

Comparative Analysis of Evolutionary Fuzzy Models for Premises Valuation Using KEEL

Marek Krzystanek¹, Tadeusz Lasota², and Bogdan Trawiński¹

¹ Wrocław University of Technology, Institute of Informatics,
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

² Wrocław University of Environmental and Life Sciences, Dept. of Spatial Management
Ul. Norwida 25/27, 50-375 Wrocław, Poland

marek.krzystanek@gmail.com, tadeusz.lasota@wp.pl,
bogdan.trawinski@pwr.wroc.pl

Abstract. The experiments aimed to compare evolutionary fuzzy algorithms to create models for the valuation of residential premises were conducted using KEEL. Out of 20 algorithms divided into 5 groups to final comparison five best were selected. All models were applied to actual data sets derived from the cadastral system and the registry of real estate transactions. A dozen of predictive accuracy measures were employed. Although statistical tests were not decisive, final evaluation of models could be done on the basis of the measures used.

Keywords: genetic fuzzy systems, real estate appraisal, KEEL.

1 Introduction

Applying the most popular approach to determining the market value of a property, i.e. sales comparison approach, it is necessary to have transaction prices of the properties sold with attributes similar to the one being appraised. If good comparable transactions are available, then it is possible to obtain reliable estimates. Prior to the evaluation the appraiser must conduct a thorough study of the appraised property using available sources of information such as cadastral systems, transaction registers, performing market analyses, accomplishing on-site inspection. His estimations are usually subjective and are based on his experience and intuition.

Automated valuation models (AVMs), devoted to support appraisers' work, are based primarily on multiple regression analysis [17], [26], soft computing and geographic information systems (GIS) [28]. Many intelligent methods have been developed to support appraisers' works: neural networks [27], fuzzy systems [10], case-based reasoning [22], data mining [21] and hybrid approaches [16].

In our previous works [14], [15] we investigated different machine learning algorithms, among others genetic fuzzy systems devoted to build data driven models to assist with real estate appraisals using MATLAB and KEEL tools. In this paper we report the results of experiments conducted with KEEL aimed at the comparison of several evolutionary fuzzy algorithms with respect to a dozen performance measures,

using actual data taken from cadastral system in order to assess their appropriateness to an internet expert system assisting appraisers' work. Compared to our earlier works we tried to test all appropriate genetic fuzzy algorithms for regression included in KEEL and our preliminary experiment comprised 20 of them.

2 Cadastral Systems as the Source Base for Model Generation

The concept of a data driven models for premises valuation, presented in the paper, was developed based on the sales comparison method. It was assumed that whole appraisal area, that means the area of a city or a district, is split into sections (e.g. clusters) of comparable property attributes. The architecture of the proposed system is shown in Fig. 1. The appraiser accesses the system through the internet and chooses an appropriate section and input the values of the attributes of the premises being evaluated into the system, which calculates the output using a given model. The final result as a suggested value of the property is sent back to the appraiser.

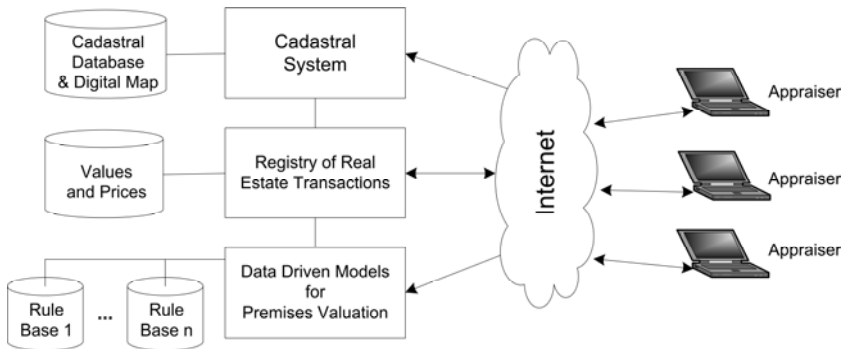


Fig. 1. Information systems to assist with real estate appraisals

Actual data used to generate and learn appraisal models came from the cadastral system and the registry of real estate transactions referring to residential premises sold in one of the big Polish cities at market prices within two years 2001 and 2002. They constituted original data set of 1098 cases of sales/purchase transactions. Four attributes were pointed out as price drivers: usable area of premises, floor on which premises were located, year of building construction, number of storeys in the building, in turn, price of premises was the output variable.

