Chapter 5
The Quantum-like Brain

In Chapter 3, the contextual probabilistic model was invented: the Växjö model. Now it is applied to the description of mental processes. This description is based on QL representations – by probability amplitudes – in cognitive, social and political sciences, psychology, and economics. In particular, this model suggests interesting cognitive experiments to check QL structures of mental processes. The crucial role is played by interference of probabilities for *mental observables.* Recently, such experiments based on recognition of *ambiguous images* have been performed by Conte et al. [66, 67]. These experiments confirmed my prediction [173, 180] of the QL behavior of mind. In the Växjö approach "quantumness of mind" has no direct relation to the fact that the brain (as any physical body) is composed of quantum particles. A new terminology *quantum-like mind* is used. Cognitive QL behavior is characterized by a nonzero coefficient of interference (supplementarity) λ, see Section 3.2. It can be found on the basis of statistical data. The hypothesis of QL mind can be tested experimentally!

The Växjö model predicted, see Chapter 4, not only $\cos\theta$ interference of probabilities, but also hyperbolic $\cosh\theta$ interference. The latter type of interference has never been observed for physical contexts, but such a possibility cannot be excluded for cognitive systems, see [275] and Chapter 7 for more details.

In this chapter, a model of the brain's functioning as a *QL computer* is proposed; the difference between quantum and QL computers is discussed.

5.1 Quantum and Quantum-like Cognitive Models

The idea that the description of the brain's functioning, cognition, and consciousness cannot be reduced to the theory of neural networks and dynamical systems (see Ashby [19], Hopfield [150], Amit [15], Strogatz [285], van Gelder [296], van Gelder and Port [297]), and that quantum theory may play an important role has been discussed in a huge variety of forms, see, e.g., Whitehead [303] , Orlov [247], Albert and Loewer [10], Albert [11], Healey [138], Lockwood [229], Penrose [249, 250], Donald [91– 93], Jibu and Yasue [154], Bohm and Hiley [40], Stapp [284], Hameroff [128, 129], Loewer [230], Hiley and Pylkkänen [144],

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Deutsch [86], Barrett [29], Khrennikov [161, 173, 175, 176, 180, 198], Vitiello [298] and literature therein.

One dominant approach to the application of QM formalism to the description of brain functioning is *quantum reductionism,* see e.g. Hameroff [128, 129] and Penrose [249, 250]. This was a new attempt at physical reduction of mental processes, cf. Ashby [19], Hopfield [150], Amit [15]. This is an interesting project of great complexity and it is too early to draw any conclusions about its future. One important contribution of quantum reductionism is critique of the classical reductionist approach (neural networks and dynamical systems approach) and artificial intelligence, see especially Penrose [249, 250]. On the other hand, quantum reductionism has been strongly criticized by neurophysiologists and cognitive scientists, who assume that the *neuron is the basic unit of processing of mental information.*

We mention the *quantum logic* approach: mind cannot be described by classical logic and the formalism of quantum logic should be applied. Orlov [247] published the first paper in which this idea was explored. It is important to remark that he discussed interference within a single mind. Such an interference was also discussed by Deutsch [86]. We point to extended investigations based on the *many-minds* approach, see Healey [138], Albert and Loewer [10], Albert [11], Lockwood [229], Donald [91– 93], Loewer [230], Barrett [29], etc. Finally, we mention attempts to apply *Bohmian mechanics* to describe mental processes – Bohm and Hiley [40], Hiley and Pylkkänen [144], Khrennikov [161], Choustova [54–62].

In [198] I developed the theory of "quantum-like mind", which is presented in this chapter.[1] As was already emphasized, the QL approach has nothing to do with *quantum reductionism.* Of course, I do not claim that my approach implies that quantum physical reduction of mind is totally impossible. However, I can explain the main QL feature of mind – *interference of minds* – without reduction of mental processes to quantum physical processes. Consequently my QL model does not face such horrible problems of QM as nonlocality or death of realism.[2] One may ask:

Why is it so important to combine realism with quantum probabilistic features in neurophysiology, cognitive sciences, psychology and sociology?

A fundamental consequence of the possibility of such a combination is that macroscopic neuronal structures (in particular, a single neuron) as well as cognitive and psychological contexts can exhibit QL features. It is possible to eliminate the fundamental problem disturbing adherents of quantum physical reductionism:

How can one combine the neuronal (macroscopic) and quantum (microscopic) models?

