A Review of Goal Programming for Portfolio Selection

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Abstract Goal Programming (GP) is the most widely used approach in the field of multiple criteria decision making that enables the decision maker to incorporate numerous variations of constraints and goals, particularly in the field of Portfolio Selection (PS). This paper gives a brief review of the application of GP and its variants to Portfolio Selection and analysis problems. The paper firstly discusses the Multi-Criteria Decision Analysis in PS context in which GP is introduced as an important approach to PS Problems. An overview of performance measurement in portfolio selection context is also provided. Amongst the concluding remarks many issues in PS that may be addressed by GP such as multi-period, different measures of risk, and extended factors influencing portfolio selection are listed.

1 Introduction

Finance theory has produced a variety of models that attempt to provide some insight into the environment in which financial decisions are made. By definition, every model is a simplification of reality. Hence, even if the data fail to reject the model, the decision maker may not necessarily want to use the model as a dogma. At the same time, the notion that models implied by finance theory could entirely be worthless seems rather extreme. Hence, even if the data reject the model, the decision maker may still want to use the model at least to some degree (Pastor 2000).

Some researchers involved in the mean-variance analysis of Markowitz (1952) for Portfolio Selection (PS) have only focused on PS as risk adjusted return with little or no effort being directed to the inclusion of other essential factors. Therefore, the usual portfolio analysis assumes that investors are interested only with returns attached to specific levels of risk when selecting their portfolios. In a wide variety of

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applications, neither part of this restriction is desirable or important. Consequently, a portfolio analysis model that includes more essential factors in the analysis of portfolio problems is a more realistic approach. Some of these factors include liquidity, asset class, asset region, micro economics, macro economics and market dynamics.

Original PS problems, with risk and return optimisation can be viewed as a GP with two objectives. Additional objectives representing other factors can be introduced for a more realistic approach to PS problems.

Charnes et al. developed GP in 1955. GP is a multi-objective programming technique. The ethos of GP lies in the Simonan concept of satisficing of objectives (Tamiz et al. 1998). Simon introduced the concept of satisficing, a word that originated in Northumbria[1] where it meant "to satisfy". Satisficing is a strategy for making decisions in the case that one has to choose among various alternatives which are encountered sequentially, and which are not known ahead of time (Reina 2005).

GP is an important technique for decision making problems where the decision maker aims to minimize the deviation between the achievement of goals and their aspiration levels. It can be said that GP has been, and still is, the most widely used multi-objective technique in management science because of its inherent flexibility in handling decision-making problems with several conflicting objectives and incomplete or imprecise information (Romero 1991; 2004; Chang 2007).

The remaining parts of this paper are organized as follows. Section 2 discusses the literature on Multi-Criteria Decision in PS context as well as the importance of GP applications to portfolio problems. Section 3 outlines the available research papers on GP for PS. An overview of GP variants for PS is given in Sect. 4. An outline of performance measurement in portfolio selection context is provided in Sect. 5. Section 6 develops arguments for further exploitation of GP in addressing some issues in PS, and the concluding remarks are provided in Sect. 7.

2 The Use of Multi-Criteria Decision Analysis in Portfolio Selection and the Importance of Goal Programming

Optimisation is a process by which the most favourable trade-off between competing interests is determined subject to the constraints faced in any decision making process. Within the context of portfolio management, the competing interests are risk reduction and return enhancement among the other interests (Kritzman 2003).

Present-day theory of portfolio analysis prescribes a way of thinking about opportunities for investment. Instead of extensive evaluation of a single asset in isolation, the theory prescribes that investment policy can be formulated in a manner in which a purchase of an asset is done if and only if it will cause a rise in the overall

[1] A region in England on the Scottish boarder.

personal satisfactions. A rise may come about via one of three schemes as follows (Renwick 1969):

1. The new asset can cause a net increase in total present expected return on the portfolio.
2. The new asset can cause a net decline in total risk exposure on the entire portfolio.
3. There can be some subjectively acceptable trade off between change in total risk and change in total expected return on the portfolio.

The first two are the traditional and direct schemes for selecting portfolios. While, the third one is quite open to many possibilities and consequently has stimulated many studies in search for better PS for investors.

