# Controlling Unstable Non-Minimum-Phase Systems with Fuzzy Logic: The Perturbed Case

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**Abstract.** In this paper Fuzzy Logic Systems (FLS) for controlling non-minimum-phase systems are proposed. A generalized Proportional Integral Derivative (PID) Fuzzy Logic Controller (FLC) for a benchmarking second order problem with an unstable zero is presented. The same Fuzzy Rule Base (FRB) of the PID FLC is used in a PD FLC to regulate a plant consisting in a non-minimum-phase servomechanism with nonlinear backlash. Simulations demonstrate that the proposed FLC can be used to handle the non-minimum-phase systems.

## 1 Introduction

There exist several papers dealing with the stabilization of non-minimum-phase systems. For example in [7], a simple design method by means of which it is possible to robustly stabilize, using output feedback, a significant class of uncertain nonlinear systems whose zero dynamics are unstable, the proposed procedure leads to the construction of a dynamic controller yielding robust, semi-global practical stability.

In [1] nonlinear $H_\infty$ control synthesis is extended to an output regulation problem for a servomechanism with backlash. The problem in question is similar to one of the problems proposed in this paper, the design a feedback controller so as to obtain the closed-loop system in which all trajectories are bounded and the load of the driver is regulated to a desired position while also attenuating the influence of external disturbances.

On the other hand, Computational Intelligent (CI) strategies have been used to in order to solve the non-minimum-phase stabilization control problem. For example in [14] a fuzzy logic controller (FLC) is developed for the nonminimum phase system. The well-designed fuzzy rules are exploited to resolve the undershoot problem caused by the unstable zeros. A 3rd-order plant with two unstable zeros is used to verify the performance of the fuzzy controlled system.

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A fuzzy logic based approach is proposed in [2] for the control of non-minimum phase systems. In this approach, a variable structure controller can be designed using fuzzy logic and linguistic control rules implementation.

In [3] an adaptive fuzzy logic system is incorporated with the Variable Structure Control (VSC) system for the purpose of improving the performance of the control system. A sliding surface with an additional tunable parameter is defined as a new output based on the idea of output redefinition, as a result the overload system of missile with the characteristic of non-minimum phase can be transformed into minimum-phase system by tuning the parameters of the sliding surface, and a sliding-mode controller can be designed.

In [5] and [8], an hybridization of FLS and Genetic Algorithms (GA) [6] is used in order to control non-minimum-phase systems, by finding the optimal parameter, of each MF in a FLS.

This paper addresses the problem of designing FLS to control non-minimum-phase systems, first for non-minimum-phase Linear Time Invariant (LTI) [13] class of systems, and then for the design a feedback controller so as to obtain the closed-loop system in which all trajectories are bounded and the load of the driver is regulated to a desired position while also attenuating the influence of external disturbances, where the provided servomotor position is the only measurement available for feedback. Performance issues of the proposed FLC output regulator constructed are illustrated in a simulation study. In the rest of this paper a non-minimum-phase system will be considered as defined in [13]: a system that has at least one positive (unstable) zero.

The paper is organized as follows: Section 2 present FLS theoretical aspects, and the designing of a FLC for controlling non-minimum-phase systems, Section 3 presents the configuration as PID FLC of the FLC designed in Section 2 to control a non-minimum-phase LTI system with an unstable zero, Section 4 presents the configuration as PD FLC of the FLC designed in Section 2 for the output regulation of a servomechanism with non-minimum-phase backlash, and finally, in Section 5 we present our conclusions.

## 2 Fuzzy Logic Control

A FLS is a numerical system that makes a nonlinear mapping from input to output data. A FLS consists of four basic elements (see Fig.1): the ***fuzzifier***, the ***fuzzy rule-base***, the ***inference engine***, and the ***defuzzifier***. The ***fuzzy rule-base*** is a collection of rules in the form of (2), which are combined in the ***inference engine*** to produce a fuzzy output. The ***fuzzifier*** maps the crisp input into Fuzzy Sets (FS), which are subsequently used as inputs to the ***inference engine***, whereas the ***defuzzifier*** maps the FS produced by the ***inference engine*** into crisp numbers.