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[1] Recently, Busemeyer, a professor of psychology, has explained some paradoxical features of psychological behavior by using a QL model that was based on an approach very similar to that developed in the author's papers, namely, on the QL deformation of the classical formula of total probability, see e.g. [48, 49]. It is amazing that people working in such different domains of science as foundations of probability theory and psychology arrive at similar models.

[2] I reject the idea of using quantum nonlocality in cognitive science as totally absurd.

It is a terrible problem for everybody who tries to proceed with quantum reductionism, e.g., for Penrose [250]: "It is hard to see how one could usefully consider a quantum superposition consisting of one neuron *firing,* and simultaneously *non-firing.*"

In the Växjö model it is possible to operate with QL probabilities without appealing to such a notion as *superposition of states of a single system,* see Chapter 4. All distinguishing probabilistic features of quantum mechanics can be obtained without it. This implies that, unlike quantum reductionism, there is no need to look for the microscopic basis of mental processes.[3] In my model *"mental interference"* is not based on superposition of individual quantum states. Mental interference is described in a classical (but contextual) probabilistic framework. A *mental wave function* represents not a mental state of an individual cognitive system, but a neurophysiological, cognitive or psychological context C, see Chapter 4.[4]

In particular, Växjö model can be applied to the description of mental observations in the QL terms. We start with *mental interference*, which is defined as interference of probability distributions of two *supplementary mental observables*, see Definition 3.2, Section 3.2. For example, in psychology such observables can be realized in the form of two supplementary questions that are asked to people participating in a test. A condition of supplementarity can be checked easily on the basis of experimental statistical data collected in the form of "yes-no" answers to questions. The magnitude of mental interference is characterized by a coefficient of interference (or supplementarity) λ. Depending on this magnitude we obtain different representations of probabilities in experiments with cognitive systems. In particular, we obtain the QL representation of cognitive (or social, or economic) contexts in complex (or maybe even hyperbolic) Hilbert space, by using the representation algorithm, QLRA, given in Chapter 4. This approach should be justified experimentally. A priori there is no reason for cognitive systems to exhibit QL probabilistic behavior, in particular, nontrivial interference. We present the detailed description of a few experimental tests to check the hypothesis of QL probabilistic behavior. We hope that a variety of such tests will be performed in various domains of science: psychology, cognitive science and sociology, economics, see [66, 67].

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[3] Reductionists should do this and go to the deepest scales of space and time to find some reasonable explanation of superposition and interference (e.g., go inside *microtubules* or to scales of *quantum gravity*).

[4] My comparison of the contextual approach and quantum reductionism cannot be used as an argument against the latter. One could not exclude the possibility that mental processes could be reduced to quantum physical processes, e.g., in microtubules, or that the act of consciousness is really induced by the collapse of the wave function of superposition of two mass states. However, the Växjö model makes it possible to use quantum mathematical formalism in neurophysiology, cognitive science, psychology, and sociology without all those tricky (quantum physical) things that are so important in the reductionist approach.

5.2 Interference of Minds

5.2.1 Cognitive and Social Contexts; Observables

We consider examples of cognitive contexts and observables that can be measured for these contexts.

1) C is a procedure of selection of a specific group S_C of people or animals (creation of an ensemble of cognitive systems). Context C is represented by this group S_C. For example, a group $S_{\text{prof.math.}}$ of professors of mathematics is selected. Then one can perform "mental measurements" by asking questions or giving tasks. In the simplest experiment, to check interference of minds, two questions, say a and b, are asked. We can select a group of people of a particular age or a group of people having a specific mental state: for example, people in love, hungry or depressed.

2) C is a learning procedure that is used to create some specific group of people or animals. For example, rats are trained to react to a special stimulus. Students are trained in probability theory. Here a and b are two supplementary questions (Definition 3.4) given in the exam. For instance, a is a theoretical question or task, e.g., to prove the central limit theorem (CLT), and b is a practical question, e.g., to find the average with respect to a given probability distribution.[5] In this example, post-measurement condition (3.5) holds (Section 3.1.1: "projection postulate"). Suppose that a student proved CLT. Ask him to do the same, within a reasonable period of time. We can be practically sure (up to small statistical deviations) that he will prove it again. Thus

$$\mathbf{P}(\text{to prove CLT}|\text{CLT was proven}) = 1$$

as well as $\mathbf{P}(\text{not prove CLT}|\text{CLT was not proven}) = 1$.

3) C is a collection of paintings, C_{paint} (e.g., the collection of the Hermitage in St. Petersburg) and people interact with C_{paint} by looking at the pictures. Mental measurements are based on questions which those people are asked about this collection.

