Describing Evolutions of Multi-Agent Systems

Sergey Babenyshev and Vladimir Rybakov

Department of Computing and Mathematics,
Manchester Metropolitan University,
John Dalton Building, Chester Street, Manchester M1 5GD, U.K.
and
Institute of Mathematics, Siberian Federal University,
79 Svobodny Prospect, Krasnoyarsk, 660041, Russia
Sergey.Babenyshev@gmail.com, V.Rybakov@mmu.ac.uk

Abstract. This paper[1] focuses on the issue of the formal logical description of evolutions of multi-agent systems (MAS). By evolution of a MAS we mean the change of inner states of the combined MAS caused by interaction of participating agents. We introduce a general scheme of combining propositional modal languages and respective logics into a single language suitable for such descriptions. The method is based on the representation of multi-agent systems by Kripke-Hintikka models. The obtained description allows to study the question of verifiable specifications.

Keywords: multi-agent systems, multi-modal logics, decision algorithms, satisfiability, Kripke semantics.

1 Introduction

Numerous attempts at combining logics — the hybrid, fusion, product logics providing by far not complete list — were motivated largely by applications, which more and more often require formalisms to describe complex systems with multiple ontologies. There are two main approaches toward combining propositional logic, that assume full combination of signatures. One is product and another is fusion [1]. From the point of view of applicability to modeling evolutions of MAS, there are certain constraints inherent to both these methods. Basic fusions do not allow for interaction of modalities. Products assume that structural configuration, representing a snapshot of a multi-agent system, should be fixed once and for all. Even more challenging restriction of products is that they are undecidable even for quite simple constituent logics, like $S5$ [2].

For modeling behavior of multi-agent systems we propose a general scheme of *cluster-based fusions* that partially combines features of both fusion and product approaches, while avoiding their above-mentioned shortcomings. In particular, unlike products, cluster-based fusions lead to decidable logics, that allows, at

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least in principle, to use them for specification and verification purposes. Secondly, unlike the methods based on basic fusions, cluster-based fusions allow for far more expressive descriptive languages. The main difference with the previously used methods (cf. [3,4]), is that every state of a MAS is assumed to be contained inside a time-cluster.

The paper is structured as follows: in the next section we briefly review fusions and products of Kripke frames and discuss how our approach relates to them. In Section 3, we present the result about decidability of logics obtained by our method.

2 Cluster-Based Fusions of Kripke Frames

Let us recall the basics of multi-modal Kripke semantics.

A *Kripke multi-modal frame* F of the *Kripke signature* $\langle R_1, \ldots, R_m \rangle$ is a tuple $\langle W, R_1, \ldots, R_m \rangle$, where $W \neq \emptyset$ is the set of *worlds* or *states*, and every R_i is a binary relation on the elements of W (i.e., each $R_i \subseteq W \times W$). The set W is also called the *universe* of F, and the R_is are called *accessibility relations*.

Later on, we will be dealing mainly with two types of Kripke frames and Kripke signatures. The first type, uni-modal, is used to imitate the flow of time. The second, multi-modal, is used to represent particular configurations of multi-agent system at given moments of time. Combined together, they represent an evolution of a MAS as a change of structural configurations in (possibly non-deterministic) time. Both type of Kripke frames will be assumed to satisfy certain structural conditions, that can be defined by modal formulas.

Let $F = \langle W, R, R_1, \ldots, R_m \rangle$ be a multi-modal Kripke frame, where R is a reflexive, transitive relation. A *R-cluster* of F is a maximal under inclusion subset C of W, such that the restriction of R to C is an equivalence relation. We denote by $Cl_R(F)$ the set of all R-clusters of F. If the relation R is clear from context, we routinely drop the related subscripts or qualifications. Usually in this paper, the first relation in a combined Kripke signature will represent time, therefore we call the respective clusters — *time clusters.* The union of all clusters covers the frame and distinct clusters do not intersect. Therefore for every $w \in W$, we can define by $C(w)$ the unique cluster of F that contains w. In particular, any cluster of F is a cluster of the type $C(w)$ for some $w \in W$.

