Genetic Optimization for the Design of Walking Patterns of a Biped Robot

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Abstract. This paper presents an Genetic Algorithm (GA) for the design of walking patterns of a 3-DOF biped robot formulated as a system with impulse effects. The GA optimizes the coefficients of a polynomial which represents the desired behavior of the walking which is included into the dynamics of the biped robot to obtain periodic motions while fulfills a minimal energy consumption over a complete walking cycle assumed as single support and instantaneous double support phases. Optimization results are presented showing walking patterns with low energy consumption and periodic motions.

1 Introduction

In the last years several results has been provided for the stabilization of the biped locomotion, which usually are based on find a suitable reference trajectory i. e. walking pattern such that guarantee to perform stable steps while obey a certain criteria such as walking velocity, step length, step frequency, including energy minimization [16], [17]. Recent methods proposed for the walking pattern synthesis incorporate the zero moment point approach (ZMP) [18] as stability criteria. Qiang et. al. in [9] considering the ground condition and walking patterns with small torque and velocity of the joint actuators are obtained. Capi et. al. in [3] generate the joint trajectories for walking and going up-stairs based on consumed energy and torque change using evolutionary computation methods. Denk et. al. in [5] propose a systematic approach for the design of a walking pattern database using optimal control techniques and ZMP for prevention of the foot rotation and ensure the feasibility of the walking; Kajita et. al. in [11] introduce a method for a walking pattern generation by using a preview control of ZMP that uses reference future.

On the other hand the formulation for the walking pattern synthesis is done as a optimal control problem and optimization techniques are used. Bessonnet et al. in [2], by quadratic programming define the generalized coordinates as spline functions such that fulfill the minimization of the amount of the driving torques.

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A Genetic Algorithm is used as optimization method by Sang-Ho et. al. [4] to determine the shape of velocities and accelerations trajectories that minimizes the sum of deviation of each one through the search of suitable via points for obtain smooth and stable walking. Likewise, Park et. al. in [15] obtain an optimal locomotion pattern with minimal energy consumption and optimal locations of the center of mass of the links.

This paper is concerned in the energy optimal walking pattern design for a biped robot modeled as a system with impulse effects. The joint trajectories are parameterized as an polynomial (as are reported in [15]), which are included into the dynamics of the biped robot. A real coded Genetic Algorithm (GA) is used as optimization method to find the compatible polynomial coefficients which minimize the required input energy during the walking cycle while holding a stable walking motions. The optimization process is based on the evaluation of the biped robot in closed-loop, considering a set of constraints which guarantee no violation of the biped model assumptions and reach periodic walking motions. The difference with related publications is the generation of walking patterns whose motions are restricted to be described as periodic orbits and the design is not based on the ZMP approach, thus the reported walking patterns, allow more maneuvers than in this work is so far of our goal.

The paper is organized as follows: Section 2 presents the mathematical model of an 3-DOF biped robot. In Section 3, the problem formulation is summarized. Section 4, presents the GA implementation details used for the problem optimization established in previous section. Section 5, presents the results of numerical simulations. Section 6 provide the conclusions.

2 Biped Robot Model

In this section the dynamical model of a planar biped robot is introduced. The biped robot consists of a torso, hips, and two legs of equal length without ankles and knees. Two torques are applied between the legs and torso, so the system is under actuated. The definition of the angular coordinates and the disposition of the masses of the torso, hips and legs of the biped robot are shown in Figure 1, note that positive angles are computed in clockwise with respect to the indicated vertical lines and all masses of the links are lumped.

The walking cycle takes place in sagittal plane and on a level of surface, and it is assumed as successive phases where only one leg (stance leg) touching the walking surface (swing phase), with the transition from one leg to another taking place in a smaller length of time. During swing phase, the stance leg is modeled like a pivot and the swing leg is assumed move into the frontal plane [13] and to renter the plane of the motion when the angle of the stance leg attains a given desired value. The assumptions concerning to the walking cycle, define the biped robot model in two parts: the model that describe the swing phase and another one that describe the contact event of the swing leg with the walking surface; which are presented below.

