Enhanced Empirical Data for the Fundamental Diagram and the Flow Through Bottlenecks

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Summary. In recent years, several approaches for modeling pedestrian dynamics
have been proposed and applied e.g. for design of egress routes. However, so far not
much attention has been paid to their *quantitative* validation. This unsatisfactory
situation belongs amongst others on the uncertain and contradictory experimental
data base. The fundamental diagram, i.e. the density-dependence of the flow or ve-
locity, is probably the most important relation as it connects the basic parameter to
describe the dynamic of crowds. But specifications in different handbooks as well as
experimental measurements differ considerably. The same is true for the bottleneck
flow. After a comprehensive review of the experimental data base we give an survey
of a research project, including experiments with up to 250 persons performed under
well controlled laboratory conditions. The trajectories of each person are measured
in high precision to analyze the fundamental diagram and the flow through bot-
tlenecks. The trajectories allow to study how the way of measurement influences
the resulting relations. Surprisingly we found large deviation amongst the methods.
These may be responsible for the deviation in the literature mentioned above. The
results are of particular importance for the comparison of experimental data gained
in different contexts and for the validation of models.

1 Introduction

The number of models for pedestrian dynamics has grown in the past years,
but the experimental data to test them and to discriminate between these
models is still to a large extent uncertain and contradictory (see e.g. [1]).
In most models, pedestrians are considered to be autonomous mobile agents,

hopping particles in a cellular automaton or self-driven particles in a continuous space. If the objective is to make quantitative predictions, like evacuation or travel times, the model has to be calibrated with empirical data.

One of the most important characteristics of pedestrian dynamics is the fundamental diagram giving the relation between pedestrian flow and density. Beside its importance for the dimensioning of pedestrian facilities it is associated with every qualitative self-organization phenomenon, like the formation of lanes or the occurrence of congestions. However, specifications of different experimental studies, guidelines and handbooks, all display non negligible differences even for the most relevant characteristics like maximal flow values, the corresponding density and the density where the flow is expected to become zero due to overcrowding. The connection between fundamental diagram and bottleneck flow is important as well and not really understood. In particular the maxima of fundamental diagrams are significantly lower than maximal flow values measured at bottlenecks.

Although a large variety of models for pedestrian dynamics has been proposed, so far there have been only limited attempts to calibrate and validate these approaches. One reason is the unclear situation of the empirical data, as described above. This situation is very unsatisfactory and poses serious limitations on the use of such models e.g. in the area of safety planning. To improve the current state of affairs it is necessary to have more reliable data that can be used as basis for validation and calibration which then would allow to make quantitative predictions based on computer simulations.

In Sect. 2 we give a review of empirical results and discuss their discrepancies by comparing various experimental data and specifications from the literature. To resolve some of the contradictions we initiated a research project including experiments with up to 250 persons under well controlled laboratory conditions, see Sect. 3. Great emphasis was given to the method of data recording by video technique and careful preparation of the experimental setups. This enables the accurate determination of all trajectories providing a microscopic insight into pedestrian dynamics, see [2]. In Sect. 4 we analyze how the measurement method influences the resulting outcomes.

2 Review of Empirical Results

2.1 Fundamental Diagram

The fundamental diagram describes the empirical relation between density ρ and flow J (or specific flow per unit width $J_s = J/w$). The name already indicates its importance and naturally it has been the subject of many investigations. Due to the hydrodynamic relation $J = \rho v w$ there are three equivalent forms: $J_s(\rho)$, $v(\rho)$ and $v(J_s)$. In applications the relation is a basic input for engineering methods developed for the design and dimensioning of pedestrian
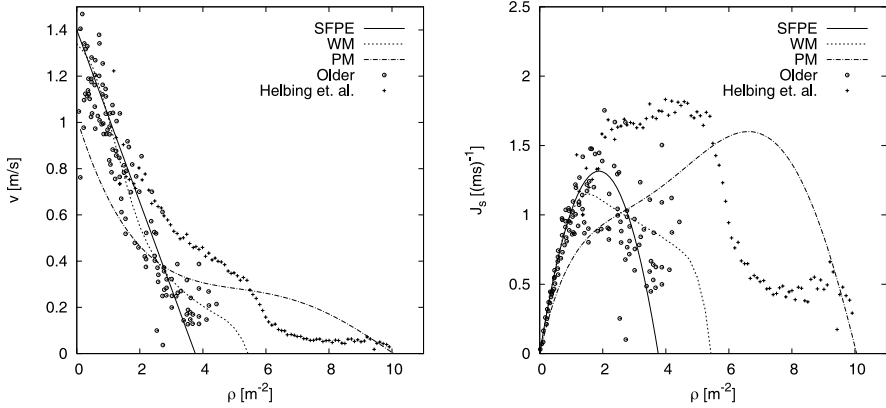
Fig. 1. Fundamental diagrams for pedestrian movement in planar facilities. The lines refer to specifications according to planing guidelines (SFPE Handbook [6], PM: Predtechenskii and Milinskii [3], WM: Weidmann [5]). Data points give the range of experimental measurements [7, 8].