Markowitz (1952) suggests that investors should consider risk and return together and determine the allocation of funds among investment alternatives on the basis of the trade-off between them. Later on the recognition that many investors evaluate performance relative to a benchmark led to the idea of PS based on return and relative risk (Cremers et al. 2005). For many investors, both approaches fail to yield satisfactory results. Chow (1995) emphasizes that the portfolio optimisation techniques can assist in the search of portfolio that best suits each investor's particular objectives.

An alternative to Markowitz model is the Mean-Absolute Deviation (MAD) model, proposed by Konno and Yamazaki (1991). While Markowitz model assumes normality of stock returns, the MAD model does not make this assumption. The MAD model also minimizes a measure of risk, where the measure is the mean absolute deviation (Kim et al. 2005; Konno and Koshizuka 2005). Konno and Yamazaki (1991) further developed the MAD model into an equivalent GP model.

Konno and Kobayashi (1997) propose a new model for constructing an integrated stock-bond portfolio, which serves as an alternative to the popular asset allocation strategy. The fund is first allocated to indexes corresponding to diverse asset classes and then allocated to individual assets using appropriate models for each asset class.

Their model (Konno and Kobayashi 1997) determines the allocation of the fund to individual assets in one stage by solving a large scale mean-variance or mean-absolute deviation model using newly developed technologies in large scale quadratic programming and linear programming analysis, respectively. Computational experiments show that the new approach can serve as a more reliable and less expensive method to allocate the fund to diverse classes of assets.

Konno (2003) shows that there is a possibility to apply standard portfolio optimisation methods to the management of small and medium scale fund, where transaction cost and minimal transaction unit constraints are negligible. He shows that the use of mean-absolute deviation model can handle concave transaction cost and minimal transaction unit constraints in an efficient manner using branch and bound algorithm. Transaction cost is still not negligible for the majority of standard investors.

Parra et al. (2001) amongst other authors, claim that there has been a growing interest in incorporating additional criteria beyond risk and return into the PS process. Multiple criteria PS problems normally stem from multiple-argument investor utility functions. For investors with additional concerns steps can be taken to integrate them into the portfolio optimisation process more in accordance with their criteria status.

Chow (1995) mentions that investment practitioners have implicitly sent a message that optimisation models have limited relevance in real world investment decisions. One of the best arguments for this assertion is that few investors allocate their assets in the proportions indicated by an optimisation model.

Furthermore, Christiansen and Varnes (2008) present a framework for understanding how portfolio decision making is shaped through appropriate decision making. They find that the identity of decision maker is shaped and influenced by four factors, which are the formal system and rules, observations of others, the organizational context, and organizational learning. In practice, the decision maker must deal with multiple factors and criteria that make it difficult to carry out a traditional rational decision-making process.

In addition, Rifai (1996) highlights the fact that the survival of any US firm in the national and international markets depends on the use of scientific techniques in their decision-making processes. The utilization of scientific techniques requires certain steps to be followed. The most important steps are identifying, quantifying and solving the problem. He described GP as a very powerful quantitative model, which, if used properly, can be an excellent tool, particularly for investment decisions.

Cremers et al. (2005) emphasize the importance of using more approaches to portfolio formulation, particularly the mean-variance optimisation, and the full-scale optimisation approaches. They argue that institutional investors typically use mean-variance optimisation in PS, in part because it requires knowledge of only the expected returns, standard deviations, and correlations of portfolio's components, while other investors prefer to use full-scale optimisation as an alternative to mean-variance optimisation since computational advances now allow us to perform such full-scale optimisations. Under this approach, PS process considers as many asset mixes as necessary in order to identify the weights that yield the highest expected utility, given any utility function.

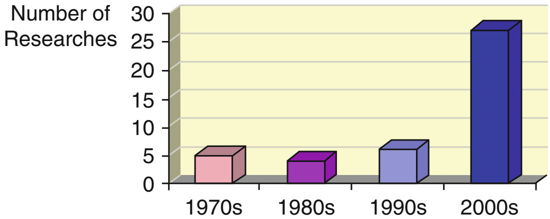
Renwick (1969) mentions that investment portfolio behaviour can be characterized and classified using combinations of the four interrelated variables, which are rate of return on total assets, rate of growth of output, capital structure and rate of retention of available income.