4) C is "context of classical music", $C_{\text{clmus.}}$, and people interact with $C_{\text{clmus.}}$ by listening to this music. In principle, we need not use an ensemble of different people. It can be one person of whom we ask questions each time after he has listened to a CD (or radio program) of classical music.

The last two examples illustrate why we started with the contextual approach and not simply ensembles of systems. A cognitive context need not be identified with an ensemble of cognitive systems representing this context. For us C_{paint} and $C_{\text{clmus.}}$ and not ensembles of people representing them, $S_{C_{\text{paint}}}$ and $S_{C_{\text{clmus.}}}$, are basic.

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[5] The problem of supplementarity is very delicate. For example, if the first task was to prove CLT and the second to find the average with respect to a concrete Gaussian distribution, then a and b are definitely not supplementary: $\mathbf{P}(b = +|a = +) = 1$ and $\mathbf{P}(b = -|a = +) = 0$. However, if the first question was on the Poisson distribution and the second to find the average with respect to the Gaussian distribution, then they can be considered as supplementary: $\mathbf{P}(b = \pm|a = \pm) > 0$.

We can also consider *social contexts*, for example, social classes: proletariat and bourgeois contexts; or war and revolution contexts, financial crises context, poverty and welfare contexts, and so on.[6]

5.2.2 Quantum-like Structure of Experimental Mental Data

We describe a mental interference experiment. Let $a \in X_a = \{\alpha_1, \alpha_2\}$ and $b \in X_b = \{\beta_1, \beta_2\}$ be two dichotomous mental observables, e.g., two questions: α_1='yes', α_2='no', β_1='yes', β_2='no'. We use these two fixed reference observables for the probabilistic and then QL representations of cognitive reality given by some context C.[7] This context is assumed to be reproducible such that repeatable measurements of both reference observables can be performed. It can be very sensitive and each measurement may change it essentially.

We perform observations of b under C and obtain frequencies

$$\nu_C^b(\beta) = \frac{\text{the number of results } b = \beta}{\text{the total number of observations}}, \quad \beta \in X_b.$$

When the total number of observations $N \to \infty$, the frequencies $\nu_C^b(\beta) \equiv \nu_C^b(\beta; N)$ approaches the probability $p_C^b(\beta)$ of getting the result β for the b-observation. We also define frequencies $\nu_C^a(\alpha)$ and probabilities $p_C^a(\alpha)$ for the a-observation.[8]

As was supposed in Section 3.1, selection-contexts are given, e.g., $C_\alpha, \alpha \in X_a$. They are created in the following way. Measurements of a are performed (the question a is asked to all cognitive systems selected for this experiment). Cognitive systems who answered $a = \alpha$ are selected. The C_α produces an ensemble of cognitive systems, say S_{C_α}.[9] Now b-measurements are performed under cognitive context C_α – for the ensemble S_{C_α}. We find frequencies $(\beta \in X_b, \alpha \in X_a)$:

$$\nu_{\beta|\alpha} = \frac{\text{the number of the result } b = \beta \text{ under context } C_\alpha}{\text{the total number of observations under context } C_\alpha},$$

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[6] The Växjö model can be used in social and political sciences and even in history. One can try to find QL corresponding data. It would be amazing to show that the historical process can exhibit QL features.

[7] In general by choosing another pair of reference observables we shall obtain another representation of cognitive contextual reality. Can we find two fundamental mental observables? This is a very difficult question. In physics the answer is well known: the position and the momentum form the fundamental pair of reference observables. Which mental observables can be chosen as mental analogons of the position and the momentum?

[8] Observables, e.g., questions, should be supplementary, Definition 3.4. The answer, e.g., a ='yes' does not pre-determine (statistically) the answer to the subsequent question b. Moreover, the questions should satisfy post-measurement condition (3.5), Section 3.1.1: "projection postulate".

[9] In the general case the situation is more complicated, Section 5.2.3. However, we restrict considerations to the mentioned scheme of creation of selection contexts. In any event it was used in experiments [66].

and the corresponding probabilities $p_{\beta\alpha}$. Data to find the interference (supplementarity) coefficient (3.17) are collected. We operate with frequencies

$$\lambda_{\text{ex}}(b=\beta|a,C)=\frac{\nu_C^b(\beta)-\nu_C^a(\alpha_1)\nu_{\beta|\alpha_1}-\nu_C^a(\alpha_2)\nu_{\beta|\alpha_2}}{2\sqrt{\nu_C^a(\alpha_1)\nu_{\beta|\alpha_1}\nu_C^a(\alpha_2)\nu_{\beta|\alpha_2}}}. \tag{5.1}$$

An empirical situation with $\lambda_{\text{ex}}(b=\beta|a,C)\neq 0$ would yield evidence for QL behavior of cognitive systems. In this case, starting with the (experimentally calculated) coefficient of interference $\lambda_{\text{exp}}(b=\beta|a,C)$ we can proceed either to the conventional Hilbert space formalism (if this coefficient is bounded by 1) or to so-called hyperbolic Hilbert space formalism (if this coefficient is larger than 1), see Chapter 4 and more in the book [214].