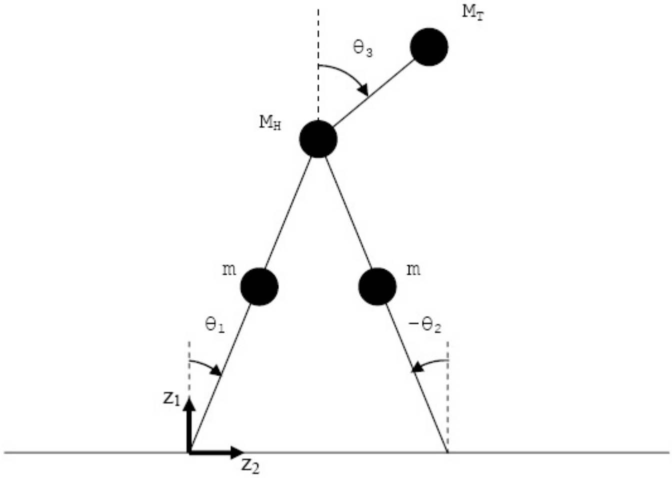

Fig. 1. 3-linkbiped robot.

2.1 Dynamic Model

The dynamical model of the robot during the swing phase is a second order system derived of Lagrange method [12]:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = Bu, \tag{1}$$

where $\theta = [\theta_1, \theta_2, \theta_3]^T$ are the generalized coordinates, θ_1 parameterizes the stance leg, θ_2 the swing leg and θ_3 the torso; $u = [u_1, u_2]^T$ are the input; $M(\theta)$ is a positive-definite inertia matrix, $C(\theta,\dot{\theta})\dot{\theta}$ is the vector of the centripetal and Coriolis forces; and $G(\theta)$ is the vector of the conservative forces and B is the input matrix.

Transforming the second order equation (1) to state-space form by defining

$$\begin{aligned}\dot{x} &:= \frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\dot{\theta} \\ M^{-1}(\theta)[-C(\theta,\dot{\theta})\dot{\theta} - G(\theta) + Bu]\end{bmatrix} \\ &:= f(x) + g(x)u. \end{aligned} \tag{2}$$

2.2 Impact Model

The impact between the swing leg and the ground is modeled like a contact between two rigid bodies. The main objective is to obtain the velocity of the generalized coordinates after the impact of the swing leg with the walking surface in terms of the velocity and position before the impact. The impact model for the biped robot is based in the rigid impact model of [10] and assume the case where the contact of the swing leg with de walking surface produce either no rebound nor slipping of the swing leg, and the stance leg lifting the walking surface without interaction. The conditions in [10] for these assumptions to be valid are that:

1. the impact takes place over an infinitesimally small period of time;
2. the external forces during impact can be represented by impulses;
3. impulsive forces may results in instantaneous change in the velocities of the generalized coordinates, but positions remain continuous;
4. the torques supplied by the actuators are not impulsional.

Taking into account the previous assumptions, the impact model is expressed with the equation [19]:

$$\begin{bmatrix} M_e & -E^T \\ E & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_e^+ \\ F \end{bmatrix} = \begin{bmatrix} M_e \dot{\theta}_e^- \\ 0 \end{bmatrix}, \tag{3}$$

where $\theta_e = [\theta_1, \theta_2, \theta_3, z_1, z_2]^T$ are the generalized coordinates resulting of the add the Cartesian coordinates $[z_1, z_2]^T$ of the end of the stance leg as shown in Figure 1; M_e is the positive-definite inertia matrix of the biped model with the new generalized coordinates; $F = [F_T, F_N]^T$ is the tangential and normal forces applied at the end of the swing leg; $\dot{\theta}_e^+$ is the velocity after the impact; $\dot{\theta}_e^-$ is the velocity before the impact and

$$E := \frac{\partial Y}{\partial \theta_e} = \begin{bmatrix} r\cos(\theta_1) & -r\cos(\theta_2) & 0 & 1 & 0 \\ -r\sin(\theta_1) & r\sin(\theta_2) & 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

obtaining $\dot{\theta}_e^+$ of the equation (3) must be done a change of coordinates and reinitialize the model (2). The change of coordinates is an expression for $x^+ := (\theta^+, \dot{\theta}^+)$ in terms of $x^- := (\theta^-, \dot{\theta}^-)$, which is given for the mapping function:

$$x^+ = \Delta(x^-) := \begin{bmatrix} \theta_2^- & \theta_1^- & \theta_3^- & \dot{\theta}_2^+(x^-) & \dot{\theta}_1^+(x^-) & \dot{\theta}_3^+(x^-) \end{bmatrix}. \tag{5}$$