The evidence which Renwick presented in his paper (1969) supports the view that dividend policy is relevant to the investment decision as well as that finance does matter for the valuation of corporate assets. Both current and anticipated future returns on investment, along with the various types of risks associated with those returns, all interact to determine and characterize the empirical behaviour and performance of investor portfolios.

Despite the volume of research supporting standard PS, there has always been a slight undercurrent of multiple objectives in PS, but this is changing.

Generally, in PS problems the decision maker considers simultaneously conflicting objectives such as rate of return, risk and liquidity. Multi-objective programming techniques, such as GP, are used to choose the portfolio best satisfying the decision maker's aspirations and preferences.

The following figure illustrates the number of publication of research papers in the area of PS using GP:



Significant advances have taken place in recent years in the field of GP. A higher level of computerized automation of the solution and modelling process has brought use of the already existing and the new analysis techniques within reach of the average practitioner (Tamiz and Jones 1998).

3 Portfolio Selection Using Goal Programming: Theoretical and Practical Developments

The ultimate objective of optimal PS is to determine and hold a portfolio which offers the minimum possible deviation for a given or desired expected return. But this objective is assuming a stable financial environment. In a world in which an investor is certain of the future, the optimal PS problem is reduced to that of structuring a portfolio that will maximize the investor's return.

Unfortunately, the future is not certain, particularly now like never before and consequently the solution to the optimal PS problem will depend upon the following elements (Callin 2008):

(a) A set of possible future scenarios for the world.
(b) A correspondence function, linking possible future scenarios to the returns of individual securities.
(c) A probabilities function of the likelihood of each of the possible future scenarios of the world.
(d) A way to determine whether one portfolio is preferable to another portfolio.

These elements are considered under different assumptions based on investors' strategy and their analysis is achievable through GP approach.

Kumar et al. (1978) highlight the fact that standard PS techniques are typically characterized by motivational assumptions of unified goals or objectives. Therefore, their immediate relevance to real-world situations, usually marked by the presence of several conflicting goals, is at best limited.

Nevertheless, with appropriate extensions the standard techniques can form the basis for accommodating multiple goals. Kumar et al. (1978) address the problem of goal conflicts in the PS of Dual-Purpose Funds, and suggest an extension of standard methodology, in terms of the development of a GP model in conceptual form, which can be applied for the resolution of inherent clash of interests.

GP in PS context is an analytical approach devised to address financial decision making problems where targets have been assigned to the attributes of a portfolio and where the decision maker is interested in minimising the non-achievement of the corresponding goals.

During the recent years many models concerning PS using GP have been developed. Amongst the research papers that introduce such models are:

Category	Year	Author	Portfolio selection using goal programming (GP variants and applications)
1970s	1973	Lee and Lerro	LGP (PS for mutual funds)
	1975	Stone and Reback	Other variant (nonlinear GP-portfolio revisions)
	1977	Booth and Dash	Other variant (nonlinear GP-bank portfolio)
	1978	Kumar et al.	LGP (dual-purpose funds)
		Muhlemann et al.	LGP (portfolio modelling)
	1979	Kumar and Philippatos	LGP (dual-purpose funds)
1980s	1980	Lee and Chesser	LGP (PS)
	1984	Levary and Avery	LGP (weighting equities in a portfolio)
	1985	Alexander and Resnich	LGP (bond portfolios)
1990s	1992	Konno and Yamazaki	MAD, PS
	1994	Byrne and Lee	Spreadsheet optimizer (real estate portfolio)
	1996	Tamiz et al.	Two staged GP model for portfolio selection
	1997	Tamiz et al.	Comparison between GP and regression analysis for PS
		Watada	Fuzzy portfolio selection
	1998	Kooros and McManis	Multiattribute optimisation for strategic investment decisions
2000s	2000	Deng et al.	Criteria, models and strategies in portfolio selection
		Inuiguchi and Ramik	Fuzzy GP and other variant
		Ogryczak	Linear programming model for portfolio selection