5.2.3 Contextual Redundancy

We remark that in general transition probabilities $p_{\beta\alpha}$ can depend on the original cognitive context C :

$$p_{\beta|\alpha}=p_C(\beta|\alpha)$$

To perform the $[a=\alpha]$-selection, one should first perform measurement of a for some initial context C. In general, there is no reason to hope that after subsequent measurement of another (even supplementary) observable, denoted by b, dependence on C will disappear.

Let us consider the very special case when dependence of the transition probabilities $p_C(\beta|\alpha)$ on C is redundant. For example, students belonging to the group S_C (which was trained under the mental or social conditions C) should answer the question a. After this we select a new ensemble S_{C_α} of students who have answered $a=\alpha$. If this question is so important for a student that he totally forgets about the previous C-training and remembers only the previous answer $a=\alpha$, then the transition probabilities do not depend on C and the index C can be omitted:

$$p_C(\beta|\alpha)\equiv p_{\beta|\alpha}. \tag{5.2}$$

We call (5.2) the condition of *contextual redundancy*. Condition of contextual redundancy is similar to condition of *Markovness* in classical probability theory.

The total destruction of memory of the previous context C (i.e., learning procedure) is too strong a metaphor. It is better to consider an *essential state update*. Thus the memory on C is still present, but the experience generated by an interaction with the question a will dominate in interaction with a subsequent question b. In the example with the exam in probability theory, see Sect. 5.2.1, by proving CLT a student does not destroy the memory of the course in probability theory. However, his state of mind was essentially updated in the process of proving CLT. Consider a number of contexts $C, C', \ldots$ corresponding to courses in probability theory at various universities. Consider $[a = +]$-selection contexts

$$C_+^a(C), C_+^a(C'), \ldots, \tag{5.3}$$

a selection of students who proved CLT in the exams on probability theory. For a sufficiently large spectrum of supplementary questions, the condition of contextual redundancy (5.2) holds. Thus contexts (5.3) can be identified and considered as one context, selection context C_+^a. Of course, condition (5.2) cannot hold for all possible (supplementary) questions b. However, in this book we typically operate only with a pair of supplementary questions.

We remark that contextual redundancy takes place in QM for observables with *nondegenerate spectra.* Here the transition probabilities do not depend on the original context C, the preparation procedure for a quantum state ψ, see formula (2.50), Sect. 2.4. One can (but need not!) also appeal to von Neumann's projection postulate, Sect. 12.3. If quantum observable a is represented by the operator $\widehat{a}$ having nondegenerate spectrum, then the post-measurement state is just one of the eigenvectors of $\widehat{a}$. Memory about the pre-measurement state ψ is completely destroyed by a-measurement. We remark that QM can be considered as a contextual model: contexts are given by quantum states: $C \equiv C_\psi$, see Sect. 12.4. Thus under the condition of contextual redundancy we obtain a class of Växjö models that is the closest to QM (for observables with nondegenerate spectra).

However, we do not want to restrict our considerations to this class of models. How can we proceed in the general case? Some context, say $\Omega \in \mathcal{C}$, should be chosen as a "basic context". Corresponding contexts $C_\alpha^a(\Omega)$ are declared as C_α^a-contexts of the model, cf. with Kolmogorovian contextual models in Sect. 12.4. In the latter case the total space of elementary events Ω is considered as the basic context, and here $C_\alpha^a \equiv C_\alpha^a(\Omega) = \{\omega \in \Omega : a(\omega) = \alpha\}$.

The problem of finding of an adequate basic context $\Omega \in \mathcal{C}$ is very complicated. In fact, transition probabilities encode correlations between observables, see (3.9), Section 3.1.4. Therefore the basic context Ω should be selected to represent the pure (as much as possible) correlation effect between observables a and b. Of course, it depends of the concrete pair of reference observables a and b, i.e., $\Omega = \Omega(a, b)$.