3 KEEL as the Tool for Data Driven Model Exploration

KEEL is a non-commercial Java software tool [1] designed to assess evolutionary algorithms for data mining problems. It enables problem solving through regression, classification, clustering, and data mining. Genetic fuzzy algorithms based on different approaches such as Pittsburgh, Michigan, IRL (iterative rule learning), and GCCL (genetic cooperative-competitive learning), are encapsulated into one system. KEEL is designed for different users with different expectations and provides three

main functionalities: *Data Management*, which is used to set up new data, data import and export, data edition and visualization, apply data transformations and partitioning etc.; *Experiments*, which is used to design and evaluate experiments with use of selected data and provided parameters; *Education*, which is used to design an experiment and run it step-by-step in order to display learning process.

KEEL algorithms for building, learning and tuning fuzzy models employed to carry out the experiments are listed in Table 1, where references to source articles are shown. Details of the algorithms can also be found on KEEL web site: www.keel.es.

Table 1. Evolutionary fuzzy algorithms used in study

| Group | Code | KEEL name | Description |
|-------|------|--|--|
| A | MOG1 | Regr-Fuzzy-MOGUL-IRL | MOGUL: Iterative Rule Learning of Descriptive Mamdani Rules [6] |
| | MOG2 | Regr-Fuzzy-MOGUL-IRLHC | MOGUL: Iterative Rule Learning of Mamdani Rules - High Constrained Approach [8] |
| | MOG3 | Regr-Fuzzy-MOGUL-IRLSC | MOGUL: Iterative Rule Learning of Mamdani Rules - Small Constrained Approach [6] |
| | MOG4 | Regr-Fuzzy-MOGUL-TSK | Local Evolutionary Learning of TSK fuzzy rule-based system (LEL-TSK) [2] |
| B | WMT1 | Regr-Fuzzy-WM & Post-G-G-Tuning-FRBSs | Wang-Mendel algorithm tuned using Global Genetic Tuning of the Fuzzy Partition of Linguistic FRBSs [6], [25] |
| | WMT2 | Regr-Fuzzy-WM & Post-A-G-Tuning-FRBSs | Wang-Mendel algorithm tuned using Approximative Genetic Tuning of FRBSs [9], [25] |
| | WMT3 | Regr-Fuzzy-WM & Post-G-T-Weights-FRBSs | Wang-Mendel Algorithm tuned using Genetic Tuning of FRBSs Weights [3], [25] |
| | WMT4 | Regr-Fuzzy-WM & Post-G-S-Weight-RRBS | Wang-Mendel Algorithm tuned using Genetic Selection of rules and rule weight tuning [3] |
| C | EFR1 | Regr-COR_GA | Genetic Fuzzy Rule Learning, COR algorithm inducing cooperation among rules [5] |
| | EFR2 | Regr-Thrift | Genetic Fuzzy Rule Learning, Thrift Algorithm [23] |
| | EFR3 | Regr-GFS-RB-MF | Genetic-Based Fuzzy Rule Base Construction and Membership Functions Tuning [6], [12] |
| | EFR4 | Regr-Fuzzy-P_FCS1 | Pittsburgh Fuzzy Classifier System #1 [4] |
| D | SYM1 | Regr-Fuzzy-GAP-RegSym | Symbolic Regression for fuzzy-Valued Data, Grammar-based GAP Algorithm [19] |
| | SYM2 | Regr-Fuzzy-SAP-RegSym | Symbolic Regression for fuzzy-valued data, Grammar-GP based operators and Simulated Annealing-based algorithm [19], [20] |
| | SYM3 | Regr-SAP | Symbolic Regression, Grammar-GP based operators and Simulated Annealing-based algorithm [19], [20] |
| | SYM4 | Regr-GAP | Symbolic Regression, Grammar-based GAP Algorithm [19] |
| E | FUZ1 | Regr-Fuzzy-SAP | Fuzzy Rule Learning, Grammar-GP based operators and Simulated Annealing-based algorithm [20] |
| | FUZ2 | Regr-Fuzzy-SEFC | SEFC: Symbiotic-Evolution-based Fuzzy Controller design method [13] |
| | FUZ3 | Regr-FRSBM | Fuzzy and Random Sets based Modeling [18] |
| | FUZ4 | Regr-Fuzzy-TSK-IRL | Iterative Rule Learning of TSK Rules [7] |

