Sensitivity Analysis of a Tax Evasion Model Applying Automated Design of Experiments

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Abstract. Risk, audit frequency, expected utility, decision on the rate of tax evasion: probably these words occur to the reader first about tax evasion modeling. However, it can easily turn out in the real world that the 'everyday evader' hasn't got reliable information about the risks of evasion, about the possible amount of fine, or about the efficiency of the tax authorities. The TAXSIM agent-based tax evasion model was developed to understand the macro-level taxpayer behavior better with its help. The model and first simulation results were presented on the ESSA 2008 conference. The aim of this article is to present a sensitivity analysis of the model. We applied Design of Experiments method to reveal the main parameter-response correlations on a selected parameter domain and used two extreme parameter sets to examine on what level the contradictory factors can compensate each other.

Keywords: Agent-based computational economics, agent-based simulation, tax modeling, tax evasion.

1 Introduction

In our paper at ESSA 2008 [14] we introduced the tax evasion model in its complexity. We also presented simulation results of taxpayer behavior in life inspired scenarios in which i) the quality of governmental services increases permanently, ii) a market leader unilaterally adopts the legal position, and iii) multi-national companies with tax allowances enter the market. The first experimental results showed that TAXSIM might be a useful tool to understand the tax evasion phenomena better. However many questions remained about the system behavior. Two type of model 'investigations' were planned to test the model usability: one was to reveal the parameter-response correlations and the other was to reproduce statistical data collected on actual tax evasion. This article discusses the parameter-response correlations of the TAXSIM model. We also publish results on compensating the tax authority activities.

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Agent-based models brought important new results on tax evasion. Altering the compliance strategy of the inhomogeneous agents' periodically (honest, 'imitative', 'free rider' [11] or honest, 'susceptible' and 'evader' [7] strategies) as environment changes can better explain how low number of audits can result a compliant population [5]. Balsa et al. created a generative set of models those demonstrate how can some extensions raise the predictive power of the standard theory [1][3] of tax evasion [2]. These extensions were: agent individuality and adaptivity, social perceptions and social interactions. Korobow et al. found that even in case of almost chaotic dynamics on the agent-level a predominantly compliant equilibrium can emerge. They also demonstrated that when agents weight neighbors' compliance strategy payoffs more heavily in their decisions, noncompliance tends to increase [9].

In these models the urge for evasion is a built-in part of taxpayer strategies. On the contrary law enforcement (i.e. tax audits) is the direct and – because of agents' awareness to their environment - indirect motivation for compliance (that is affected by other factors: e.g. the weight of neighbors' behavior affects the taxpayer strategy). These models help to understand how agents react to different enforcement techniques applied on various levels assuming realistic taxpaying strategies and inhomogeneous agents. Still, some assumptions might be useful to take in account too.

All regulation is more than a collection of arbitrary rules. For example a driver may respect a stop sign not because she wants to avoid being prosecuted committing a crime, but because of her own interest in avoiding a car accident. On the other hand even those who want to exceed the speed limit can be prevented from speeding easily by a traffic jam. Analogue ideas motivated the development of the TAXSIM model: we made an effort to take the taxpayer's reasonability (or her interests beyond maximizing the wage) and opportunities into account. This approach has the promise to gain information about how can be the taxpayers motivated to comply when the aggravation of enforcement is not viable.

In the following we will discuss some experimental results of the TAXSIM model that aggregates taxpayer motivations, market labor characteristics and authority activities. In the previous paper at ESSA [14] we presented the TAXSIM's architecture in details. It [14] also includes simulation results of some realistic scenarios. After testing the model as a whole we decided to run a sensitivity analysis.

This paper is structured as follows. Section 2 overviews the model. Section 3 describes the sensitivity analysis: subsection 3.1 presents the method of the experiment, while the experimental results are demonstrated in the following subsection. Section 4. The last section concludes the paper.

2 Short Overview of the Model

The TAXSIM model is a complex agent-based approach for tax evasion (and tax compliance) simulations. The novelty of TAXSIM is that taxpayer compliance is strongly affected by the environment of the agent. An agent who decides to evade tax on a certain level has to find a job offer that meets her preferences: if she was unable to find one, she will make a compromise and accept an available offer that closest to her decision. The other novelty is that agents accept the need for taxes in TAXSIM. That is, taxpayers experience taxes as the price of services they resort (e.g. courts, education, etc.).