facilities [3–6]. In this section we will concentrate on planar facilities like sidewalks, corridors or halls. For various facilities like floors, stairs or ramps the shape of the diagrams differ, but in general it is assumed that the fundamental diagrams for the same type of facilities but different widths merge into one diagram for the specific flow J_s.

Figure 1 shows various fundamental diagrams used in planing guidelines plus the measurements of two selected empirical studies representing the overall range of the data. The comparison reveals that specifications and measurements disagree considerably. In particular the maximum of the function giving the capacity $J_{s,\max}$ ranges from 1.2 $(\mathrm{ms})^{-1}$ to 1.8 $(\mathrm{ms})^{-1}$, the density ρ_0 where the velocity approaches zero due to overcrowding ranges from 3.8 m^{-2} to 10 m^{-2} and, most notably, the density value where the maximum flow is reached ρ_c ranges from 1.75 m^{-2} to 7 m^{-2}. Several explanations for these deviations have been suggested, including cultural and population differences [8], differences between uni- and multidirectional flow [9, 10], short-ranged fluctuations [10], influence of psychological factors given by the incentive of the movement [3] and, partially related to the latter, the type of traffic (commuters, shoppers) [11].

The most elaborate fundamental diagram has been given by Weidmann, who collected 25 data sets. An examination of the data which were included in Weidmann's analysis shows that most measurements with densities larger than $\rho = 1.8$ m^{-2} are performed on multidirectional streams. Weidmann neglected differences between uni- and multidirectional flow in accordance with Fruin, who states in his often cited book [4] that the fundamental diagrams of multidirectional and unidirectional flow differ only slightly. This disagrees with results of Navin and Wheeler [9] who found a reduction of the flow in de-

pendence of directional imbalances. Here lane formation in bidirectional flow has to be considered. Bidirectional pedestrian flow includes unordered streams as well as lane-separated and thus quasi-unidirectional streams in opposite directions. Another explanation is given by Helbing et al. [8] who argue that cultural and population differences are responsible for the deviations between Weidmann and their data. In contrast to this interpretation the data of Hanking and Wright [12] gained by measurements in the London subway (UK) are in good agreement with the data of Mori and Tsukaguchi [13] measured in the central business district of Osaka (Japan), both on strictly uni-directional streams. This brief discussion clearly shows that up to now there is no consensus about the origin of the discrepancies between different fundamental diagrams and how one can explain the shape of the function.

However, all diagrams agree in one characteristic: velocity decreases with increasing density. As the discussion above indicates there are many possible reasons and causes for the velocity reduction. For the movement of pedestrians along a line a linear relation between speed and the inverse of the density was measured in [14]. The speed for walking pedestrians depends also linearly on the step size [5] and the inverse of the density can be regarded as the required length of one pedestrian to move. Thus it seems that smaller step sizes caused by a reduction of the available space with increasing density is, at least for a certain density region, one cause for the decrease of speed. However, this is only a starting point for a more elaborated modeling of the fundamental diagram.

2.2 Bottleneck Flow

One of the most important practical questions is how the capacity of the bottleneck increases with rising width. Studies of this dependence can be traced back to the beginning of the last century [15, 16] and are up to now discussed controversially. At first sight, a stepwise increase of capacity with the width appears to be natural if lanes are formed. For independent lanes, where pedestrians in one lane are not influenced by those in others, capacity increases only if an additional lane can be formed.

In contrast, the study [17] found that the distance of lanes and the speed in a lane increases with the bottleneck width until a new lane is formed, when the lanes come closer together again. This variation of lane distance leads to a very weak dependence of the density and velocity inside the bottleneck on its width. Thus in reference to $J = \rho v w$ the flow does not directly depend on the number of lanes. To find a conclusive judgment whether the capacity grows continuously with the width the results of different laboratory experiments [18–22] are compared in [22].