All the algorithms were divided into five groups for four methods: group A contains all kinds of MOGUL algorithms, group B - the Wang-Mendel algorithm for fuzzy rule learning tuned by means of evolutionary post-processing algorithms, group C - simple evolutionary fuzzy rule based systems for regression, group D - evolutionary symbolic regression algorithms, and other fuzzy systems were assigned to group E.

4 Experiment Description

The main goal of our investigations was to carry out the comparative analysis of 20 evolutionary fuzzy algorithms for regression implemented in KEEL, which task was to create and learn data driven models for premises property valuation. Our study consisted of two stages: the group one and the final comparison one (like in tournament). The first one aimed at selecting from each group one algorithm, which produces the best evolutionary fuzzy model. The second one was final contest between groups winners. All experiments were run for data described in section 2 using 10-fold cross validation from KEEL. In order to obtain comparable results, the normalization of data was accomplished using the min-max approach. As fitness function the mean square error (MSE) implemented in KEEL was applied. Within each group preliminary tuning was performed using empirical trial and error method in order to choose the values of parameters of individual algorithms providing the best prediction accuracy in terms of MSE.

At final stage, a dozen of commonly used performance measures [11], [26] was applied to evaluate models built by respective algorithms. The values of the measures were calculated using actual and predicted prices, obtained for testing sets, and which were saved as the results of experiments in one of KEEL experiment folders. These measures are listed in Table 2 and expressed in the form of following formulas below, where y_i denotes actual price and \hat{y}_i - predicted price of i -th case, $avg(v)$, $var(v)$, $std(v)$ - average, variance, and standard deviation of variables v_1, v_2, \dots, v_N , respectively and N - number of cases in the testing set.

Table 2. Performance measures used in study

| Denot. | Description | Dimension | Min value | Max value | Desirable outcome | No. of form. |
|----------------------|--|-----------|-----------|-----------|-------------------|--------------|
| <i>MSE</i> | Mean squared error | d^2 | 0 | ∞ | min | 1 |
| <i>RMSE</i> | Root mean squared error | d | 0 | ∞ | min | 2 |
| <i>RSE</i> | Relative squared error | no | 0 | ∞ | min | 3 |
| <i>RRSE</i> | Root relative squared error | no | 0 | ∞ | min | 4 |
| <i>MAE</i> | Mean absolute error | d | 0 | ∞ | min | 5 |
| <i>RAE</i> | Relative absolute error | no | 0 | ∞ | min | 6 |
| <i>MAPE</i> | Mean absolute percentage error | % | 0 | ∞ | min | 7 |
| <i>NDEI</i> | Non-dimensional error index | no | 0 | ∞ | min | 8 |
| <i>r</i> | Linear correlation coefficient | no | -1 | 1 | close to 1 | 9 |
| <i>R²</i> | Coefficient of determination | % | 0 | ∞ | close to 100% | 10 |
| <i>var(AE)</i> | Variance of absolute errors | d^2 | 0 | ∞ | min | 11 |
| <i>var(APE)</i> | Variance of absolute percentage errors | no | 0 | ∞ | min | 12 |