The model is concerned with the operations of a single market sector, where there are four kinds of agents involved: employee, employer, (tax) authority and government. The economic well-being of employees depend on their net wages, while that of the employers' is a function of the market demand and the level of gross wages they are forced to pay. The rate of tax evasion is an agreement between an employer and an employee that is made when the employee occupies a new job. As the agreed employment type determines the income of the employee and the (producing) costs of the employer, both participating agents have a motivation to evade.

The government and the tax authority have service providing and regulatory roles, respectively. Since the market demand is modeled as an exogenous component and employers and employees are assumed to be homogeneous in technological and productive ability, competitiveness is determined by the agents' approach to taxes.

In this model tax evasion is a technique to reduce costs (and to raise wages). Therefore a more refined measure of level of the evasion fits better our purposes than the classical binary or ternary choice (e.g., complier/evader, or complier/evader/skeptic). Thus, we used the five types of income (those found empirically most common in Hungary – both legal and illegal ones) to create employment types. An employment type is the combination of reported wage, fringe benefits, ad hoc engagement agreement, unreported wage and payment in kind. The employment types are grouped so that when there's no reported wage it is termed *illegal* (or *hidden*) and it is called *legal* when there's only reported wage and fringe benefits. The remaining combinations belong to the group of *mixed* employment.

The agents have no perfect information about the policies of the government and of the accuracy of the tax authority. They learn from these previous experiences and from interactions within their social network. Thus, in addition to the agents, the last major component of the model is the social network of both the employees and the firms. The employees and employers use their knowledge during the so called negotiation process that takes place when an employee occupies a new job. During the negotiation procedure both the employer's and the employee's expectations depend on their respective satisfaction with the government and on the estimated costs and benefits of evasion. Previous interactions with the authority agent (audits) and information derived from the social network determine cost and benefit estimations. It is also assumed that all agents utilize some services provided by the government (e.g. a company wants to register a trademark, or a person wants to get a passport). These interactions (the experienced effectiveness, corruption, etc.) determine the contentment of the agent.

The model of the market sector is kept as simple as possible. Companies (employers) producing the cheapest goods sell first. When demand is less than the actual productivity, companies producing most expensive goods will meet losses that may force them towards evasion. A similar force is faced by the employees: after a period of unemployment (the length depending on the agents reserves that in turn, depend on the length of previous employment) an employee decreases its expectations and will eventually accept any job offer. An employee becomes unemploymed when its employer fires her due to financial reasons.

Employer agents start to operate by hiring employees and selling products. Costs are the wages, while income is the price of the sold products (the price of a single product depends on the employer's average wage cost and the profit margin, the latter being a model parameter). Employers operate until becoming bankrupt.

As discussed above, employers have an implicit strategy to produce as cheap as possible, which is realized by making tax evasion deals with employees. The key this is the negotiation process, in which employers make job offers. (A job offer is a pair of a wage and an employment type.) The agents try to optimize the following function:

$$V(B, a_2) = B \cdot (a_1 - a_2) - p \cdot q \cdot f \cdot B \cdot (a_1 - a_2) \rightarrow \max(a_2) \ . \quad (1)$$

Where B is the gross salary (constant within a simulation), a_1 is the tax rate (constant within a simulation), a_2 is the actual tax rate paid, p is the chance of audit, q is the chance of being caught during an audit in case of evasion (accuracy of the authority) and f is the fine rate (constant within a simulation). The agent learns the value of p and q.

However this function is constrained by the employer's contentment level, derived from governmental interactions that determine the minimum level of compliance (that can be zero). Moreover, the produced new offer needs to match the employee's preferences. Feedback from employees modifies the employer's strategy when no one accepts the offer for a period of time.