A FS can be interpreted as a MF $U_x$ that associates with each element of $x$ of the universe of discourse, $U$, a number $\mu_X(x)$ in the interval [0,1]:

$$\mu_x : U \to [0,1]. \tag{1}$$

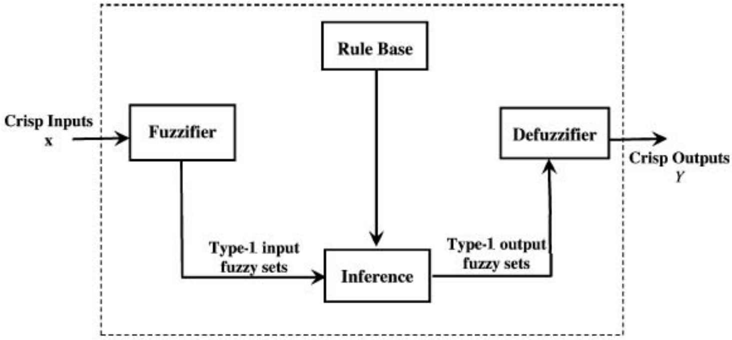


**Fig. 1.** Structure of Fuzzy Logic System.

### 2.1 FLC Design

To solve the regulation problem for non-minimum-phase systems, we propose two-input one-output rules in the formulation of the knowledge base. The IF-THEN rules are according with the Mamdani type of Fuzzy Inference Systems [10][9][11]:

$$\text{IF } y_1 \text{ is } A_1^l \text{ AND } y_2 \text{ is } A_2^l \text{ THEN } y_3 \text{ is } B^l \tag{2}$$

where $[y_1, y_2]^T = y \in U = U_1 \times U_2 \subset \mathbb{R}^2$, where $y_1$ is the fuzzified value of a measured variable, $y_2$ is the fuzzified value of the derivative of the measured variable and $y_3 \in V \subset \mathbb{R}$. For each input fuzzy set $A_k^l$ in $y_k \subset U_k$ with $k = 1,2$; and output fuzzy set $B^l$ in $y_3 \subset V$ exists an input membership function $\mu_{A_k^l}(y_k)$ whit $k = 1,2$, and output membership function $\mu_{B^l} \in y_3 \subset V$, respectively, with $l$ being the number of membership functions associated to the input $k$.

The particular choice of each $\mu_{B^l}(y_3)$ will depend on the heuristic knowledge of the experts over the plant.

We select triangular membership functions for each input (*error* and *change of error*) and output (*control*) variables, granulating each one of these three variables

**Table 1.** Fuzzy rules

| No. | *error* | *change of error* | *control* |
|---|---|---|---|
| 1 | n | n | p |
| 2 | n | z | p |
| 3 | n | p | z |
| 4 | z | n | z |
| 5 | z | z | z |
| 6 | z | p | z |
| 7 | p | n | z |
| 8 | p | z | n |
| 9 | p | p | n |