2001	Jobst et al.	Other variant (alternative portfolio selection models)
	Parra et al.	FGP (portfolio selection)
2002	Wang and Zhu	Fuzzy portfolio selection
	Zopounidis and Doumpos	Multi-criteria decision aid in financial decision making
2003	Allen et al.	Other variant-portfolio optimisation
	Prakash et al.	Other variant-polynomial GP (selecting portfolio with skewness)
	Rostamy et al.	Other variant
	Sun and Yan	Other variant-skewness & optimal portfolio selection
2004	Kosmidou and Zopounidis	GP, simulation analysis & bank asset liability management
	Pendaraki et al.	GP (equity mutual fund portfolios)
2005	Dash and Kajiji	Other variant (nonlinear GP for asset-liability management)
	Davies et al.	Other variant (polynomial GP-fund of hedge funds PS)
	Deng et al.	Minimax portfolio selection
	Pendaraki et al.	GP-construction of mutual funds portfolios
	Tektas et al.	GP-asset and liability management
2006	Bilbao et al.	GP-portfolio selection with expert betas
	Sharma and Sharma	LGP for mutual funds
2007	Abdelaziz et al.	Other variant
	Bilbao et al.	GP
	Gladish et al.	Interactive three-stage model-mutual funds portfolio selection
	Li and Xu	Other variant (nonlinear GP-portfolio selection)
	Sharma et al.	Other variant (credit union portfolio management)
	Wu et al.	GP-index investing

Romero claims in a recent paper (2004) that most of GP applications, which are reported in the literature, use weighted or lexicographic achievement function. He explains that this election is usually made in a rather mechanistic way without theoretical justification and if the election of the achievement function is wrong, then it is very likely that the decision maker will not accept the solution.

The majority of research papers prior to 2000 develop PS models utilising weighted and/or lexicographic GP variants. This trend has since changed to the fuzzy goal programming variant. When attributes and/or goals are in an imprecise environment and they cannot be stated with precision, it is appropriate to use fuzzy GP. A primary challenge in today's financial world is to determine how to proceed in the face of uncertainty, which arises from incomplete data and from imperfect knowledge. Volatility is an important challenge too, since estimates of volatility allow us to assess the likelihood of experiencing a particular outcome.

4 Goal Programming Variants for Portfolio Selection

Romero (2004) mentions that a key element of a GP model is the achievement function, which measures the degree of minimization of the unwanted deviational variables of the model's goals. Each type of achievement function leads to a different GP variant as follows.

4.1 Weighted Goal Programming in Portfolio Selection Models

The weighted GP for PS model usually lists the unwanted deviational variables, each weighted according to their importance.

Weighted Goal Programming (WGP) attaches weights according to the relative importance of each objective as perceived by the decision maker and minimises the sum of the unwanted weighted deviations (Tamiz and Jones 1995).

For example, the objective function in WGP model for PS seeks to minimise risk and maximise return by penalising excess risk and shortfalls in return, relative to the respective targets. Therefore, lower levels of risk and higher levels of return are not penalised. Additional objectives specifying other portfolio's attributes, such as liquidity, cost of rebalancing and sectors allocation can be included in the WGP model.

4.2 Lexicographic Goal Programming in Portfolio Selection Models

The achievement function of the lexicographic GP model to PS is made up of an ordered vector whose dimension coincides with the q number of priority levels established in the model. Each component in this vector represents the unwanted deviational variables of the goals placed in the corresponding priority level.

Lexicographic achievement functions imply a non-compensatory structure of preferences. In other words, there are no finite trade-offs among goals placed in different priority levels (Romero 2004).

The priority structure for the model can be established by assigning each goal or a set of goals to a priority level, thereby ranking the goals lexicographically in order of importance to the decision maker. When achieving one goal is equally important as achieving other goals, such as the goals of risk and return, then they may be included at the same priority level, where numerical weights represent the relative importance of the goals at the same priority level (Sharma and Sharma 2006).

Therefore, LGP could deal with many priority levels in PS problem, in which goal constraints are included according to their importance of achievement in the model. For example, a PS model using LGP may include the following priority structure:

1. Maximising the portfolio's expected return, while minimising some measurement of portfolio's risk.
2. Minimising other portfolio's risks (e.g. the systematic risk as measured by Beta coefficient).
3. Minimising the portfolio's cost of rebalancing.

+ Other priority levels.