TAXSIM employees attempt to get a job of their preferences, or any job possible if they are unemployed for a given amount of time. Employees try to maximize their income by avoiding taxes, counting in the potential drawbacks of evasion (e.g. lower expected pension). Note that a greater take-home wage doesn't necessarily imply a greater expected income automatically due to additional estimated costs. Expected income is calculated by the following function:

$$v(N, \Delta_1, \Delta_2) = N - p_1 \cdot N \cdot \Delta_1 \cdot f - p_2 \cdot \Delta_2 \cdot k \ . \quad (2)$$

Where N is the take-home wage, Δ_1 is the evaded tax percent, Δ_2 is the evaded medical insurance (in percent), p_1 is the chance of being caught, p_2 is the chance of illness, f is the fine rate, and k is the medical cost. The agent learns the value of p_1 and p_2. When an employee looks for a job she will evaluate more than one offers using (2). If all of the offers are too illegal (compared to the agent's minimum level of compliance) and the employee has savings (practically, she has not been unemployed for a long period) then she won't accept any of the offers but keep searching. She will accept the best offer otherwise. (Unemployed agents do not pay any taxes.)

Employees live forever: there is no ageing or any fluctuation in the population of the employees. The financial status of the employee has no effect on her work abilities, but it shortens the period she looks for a desirable job.

An employee (or an employer) evaluates (1) (or (2)) only when a decision is to be made: e.g. when looking for a new job (or wants to hire a new employee). That means agents don't change their compliance level periodically. Employers apply no radical changes on their compliance as they alter their offers by shifting the current compliance level (that is between 0 and the tax rate) one step towards the profitable direction.

TAXSIM includes two distinct social networks of agents transmitting information between neighbors: one connects the employers and the other connects the employees. For simplicity, these are modeled by Erdős-Rényi random graphs [8] in the current version. Erdős-Rényi graphs have small agent-agent distances, an important property of real-world social networks. (On the other hand, they don't match other social network attributes like the clustering coefficient or degree distribution). Both employers and employees transmit information to their neighbors about the experienced accuracy of the authorities and their level of satisfaction. The average of the data received from neighbors affects the estimated accuracy of the authorities and the level of satisfaction respectively.

The tax authority audits employees via employers: employers are picked randomly for audit in each round depending on the audit frequency parameter. During a particular audit the authority checks the employer's employees randomly (the probability is equal to the 'authority accuracy' parameter). If the authority finds mixed or illegal employment both the employer and the employee is fined, proportionally to the tax evaded, and the employee is forced to quit the illegal job.

In return of taxes paid, agents expect advantages from the government agent (e.g. health care for employees, legal certainty for employers, etc.) by 'requesting' services. The levels of quality (for employees and for employers, respectively) are parameters of the model. The services are requested in every round by the agents according to probabilities set in parameters. The response is drawn form a distribution determined by the quality of service variable. Agents update their minimum level of compliance by calculating the weighted average of current and past experiences.

Table 1. Factors of the experiment

Parameter name	Low value	High value	Code
Authority accuracy	20%	40%	A
Chance of a new employer	5%	25%	B
Chance of illness (employees)	10%	30%	C
Cost of health care	5	10	D
Employee network density	5%	25%	E
Employee service quality	5%	25%	F
Employer network density	5%	25%	G
Employer service quality	20%	40%	H
Need supply (employers)	5%	25%	I
Number of requested goods	380	450	J
Probability of audit (each time step)	5%	25%	K
Income (percentage of investment)	105%	125%	L
Rate of fine (percentage of evaded tax)	140%	160%	M
Search for a new job (probability)	5%	25%	N
Tax rate	35%	55%	O

3 Sensitivity Analysis

3.1 Applying Design of Experiments

We decided to use designed experiments to analyze the model: a strong motivation for applying designed experiments instead of the one-factor-at-a-time (OFAT) method is in [6]. Amongst others the application of Design of Experiments (DoE) ensures a more exhaustive investigation and therefore more relevant results. A large body of literature is available on DoE: a good summary can be found in [12]; wide range of applications is in [5].

A *screening method* seemed to be appropriate for our purposes. The *two-level fractional factorial* method is able to calculate the effect of model parameters (or *factors* in DoE terminology) on the simulation results by running simulations with a *high* and a *low* value (*levels*) of the factors [12]. See Table 1. for the examined 15 factors and their levels. In the experiment we studied the model behavior in a small environment of an illegal market equilibrium that was used previously for simulating three scenarios [14]. A small subspace was selected as not all possible parameter values can be addressed to any real world phenomena. For example having the maximum value of the audit probability (which is 1 that means every employer is being audited in each time step) is an unrealistic option for the tax authorities.