Thus, the overall model of the biped walking is the combination of the swing phase model and impact model, as follows:

$$\Sigma : \begin{cases} \dot{x} = f(x) + g(x)u & x^- \notin S \\ x^= \Delta(x^-) & x^- \in S \end{cases} \tag{6}$$

where

$$S := \{(\theta, \dot{\theta}) | z_1 > 0, z_2 = 0, \theta_1 = \theta_1^d\} \tag{7}$$

3 Problem Formulation

This paper is based on the feedback controller for walking proposed by Grizzle et al. in [19] which designs a feedback for posture control and swing leg advancement of the biped robot (6), where the posture control means maintaining the torso angle at a constant value θ_3^d and swing leg advancement consists in driving the swing leg to behave as mirror image of the stance leg ($\theta_2 = \theta_1$). Thus, the behavior is encoded into the dynamics of the robot by defining the following output function:

$$y := \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} := \begin{bmatrix} h_1(\theta, a) \\ h_2(\theta, a) \end{bmatrix} := \begin{bmatrix} \theta_3 - h_{d,1}(\theta_1, a) \\ \theta_2 - h_{d,2}(\theta_1, a) \end{bmatrix} \tag{8}$$

where

$$\begin{array}{rl} h_{d,1}(\theta_1,a) := & a_1^0+\cdots+a_1^3(\theta_1)^3 \\ h_{d,2}(\theta_1,a) := & -\theta_1+(a_2^0+\cdots+a_2^3(\theta_1)^3)\times(\theta_1+\theta_1^d)\times(\theta_1-\theta_1^d) \end{array} \tag{9}$$

with the control objective being to zeroing the outputs (8), they define the feedback

$$u(x) := (L_gL_fh(x))^{-1}(\Psi(h(x),L_fh(x))-L_f^2h(x)) \tag{10}$$

and

$$\Psi(y,\dot{y}) := \begin{bmatrix} \frac{1}{\varepsilon^2}\psi_\alpha(y_1,\varepsilon\dot{y}_1) \\ \frac{1}{\varepsilon^2}\psi_\alpha(y_2,\varepsilon\dot{y}_2) \end{bmatrix} \tag{11}$$

$$\psi_\alpha(x_1,x_2) := -sign(x_2)|x_2|^\alpha - sign(\phi_\alpha(x_1,x_2))|\phi(x_1,x_2)|^{\frac{\alpha}{2-\alpha}} \tag{12}$$

$$\phi_\alpha := x_1+\left(\frac{1}{2-\alpha}\right)sign(x_2)|x_2|^{2-\alpha} \tag{13}$$

The problem statement is described as follows: Given the biped robot model (6) and the feedback (10), find the parameters a_i^j of the function in (9) such that it obtains a walking pattern which produce periodic walking motions with low energy consumption.

4 Optimization Strategy

According to the problem, a Genetic Algorithm (GA) is proposed to solve the optimization problem. GA are search algorithms based on natural selection and natural genetic principles [8], apply them to optimization problems differ from traditional optimization methods that they use: payoff information (based on a objective function), no derivatives or other auxiliary knowledge; a coding of the parameters, not the parameters themselves and probabilistic rules as a tool to guide the search [14].

The implementation of the GA begins with the election of consistent decision variables which to solve the problem continuing with the coding of them into a string called individual, as well as an objective function which provide a numerical value that indicate the goodness degree or fitness of the individual to solve the problem. The GA operation consist of several stages, that to imitate the natural principles in which are based: The *initialization* stage where a set of individuals called population is created randomly; the *selection* stage in which each individual is evaluated through the objective function, the values of the evaluation are used by a selection algorithm that identifies and choose the fittest individuals that to serve as the parents for the creation of new individuals; The new individuals are created by applying the genetic operations to the individuals selected in the previous stage. This operations are: crossover, which with certain probability, combine the parameters encoded between pairs of individuals producing two individuals with information of both parents and the mutation, which is the alteration of the value of one parameter encoded in the individual, its function is to introduce diversity in the population, this stage is called *reproduction*; the last stage is the *replacement*, it consist in the creation of the new population with the new individuals and the parents. The overall stages is performed of iterative manner, each iteration is called generation. The GA operation explained above is expressed through an algorithm as shown below:

1. Begin
2. $i \leftarrow 0$
3. Create a population P_i of N individuals
4. Evaluate fitness of all individuals of the population: $P_i^e \leftarrow Fitness(P_i)$
5. Repeat the steps until the conditions are reached
   a) Select the parents for matting: $P_i^s \leftarrow Select(P_i, P_i^e)$
   b) Apply crossover and mutation: $P_i^g \leftarrow Genetic\_Operation(P_i^s)$
   c) $i \leftarrow i+1$
   d) Create the new population: $P_i \leftarrow New\_Pop(P_i^g, P_{i-1})$
   e) Evaluate fitness of new individuals: $P_i^e \leftarrow Fitness(P_i)$
6. Print out the best solution
7. End

Thus, the details of our implementation of the each stage are provided below.

4.1 Individual Representation

The individual representation selected is the real coding scheme, whose genotype is composed by eight real numbers such that represent to each parameter a_i^j (decision variables) of the function in (9).

4.2 Objective Function

The goal of the GA is the minimization of the cost function which represents the approximation to the average energy consumed over the walking cycle:

$$\hat{J}(a) = \int_0^T (u_1^2 + u_2^2)dt \tag{14}$$

where T is such that $\theta_1(T) - \theta_1^d = 0$, and $u(t)$ is obtained of applying (10) into the closed loop with the biped robot model (2) and (8).

This cost function is subject to the following constraints taking of [7]:

$$\begin{array}{ll} \hat{\dot{\theta}}_1^- - 0.99\lambda(\hat{\dot{\theta}}_1^-) & \leq 0 \\ \|y(\bar{t})\| & \leq 0 \\ |\frac{F_T}{F_N}| - \mu & \leq 0 \\ -\dot{z}_2^+ & \leq 0 \end{array} \tag{15}$$

The first constraint assures the existence of the one fixed point, the second one assures that the controller has converged before the impact and the last two constraints verify the validity of the impact model.

4.3 Genetic Operations

The genetic operations comprise the classical one point crossover and non-uniform mutation [6][14]. Non-uniform mutation is defined as follows: if $s_v^t = [v_1, ..., v_m]$ is a individual (with t, which means the generation number) and the element v_k was selected for this mutation, the result is a vector $s_v^t = [v_1, ..., v_k^{'}, ..., v_m]$ where:

$$v_k^{'} = \begin{cases} v_k + \Delta(t, UB - v_k) & \text{if a random digits is 0} \\ v_k - \Delta(t, v_k - LB) & \text{if a random digit is 1} \end{cases} \tag{16}$$

and LB and UB are the lower an upper domains bounds of the variable v_k the function $\Delta(t,y) = y\left(1 - r\left(1 - \frac{t}{T}\right)^b\right)$ return a value in the range $[0, y]$; r is a random number, T is the maxima generation number and b is the dependency degree on the iteration number.

5 Simulation Results

The GA was developed in MatLab with the characteristics described in Section III, the individual evaluation is during one walking cycle and uses the implementation in software of 3-DOF model (6) with the controller (10) available in [1]. The physical parameters of the biped model are shown in the Table 1, pre-impact angular velocity is taken as $\dot{\theta}_1^- = 1.55$, the friction coefficient $\mu \geq \frac{2}{3}$, and the controller parameters for equations (11) and (12) are $\varepsilon = 0.1$ and $\alpha = 0.9$ respectively. Details of the biped model can be found in [19].