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 \tag{1}$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2} \tag{2}$$

$$RSE = \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (y_i - avg(y))^2} \tag{3}$$

$$RRSE = \sqrt{\frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (y_i - avg(y))^2}} \tag{4}$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i| \tag{5}$$

$$RAE = \frac{\sum_{i=1}^N |y_i - \hat{y}_i|}{\sum_{i=1}^N |y_i - avg(y)|} \tag{6}$$

$$MAPE = \frac{1}{N} \sum_{i=1}^N \frac{|y_i - \hat{y}_i|}{y_i} * 100\% \tag{7}$$

$$NDEI = \frac{RMSE}{std(y)} \tag{8}$$

$$r = \frac{\sum_{i=1}^N (y_i - avg(y)) (\hat{y}_i - avg(\hat{y}))}{\sqrt{\sum_{i=1}^N (y_i - avg(y))^2} \sqrt{\sum_{i=1}^N (\hat{y}_i - avg(\hat{y}))^2}} \tag{9}$$

$$R^2 = \frac{\sum_{i=1}^N (\hat{y}_i - avg(y))^2}{\sum_{i=1}^N (y_i - avg(y))^2} * 100\% \tag{10}$$

$$var(AE) = var(|y - \hat{y}|) \tag{11}$$

$$var(APE) = var\left(\frac{|y - \hat{y}|}{y}\right) \tag{12}$$

5 Results of Experiments

Results of preliminary stage are shown in Fig. 2-4. It can be observed that MSE and MAPE differentiate the algorithms similarly. The algorithms which created models with the lowest MSE within individual groups are listed in Table 3.

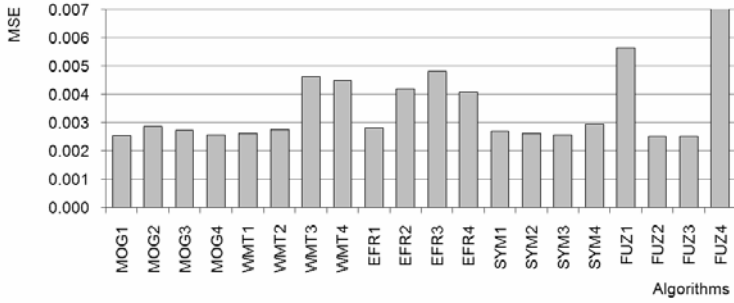


Fig. 2. Comparison of MSE values at preliminary stage

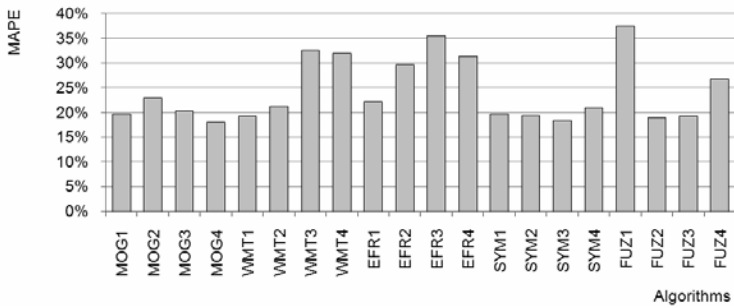


Fig. 3. Comparison of MAPE values at preliminary stage

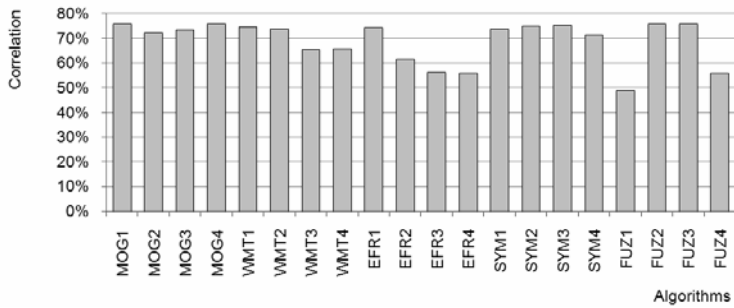


Fig. 4. Comparison of correlation between actual and predicted prices at preliminary stage

Table 3. Best algorithms within respective groups of algorithms

| Group | Final stage | Group winner | KEEL name |
|-------|-------------|--------------|---------------------------------------|
| A | MOG | MOG1 | Regr-Fuzzy-MOGUL-IRL |
| B | WMT | WMT1 | Regr-Fuzzy-WM & Post-G-G-Tuning-FRBSs |
| C | EFR | EFR1 | Regr-COR_GA |
| D | SYM | SYM3 | Regr-SAP |
| E | FUZ | FUZ2 | Regr-Fuzzy-SEFC |

Final stage of study contained comparison of algorithms listed in Table 3, using all 12 performance measures enumerated in pervious section. The results of respective measures for all models are shown in Fig. 5-16, it can be easily noticed that relationship among individual models are very similar for some groups of measures.