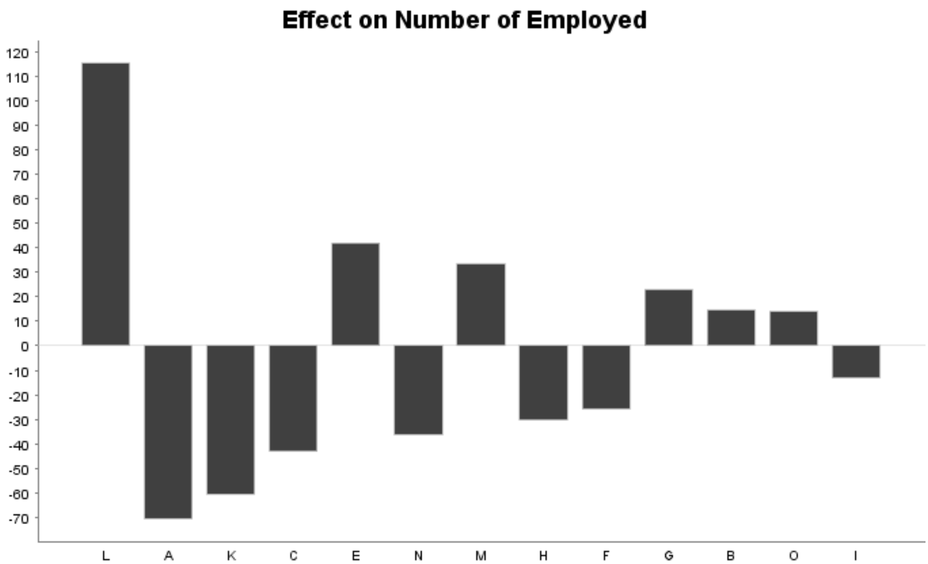

Fig. 1. Factor effects on number of employed. Category codes are in Table 1. The vertical axis shows the factor effect in the number of affected employments.

To perform a two-level fractional factorial design two levels (or "versions") of each of a number of factors (here model parameters) have to be selected and then the experiment is run in carefully chosen combinations defined by the design. In this experiment we used quantitative factors only.

To calculate the effect of a particular factor the measured response values are divided to two sets: the first set contains the response values measured at the low value of the factor; the other set contains the rest. The difference between the averages of the members of the first and the second sets is the factor effect.

DoE methods are usually applied in experiments using 2-6 parameters (in physics, chemistry, etc.), but bigger designs exist too. We applied a factorial method that reduced the number of 2^{15} runs of the full factorial method to 2^4 runs (the design specification is 2^{15-11}, resolution III: see [12] for details). The design was randomized so the experiment had a total of 240 simulation runs.

The experiments with the Repast J [13] model were run using the Model Exploration Module (MEME) [10] software tool. We also used MEME design the experiment and to process simulation data automatically.

There were initially 50 employers and 500 employees in the modeled sector (while the number of employers may change the number of employees is constant during each run). The employers' initial offers are drawn from uniform distribution. Market demand was 450 units.