Many authors developed lexicographic GP models for PS, particularly during the 1970s and 1980s (for example, Lee and Lerro 1973; Kumar et al. 1978; Levary and Avery 1984). Other applications of LGP for PS are developed recently within the mutual funds industry (Sharma and Sharma 2006).

4.3 MINMAX (Chebyshev) Goal Programming in Portfolio Selection Models

The achievement function of a Chebyshev GP model implies the minimization of the maximum deviation from any single goal. Moreover, when some conditions hold the corresponding solution represents a balanced allocation among the achievement of the different goals (Romero 2004).

The model of MinMax, Chebyshev, GP Portfolio Selection usually seeks the minimisation of the maximum deviation from any single goal in PS. In other words, it seeks the solution that minimizes the worst unwanted deviation from any single goal.

Some authors focus historically in developing PS models using MinMax GP variant. Deng et al. (2005), amongst others, develop MinMax GP model for PS.

4.4 Fuzzy Goal Programming in Portfolio Selection Models

While the weighted, lexicographic and MinMax forms of the achievement function are the most widely used, other recently developed variants, like Fuzzy Goal

Programming, may represent the decision makers' preferences or the decision making circumstances with more soundness.

Fuzzy mathematical programming is developed for treating uncertainties in the setting of optimisation problems. The fuzzy mathematical programming can be classified into three categories with respect to the kind of uncertainties treated in the method (Inuiguchi and Ramik 2000):

1. Fuzzy mathematical programming with vagueness.
2. Fuzzy mathematical programming with ambiguity.
3. Fuzzy mathematical programming with combined vagueness and ambiguity.

Vagueness is associated with the difficulty of making sharp or precise distinctions in the world; that is, some domain of interest is vague if it cannot be delimited by sharp boundaries, while ambiguity is associated with one-to-many relations, that is, situations in which the choice between two or more alternatives is left unspecified (Inuiguchi and Ramik 2000).

In fuzzy GP Portfolio Selection model, the decision maker is required to specify an aspiration level for each objective in the model in which aspiration levels are not known precisely. In this case, an objective with an imprecise level can be treated as a fuzzy goal (Yaghoobi and Tamiz 2006).

The use of fuzzy models not only avoids unrealistic modelling but also offers a chance for reducing information costs. Fuzzy sets are used in fuzzy mathematical programming both to define the objective and constraints and also to reflect the aspiration levels given by the decision makers (Leon et al. 2002).

In this context, Watada (1997) argues that Markowitz's approach to PS has difficulty in resolving the situation where the aspiration level and utility given by the decision makers cannot be defined exactly. Therefore, he proposes a fuzzy PS to overcome such difficulty. The fuzzy PS enables obtaining a solution which realizes the best within a vague aspiration level and the goal given as a fuzzy number, which is obtained from the expertise of the decision makers.

Gladish et al. (2007) argue that PS problem is characterized by imprecision and/or vagueness inherent in the required data, in which they proposed a three stage model, in order to mitigate such problems, based on a multi-index model and considering several market scenarios described in an imprecise way by an expert.

Gladish et al. (2007) discuss how the proposed fuzzy model allowed the decision maker to select a suitable portfolio taking into account the uncertainty related to the market scenarios and the imprecision and/or vagueness associated with the model data.

On the other hand, Leon et al. (2002) focus on the infeasible instances of different models, which are suppose to select the best portfolio according to their respective objective functions. They propose an algorithm to repair infeasibility. Such infeasibility, which usually provoked by the conflict between the desired return and the diversification requirements proposed by the investor, could be avoided by using fuzzy linear programming techniques.

Parra et al. (2001) deal with the optimum portfolio for a private investor with emphasis on three criteria, which are expected return of the portfolio, the variance return of the portfolio, and the portfolio's liquidity measured as the possibility of converting an investment into cash without any significant loss in value. They formulated these three objectives as a GP problem using fuzzy terms since they cannot be defined exactly from the point of view of the investors. Parra et al. (2001) propose a method to determine portfolios with fuzzy attributes that are set equal to fuzzy target values. Their solution is based on the investor's preferences and on the GP techniques.