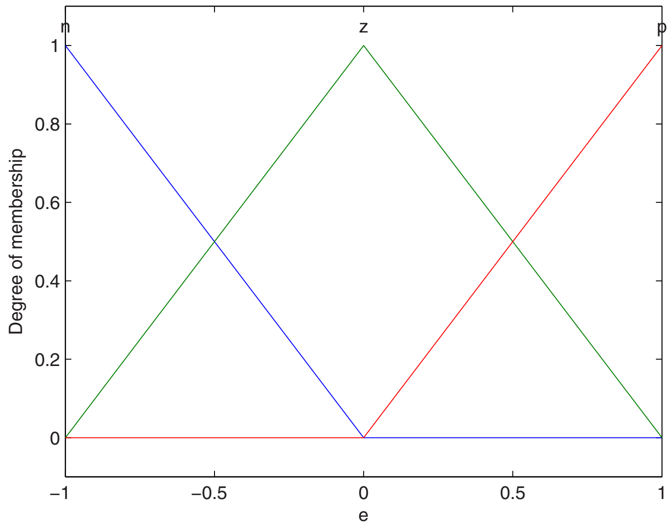


**Fig. 2.** Input variable *error*.

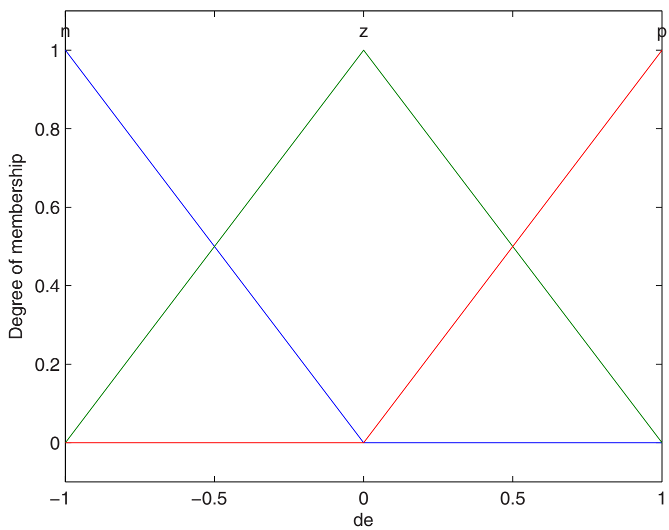


**Fig. 3.** Input variable *change of error*.

in three fuzzy sets: *negative* (**n**), *zero* (**z**) and *positive* (**p**). The shape of the variables can be seen in Fig. 2, 3 and 4 respectively.

These input and output variables are combined in a FRB in the form of (2), and we select the fuzzy rules shown in Table 1.

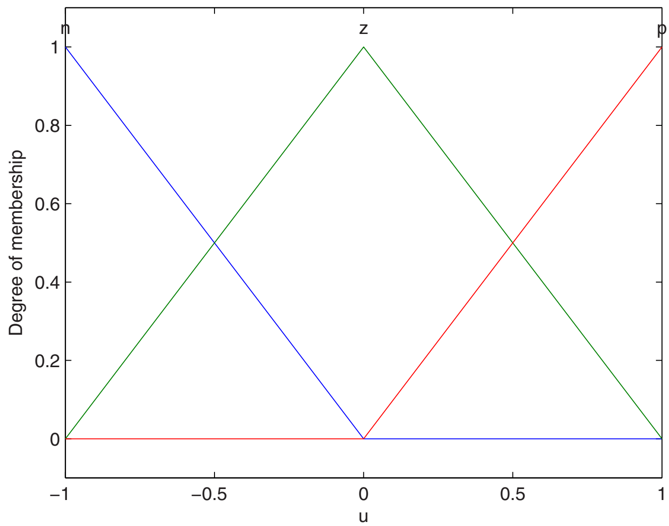


**Fig. 4.** Input variable *control*.

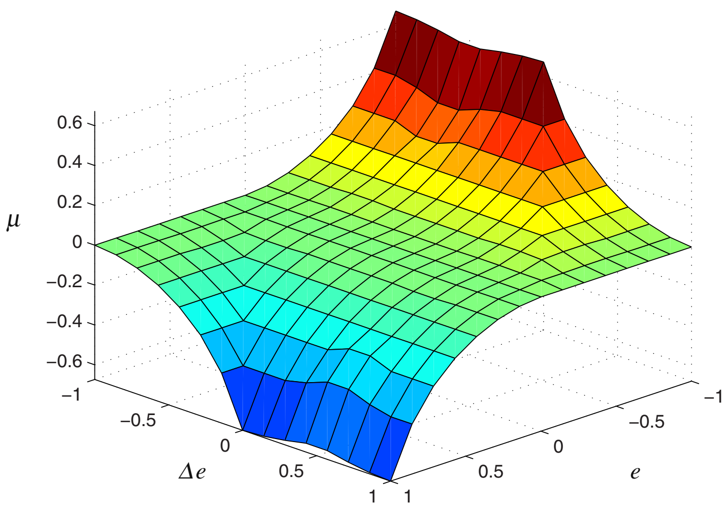


**Fig. 5.** Surface of control.

For the inference process, we implement the Mamdani [10][9] type of Fuzzy Inference, with *minimum* as disjunction operator, *maximum* as conjunction operator, *minimum* as implication operator, *maximum* as aggregation operator and Centroid (COA) [4] as our defuzzification method.