Finally, we come back once again to the example with the exam in probability theory, see Sect. 5.2.1. To prove CLT, a student should invest a lot of effort, in particular, this (very complicated) proof takes time. Thus, as in QM, the process of measurement is a complex process of interaction between a system and a measurement device. In the present example, systems are students, but the a-measurement device is CLT, i.e., a mental structure.[10]

[10] Well, there are also teachers in this exam.

5.2.4 Mental Wave Function

The algorithm (QLRA): $C \to \psi_C$, Chapter 4, represents cognitive, social, psychological, and economic contexts by complex and hyperbolic amplitudes. To obtain a closer analogy with QM, one can speak about the *mental wave function.* One need not imagine "mental waves." In the contextual approach the mental wave function $\psi \equiv \psi_C$ is simply a special representation of probabilistic data collected about context C with the aid of two (specially selected) reference observables a and b.

I speculate that some *cognitive systems developed (in the process of evolution) the ability to operate with mental wave functions,* i.e., to represent probabilistic data in linear space. Roughly speaking, such a system does not feel individual counts, but the general statistics encoded in the ψ_C. In this sense the mental wave function ψ_C is an element of mental reality. Encoding by ψ_C provides a possibility for linear processing of data.

5.3 Quantum-like Projection of Mental Reality

The QL representation for mental processes is a projection of the neuronal model to the complex (or hyperbolic) Hilbert space model. It induces huge loss of information produced by the neurons.

5.3.1 Social Opinion Poll

Let us consider a family of social contexts $\mathcal{C}$ such that each context corresponds to the society of some country: $C_{\mathrm{USA}}, C_{\mathrm{GB}}, C_{\mathrm{FR}}, ..., C_{\mathrm{GER}}, ...$ and let us consider two reference observables given by the questions

a) "Are you against pollution?" and
b) "Would you like to have lower prices for gasoline?"

It is supposed that observables a and b are supplementary:

$$\mathbf{P}(b = \mathrm{yes}|\mathrm{a} = \mathrm{no}) \neq 0, \quad \mathbf{P}(\mathrm{b} = \mathrm{no}|\mathrm{a} = \mathrm{no}) \neq 0,$$
$$\mathbf{P}(b = \mathrm{yes}|\mathrm{a} = \mathrm{yes}) \neq 0, \quad \mathbf{P}(\mathrm{b} = \mathrm{no}|\mathrm{a} = \mathrm{yes}) \neq 0.$$

Moreover, the transition probabilities $\mathbf{P}(b = \beta|a = \alpha)$ do not depend on a society C, condition of contextual redundance holds. For example, the proportion of people who are against pollution among people who are satisfied by prices for gasoline is the same in the USA, Great Britain, France, and so on. Of course, this is a rather strong assumption.

In our QL-model societies are represented by complex (or maybe hyperbolic?) probability amplitudes

$$\psi_{\text{USA}}, \quad \psi_{\text{GB}}, \quad \psi_{\text{FR}}, \ldots, \quad \psi_{\text{GER}}, \ldots$$

These mental wave functions can be used to describe the dynamics of these societies. However, answers to the questions a and b do not completely characterize a society. Thus this QL representation induces a huge loss of information about the society.

5.3.2 Quantum-like Functioning of Neuronal Structures

Let us consider two coupled neural networks G_1 and G_2. They interact with a family of contexts $\mathcal{C} = \{C\}$, which are given by input signals into both networks. For example, contexts $\mathcal{C} = \{C\}$ can be visual images and networks G_1 and G_2 contribute to recognition of these images, e.g., G_1 is responsible for contours and G_2 for colors. I emphasize from the very beginning that in my model an image in the brain is not created by networks. It is the result of the QL representation of statistics of signals produced by networks.

We use the so-called *frequency-domain approach*, see for example *Hoppensteadt* [151], and assume that cognitive information is presented by frequencies of firing of neurons. We recall that in the process of interaction with the cognitive context frequencies of firing of neurons in, e.g., the network G_1 are synchronized. It is possible to speak about the *"network frequency."* Finally, we point out that each network can be widely distributed in the brain. Thus spatially separated neurons fire synchronously.

Typically a network has a hierarchic structure and the network's frequency can be identified with the frequency of firing of the network's *conductor.* Denote conductors of G_1 and G_2 by symbols c_{G_1} and c_{G_1}. We are aware that the question of the presence of a hierarchic structure of neural networks in the brain and, in particular, the existence of neuron-conductors [14], *"grandmother neurons",* is still a source of intense debate in the neurophysiological community, see, e.g., [232] on experimental results in favor of the neural hierarchy. Therefore later we will attempt to exclude such conductors from our model. However, the use of them makes the model more illustrative.

