
a_9				FSD						
a_{10}	FSD			FSD			SSD		FSD	

Table 5 Data presented to the DM at iteration 1

	Mean			
	X^1	X^2	X^3	X^4
a_6	5,714	5,000	7,714	7,571
Max	8,143	8,429	8,429	9,000

Table 6 Iteration 1 – SD relations between trade-offs for attributes (X_2, X_3)

	T_{16}^{23}	T_{46}^{23}	T_{56}^{23}	T_{76}^{23}	T_{96}^{23}	T_{106}^{23}
T_{16}^{23}		SSD	FSD		FSD	FSD
T_{46}^{23}			SSD		FSD	FSD
T_{56}^{23}					SSD	
T_{76}^{23}		FSD	FSD		FSD	FSD
T_{96}^{23}						
T_{106}^{23}					SSD	

11. As relations between trade-offs for each pair of attributes have been analyzed and the set of potential new proposals still consists of more than one alternative, we analyze relations between the alternatives a_4 and a_7 with respect to the attribute that should be improved, that is, X_2. As $X_1^2 \succ_{\mathrm{FSD}} X_7^2$, we assume the alternative a_1 to be a new proposal.

The procedure is continued in the same way, until the DM is satisfied with the proposal.

6 Conclusions

In many cases, the DM faced with a candidate solution is able to answer the simplest questions only: which attribute should be improved and which attributes can be decreased. In such a situation trade-offs can be used for generation of a new proposal. When the evaluations of alternatives with respect to attributes are characterized by random variables, a point-to-point trade-off is characterized by a random variable as well.

In this paper a new interactive procedure based on the treatment of trade-offs has been proposed. The procedure requires a limited amount of preference information from the DM.

The procedure presented in this work can also be applied for mixed problems, i.e. problems in which evaluations with respect to some attributes take the form of probability distributions, while the remaining ones are deterministic.

The proposed technique may be useful for various types of problems in which uncertain outcomes are compared. It has been designed for problems with up to moderate number of discrete alternatives (not more than hundreds) and can be applied in such areas as, for example, inventory models, evaluation of investment projects, production process control, and many others.

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