In the following we discuss the data of flow measurement collected in Fig. 2. The data by Muir et al. [19], who studied the evacuation of airplanes, seem to support the stepwise increase of the flow with the width. They show constant flow values for $w > 0.6$ m. But the flow there does not increase much up to
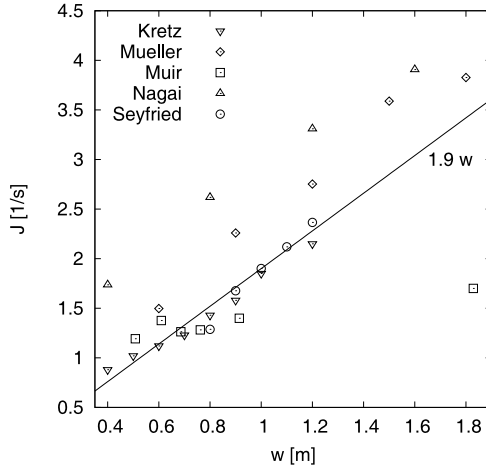
Fig. 2. Influence of the width of a bottleneck on the flow. Experimental data (Müller [18]; Muir et al. [19]; Nagai et al. [20]; Seyfried et al. [22]) of different types of bottlenecks and initial conditions. All data are taken under laboratory conditions where the test persons are advised to move normally.

$w = 1.8$ m, which indicates that in this special setup the flow is limited by some other process, e.g. reaching the corridor. Thus all data collected from flow measurements in Fig. 2 are compatible with a continuous and almost linear increase with the bottleneck width for $w > 0.6$ m. Surprisingly the data in Fig. 2 differ considerably in the values of the bottleneck capacity. In particular the flow values of Nagai [20] and Müller [18] are much higher than the maxima of empirical fundamental diagrams. The comparison of the different experimental setups shows that the exact geometry of the bottleneck is of only minor influence on the flow while a high initial density in front of the bottleneck can increase the resulting flow values. This leads to another interesting question, that is to say how the bottleneck flow is connected to the fundamental diagram. General results for driven diffusive systems [23] show that boundary conditions only *select* between the states of the undisturbed system instead of creating completely different ones. Therefore it is surprising that the measured maximal flow at bottlenecks can exceed the maximum of the empirical fundamental diagram. These questions are related to the common jamming criterion. Generally it is assumed that a jam occurs if the incoming flow exceeds the capacity of the bottleneck. In this case one expects the flow through the bottleneck to continue with the capacity (or lower values). The data presented in [22] show a more complicated picture, we refer to the contribution of A. Winkens and T. Rupprecht in these proceedings. While the density in front of the bottleneck amounts to $\rho \approx 5.0$ (± 2) m^{-2}, the density inside the bottleneck tunes around $\rho \approx 1.8$ m^{-2}.

3 Research Project—Overview

The research project is funded by the DFG and based on cooperation between the Bergische Universität Wuppertal, the Universität zu Köln and Forschungszentrum Jülich GmbH. It covers the execution of large scale experiments, the data collection via automated determination of trajectories with high accuracy, microscopic and macroscopic data analysis and the development of models to describe the dynamic of pedestrians quantitatively. In this section we give an overview of the experiments performed.

As outlined in the previous section, there are a lot of possible influences on the characteristics of pedestrian crowd movement. To reduce as much as possible uncontrollable influences we decided to use a homogeneous group of test persons and to perform the experiments under well controlled laboratory conditions. It is obvious that results performed under special conditions are not suited for design recommendations of e.g. escape routes. However such types of experiments make it possible to study the influence of single parameters, like the bottleneck width, and thus to resolve whether the capacity of a bottleneck increases linearly or step wise. Moreover the determination of the trajectories of all persons with high accuracy allows a microscopic insight into pedestrian dynamics and thus to provide a secure data base for the development and microscopic verification of models. Concerning the determination of trajectories we refer to [2].