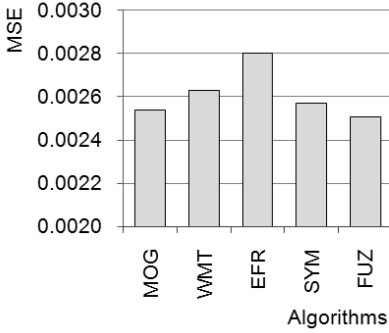


Fig. 5. Comparison of MSE values

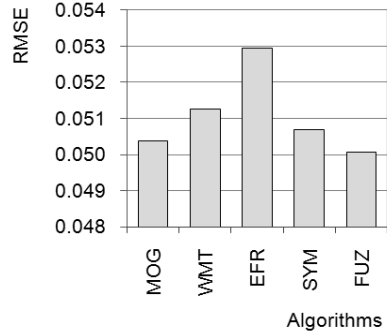


Fig. 6. Comparison of RMSE values

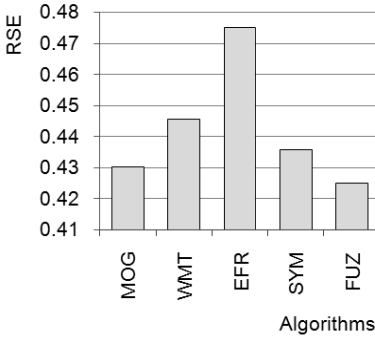


Fig. 7. Comparison of RSE values

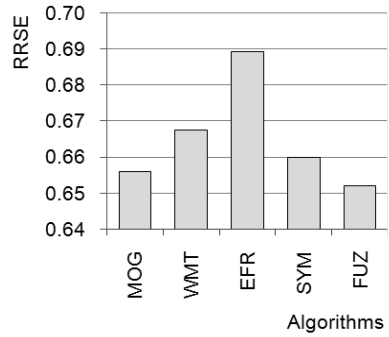


Fig. 8. Comparison of RRSE values

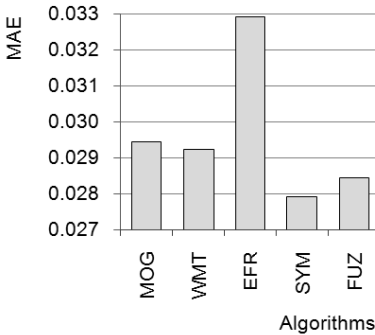


Fig. 9. Comparison of MAE values

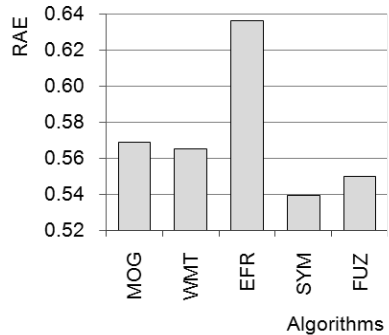


Fig. 10. Comparison of RAE values

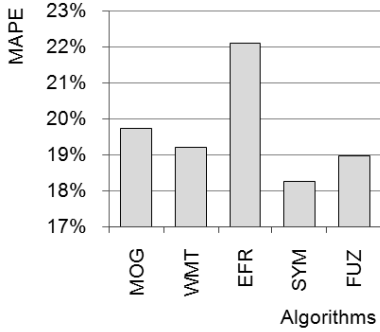


Fig. 11. Comparison of MAPE values

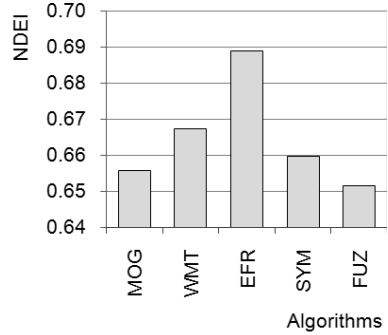


Fig. 12. Comparison of NDEI values

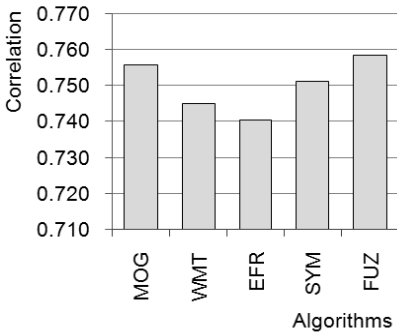


Fig. 13. Comparison of correlation coefficient (r) values

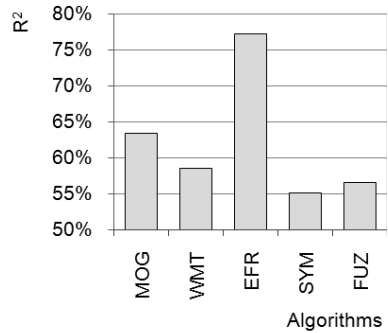


Fig. 14. Comparison of determination coefficient (R2) values

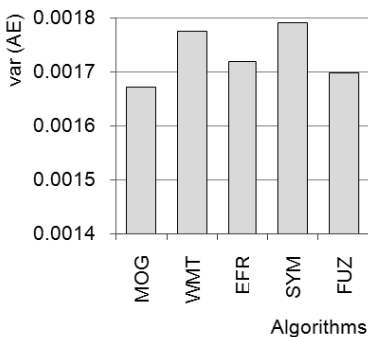


Fig. 15. Comparison of var(AE) values

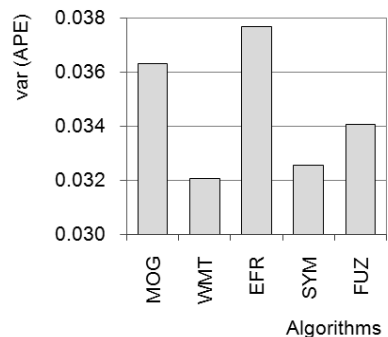


Fig. 16. Comparison of var(APE) values

Fig. 11 depicts that the values of MAPE range from 18.2% to 22.1%, what can be regarded as fairly good, especially when you take into account that not all price drivers were available in our sources of experimental data.

Fig. 13 shows there is high correlation, i.e. about 0.75, between actual and predicted prices for each model. In turn, Fig.14 illustrating the coefficients of determination indicates that from 55.2% to 77.2% of total variation in the dependent variable (prices) is accounted for by the models. This can be explained that data derived from the cadastral system and the register of property values and prices could cover only some part of potential price drivers.