Consider two reference observables a, b. Here $a = +$ if the neuron c_{G_1} is firing, and $a = -$ if the neuron c_{G_1} is non-firing, and $b = +$ if the neuron c_{G_2} is firing, and $b = -$ if the neuron c_{G_2} is non-firing. Probabilities $p_C^a(\pm)$, $p_C^b(\pm)$ are defined by frequencies of firing. Consider a possible mechanism of production of frequency probabilities:

Two *time scale* parameters, depending on the cognitive system, are given: Δ is the time scale of production of probabilities ("probabilistic images"), δ is the duration (average) of a pulse from a neuron. Set $\tau = \delta/\Delta$. Let $n_C^a(+)$ be the number of pulses produced by G_1 during the interval Δ (in the process of interaction with cognitive context C). Then probability is given by

$$p_C^a(+) = \tau n_C^a(+), \quad p_C^a(-) = 1 - p_C^a(+).$$

Probabilities $p_C^b(\pm)$ are defined in the same way. These probabilities are easily expressed in networks' frequencies. Let G_1 oscillate (synchronously) with frequency $f_C^{G_1}$ oscillations per second. Then $n_C^a(+) = f_C^{G_1} \Delta$ and

$$p_C^a(+) = f_C^{G_1} \delta. \tag{5.4}$$

Thus it is possible to define probabilities even without involving hierarchic structures and conductor neurons. It is enough to know the frequencies of synchronized (in the process of interaction with C) firings for the corresponding networks. These probabilities provide partial information on the neuronal representation of context C.

Transition probabilities are defined in the following way. First, we should find an appropriate basic context $\Omega = \Omega(G_1, G_2)$, see the very end of Section 5.2.3. As was pointed out, it should be the basis of estimation of pure correlations between two networks G_1 and G_2. So, the specific influence of concrete cognitive context C should be eliminated, as much as possible. One can speculate that Ω corresponds to the state of relaxation. For example, G_1 and G_2, performing the image recognition are not excited by interaction with images.

Denote by $n_{+|+}$ the number of c_{G_2} firings during the periods of c_{G_1} firing, i.e., the number of "matched firings." Then

$$p_{+|+} = \tau n_{+|+}, \quad p_{-|+} = 1 - p_{+|+}.$$

It is also clear how to find probabilities $p_{\pm-}$. Thus the matrix of transition probabilities is created in advance in the state of relaxation.[11]

The brain can now execute QLRA (and we assume that it really can do this) and represent context C (e.g., an image C) by the amplitude ψ_C. This vector in Hilbert space is the mental image of context C.

Of course, ψ_C provides only a rough projection of the neuronal image of the context C. However, we cannot exclude that cognition (and especially consciousness) is really based on such a QL-projecting of neuronal states. The brain makes its decisions by operating with mental wave functions and not with frequencies of firings. In cognitive literature, the problem of the *neural code* is widely discussed. My conjecture is that the neural code is given by QLRA, transforming frequencies of firings into probability amplitudes.

Denote by κ the average time for processing of QLRA, i.e., the time that is required to produce ψ_C on the basis of probabilistic data, namely $W(a, b, C) = \{p_C^a(\pm), p_C^b(\pm)\}$, collected on C. Intervals of time which are less than $\Delta_{\text{cogn}} = \Delta + \kappa$

[11] By coupling our model with EEG studies of the brain, we can say that the latter state is characterized by frequencies of α-waves in the brain. The states of active interaction with sufficiently complex cognitive contexts are characterized by frequencies of β- and γ-waves. By (5.4) probabilities $p_C^a(+)$ increase with increasing brain-wave frequencies. In contrast, transition probabilities do not vary; they are rigidly coupled to the α-waves.

has no cognitive meaning. So, the right *cognitive scale* is given by $\Delta_{\rm cogn}$. This scale corresponds to the dynamics of the mental wave function, $t \mapsto \psi(t)$.

We mention experimental evidence that a) cognition is not based on continuous-time processes (a moment in "cognitive time" correlates with $\Delta_{\rm cogn} \approx 100$ ms of physical time); b) different *psychological functions* based on groups of neural networks performing specific cognitive tasks operate on different scales of physical time. In [173, 176] mental time was described mathematically by using p-adic hierarchic trees; see also [157–160] for applications of p-adic numbers in mathematical physics.