Defuzzification refers to the way a numeric value is extracted from a fuzzy set as a representative value. In general, the defuzzification methods take a fuzzy set *B* of a universe of discourse *V*, where *B* is usually represented by an aggregated output membership function.

The COA defuzzication method is expressed as

$$\tau_m = \frac{\int_{y_3} \mu_B(y_3) y_3 dy_3}{\int_{y_3} \mu_B(y_3) dy_3}, \tag{3}$$

where $\mu_B(y_3)$ is the aggregated output MF. This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values in probability distributions.

The FLS is builded with the FRB of Table 1 and triangular MF of Figs. 2, 3 and 4, this FLS outputs the control surface of Fig. 5.

## 3 Controlling a LTI System with an Unstable Zero

The LTI [13] systems, are systems that can be represented by ordinary differential equations.

A LTI system that has a positive zero is called a non-minimum-phase system, the fact that a system has a positive zero means that the zero is unstable.

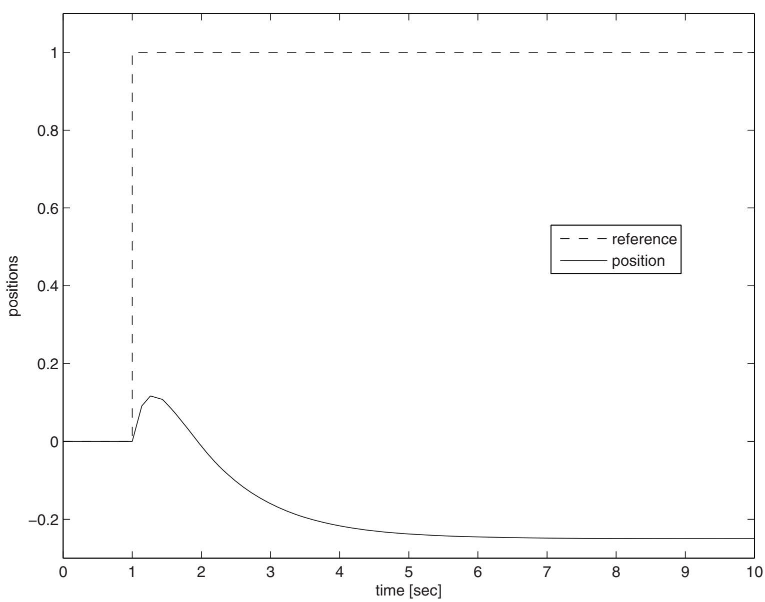


**Fig. 6.** System (4) response to the unitary step.

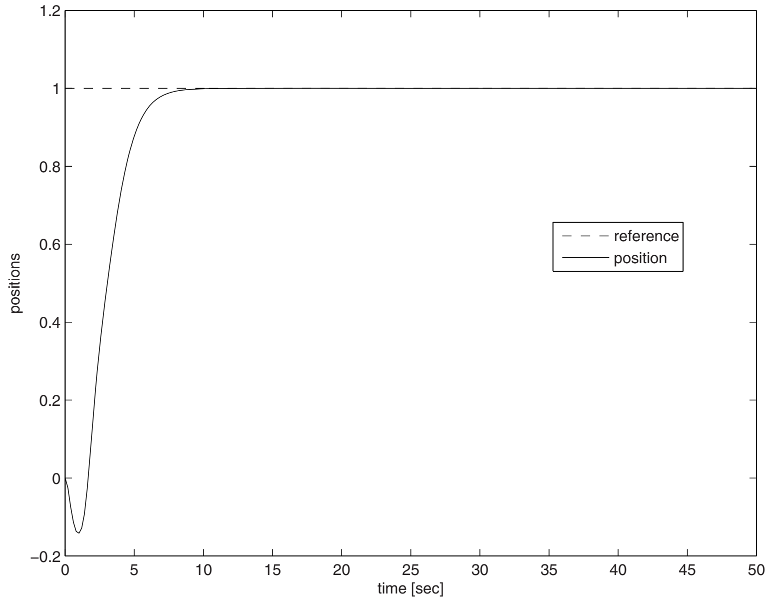


**Fig. 7.** Closed-loop system response for system (4).

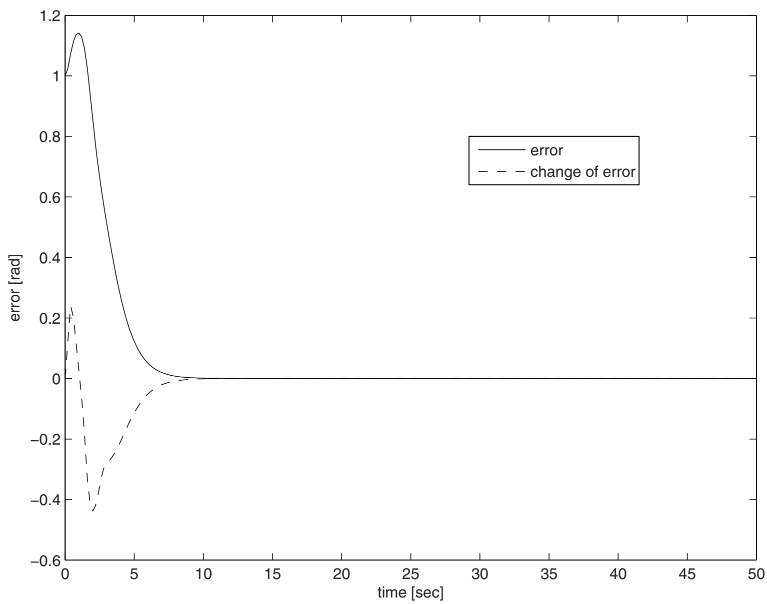


**Fig. 8.** Behavior of input variables *error* and *change of error* in the closed-loop system for system (4).

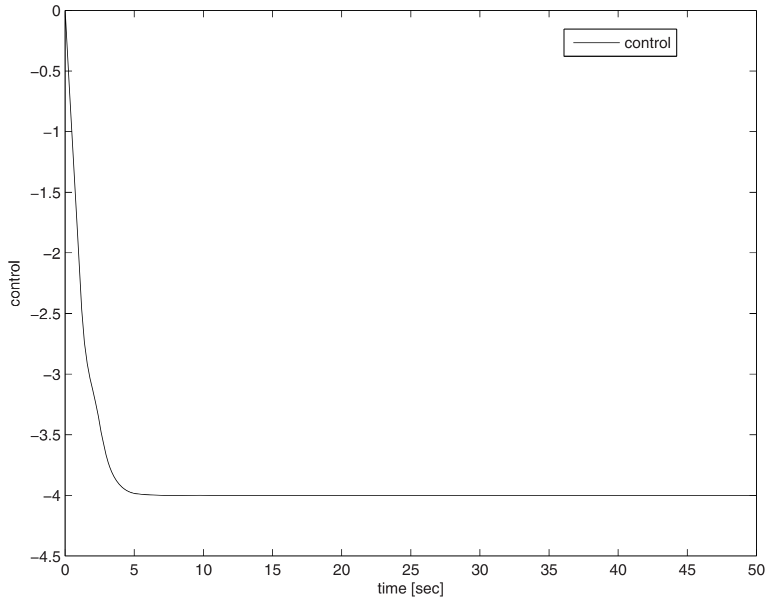


**Fig. 9.** Control signal applied to system (4) in the closed-loop.

Let us consider the following non-minimum-phase system:

$$H(s) = \frac{(s-1)}{s^2+5s+4}. \tag{4}$$

The open-loop system response of (4) to an unitary step is depicted in Fig. 6.