The experiments were arranged 2006 in the wardroom of the 'Bergische Kaserne Düsseldorf'. The group of test persons was composed of soldiers. Fig. 3 (left) shows a sketch of the experimental setup to determine the fundamental diagram. We performed runs for different widths w as well as uni- and bidirectional flows. To scan the whole density regime the number of the pedestrians inside the corridor was changed. The right figure shows the sketch of the experimental setup to analyze the flow through bottlenecks. We performed runs for different bottleneck widths w, corridor widths w_c, bottleneck length l, number of pedestrians N and distances to the entrance d. To ensure an equal initial density for every run, holding areas were marked on the floor (dashed regions). All together 99 runs with up to 250 people distributed over five days were performed.
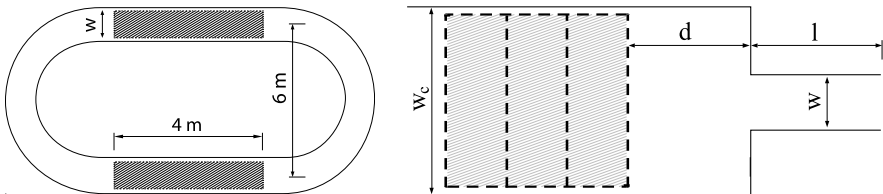
Fig. 3. Sketch of the experimental setup to determine the fundamental diagram (*left*) and to analyze the flow through bottlenecks (*right*).
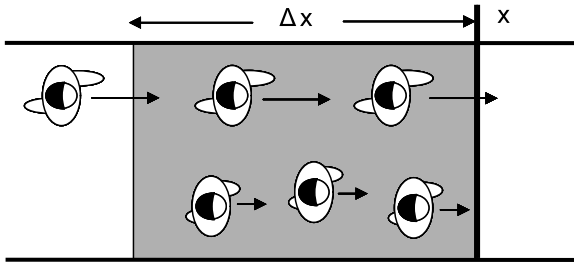
4 Influence of the Measurement Method

The discussion outlined in Sect. 2 is put into perspective by two observations. First we note that in the majority of cases the data come without fluctuations and error margins and thus, strictly speaking, there is no contradiction. Second it is well known in vehicular traffic that different measurement methods can lead to deviations for the fundamental diagram [24, 25]. In previous experimental studies of pedestrian traffic, different kinds of measurement methods are used, and often a mixture of time and space averages are realized due to cost reasons. But in case of spatial and temporal inhomogeneities it cannot be excluded that the averaging over different degrees of freedom leads to non comparable results. In this section we analyze how large the deviations due to different measurement methods are. For this purpose we choose the most ordered and controlled system examined in the project, namely the fundamental diagram for the movement of pedestrians along a line under closed boundary conditions during a stationary state. Due to the controlled character of the movement it can be expected that deviations caused by inhomogeneities give a lower bound for deviations in more disordered systems.

In the following we introduce the basic quantities and the flow equation along the measurement methods. The discussion follows the explanation in text books for vehicular traffic [24, 25] and is adapted to pedestrian characteristics. The sketch in Fig. 4 illustrates two principle possibilities to measure the observable like flow, velocity and density.

Method A: local measurement of the observable O at a certain location x averaging over a time interval Δt. We refer to this by $\langle O \rangle_{\Delta t}$. Measurements at a certain location allow a direct determination of the flow J and the velocity v.

$$\langle J \rangle_{\Delta t} = \frac{N}{\Delta t} = \frac{1}{\langle \Delta t_i \rangle_{\Delta t}} \quad \text{and} \quad \langle v \rangle_{\Delta t} = \frac{1}{N} \sum_{i=1}^{N} v_i. \tag{1}$$

Fig. 4. Illustration of different measurement methods to determine the fundamental diagram. It has to be distinguished between local measurements at cross-section with position x averaged over a time interval Δt and measurements at certain time averaged over space Δx.

The flow is given as the number of persons N passing a specified cross-section at x per unit time. Usually it is taken as a scalar quantity since only the flow normal to the cross-section is considered. To relate the flow with a velocity one measures the individual velocities v_i at location x and calculates the mean value of the velocity $\langle v \rangle_{\Delta t}$ of the N pedestrians. In earlier studies normally the velocity of a single pedestrian was considered and only the number of pedestrians N passing the cross-section in the time interval Δt are counted [3, 12, 26]. In principle it is possible to determine the velocities v_i and crossing times t_i of each pedestrian and to calculate the time gaps $\Delta t_i = t_{i+1} - t_i$ defining the flow as the inverse of the mean value of time gaps over the time interval Δt.

Method B is to average the observable O over space Δx at a specific time t_k which gives $\langle O \rangle_{\Delta x}$. The introduction of an observation area with extend $w\Delta x$ allows to determine directly the density ρ and the velocity v:

$$\langle \rho \rangle_{\Delta x} = \frac{N'}{w\Delta x} \quad \text{and} \quad \langle v \rangle_{\Delta x} = \frac{1}{N'} \sum_{i=1}^{N'} v_i. \tag{2}$$

This method was used in combination with time-lapse photos. Often and due to cost reasons only the velocity of single pedestrians and the mean value of the velocity during the entrance and exit times were considered [7, 9].