The main construction of this paper is given by

Definition 1. *A multi-modal frame $\langle W, R, R_1, \ldots, R_m \rangle$ is called a cluster-based fusion (CB-fusion) if*

- *R is a reflexive, transitive binary relation on W (R represents time);*
- *R_i are arbitrary relations on W (each R_i represents informational channels available to the i-th agent);*
- *each $R_i \subseteq R$ (i.e., all informational transactions are aligned with the time);*
- *each $R_i \subseteq R^{-1}$ (i.e., all informational transactions are determined by the current state of the MAS in question).*

A *Kripke model* with the signature $\langle R_1,\dots,R_m\rangle$ is a tuple $\langle W,R_1,\dots,R_m,V\rangle$, where $\langle W,R_1,\dots,R_m\rangle$ is a Kripke frame with the signature $\langle R_1,\dots,R_m\rangle$, and $V : X \to \mathcal{P}(W)$ is a *valuation* of some subset $X \subseteq Var$ of variables. For a valuation V, $V(x_i)$ denotes the set of all worlds of W, where the basic fact, represented by the propositional variable x_i, is true.

For describing properties of multi-modal Kripke frames of the signature $\langle R_1,\dots,R_m\rangle$ the following modal propositional language is usually used:

$$\Lambda = \langle \wedge, \vee, \to, \neg, \Diamond_1, \dots, \Diamond_m\rangle.$$

We fix an enumerable set $Var := \{x_1, x_2, x_3, \dots\}$ of *propositional* variables. *Well-formed formulas of the language* Λ (*Λ-formulas*) are defined by the following grammar

$$\alpha ::= x_i \mid \alpha_1 \wedge \alpha_2 \mid \alpha_1 \vee \alpha_2 \mid \alpha_1 \to \alpha_2 \mid \neg\alpha \mid \Diamond_i \alpha \ (i \in I)$$

The set of all Λ-formulas is denoted by For_Λ. For a formula α, the *set of variables* $Var(\alpha)$ *of* α is defined inductively:

$$Var(x_i) := \{x_i\},\ Var(*\beta) := Var(\beta),\ Var(\beta * \gamma) := Var(\beta) \cup Var(\gamma)\ .$$

We will use the following shortcuts: $\Box_i\phi := \neg\Diamond_i\neg\phi$, $\top := p \vee \neg p$, $\bot := p \wedge \neg p$.

By Λ we denote the *standard* multi-modal language $\langle \wedge, \vee, \to, \neg, \Diamond_1, \dots, \Diamond_m\rangle$, where all $\Diamond_i$ are assumed to have the *standard* interpretation, i.e., given a Kripke model $\mathcal{M}$ of the signature $\langle R_1,\dots,R_m\rangle$, for all $\phi \in For_\Lambda$, $w \in W$ and $i \in I$

$$(\mathcal{M}, w) \vDash \Diamond_i\phi \Leftrightarrow \exists u : wR_iu \,\&\, (\mathcal{M}, u) \vDash \phi,$$

where we write $(\mathcal{M}, w) \vDash \alpha$ to say that the formula α is *true* or *holds* in the model $\mathcal{M}$ at the world w. We will call the standard modalities (or their duals: $\Box_i := \neg\Diamond_i\neg$) with the presumed standard interpretation, *Kripke modalities.*

A formula α is *valid in a frame* F, if, for any valuation V of variables $Var(\alpha)$, $F \vDash_V \alpha$. If a formula α is not valid on F, then there is a valuation V such that $F \nvDash_V \alpha$. In that case we say that α is *refuted* in F (by V).

Suppose $\mathcal{K}$ is a class of Kripke frames of $\langle R_1,\dots,R_m\rangle$-signature. Let Λ be the respective modal language. We denote by $Log(\mathcal{K})$ the logic

$$\{\alpha \in For_\Lambda \mid \mathcal{K} \vDash \alpha\}.$$

If $\mathcal{L} = Log(\mathcal{K})$ we say that $\mathcal{L}$ is *generated* by $\mathcal{K}$.

If we have a multi-modal logic $\mathcal{L}$, let $Fr(\mathcal{L})$ be the class of all frames F of the respective Kripke signature, such that all theorems of $\mathcal{L}$ are valid in F. A frame from $Fr(\mathcal{L})$ we will call *adequate for* $\mathcal{L}$ or an *$\mathcal{L}$-frame.*

The simplest way of combining logics is the *fusion.* If $\mathcal{L}$ is a Kripke complete unary modal logic, we denote $\mathcal{L}_N$ the *N-fusion* of $\mathcal{L}$, i.e. the logic generated by the class of frames $\mathcal{K}$ such that for every frame $F = \langle W, R_1, \dots, R_N\rangle \in \mathcal{K}$ and all $i \in \{1,\dots,N\}$, the frame $\langle W, R_i\rangle$ is an $\mathcal{L}$-frame.