In order to regulate system (4) in a set-point of 1, we implement a Fuzzy PID Controller with the control law (3), with (2), FRB of Table. 1 and the MF parameters proposed in Section 2. The closed-loop systems' response is in Fig. 7, the behavior of the input variables *error* and *change of error* is in Fig. 8, and the control signal applied is in Fig. 9. Considering a settling criterion of $\pm 1\%$, the closed-loop system achieves the control objective in 5.4 seconds.

## 4 Controlling a Servomechanism with Non-Minimum-Phase Backlash

In order to prove the robustness of the proposed FLC a different problem is proposed: *the output regulation problem of a servomechanism with non-minimum-phase backlash*.

### 4.1 Dynamic Model

The dynamic models of the angular position $q_i(t)$ of the DC motor and the $q_0(t)$ of the load are given according to

$$\begin{aligned} J_0 N^{-1}\ddot{q}_0 + f_0 N^{-1}\dot{q}_0 &= T + w_0 \\ J_i\ddot{q}_i + f_i\dot{q}_i + T &= \tau_m + w_i \end{aligned} \tag{5}$$

hereafter, $J_0$, $f_0$, $\ddot{q}_0$ and $\dot{q}_0$ are, respectively, the inertia of the load and the reducer, the viscous output friction, the output acceleration, and the output velocity. The inertia of the motor, the viscous motor friction, the motor acceleration, and the motor velocity denoted by $J_i$, $f_i$, $\ddot{q}_i$ and $\dot{q}_i$, respectively. The input torque $\tau_m$ serves as a control action, and $T$ stands for the transmitted torque. The external disturbances $w_i(t)$, $w_0(t)$ have been introduced into the driver equation (5) to account for destabilizing model discrepancies due to hard-to-model nonlinear phenomena, such as friction and backlash.

The transmitted torque $T$ through a backlash with an amplitude $j$ is typically modeled by a dead-zone characteristic [12, p.7]:

$$T(\Delta q) = \begin{cases} 0 & |\Delta q| \le j \\ K\Delta q - Kj\,\mathrm{sign}(\Delta q) & \text{otherwise} \end{cases} \tag{6}$$

with

$$\Delta q = q_i - N q_0, \tag{7}$$

where $K$ is the stiffness, and $N$ is the reducer ratio. Such a model is depicted in Fig. 10. Provided the servomotor position $q_i(t)$ is the only available measurement on the system, the above model (5)-(7) appears to be non-minimum-phase because along