Flow equation: To connect these methods and to change between different representations of the fundamental diagram the hydrodynamic flow equation $J = \rho v w$ is used. It is possible to derive the flow equation from the definition of the observables introduced above by using the distance $\Delta \tilde{x} = \Delta t \langle v \rangle_{\Delta t}$. Thus one obtains

$$J = \frac{N}{\Delta t} = \frac{N}{\Delta \tilde{x} w} \frac{\Delta \tilde{x} w}{\Delta t} = \tilde{\rho} \langle v \rangle_x w \quad \text{with } \tilde{\rho} = \frac{N}{\Delta \tilde{x} w}. \tag{3}$$

At this point it is crucial to note that the mean values $\langle v \rangle_x$ and $\langle v \rangle_t$ do not necessarily correspond. This can already be seen by examination of Fig. 4. Thus a density calculated by $\tilde{\rho} = \langle J \rangle_{\Delta t} / \langle v \rangle_{\Delta t}$ may differ from a direct measurement of the density via $\langle \rho \rangle_{\Delta x}$. We come back to this point later.

As already mentioned above we choose the most simple system to get an estimation for the lower bound of deviation resulting from different measurement methods. To measure the fundamental diagram of the movement along a line we performed 12 runs with varying number of pedestrians, $N = 17$ to $N = 70$. Figure 5 shows the projection of the trajectories to the (x, t)-plane for the runs with $N = 45$, 56 and 62. For the movement along a line we set $w = 1$ in the equations introduced above. We note again that the different measurements shown in the next figures are based on the same set of trajectories determined automatically from video recordings of the measurement area with high accuracy ($x_{\text{err}} \pm 0.02$ m). The data analysis is restricted to the stationary state.
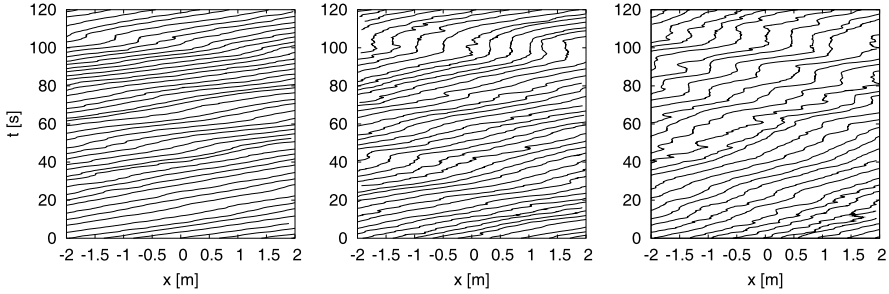
Fig. 5. Projection of the trajectories to the (x, t)-plane of the movement along a line for the runs with $N = 45, 56, 62$ (from left to right). For increasing N the dynamics becomes more unordered and the trajectories show intermittent stopping by a constant x-values in time.
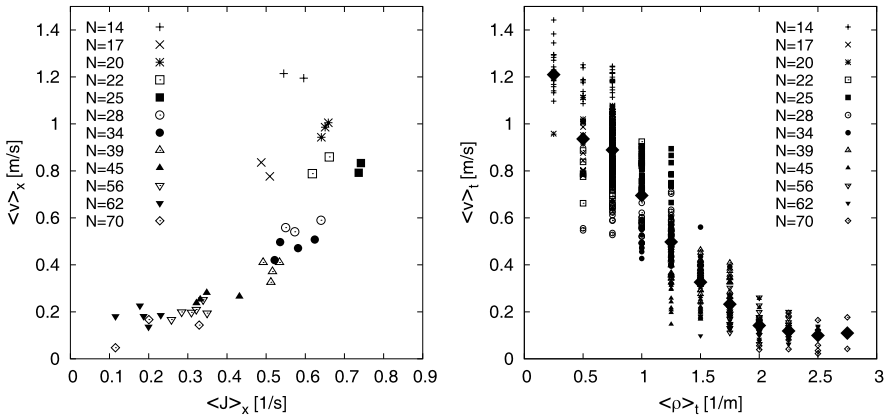
Fig. 6. Fundamental diagrams measured at the same set of trajectories but with different methods. *Left*: Measurement at a certain cross-section averaging over time interval (Method A). *Right*: Measurement at a certain point in time averaging over space (Method B). *Large diamonds* give the over all mean value of the velocity for one density value.