The *product* $\mathcal{L}_1 \times \mathcal{L}_2$ of logics $\mathcal{L}_1$ and $\mathcal{L}_2$ is generated by *products of frames.* The product of $\langle W_1, R_1\rangle$ and $\langle W_2, R_2\rangle$ is the frame $\langle W_1 \times W_2, R_h, R_v\rangle$, where

(i) $\langle u_1, v_1\rangle R_h \langle u_2, v_2\rangle$ iff ($u_1 R_1 u_2$ and $v_1 = v_2$); (ii) $\langle u_1, v_1\rangle R_v \langle u_2, v_2\rangle$ iff ($v_1 R_2 v_2$ and $u_1 = u_2$). This construction can be iterated.

The combining scheme we propose is based on cluster-based fusions.

Definition 2. *Suppose we have two Kripke-complete logics $\mathcal{B}$ and $\mathcal{S}$, of Kripke signatures $\langle R\rangle$ and $\langle R_1, \ldots, R_m\rangle$, respectively, where $\mathcal{B}$ is transitive. We call a* $\mathcal{B}_\mathcal{S}$-frame, *every frame $F = \langle W, R, R_1, \ldots, R_m\rangle$, such that (i) F is a CB-fusion, (ii) $\langle W, R\rangle$ is a $\mathcal{B}$-frame, (iii) for every $C \in Cl_R(F)$, $\langle C, R_1, \ldots, R_m\rangle$ is an $\mathcal{S}$-frame.*

We define the cluster-based fusion *of $\mathcal{B}$ by $\mathcal{S}$, denoted by $\mathcal{B}_\mathcal{S}$, the logic*

$$\{\alpha \in For_\Lambda \mid \textit{ for all } \mathcal{B}_\mathcal{S}\textit{-frames } F : F \vDash \alpha\},$$

where $\Lambda = \langle \wedge, \vee, \rightarrow, \neg, \Diamond, \Diamond_1, \ldots, \Diamond_m\rangle$.

Let $\Lambda = \langle \neg, \wedge, \Diamond_1, \ldots, \Diamond_m\rangle$ be a given modal language. A *clause* (or *type*) over variables $x_1, \ldots, x_n$ is a formula of the kind

$$\bigwedge_{k=1}^{n} x_k^{t(0,k)} \wedge \bigwedge_{i=1}^{m} \bigwedge_{k=1}^{n} (\Diamond_i x_k)^{t(i,k)},$$

where x_i are variables, $t(i,k) \in \{0,1\}$, and for any formula α, $\alpha^0 := \neg\alpha$, $\alpha^1 := \alpha$.

It is easy to see that there are only $2^{n(m+1)}$ distinct clauses over a set of n variables. We denote the set of all clauses over variables $x_1, \ldots, x_n$ by $\Theta(x_1, \ldots, x_n)$.

For every $\theta \in \Theta(X)$ and $i \in \{1, \ldots, m\}$, we denote

$$\mu^i(\theta) := \{x_k \in X \mid t(i,k) = 1\}.$$

We say that a clause θ is *realized* in a model $\mathcal{M}$, if there is a world $w \in \mathcal{M}$, such that $(\mathcal{M}, w) \vDash \theta$.

Given a Kripke model $\mathcal{M}$ of the signature $\langle R_1, \ldots, R_m\rangle$ with the finite $dom\, V$, every world $w \in W$ generates a unique clause $\theta_\mathcal{M}(w) \in \Theta(dom\, V)$, defined as

$$\theta_\mathcal{M}(w) := \bigwedge_{(\mathcal{M},w)\vDash x_k^{t(0,k)}} x_k^{t(0,k)} \wedge \bigwedge_{(\mathcal{M},w)\vDash(\Diamond_i x_k)^{t(i,k)}} (\Diamond_i x_k)^{t(i,k)},$$

where all $t(i,k) \in \{0,1\}$. We will omit the subscript in $\theta_\mathcal{M}(w)$, whenever the model is clear from context. Also, we write for every $i \in \{0, 1, \ldots, m\}$

$$\mu^i(\mathcal{M}) := \bigcup_{w\in W} \mu^i(\theta_\mathcal{M}(w)).$$

In particular, $\mu^0(\mathcal{M})$ is the set of the variables, that hold at least at one world in the model $\mathcal{M}$.