In the Table 3 presents the parameters for the function (9) and the energy consumption as results of five independent GA executions using the GA parameters

Table 1. Biped model parameters

Parameter	Value
Mass of Hips (M_H)	15 kg
Mass of Torso (M_T)	10 kg
Mass of Legs (m)	5 kg
Length of legs (r)	1 m
Length of torso (l)	0.5 m

Table 2. Genetic algorithm parameters

Parameter	Value
Number of individuals	100
Number Maximum of Generations	400
Crossover Rate	0.9
Mutation Rate	0.2
Selection Rate	0.4

Table 3. Parameter a_i^j and energy magnitude results with the GA

Number	Parameters								$\hat{J}(a)$
	a_1^0	a_1^1	a_1^2	a_1^3	a_2^0	a_2^1	a_2^2	a_2^3	
1	0.5893	0.4644	1.5558	-0.0367	-1.2656	1.0308	1.1086	3.7436	719.32
2*	0.5120	0.0730	0.0350	-0.8190	-2.2700	3.2600	3.1100	1.8900	761.00
3	0.7098	0.0581	0.0536	-1.0280	-1.9803	2.5752	3.6109	2.0000	1225.40
4	0.3992	0.7515	-2.5759	2.5365	1.8022	0.5344	0.2620	6.0033	1679.20
5	1.0730	-0.4986	-0.4127	1.6194	2.7029	-1.4978	1.6201	-0.8384	2541.70
6	0.2892	1.9435	-2.8760	0.4328	2.0252	4.5643	-0.5546	-4.1460	2980.50

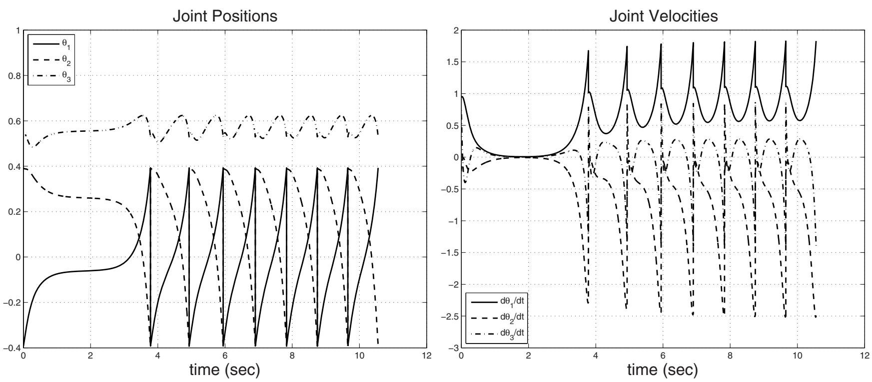

Fig. 2. Joint positions and velocities trajectories corresponding to closed-loop system execution with the parameters of the first entry of the Table 3

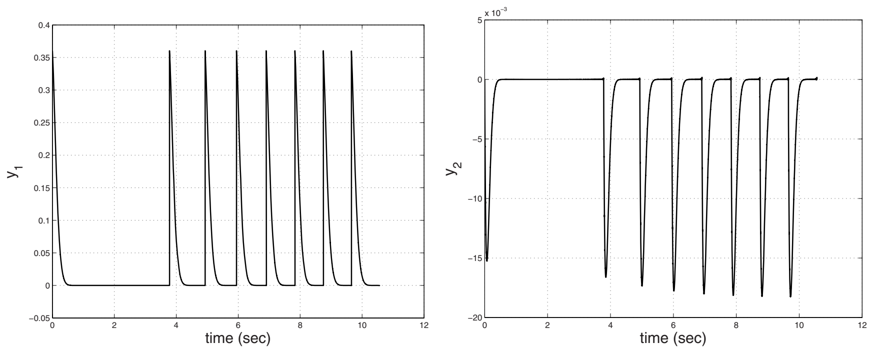

Fig. 3. Output function trajectories corresponding to closed-loop system execution with the parameters of the first entry of the Table 3

showed in the Table 2. The second entry on the table 3 include the parameters obtained by Grizzle et. al. in [19] with the same biped robot model using a traditional optimization method.

The results summarized in the Figures 2 to 5, corresponds to the closed-loop system simulations using the walking patterns defined by parameters of the first entry of the Table 3, and Figure 6 shows the fitness values of the best individual in each generation, finishing the optimization process about 140^{th} generation. Overall results depict the execution of the closed-loop system performing eight steps. Note that the energy magnitude obtained with GA is slightly lower than the reported in [19]. The joint trajectories in the Figure 2 shows that the time needed to execute the first step is longer due that minimal change in the input torques succeeded, hence