The nonparametric Wilcoxon signed-rank tests were carried out for three measures: MSE, MAPE, and MAE. The results are shown in Tables 4, 5, and 6. In each cell results for a given pair of models were placed, in upper halves of the tables – p-values, and in lower halves - final outcome, where N denotes that there are no differences in mean values of respective errors, and Y indicates that there are statistically significant differences between particular performance measures. For EFR algorithm all tests resulted in significant differences, thus it can be stated this algorithm provides the worst results. However, for other algorithms the Wilcoxon signed-rank tests did not provide any decisive result.

Table 4. Results of Wilcoxon signed-rank test for squared errors comprised by MSE

| MSE | MOG | WMT | EFR | SYM | FUZ |
|------------|------------|------------|------------|------------|------------|
| MOG | | 0.374 | 0.000 | 0.048 | 0.438 |
| WMT | N | | 0.001 | 0.161 | 0.885 |
| EFR | Y | Y | | 0.000 | 0.000 |
| SYM | Y | N | Y | | 0.077 |
| FUZ | N | N | Y | N | |

Table 5. Results of Wilcoxon test for absolute percentage errors comprised by MAPE

| MAPE | MOG | WMT | EFR | SYM | FUZ |
|-------------|------------|------------|------------|------------|------------|
| MOG | | 0.247 | 0.000 | 0.000 | 0.208 |
| WMT | N | | 0.000 | 0.039 | 0.982 |
| EFR | Y | Y | | 0.000 | 0.000 |
| SYM | Y | Y | Y | | 0.010 |
| FUZ | N | N | Y | Y | |