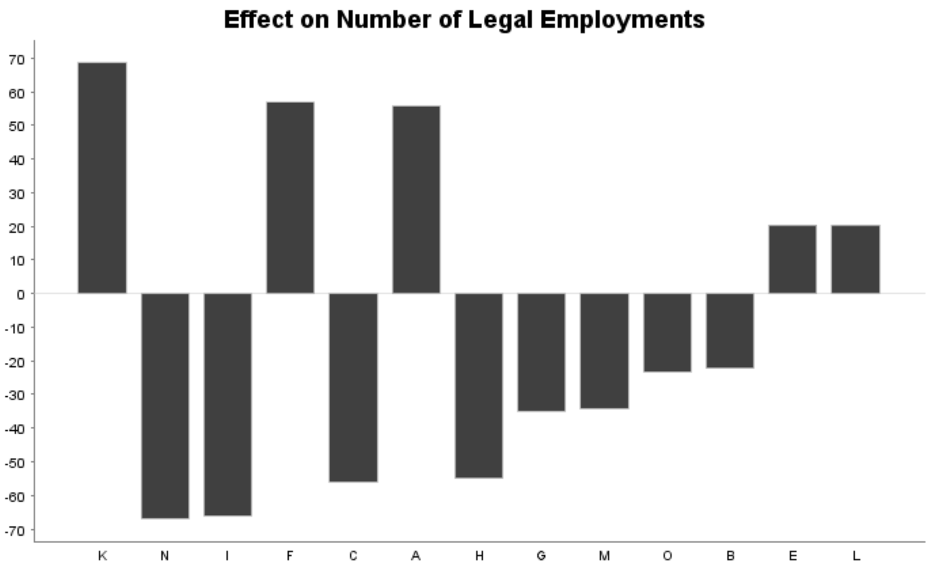

Fig. 2. Factor effects on the number of legal employments. Category codes are in Table 1. The vertical axis shows the factor effect in the number of affected employments.

3.2 Simulation Results

The following responses were considered during the experiment: number of employed, number of legal employments, number of mixed type employments and number of hidden employments. It could also be useful to focus on the aggregated tax income instead. However the selected values characterize the modeled system in a more detailed way and therefore those are more informative on the details of the results.

The two-level fractional factorial method indicates the effects of raising the factor values from the defined low values to high values (see Figures 1-4 for results). Table 1 shows the high and low values of the design factors: these factor value pairs define a parameter subspace examined during the experiments. The exact number of employments per types is unknown – only the changes are measured.

We found that income has the most powerful effect on the number of employed agents. Increasing the profit rate accelerates the simulated economy: employment rate increases in all employment types. The biggest increase is on the number of illegal employments (see factor 'L' on Figure 1-4).

An opposite effect can be observed as the accuracy and frequency of audits increase (accuracy of audits: factor 'A' and frequency of audits: factor 'K' on Figure 1-4). When the tax authority reveals an offense, the employee loses her job. Because of the high rate of illegal employments the bigger accuracy of the authority and the more frequent audits cause that the number of employed agents decreases significantly. It can be the case that the authority is too strict according to the economical environment, where the competition is very strong. Still, an accurate authority decreases the number of illegal employments significantly and legalizes the sector that can be tracked on the increasing number of legal and mixed type employments (see Figure 2-3).

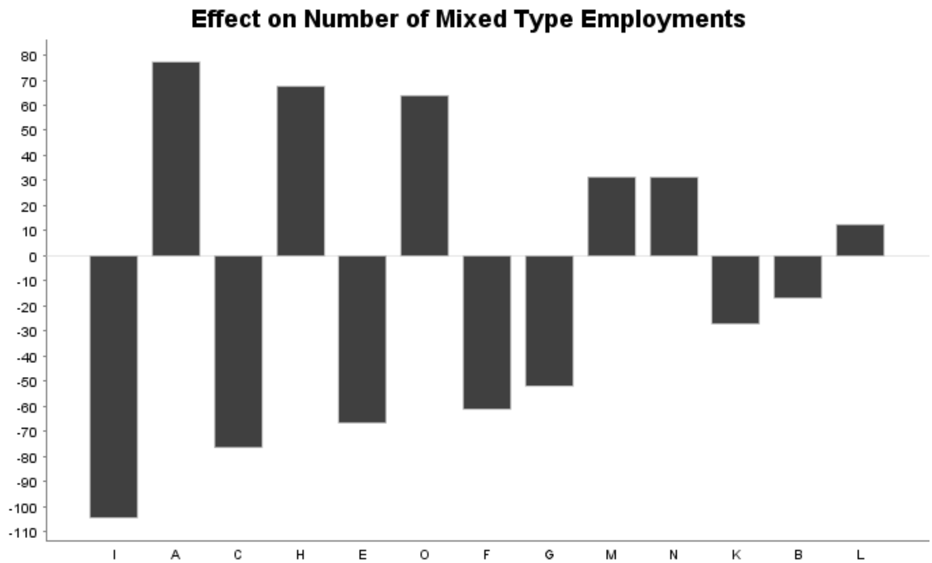

Fig. 3. Factor effects on the number of mixed type employments. Category codes are in Table 1. The vertical axis shows the factor effect in the number of affected employments.