Figure 6 shows the direct measurements according to Methods A and B. For Method A we choose the position of the cross-section $x = 0$ and a time interval of $\Delta t = 30$ s, see Fig. 5. For Method B the area ranges from $x = -2$ m to $x = 2$ m, and we performed the averaging over space each time t_k a pedestrian crossed $x = 0$. For Method B we note that the fixed length of the observation area of 4 m results in discrete density values with distance $\Delta \rho = (4 \text{ m})^{-1}$. For each density value large fluctuations of the velocities $\langle v \rangle_t$ are observed. The large diamonds in the right of Fig. 6 represent the mean values over all velocities $\langle v \rangle_t$ for one density. The flow equation (3) allows to switch the direct measurement of Methods A and B into the most common representation of the fundamental diagram $J(\rho)$.
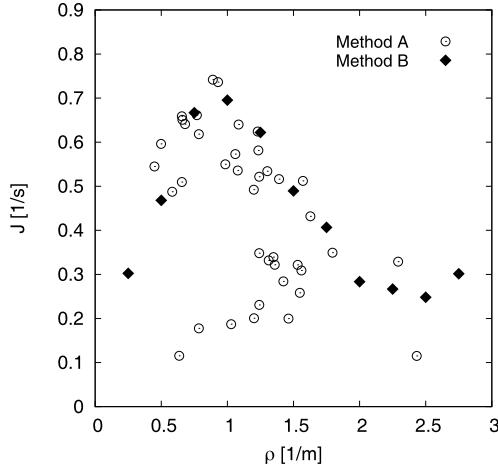
Fig. 7. Fundamental diagram determined by different measurement methods. Method A: Direct measurement of the flow and velocity at a cross-section. The density is calculated via $\rho = \langle J \rangle_{\Delta t} / \langle v \rangle_{\Delta t}$. Method B: Measurement of the density and velocity at a certain time point averaged over space. The flow is given by $J = \rho \langle v \rangle_{\Delta x}$.

Figure 7 shows a comparison of fundamental diagrams using the same set of trajectories but different measurement methods. In particular for high densities, where jam waves are present, the deviations are obvious. This is in agreement with Fig. 1 where almost all curves agree for low densities and disagree for high densities. For the high density regime the trajectories show inhomogeneities in time and space, which do not correspond, see Fig. 5. The averaging over different degrees of freedom, the time Δt for Method A and the space Δx for Method B lead to different distribution of individual velocities. Thus one reason for the deviations is that the mean values of the velocity measured at a certain location by averaging over time do not necessarily conform to mean values measured at a certain time averaged over space. However, the straightforward use of the flow equation neglects these differences. In [24] it was stated that the difference can be canceled out by using the harmonic average for the calculation of the mean velocity for Method A. We test this approach and found that the differences do not cancel out and the data are only in conformance if one takes into account the fluctuations and calculates the mean velocity by the harmonic average. But for states where congestions lead to an intermittent stopping, fluctuations of the density measured with Method A are extremely large and can span over the whole density range observed. This belongs to the fact that in Method A the density is determined indirectly by calculating $\tilde{\rho} = \langle J \rangle_{\Delta t} / \langle v \rangle_{\Delta t}$. In the high density range the flow as well as the velocity have small values causing high fluctuations for the calculated density.

5 Conclusions

This contribution summarizes open questions and differences concerning specifications of the fundamental diagram and bottleneck flow in the literature. In particular for the high density regime of the flow-density relation the discrepancies are not negligible. For the flow through bottlenecks it is an open question, why the maximal flow values through bottleneck exceed significantly the maxima of the fundamental diagrams. To dissolve these discrepancies we performed laboratory experiments with up to 250 people. The trajectories of each pedestrian are determined with high accuracy. As a first step of the analysis we investigated how the way of measurement influence the resulting relations. Surprisingly we found that even for the most regular and simplest system, namely the movement of pedestrians along a line under periodic boundary conditions, large deviations result if different measurement methods are applied. The reason for this is the averaging over different degrees of freedom in a discrete system with large inhomogeneities. Thus it cannot be excluded that the deviations discussed in Sect. 2 result from different measurement methods amongst other causes. This statement is supported by the observation based on Fig. 1, that almost all curves agree for low densities and disagree for high densities. For a systematic study and a meaningful discussion of the influence of culture or the changing population demographics on pedestrian characteristic it is necessary to assure that the studies compared are based on the same measurement approach. This applies accordingly for the validation of model results with experimental data.

Acknowledgements

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