Allen et al. (2003) investigate the notion of fuzziness with respect to funds allocation. They found that the boundary between the preference sets of an individual investor, for funds allocation between a risk free asset and the risky market portfolio, tends to be rather fuzzy as the investors continually evaluates and shifts their positions; unless it is a passive buy-and-hold kind of portfolio.

Inuiguchi and Ramik (2000) emphasize in their paper that the real world problems are not usually so easily formulated as mathematical models or fuzzy models. Sometimes qualitative constraints and/ or objectives are almost impossible to represent in mathematical forms. In such a situation, a fuzzy solution satisfying the given mathematical requirements are very useful in a sense of weak focus in the feasible area. Inuiguchi and Ramik (2000) applies fuzzy programming in PS problems and they found that decision maker can select the final solution from the fuzzy solution considering implicit and mathematically weak requirements.

5 Performance Measurement for Portfolios

Treynor (1965), Sharpe (1966), and Jensen (1968) developed the standard indices to measure risk adjusted returns for portfolios.

Numerous studies have tested the performance of portfolios (mutual funds) compared to a certain benchmark, usually market index, based on Sharpe, Treynor and Jensen performance measures (Artikis 2002; Cresson et al. 2002; Daniel et al. 1997; Lehmann and Modest 1987; Matallin and Nieto 2002; Otten and Schweitzer 2002; Raj et al. 2003; Zheng 1999).

Bottom-line performance measurement concentrates on the question of how a portfolio did, both absolutely and relative to a benchmark.

6 Goal Programming and Portfolio Analysis: Other Issues

The traditional portfolio optimisation model by Markowitz has not been used extensively in its original form to construct a large-scale portfolio. The first reason behind this is in the nature of the required inputs for portfolio analysis, in which accuracy is needed for returns as well as the correlation of returns. The second reason is the computational difficulty associated with solving a large-scale quadratic programming problem with dense (covariance) matrix.

Several researchers have tried to alleviate these problems by using various approximation schemes to obtain equivalent linear problems (such as Steuer et al. 2007). The use of index model reduces the amount of required computation by introducing the notion of factors influencing stock prices. However, these factors are discounted because of the popularity of equilibrium models such as the Capital Asset Pricing Model (CAPM).

The CAPM states that the expected return on a security depends only on the sensitivity of its return to the market return, its market beta. However, there is evidence that market beta does not suffice to describe expected return. In addition, the CAPM fares poorly in competition with multifactor alternatives. This evidence suggests that multifactor models should be considered in any research that requires estimates of expected returns. One popular multifactor model is the Arbitrage Pricing Theory (Fama 1996).

A factor model is not an equilibrium theory, in which it represents relationships among security returns. However, when returns are generated by a factor model, equilibrium in the capital markets will result in certain relationships among the values of the coefficients of the model. The Arbitrage Pricing Theory (APT), Like Capital Asset Pricing Models, is an equilibrium theory of the relationships between security expected returns and relevant security attributes. Unlike CAPM, the APT assumes that returns are generated by an identifiable factor model. However, it does not make strong assumptions about investor preferences (Sharpe 1985).

In order to facilitate application of his own covariance approach, Markowitz first suggested, and Sharpe (1966) later developed a market model formulation in which the rates of return on various securities are related only through common relationships with some basic underlying factor (Frankfurter and Phillip 1980).

Although GP and its variants have provided a more pragmatic tool to analyse PS problems and reach good solutions in terms of the inclusion of the decision maker's factors of importance in selecting portfolios, there are still other aspects of PS problems that can benefit from the application of GP. Some of these issues are listed and explained below.

6.1 Issues Concerning Multi-Period Returns

GP can be utilised to select portfolios on not only the basis of many factors, but also based on the future multi-period returns as well as the expected utility of multi-period returns.

Modem portfolio analysis has its origin in the work of Markowitz, who specified the portfolio problem in terms of the one-period means and variances of returns. However, most portfolio problems are multi-period. The appropriateness of one-period analysis for this class of problems has been seriously questioned in recent years. As a result, several alternative decision rules and modification of the one-period analysis have been proposed.