5.4 Quantum-like Consciousness

The brain is a huge information system that contains millions of patterns of neural activation. It could not "recognize" (or "feel") all those patterns at each instant of time t. Our fundamental hypothesis is that the brain is able to create the QL-representations of neural patterns. At each instant of time t, the brain creates the QL-representation of its mental context C based on two supplementary mental *self-observables* a and b. Here $a = (a_1, ..., a_n)$ and $b = (b_1, ..., b_n)$ can be very long vectors; each of them consists of nonsupplementary dichotomous observables. The *reference self-observables* can be chosen by the brain in different ways at different instances of time. Such a change of the reference observables is known in cognitive sciences as a *change of the representation.*

A mental context C in the $a|b$-representation is described by the mental wave function ψ_C. We can speculate that the brain has the ability to feel this mental field, a field of probability amplitudes.

In such a model the state of consciousness is represented by the mental wave function ψ_C. It is a projection of neuronal mental activity. The latter forms subconsciousness. We can say that one has the classical *subconsciousness* and the QL consciousness. We remark that this is a rather unusual viewpoint. Typically the consciousness is considered as the classical part of the brain's functioning and subconsciousness as quantum.

The crucial point is that in my model the consciousness is created through neglecting an essential volume of information contained in the subconsciousness. Of course, it is not just a random loss of information. Information is selected through the algorithm QLRA: a context C is projected onto ψ_C.

The (classical) mental state of subconsciousness evolves with time $C \rightarrow C(t)$. This dynamics induces dynamics of the mental wave function $\psi(t) = \psi_{C(t)}$ in complex Hilbert space, "mental Schrödinger dynamics."

Postulate QLR. *The brain is able to create the QL representation of mental contexts,* $C \rightarrow \psi_C$, *by using the algorithm (QLRA) based on the formula of total probability with the interference term.*

5.5 The Brain as a Quantum-like Computer

We can speculate that the ability of the brain to create the QL representation of mental contexts, see Postulate QLR, induces the functioning of the brain as a QL computer.

Postulate QLC. *The brain performs computation-thinking by using algorithms of quantum computing in the complex Hilbert space of mental QL states.*

We emphasize that in our approach the brain is not a quantum computer, but a QL computer. On the one hand, a QL computer works totally in accordance with the mathematical theory of quantum computations (so by using quantum algorithms). On the other hand, it is not based on superposition of individual mental states. The complex amplitude ψ_C representing a mental context C is a special probabilistic representation of information states of the huge neuronal ensemble. In particular, the brain is a *macroscopic* QL computer. Thus the QL parallelism (unlike conventional quantum parallelism) has a natural realistic base. This is real parallelism in the working of millions of neurons. The crucial point is the way in which this classical parallelism is projected onto dynamics of QL states. The QL brain is able to solve NP-problems. But there is nothing mysterious in this ability: an exponentially increasing number of operations is performed by involving an exponentially increasing number of neurons.

5.6 Evolution of Mental Wave Function

We restrict our considerations to trigonometric mental contexts (QL contexts producing the cos-interference). The mental wave function $\psi(t)$ evolves in complex Hilbert space $\mathcal{H}$ (space of probability amplitudes). The straightforward generalization of quantum mechanics implies the *linear* Schrödinger equation, see (2.43):

$$i\frac{d\psi(t)}{dt} = \hat{H}\psi(t), \quad \psi(0) = \psi_0, \tag{5.5}$$

where $\hat{H} : \mathcal{H} \to \mathcal{H}$ is a self-adjoint operator in the Hilbert space of mental QL states.

For example, let us consider a QL Hamiltonian, cf. (2.47):

$$\hat{H} \equiv H(\hat{a}, \hat{b}) = \frac{\hat{b}^2}{2} + V(\hat{a}), \tag{5.6}$$

where $V : X \to \mathbf{R}$ is a "mental potential" (e.g. a polynomial). We call $\hat{H}$ the operator of *mental energy,* cf. [161, 175, 53]. Here $\hat{a}, \hat{b}$ are two self-adjoint operators. We recall that in the Växjö model the operator representation can be constructed for any pair of supplementary observables, see Section 4.2, (4.18), (4.10).