Definition 3. *Suppose $\mathcal{M}_1$ and $\mathcal{M}_2$ are two Kripke models of the same signature, such that $\mathcal{M}_1$ is finite and $dom\, V_1 = dom\, V_2$ is finite. Let $\Psi \subseteq \Phi \subseteq \Theta(dom\, V_1)$ be a pair of sets of clauses. We say that a mapping $f : W_1 \rightarrow W_2$ is a* clause-preserving mapping of $\mathcal{M}_1$ into $\mathcal{M}_2$ modulo $\langle \Phi, \Psi\rangle$ *if*

1. $\mathcal{M}_1 \vDash \bigvee \Phi$ *and* $\mathcal{M}_2 \vDash \bigvee \Phi$*;*
2. $\forall \theta \in \Psi \, \exists a \in \mathcal{M}_1 : (\mathcal{M}_1, a) \vDash \theta$*;*
3. $\forall \theta \in \Phi \, \forall a \in \mathcal{M}_1 : (\mathcal{M}_1, a) \vDash \theta \iff (\mathcal{M}_2, f(a)) \vDash \theta$*;*
4. $\mu^0(\mathcal{M}_1) = \mu^0(\mathcal{M}_2)$.

Thus the clause-preserving mapping $f : \mathcal{M}_1 \to \mathcal{M}_2$ modulo $\langle \Phi, \Psi \rangle$ guarantees that:

1. $\mathcal{M}_1, \mathcal{M}_2 \vDash \bigvee \Phi$,
2. all $\theta \in \Psi$ are realized in $\mathcal{M}_1$,
3. f preserves validity of all clauses from Φ.

Definition 4. *Let $\mathcal{L}$ be a Kripke complete logic and $\mathcal{K}$ is a class of frames such that $\mathcal{L} = Log(\mathcal{K})$. We say that $\mathcal{L}$ is complete under* clause-preserving mapping w.r.t class $\mathcal{K}$ *if for every model $\mathcal{M}$ over a frame $F \in \mathcal{K}$, and any sets of clauses $\Psi \subseteq \Phi \subseteq \Theta(dom\, V)$, there exists a finite model $\mathcal{M}'$ such that*

1. *the frame of $\mathcal{M}'$ is an $\mathcal{L}$-frame,*
2. *there exists a clause-preserving mapping of $\mathcal{M}'$ into $\mathcal{M}$ modulo $\langle \Phi, \Psi \rangle$.*

If, in addition, each model $\mathcal{M}'$ can be chosen so its size does not exceed $f(|\Phi|)$, for some computable function $f : \mathbb{N} \to \mathbb{N}$, then $\mathcal{L}$ is said to be complete under *f-bounded clause-preserving mappings w.r.t class $\mathcal{K}$.*

3 Decidability Results

Following [5], we will be representing formulas by inference rules.

An *(inference) rule* is a pair $\langle \alpha, \beta \rangle$ of Λ-formulas. We will usually write the rule $\langle \alpha, \beta \rangle$ in the form α/β. For a rule $r = \alpha/\beta$: $Var(r) := Var(\alpha) \cup Var(\beta)$. A rule $r = \alpha/\beta$ is *valid in a model $\mathcal{M}$* (written $\mathcal{M} \vDash r$), if $Var(r) \subseteq \mathrm{dom}(V)$ and

$$\mathcal{M} \vDash \alpha \Longrightarrow \mathcal{M} \vDash \beta.$$

A rule r is *valid i a frame F*, if, for any valuation V of variables $Var(r)$, $F \vDash_V r$. If the rule r is not valid on F, then there is a valuation V such that $F \nvDash_V r$. In that case we say that r is *refuted* on F (by V).

A rule r over the modal language $\Lambda = \langle \neg, \wedge, \vee, \to, \Diamond_1, \ldots, \Diamond_m \rangle$ is said to be in the *reduced normal form* if

$$r = \bigvee_{1 \leqslant j \leqslant s} \theta_j / x_1 \ , \tag{1}$$

and each disjunct θ_j has the form

$$\theta_j = \bigwedge_{k=1}^{n} x_k^{t(0,k)} \wedge \bigwedge_{i=1}^{m} \bigwedge_{k=1}^{n} (\Diamond_i x_k)^{t(i,k)},$$

where x_i are variables, $t(i,k) \in \{0,1\}$, and for any formula α, $\alpha^0 := \neg\alpha$, $\alpha^1 := \alpha$. Note that every disjunct of the reduced form is a clause.