- Elton and Gruber (1974) evaluate two proposals that have received wide attention in the economic literature. The first involves selecting portfolios on the basis of the geometric mean of future multi-period returns. The second involves selecting portfolios on the basis of the expected utility of multi-period returns. They found that, when portfolio revision is considered, portfolio decisions based on either the expected utility of multi-period returns or the geometric mean of multi-period returns are often different from and inferior to decisions based on consideration of returns sequentially over time. This is true even when the distribution of returns is expected to be identical in each future period.
- Li and Ng (2000) consider an analytical optimal solution to the mean-variance formulation in multi-period PS. They extend the Markowitz mean-variance approach to multi-period PS problems. The derived analytical expression of the efficient frontier for the multi-period PS would enhance investors' understanding of the trade-off between the expected terminal wealth and the risk. At the same time, the derived analytical optimal multi-period portfolio policy provides investors with the best strategy to follow in a dynamic investment environment.
- Samuelson (1969) formulates and solves a many-period generalization, corresponding to lifetime planning of consumption and investment decision in his paper of Lifetime PS by Dynamic Stochastic Programming.
- Renwick (1969) emphasize almost 40 years ago that there was rarely anything even approaching unanimous agreement on any particular point of theory or interpretation of empirical data with relevance to financial analysis.

GP, if properly utilised, could provide a good approach to PS and analysis in today's complicated financial markets with multi-period returns.

6.2 Issues Concerning Extended Factors

There are a number of issues which have been introduced into practical PS problems. These include restriction on the number of assets, transaction and rebalancing costs, and cash flow or liquidity requirements.

In practice, analysts use models with both common factors, which affect all securities to a greater or lesser extent, and sector factors, which affect only some securities within a portfolio. Identification and prediction of truly pervasive factors is an extremely difficult task. Hence, the goal should be focused on permanent and important sources of security and portfolio risk and return, not the transitory and unimportant phenomena that occur in any given period (Sharpe 1985).

Nonetheless, an extended Capital Asset Pricing Models imply that expected returns may be related to additional security attributes, such as liquidity and rebalancing costs. Some of these may, in turn, be related to sensitivities to major factors.

GP and its variants provide a practical way to incorporate an extended list of factors, other than risk and return, in portfolio analysis.

For example, Steuer et al. (2007) focus on investors whose purpose is to build a suitable portfolio taking additional concerns into account. Such investors would have additional stochastic and deterministic objectives that might include dividends, number of securities in a portfolio, liquidity, social responsibility, and so forth. They develop a multiple criteria PS formulation.

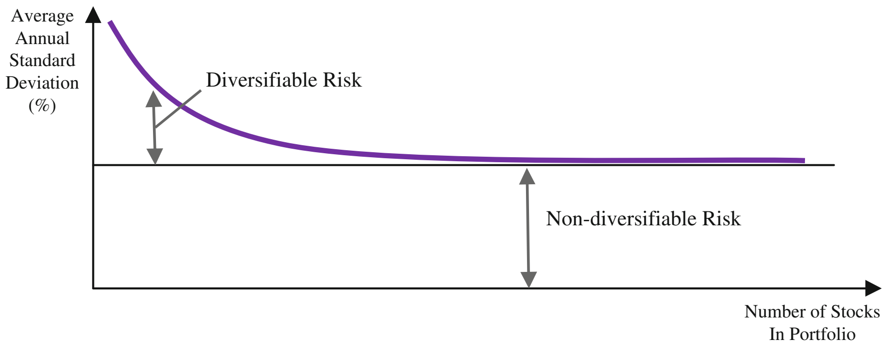
Despite the acceptance and wide-spread use of the Markowitz framework, and its numerous extensions, in practice there has been a considerable debate among academics and practitioners on the validity of including only two factors for Portfolio Selection problems and equally important the validity of variance as a representative measure of risk.

6.3 Issues Concerning the Measurement of Risk

The notion of risk has found practical application within the science of Risk Management and Risk Control. Risk Control deals with limiting or eliminating specific types of risk, in as much as this is possible by taking an active position in one or more types of risk. Deciding which types of risk to mitigate is the first dilemma of a financial institution and demands considerable attention, since focusing on one particular risk category may lead to a hedged portfolio for a particular source of risk but may result in exposure to other sources of risk.