Denote by ψ_j stationary mental QL states: $\hat{H}\psi_j = \mu_j \psi_j$. Then any mental QL state ψ can be represented as a superposition of stationary states

$$\psi = k_1\psi_1 + k_2\psi_2, \quad k_j \in \mathbf{C}, \quad |k_1|^2 + |k_2|^2 = 1. \tag{5.7}$$

One might speculate that the brain has the ability to feel the presence in the state $\psi \equiv \psi_C$ of superpositions (5.7) of stationary mental QL states. In such a case superposition would be an element of mental reality. However, it seems not to be the case. Suppose that ψ_1 corresponds to zero mental energy, $\mu_1 = 0$. For example, such a QL state can be interpreted as the state of depression. Let $\mu_2 >> 0$. For example, such a QL state can be interpreted as the state of excitement. My internal mental experience tells that I do not have a feeling of superposition of states of depression and high excitement. If I am not in one of those stationary states, then I am just in a new special mental QL state ψ and I have the feeling of this ψ (representing some mental context C, i.e., $\psi \equiv \psi_C$) and not superposition.[12] Thus it seems that the expansion (5.7) is just a purely mathematical feature of the model. Of course, the brain uses the possibility to select a basis, e.g., the eigenvectors of the operator of mental energy, and to perform self-measurements in this basis. However, as results of measurements, it will feel just these eigenfunctions and not their superposition ψ.

5.6.1 Structure of a Set of Mental States

In QM a state (wave function) ψ is represented by a vector belonging to the unit sphere S of a Hilbert space. In the two-dimensional case (corresponding to dichotomous observables, e.g., 'yes' or 'no' answers) the set of quantum states can be visualized by using the unit sphere in the three-dimensional real space $\mathbf{R}^3$, *Bloch's sphere*, see Section 4.3.

In our mental QL model, contexts producing trigonometric interference are represented by points in S. Suppose that there is given some set of cognitive contexts $\mathcal{P} \subset \mathcal{C}^{\mathrm{tr}}$, where the latter set consists of all trigonometric contexts[13] corresponding to the selected pair of two reference self-observables a and b. Let $S_{\mathcal{P}} = J^{b|a}(\mathcal{P})$, where $J^{b|a} : \mathcal{C}^{\mathrm{tr}} \to \mathcal{H}$ is the map corresponding to QLRA. Then the set of mental states is described by the $S_{\mathcal{P}}$. There is no reason to suppose that $S_{\mathcal{P}}$ coincides with the S. It is a fundamental problem to describe the set of QL metal states $S_{\mathcal{P}}$ for various classes of cognitive systems.

We might speculate that $S_{\mathcal{P}}$ depends essentially on classes of cognitive system. So $S_{\mathcal{P}}^{\mathrm{human}}$ is not equal to $S_{\mathcal{P}}^{\mathrm{leon}}$. We can even speculate that in the process of evolution

[12] We exclude abnormal behavior such as manic-depressive syndrome.

[13] Of course, the brain also could operate with non-trigonometric contexts, e.g., hyperbolic or even mixed hyper-trigonometric. We restrict modelling to trigonometric contexts to have a better analogy with conventional QM.

the set $S_{\mathcal{P}}$ has been increasing and $S_{\mathcal{P}}^{\text{human}}$ is the maximal set of mental states. It might even occur that $S_{\mathcal{P}}^{\text{human}}$ coincides with the Bloch sphere S.

5.6.2 Combining Neuronal Realism with Quantum-like Formalism

The main distinguishing feature of our QL approach to cognitive sciences is the possibility of combining neuronal realism with mathematical formalism of quantum mechanics (or its generalizations). In our model "quantum probabilistic waves" (represented in the mathematical model by complex probability amplitudes) are produced by ensembles of neurons. There is nothing mysterious in the wave-like dynamics of mental information. Such a dynamics (which we use to simulate the process of thinking) is the result of the ability of the brain to perform QL projection of the ocean of neuronal information. At each instant of (mental) time the brain selects two fundamental variables (selects a representation of the neuronal ocean[14]) and creates the image of activity of the neuronal ocean given by a complex probability amplitude (by applying QLRA producing a complex probability amplitude from the statistical data).[15] Our fundamental conjecture is that the brain operates (at least on the highest level of mental functioning) with such QL images by using algorithms of quantum computing. Thus one can call the brain a *QL computer.* Its functioning is mathematically described by the conventional theory of quantum computing, but physically it has nothing to do with the conventional quantum computer.

We can speculate that even collective cognitive systems (human societies, states, nations, groups of animals, birds, insects) are able to create QL probabilistic representations of information. One could say that such cognitive systems are driven by probabilistic QL waves. Finally, we remark that one could not exclude that such representations could be created by nonliving complex information systems. Our approach opens the way to *QL artificial intelligence.*

[14] Compare with *Solaris* by Stanislav Lem and especially with the corresponding film by Andrei Tarkovsky.

[15] Of course, it is assumed that the brain is able to collect this data. This collecting could not be performed instantaneously. Therefore we speak about moments of mental time which correspond to intervals of physical time.