Table 6. Results of Wilcoxon signed-rank test for absolute errors comprised by MAE

| MAE | MOG | WMT | EFR | SYM | FUZ |
|------------|------------|------------|------------|------------|------------|
| MOG | | 0.362 | 0.000 | 0.011 | 0.308 |
| WMT | N | | 0.000 | 0.071 | 0.727 |
| EFR | Y | Y | | 0.000 | 0.000 |
| SYM | Y | N | Y | | 0.047 |
| FUZ | N | N | Y | Y | |

Due to the non-decisive results of majority of statistical tests, rank positions of individual algorithms were determined for each measure (see Table 7). Observing median, average, minimal and maximal ranks it can be noticed that highest rank positions gained FUZ, MOG, SYM algorithms and the lowest WMT and EFR. Table 7 indicates also that some performance measures provide the same rank positions, and

Table 7. Rank positions of algorithms with respect to performance measures (1 means the best)

| | MOG | WMT | EFR | SYM | FUZ |
|----------------|------|------|------|------|------|
| MSE | 2 | 4 | 5 | 3 | 1 |
| RMSE | 2 | 4 | 5 | 3 | 1 |
| RSE | 2 | 4 | 5 | 3 | 1 |
| RRSE | 2 | 4 | 5 | 3 | 1 |
| MAE | 4 | 3 | 5 | 1 | 2 |
| RAE | 4 | 3 | 5 | 1 | 2 |
| MAPE | 4 | 3 | 5 | 1 | 2 |
| NDEI | 2 | 4 | 5 | 3 | 1 |
| r | 2 | 4 | 5 | 3 | 1 |
| R ² | 2 | 3 | 1 | 5 | 4 |
| var(AE) | 1 | 4 | 3 | 5 | 2 |
| var(APE) | 4 | 1 | 5 | 2 | 3 |
| median | 2.00 | 4.00 | 5.00 | 3.00 | 1.50 |
| average | 2.58 | 3.42 | 4.50 | 2.75 | 1.75 |
| min | 1 | 1 | 1 | 1 | 1 |
| max | 4 | 4 | 5 | 5 | 4 |

two groups of those measures can be distinguished. First one based on mean square errors contains MSE, RMSE, RSE, RRSE, NDEI, and the second one based on mean absolute errors comprises MAE, RAE, and MAPE.

6 Conclusions and Future Work

The goal of experiments was to compare several evolutionary fuzzy algorithms to create models for the valuation of residential premises, implemented in KEEL. Out of 20 algorithms divided into 5 groups to final experimental comparison five best were selected. A dozen of commonly used performance measures were applied to models generated using actual data set derived from cadastral system and the registry of real estate transactions. Although nonparametric Wilcoxon signed-rank tests were not decisive, the models could be ranked according to decreasing accuracy in following way: FUZ, MOG, SYM, WMT, EFR.

Some performance measures provide the same distinction abilities of respective models, thus it can be concluded that in order to compare a number of models it is not necessary to employ all measures, but the representatives of different groups. Of course the measures within groups differ in their interpretation, because some are non-dimensional as well as in their sensitivity understood as the ability to show the differences between algorithms more or less distinctly.

High correlation between actual and predicted prices was observed for each model and the coefficients of determination ranged from 55% to 77% .

MAPE obtained in all tests ranged from 18% do 22%. This can be explained that data derived from the cadastral system and the register of property values and prices can cover only some part of potential price drivers. Physical condition of the premises and their building, their equipment and facilities, the neighbourhood of the building, the location in a given part of a city should also be taken into account, moreover

overall subjective assessment after inspection in site should be done. Therefore we intend to test data obtained from public registers and then supplemented by experts conducting on-site inspections and evaluating more aspects of properties being appraised. This kind of data will be very interesting from the point of view of evolutionary fuzzy systems since most of them will be imprecise linguistic information. Moreover further investigations of multiple models comprising ensembles of different genetic fuzzy systems using bagging and boosting techniques is planned.

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