Two rules r_1, r_2 are *equivalent* over a Kripke class $\mathcal{K}$, if for every $F \in \mathcal{K}$:

$$F \vDash r_1 \iff F \vDash r_2 \ .$$

For a formula α, the *set of subformulas* $Sub(\alpha)$ *of* α is defined as usually. For a rule $r = \alpha/\beta$: $Sub(r) := Sub(\alpha) \cup Sub(\beta)$.

It has been shown in Rybakov [6] that any modal inference rule may be transformed to an equivalent rule in the reduced normal form. Using essentially the same technique we can transform to normal reduced forms all rules of the considered modal language.

Lemma 1. *Let* $\Lambda = \langle \neg, \wedge, \vee, \rightarrow, f_1, \ldots, f_m \rangle$ *be a language, where* f_i *are unary connectives (may be non-Kripke modalities). Suppose that for all* f_i*s holds*

$$f_i(p \leftrightarrow q) \leftrightarrow (f_i p \leftrightarrow f_i q).$$

Then any rule $r = \alpha/\beta$ *can be transformed in exponential time to an equivalent rule* r_{nf} *in the reduced normal form.*

From the definition of a normal reduced form, it is clear that under any given valuation of variables only one θ_j can hold true at a given state.

Thus, we have for every Λ-formula α and every frame F of the respective signature:

$$F \vDash \alpha \iff F \vDash x \rightarrow x/\alpha \iff F \vDash (x \rightarrow x/\alpha)_{\mathrm{nf}} \, .$$

Therefore the following lemma holds:

Lemma 2. *A formula* α *is a theorem of a logic* $\mathcal{L}$ *iff the rule* $(x \rightarrow x/\alpha)_{\mathrm{nf}}$ *is valid in all* $\mathcal{L}$*-frames.*

Further on, rules will always be of the form (1).

Lemma 3. *Suppose* $\mathcal{B}$ *and* $\mathcal{S}$ *are two Kripke complete modal logics and* S *is closed under* f*-bounded clause-preserving mappings. Then, if a rule* $r = \bigvee_{1 \leqslant j \leqslant s} \theta_j / x_1$ *is refuted on a* $\mathcal{B}_{\mathcal{S}}$*-model, then* r *is refuted on a* $\mathcal{B}_{\mathcal{S}}$*-model with the size of* R*-clusters at most* $f(s)$*.*

Definition 5. *Suppose* $\mathcal{M}$ *is a transitive Kripke model of the signature* $\langle R \rangle$*, such that* $dom\, V$ *is finite. A model* $\mathcal{N}$ *is a* clause-filtration *of the model* $\mathcal{M}$ *if*

$$W = W/\sim, \text{ where } u \sim v \iff \theta_{\mathcal{M}}(u) = \theta_{\mathcal{M}}(v).$$

$$[u]_{\sim} R_i [v]_{\sim} \iff \mu^i(\theta_{\mathcal{N}}(v)) \subseteq \mu^i(\theta_{\mathcal{N}}(u)),$$

$$V(x_k) = \{[w]_{\sim} \mid x_k \in \mu^0(\theta_{\mathcal{N}}(w))\}.$$

We say that a Kripke logic $\mathcal{L}$ *admits strong clause-filtration*, whenever for every $\mathcal{L}$-model $\mathcal{M}$ over an $\mathcal{L}$-frame, there is a clausal filtration $\mathcal{N}$ of $\mathcal{M}$, based on an $\mathcal{L}$-frame.

The strong clause-filtration property is a variant of the usual filtration property [7,8,9], modified in two respects: firstly, it adjusted for the use with the reduced normal forms, secondly, it requires the existence of a filtration model with underlying frame adequate for $\mathcal{L}$.