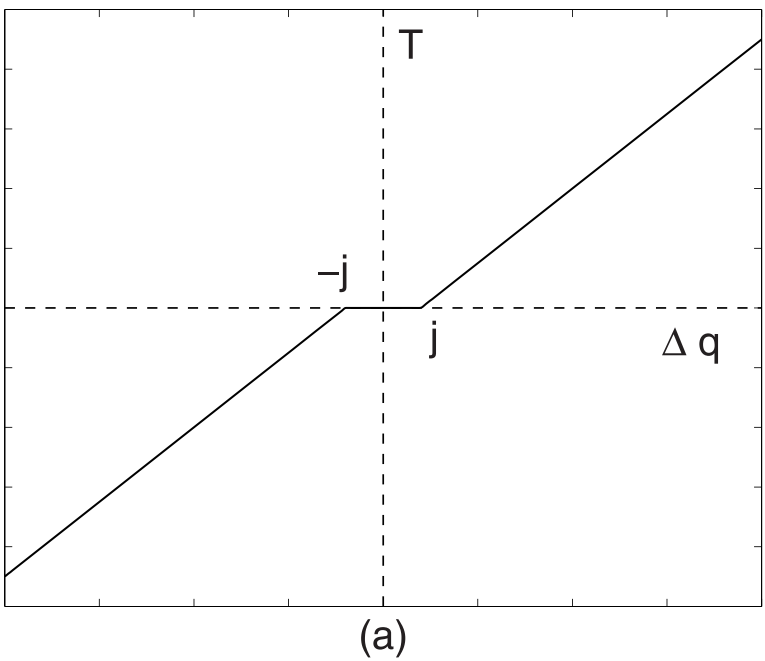


**Fig. 10.** The dead-zone model of backlash.

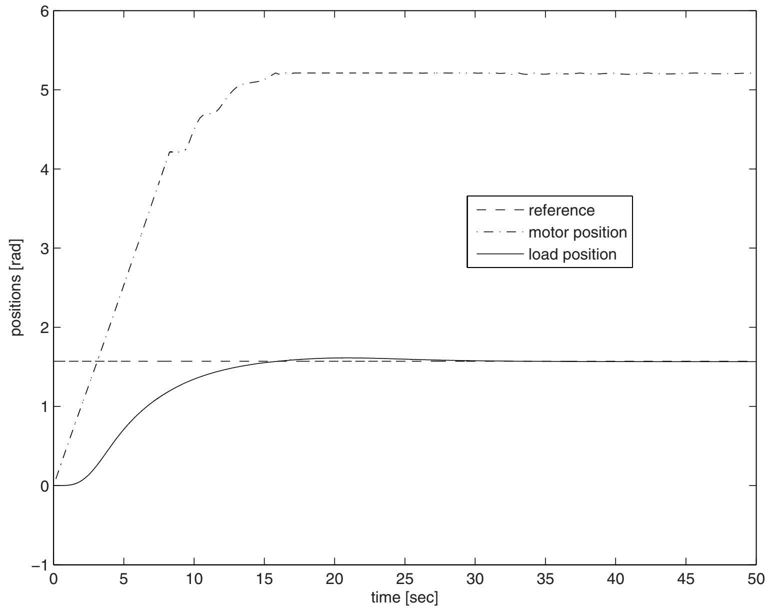


**Fig. 11.** Closed-loop system response for system (5)-(8).

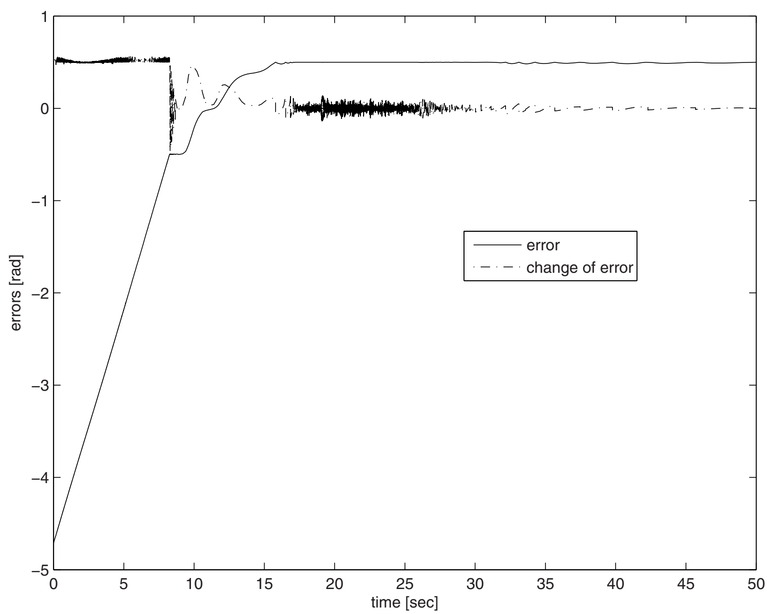


**Fig. 12.** Behavior of input variables *error* and *change of error* in the closed-loop system for system (5)-(8).

with the origin the unforced system possesses a multivalued set of equilibria $(q_i, q_0)$ with $q_i = 0$ and $q_0 \in [-j, j]$.