When the social networks' density (see factor 'E' for employees and factor 'G' for employers on Figures 1-4) is higher, the unemployment rate is lower by the increasing number of illegal employments. In an environment where evasion is the 'rational' option, the more information agents have on the expected drawback on evasion, the better picture they have on the costs of having an illegal job. This result in less payment by agents caught by authorities and therefore less bankruptcy happens, and more agents can be employed (illegally). It's a very important result: well-informed agents are able to adapt a more optimal taxpaying behavior. It also accords to the phenomena Korobow et. al. found: when agents weight neighbors' compliance strategy payoffs more heavily in their decisions, noncompliance tends to increase [9]. It is common that the aggregated knowledge of agents affects the population-level compliance negatively.

The 'search probability' parameter (factor 'N' on Figures 1-4) makes employee agents to search a better job when employed so thus apply new jobs more frequently. When employed agents search for better jobs at a higher probability, the number of agents searching jobs increases. This results in an increased competition for available jobs, and decreased chance of a successful application. This influences unemployed agents more painfully as run out savings and forced to take any (in the examined case illegal) jobs. As an additional effect the agents have to decide more frequently on available jobs, they follow the changes in environment more closely and react more aptly.

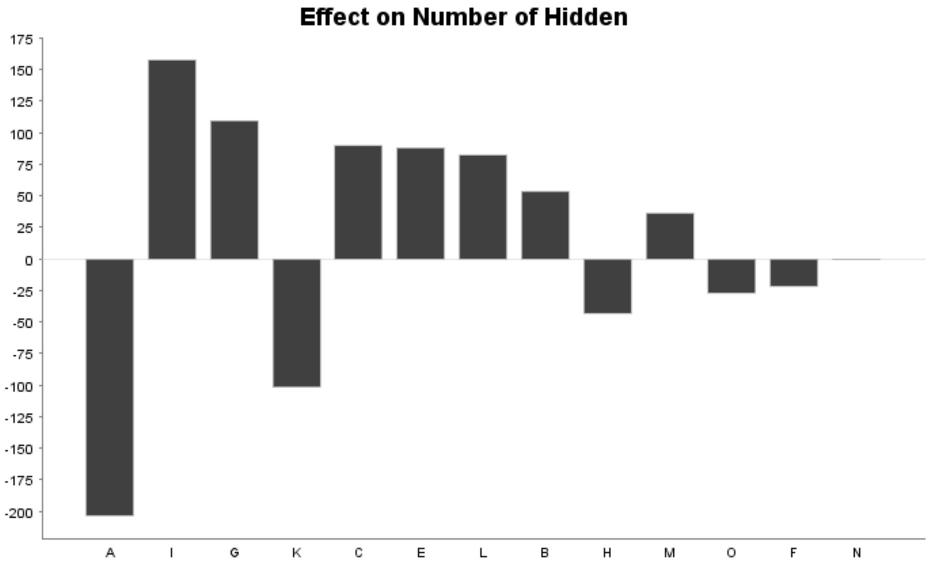

Fig. 4. Factor effects on the number of hidden employments. Category codes are in Table 1. The vertical axis shows the factor effect in the number of affected employments.

It is remarkable that a greater rate of fine alone (factor ‘M’) has a negative effect on the number of legal employments (see Figure 2): it requires further experiments to determine exactly what fine rate has a motivation for compliance.

On the one hand the better governmental services decrease the illegal employment rate. On the other hand the unemployment rate increases: in this case the legalization decreases the competitiveness of firms.

At a higher tax rate (factor ‘O’) the number of mixed type employments increases, while the other two decrease. Higher taxes result less income for employed agents that motivate them towards evasion. This moves legal agents (employees and employers) towards mixed employment types. On the other hand agent caught on not paying tax face an increased fine (due to increased amount of tax not paid). Agents anticipating on that move towards mixed type employments to reduce the cost.

4 Two Experiments in Extreme Environments

The taxpayer agents estimate the activity of the tax authority that affects their decisions strongly in an explicit way. However we wanted to demonstrate and verify that authority accuracy, the frequency of audits and the fine rate are far from being the only dominant parameters of TAXSIM on the entire parameter space. For simplicity it was examined whether taxpayer contentment (with governmental services) can compensate the abovementioned factors. No correlations between frequency of audits and taxpayer contentment were assumed; therefore agents didn't change their strategies because they experienced no punishment for evasion.