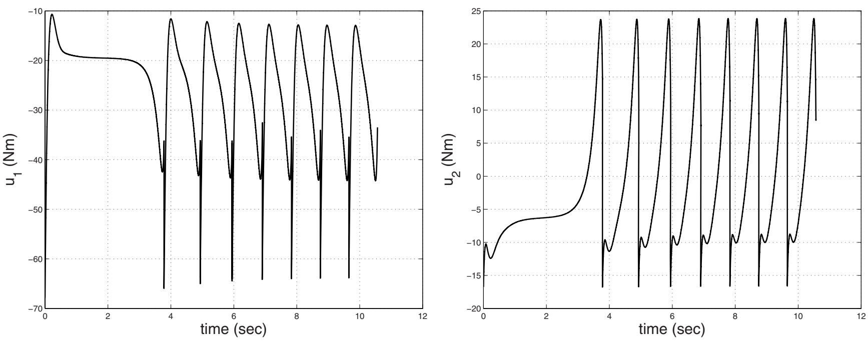

Fig. 4. Control signals corresponding to closed-loop system execution with the parameters of the first entry of the Table 3

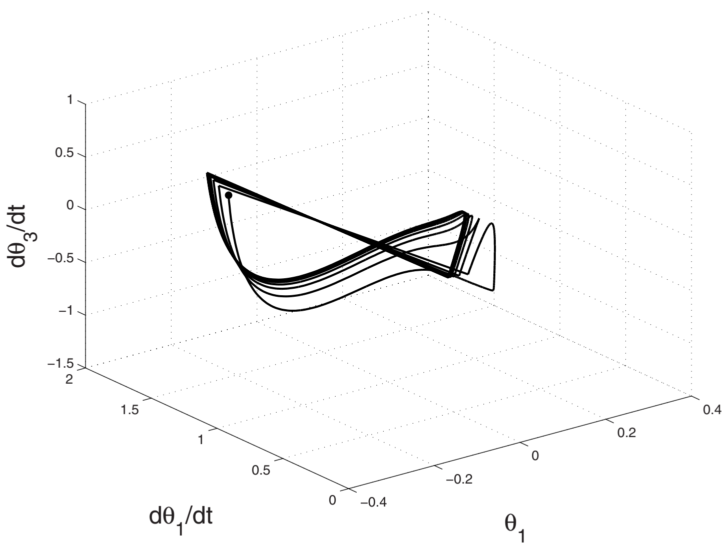

Fig. 5. Orbits obtained corresponding to closed-loop system execution with the parameters of the first entry of the Table 3

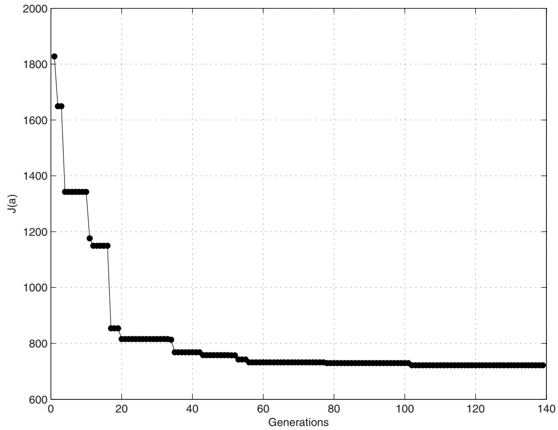

Fig. 6. Fitness values obtained through the optimization process for the first execution of the Table 3

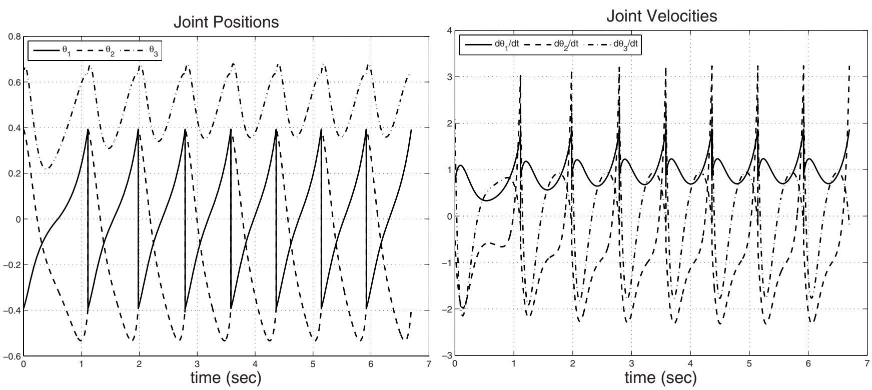

Fig. 7. Joint positions and velocities trajectories corresponding to closed-loop system execution with the parameters of the sixth entry of the Table 3