### 4.2 Problem Statement

To formally state the problem, let us introduce the state deviation vector $x = [x_1, x_2, x_3, x_4,]^T$ with

$$x_1 = q_0 - q_d$$
$$x_2 = \dot{q}_0$$
$$x_3 = q_i - Nq_d$$
$$x_4 = \dot{q}_i$$

where $x_1$ is the load position error, $x_2$ is the load velocity, $x_3$ is the motor position deviation from its nominal value, and $x_4$ is the motor velocity. The nominal motor position $Nq_d$ has been pre-specified in such a way to guarantee that $\Delta q = \Delta x$, where

$$\Delta x = x_3 - Nx_1,$$

and the output is given by

$$y = x_3. \quad (8)$$

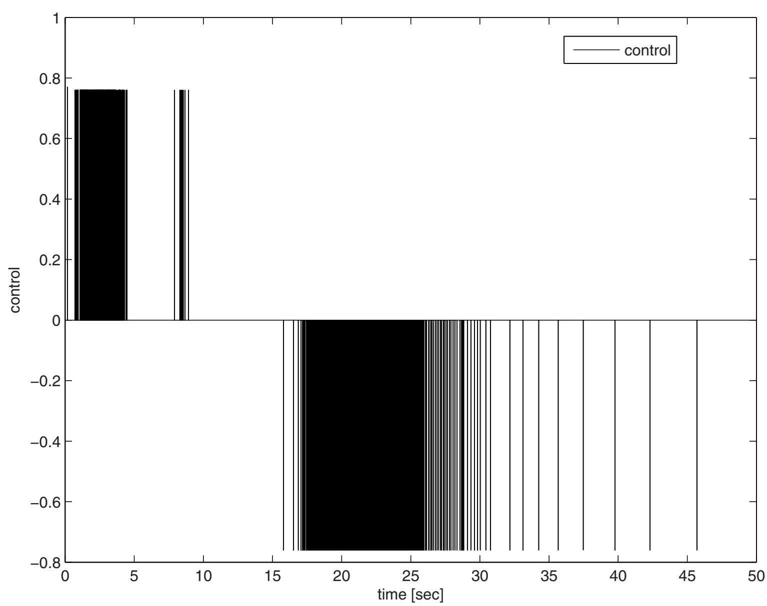


**Fig. 13.** Control signal applied to system (5)-(8) in the closed-loop.

The objective of the Fuzzy Control output regulation of the nonlinear driver system (5) with backlash (6), is thus to design a Fuzzy Controller so as to obtain the closed-loop system in which all these trajectories are bounded and the output $q_0(t)$ asymptotically decays to a desired position $q_d$ as $t \to \infty$ while also attenuating the influence of the external disturbances $w_i(t)$ and $w_0(t)$.

### 4.3 Simulation Results

In order to regulate system (5)-(8) in a desired set-point of $q_d = \pi/2$, we implement a Fuzzy PD Controller with the control law (3), with (2), FRB of Table. 1 and MF parameters proposed in Section 2. Fig. 11 shows the closed-loop systems response, the behavior of the input variables *error* and *change of error* is in Fig. 12, and the control signal applied is in Fig. 13. Considering settling criterion of $\pm 1\%$, the closed-loop system achieves the control objective in 14.5 seconds.

## 5 Conclusions

The main goal of this paper was to show that a FLC is a good strategy to control non-minimum-phase systems. This goal was achieved designing the FLC of Section 2. This FLC was configured in a close-loop system in to different control problems:

- the regulation of a LTI non-minimum-phase system, and
- the output regulation of a servomechanism with non-minimum-phase backlash.

In the first problem, regulation of a LTI non-minimum-phase system, the proposed FLC was configured as a Fuzzy PID controller in a closed-loop system, achieving the control objective satisfactory.

In the second problem of the output regulation of a servomechanism with non-minimum-phase backlash, the proposed FLC was configured as a Fuzzy PD controller in a closed-loop system, achieving the control objective in a satisfactory manner.

The simulations presented in this paper show that the proposed FLC is robust, that is, the FRB and parameters of membership functions can be configured in different ways to achieve different control objectives, furthermore, this same FLC can be used in different control problems that do not include the non-minimum-phase phenomenon.

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