Lemma 4. *Let* $\mathcal{M} = \langle W, R, R_1, \ldots, R_m, V\rangle$ *be the* $\mathcal{B}_{\mathcal{S}}$*-model obtained in Lemma 3, in particular* $\mathcal{M} \not\models r$*. Suppose also that the logic* $\mathcal{B}$ *is closed under clause-filtrations. Then* r *is refuted on a finite* $\mathcal{B}_{\mathcal{S}}$*-model of the size less or equal than* $f(s) \cdot 2^s$*, where* $s = |\Theta(r)|$*.*

A Kripke logic $\mathcal{L}$ has the *finite model property*, whenever for every formula $\alpha \notin \mathcal{L}$, there is a finite model $\mathcal{M}$ such that $\mathcal{M} \models \mathcal{L}$ and $\mathcal{M} \not\models \alpha$. If, in addition, the model $\mathcal{M}$ can be chosen to be of the size not more than $f(|\alpha|)$, for some computable function $f : \mathbb{N} \to \mathbb{N}$, then $\mathcal{L}$ has the *f-effective finite model property.*

We say that a Kripke-complete logic $\mathcal{L}$ *admits strong filtration*, whenever for every model $\mathcal{M} \not\models \alpha$ over an $\mathcal{L}$-frame, there is a finite set of formulas Σ, such that there is a model $\langle F, V\rangle \not\models \alpha$, that is a filtration of $\mathcal{M}$ modulo Σ and $F \in Fr(\mathcal{L})$. The logics that admit strong filtration form the majority of standard logics to which the *filtration method* (see [10]) can be applied. They include $K4$, $S4$, $S5$ and so on. A logic $\mathcal{L}$ that admits strong filtration modulo sets of the kind $Sub(\alpha)$, also admits strong clausal filtration.

Theorem 1. *Let*

1. $\mathcal{B}$ *be a transitive logic that admits strong filtration,*
2. $\mathcal{S}$ *be a multi-modal logic, closed under* f*-bounded clause preserving mappings,*

Then $\mathcal{B}_{\mathcal{S}}$ *has the* g*-effective finite model property, where* $g(x) = (f(x) + 1) \cdot 2^x$*.*

Proof. By Lemmas 3 and 4 □

Corollary 1. *If under conditions of Theorem 1 the class of finite models of* $\mathcal{B}_{\mathcal{S}}$ *is decidable (i.e., the set of isomorphic classes of finite* $\mathcal{B}_{\mathcal{S}}$*-models is decidable), then* $\mathcal{B}_{\mathcal{S}}$ *is decidable.*

As an easy corollary of Theorem 1 we obtain:

Corollary 2. *Suppose* $\mathcal{B}$ *is a mono-modal transitive logic that admits strong filtration. If* $\mathcal{S}$ *also admits strong filtration, then* $\mathcal{B}_{\mathcal{S}}$ *has the effective finite model property. In particular, if*

$$\mathcal{B} \in \{K4, S4, S5\}, \quad \mathcal{S} \in \{K4_N, S4_N, S5_N\},$$

then the logic $\mathcal{B}_{\mathcal{S}}$ *has the effective finite model property.*

Proof. Since the filtration property implies the clause-filtration property, therefore, by Theorem 1, the logic $\mathcal{B}_{\mathcal{S}}$ has the effective finite model property □

Corollary 3. *If, in addition to conditions of Corollary 2, logics* $\mathcal{B}$ *and* $\mathcal{S}$ *have effectively recognizable classes of finite models, then* $\mathcal{B}_{\mathcal{S}}$ *is decidable.*

Proof. Since, by Corollary 2, logic $\mathcal{B}_{\mathcal{S}}$ has the finite model property, it suffices to show that the class of finite $\mathcal{B}_{\mathcal{S}}$-frames is effectively recognizable. To check that a given frame $F = \langle W, R, R_1, \ldots, R_m\rangle$ is a $\mathcal{B}_{\mathcal{S}}$-frame we only need to check that

1. the frame $\langle W, R\rangle$ is a $\mathcal{B}$-frame,
2. the frame $\langle C, R_1, \ldots, R_m\rangle$ is $\mathcal{S}$-frame, for every $C \in Cl_R(F)$.

Both conditions can be checked effectively, and also the procedure of recognizing clusters in a finite frame is effective. Thus logic $\mathcal{B}_{\mathcal{S}}$ is decidable. □

4 Conclusion and Future Work

The paper presents a method for describing evolutions of MAS, which differs from the established approaches in several important respects. First of all, unlike the methods based on basic fusions, it allows for more expressive descriptive language, On the other hand, we prove that, unlike products of logics used as a base for describing evolutions of MAS, the proposed cluster-based approach leads to decidable logics. We demonstrate this by presenting a generic decision algorithm. This algorithm has 2EXPTIME-complexity (relative) bound, which makes it practically unfeasible. Nevertheless, the decidability of corresponding logics, opens a possibility for obtaining more practical variants of the decision algorithms, possibly using tableaux-based techniques.

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