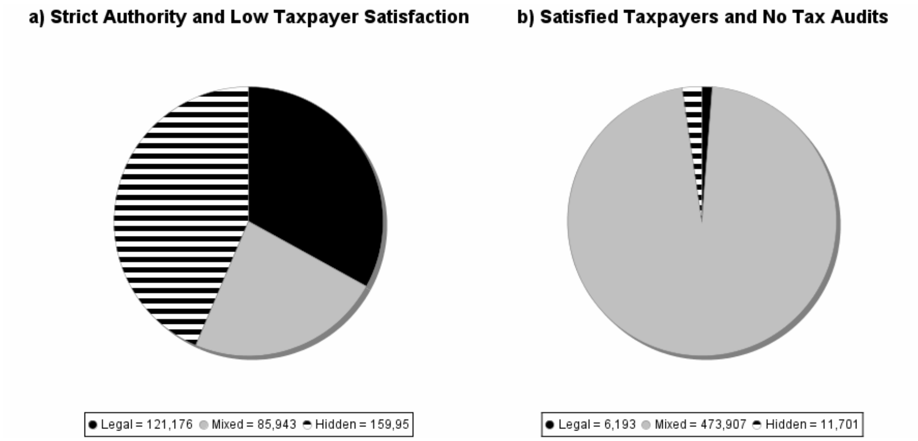

Fig. 5. Pie charts of the employment type distributions in case of a) high enforcement level and low taxpayer satisfaction and b) satisfied employers and no tax audits. Each chart depicts the time-averaged values of converged trajectories of 6000 time steps, averaged over the 216 simulation runs performed with different random number sequences.

We ran experiments to explore the taxpaying behavior in extreme environments. It was to be tested, how contradictory model factors affect the model output (the distribution of legal, mixed type and illegal employments). 500 employee agents and an initial group of 50 employers (those have dynamic lifecycles) operated in a single run. The charts below depict the time-averaged values of converged trajectories of 6000 time steps, averaged over the 216 simulation runs performed with different random number sequences.

In the first scenario a strong activity of tax authorities were present (27,5 percent of employers were audited in each turn, and discovered offenses were fined by the 250 percent of the evaded tax). On the other hand both employers' and employees' satisfaction level was extremely low (30 percent of them requested services in each turn, and only 10 percent of the responds were satisfactory). Market demand was lowered too: it was 76 percent of the modeled sector's productivity.

In the second scenario employer agents were highly compliant (90 percent of them requested services from the government in each turn, and 90 percent of the responds were satisfactory) and market demand was relatively high (only 10 percent less than the modeled sector's maximum productivity). On the other hand the agents faced no tax audits at all, and the employees' satisfaction level was relatively low (30 percent of them requested services in each turn, and only 30 percent of the responds were satisfactory).

It was trivial, that this scenario won't result a totally illegal sector, due to the parameters. The interesting result is that not just the rate of illegal, but the rate of legal employments stays marginal too (see Figure 5). This means that the low satisfaction level of employees can compensate employers' high compliance level at a certain rate.

5 Conclusions

This paper summarized the results of a sensitivity analysis of the TAXSIM model. We examined what are the most powerful motivation factors when an agent selects a certain tax compliance level in case the decision is bounded to agents' opportunities in job selection. We also assumed that agents accept the need for taxes.

We found that income has the most powerful effect on the number of employed agents: a greater profit implies lower unemployment rate in the model: it raises the number of all employment types.

Albeit an accurate authority decreases the number of illegal employments significantly and legalizes the sector, it also increase unemployment rate, if the other motivation factors for compliance are low.

We found that the more accurate information agents have on their environment the less compliant they are going to be in TAXSIM that accords to the results of Korobow et. al. [9]. They experienced that when agents weight neighbors' compliance strategy payoffs more heavily in their decisions, noncompliance tends to increase. It is common that the aggregated knowledge of agents affects the population-level compliance in a negative way.

It was also demonstrated that in the taxpayer decision mechanism of TAXSIM fine and audits are not the only dominant factors. We found that that when the taxpayer contentment is low with governmental services, the rate of illegal employment is high even in case of frequent authority audits. On the contrary a high contentment can prevent the society from total illegalization even in the lack of tax audits. It means the model has a rather complex response surface, perhaps a more complex one than to be expected from the results of initial experiments.

Acknowledgements

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