An important insight of modern financial theory is that some investment risks yield an expected reward, while other risks do not. Risks that can be eliminated by diversification do not yield an expected reward, while risks that cannot be eliminated by diversification do yield an expected reward. Thus, financial markets are somewhat fussy regarding what risks are rewarded and what risks are not (Corrado and Jordan 2005).

Diversification reduces risk, but only up to a point since some risk is diversifiable and some is not as illustrated below:



This issue becomes more challenging when optimisation models are used such as GP. For example, a GP model may result in minimisation of the risk included in the model, but the solution may be sensitive to other sources of risks that were not considered and better measured by another metric.

According to Sharpe (1966) model, the rate of return on any security is the result of two factors; a systematic component which is market related, and factors which are unique to a given security. In any application, however, concern should be not only with the alpha and beta, but with the level of uncertainty about the estimates as well.

Hu and Kercheval (2007) emphasize that portfolio optimisation requires balancing risk and return; therefore, one needs to employ some precise concept of risk. The construction of an efficient frontier depends on two inputs; a choice of risk measure, such as standard deviation, value at risk, or expected shortfall, and a probability distribution used to model returns.

Many authors provide analysis of risk measures beyond the standard deviation, such as Artzner et al. (1999), Balbas et al. (2009) and Rockafellar et al. (2006).

- For example, Mansini et al. (2007) mention that while some Linear Programs (LP) computable risk measures may be viewed as approximations to the variance (e.g., the mean absolute deviation), shortfall or quantile risk measures are recently gaining more popularity in various financial applications. Therefore, Mansini et al. (2007) study LP solvable portfolio optimization models based on extensions of the Conditional Value at Risk (CVaR) measure. The models use multiple CVaR measures thus allowing for more detailed risk aversion modeling.
- Pflug (2006) researches the measures of risk in two categories, which are risk capital measures, serve to determine the necessary amount of risk capital in order to avoid ruin if the outcomes of an economic activity are uncertain and their negative values may be interpreted as acceptability measures or safety measures, and pure risk measures, risk deviation measures, which are natural generalizations of the standard deviation.
- Rockafellar et al. (2006) study general deviation measures systematically for their potential applications to risk measurement in areas like portfolio optimization and engineering.
- Ogryczak and Ruszczynski (2002) analyse mean-risk models using quantiles and tail characteristics of the distribution. In particular, they emphasise value at risk (VAR) as a widely used quantile risk measure, which is defined as the maximum loss at a specified confidence level. Their study included also the worst conditional expectation or Tail VAR, which represents the mean shortfall at a specified confidence level.
- Artzner et al. (1999), develop a coherent measure of risk, in which they studied both market risks and nonmarket risks, and then discussed methods of measurement of these risks.

Economic analysts following the mean-variance maxim have concentrated upon the problem of portfolios as financial assets with little or no effort being directed to the inclusion of productive liabilities. Accordingly, the usual portfolio analysis assumes the absolute level of funds available for investment as fixed and concerns itself only with the distribution of that given amount over candidate opportunities. In a wide variety of applications, neither part of this restriction is either essential or desirable.

As James Tobin, the winner of the Nobel Prize in economics, showed that the investment process can be separated into two distinct steps, which are the construction of an efficient portfolio, as described by Markowitz, and the decision to combine this efficient portfolio with a riskless investment (Kritzman 2003), GP could be by far a very powerful technique for empowering the investment decision making as well as the investment process in general.

7 Conclusions

Over the last 30 years, GP for Portfolio Selection problems have been deployed extensively.

This paper has briefly reviewed many of the highlights. GP models for PS allow incorporating multiple goals such as portfolio's return, risk, liquidity, expense ratio, amongst other factors.

There is a huge capacity for future developments and applications of GP for PS issues.

In particular, GP could be used for incorporating multi-period, extended factors and different risk measures into the PS analysis. Also, the decision maker can establish target values not only for the goals but also for relevant achievement functions.

In this way, a Meta-GP model could be formulated, which allows the decision-maker to establish requirements on different achievement functions, rather than limiting their opinions to the requirements of a single variant. In this sense, this approach could be used as a second stage after GP problem for PS is being solved (Uria et al. 2002).

Future research is warrant in the area of GP applications to PS, particularly for Mutual Funds as the need for incorporating extended factors is greatly manifest.

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