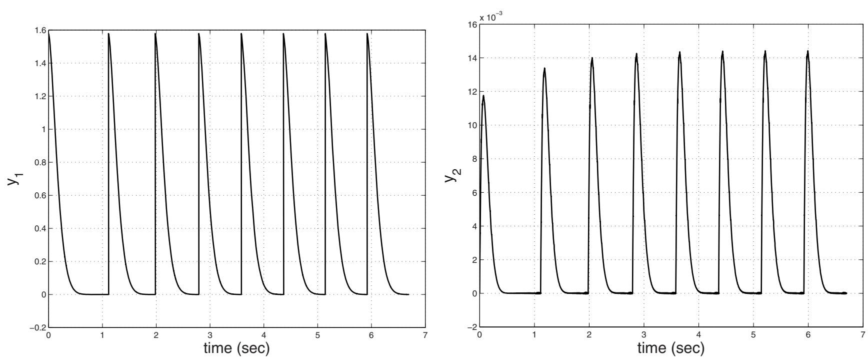

Fig. 8. Output function trajectories corresponding to closed-loop system execution with the parameters of the sixth entry of the Table 3

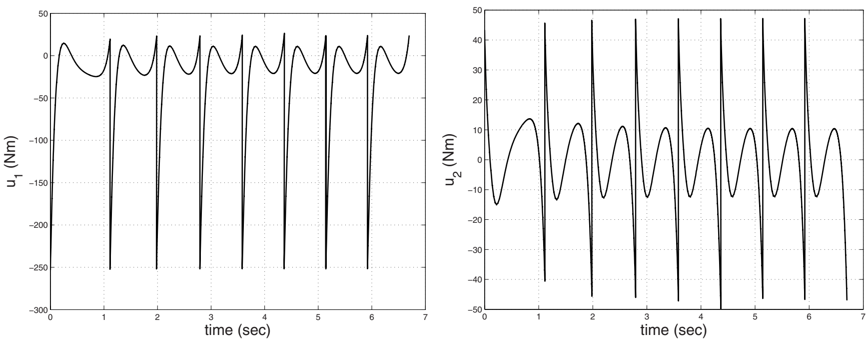

Fig. 9. Control signals corresponding to closed-loop system execution with the parameters of the sixth entry of the Table 3

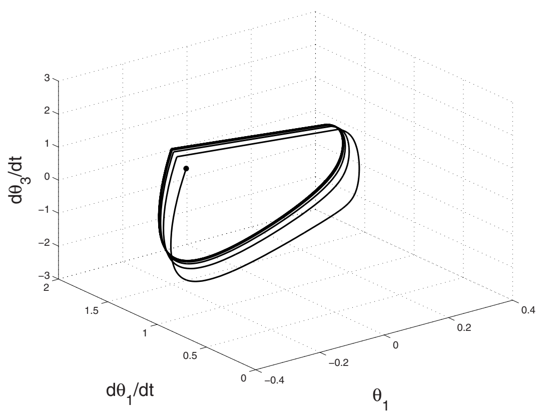

Fig. 10. Orbits corresponding to closed-loop system execution with the parameters of the sixth entry of the Table 3

the motion turning slow, after these, the step time tend to be the same through the following steps. In the Figures 7 to 10 demonstrate the results whose parameters are not optimal in energy magnitude, but still produce periodic walking motions. Moreover, in the Figures 5 and 10 present the orbits for both simulations cases, which shown the periodic nature of the walking.

6 Conclusions

In this paper a GA for the design of walking patterns of a biped robot was presented. The design approach consists in the search of suitable coefficients of a function, which describe the desired behavior of the joint trajectories such that produce periodic walking motion while keeping a minimal energy consumption. The numerical results with the function obtained by GA in the closed-loop system shows that this approach can provide a set of different walking patterns whose motions are described as periodic orbits and was able to obtain near optimal energy magnitudes.

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