The Possible and the Necessary for Multiple Criteria Group Decision

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Abstract. We introduce the principle of robust ordinal regression to group decision. We consider the main multiple criteria decision methods to which robust ordinal regression has been applied, i.e., UTA^{GMS} and GRIP methods, dealing with choice and ranking problems, $UTADIS^{GMS}$, dealing with sorting (ordinal classification) problems, and $ELECTRE^{GMS}$, being an outranking method applying robust ordinal regression to well known ELECTRE methods. In this way, we obtain corresponding methods for group decision: UTA^{GMS}-GROUP, $UTADIS^{GMS}$-GROUP and $ELECTRE^{GMS}$-GROUP.

Keywords: Robust ordinal regression, Multiple criteria choice, sorting and ranking, Additive value functions, Outranking methods, Multiple criteria group decision.

1 Introduction

In Multiple Criteria Decision Analysis (MCDA - for a recent state-of-the-art see [6]), an alternative a, belonging to a finite set of alternatives $A = \{a, b, \ldots, j, \ldots, m\}$ ($|A| = m$), is evaluated on n criteria $g_i\colon A \to \mathbf{R}$ belonging to a consistent family $G = \{g_1, g_2, \ldots, g_i, \ldots, g_n\}$ ($|G| = n$). From here on, to designate an i-th criterion, we will use interchangeably g_i or i ($i = 1, \ldots, n$). For the sake of simplicity, but without loss of generality, we suppose that evaluations on each criterion are increasing with respect to preference, *i.e.*, the more the better, defining a marginal weak preference relation as follows:

$$a \text{ is at least as good as } b \text{ with respect to criterion } i \;\Leftrightarrow\; g_i(a) \geq g_i(b).$$

There are two main approaches to construction of decision models in MCDA: Multi-Attribute Utility Theory (MAUT) [16], [5], and the outranking approach [22], [23], [8].

The purpose of MAUT is to represent preferences of a Decision Maker (DM) on a set of alternatives A by an overall value (utility) function

$$U(g_1(a), \ldots, g_n(a))\colon \mathbf{R}^n \to \mathbf{R}$$

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such that:

$$a \text{ is at least as good as } b \iff U(g_1(a), \ldots, g_n(a)) \geq U(g_1(b), \ldots, g_n(b)).$$

The goal of the outranking approach is to represent preferences of a DM on a set of alternatives A by a pairwise comparison function

$$S(g_1(a), g_1(b), \ldots, g_n(a), g_n(b)) \colon \mathbf{R}^{2n} \rightarrow \mathbf{R}$$

such that:

$$a \text{ is at least as good as } b \iff S(g_1(a), g_1(b), \ldots, g_n(a), g_n(b)) \geq 0$$

Each decision model requires specification of some parameters. For example, using multiple attribute utility theory, the parameters are related to the formulation of marginal value functions $u_i(g_i(a))$, $i = 1, \ldots, n$, while using the outranking approach, the parameters can be weights, indifference, preference, and veto thresholds for each criterion g_i, $i = 1, \ldots, n$.

Recently, MCDA methods based on indirect preference information and on the *disaggregation paradigm* [15] are considered more interesting, because they require less cognitive effort from the DM in order to express preference information. The DM provides some holistic preferences on a set of reference alternatives A^R, and from this information the parameters of a decision model are induced using a methodology called *ordinal regression.* The resulting decision model consistent with the provided preference information is used to evaluate the alternatives from set A (*aggregation stage.*) Typically, ordinal regression has been applied to MAUT models, so in these cases we speak of *additive ordinal regression.* For example, additive ordinal regression has been applied in the well-known method called UTA (see [14]). The ordinal regression methodology has been applied, moreover, to some nonadditive decision models. In this case, we speak of *nonadditive ordinal regression* and its typical representatives are the UTA like-methods substituting the additive value function by the Choquet integral (see [4], [17], [1]), and the DRSA methodology using a set of decision rules as the decision model (see [9]).

Usually, among the many sets of parameters of a decision model representing the preference information given by the DM, only one specific set is considered. We say that the set of parameters or the decision model is *compatible* with the preference information given by the DM if it is consistent with the preference information given by the DM. For example, from among the many compatible value functions only one function is selected to rank the alternatives from set A. Since the choice of one among many compatible sets of parameters is arbitrary to some extent, recently *robust ordinal regression* has been proposed with the aim of taking into account all compatible sets of parameters. The first robust ordinal regression method is the recently proposed generalization of the UTA method, called UTA^{GMS} [11]. In UTA^{GMS}, instead of only one compatible additive value function composed of piecewise-linear marginal functions, all compatible additive value functions composed of general monotonic marginal functions are taken into account.

As to the preference information, the UTA^{GMS} method requires from a DM to make some pairwise comparisons on a set of reference alternatives $A^R \subseteq A$. The set of all compatible decision models defines two relations in set A: the *necessary* weak preference relation, which holds for any two alternatives $a, b \in A$ if and only if all compatible value functions give to a a value greater than the value given to b, and the *possible* weak preference relation, which holds for this pair if and only if at least one compatible value function gives to a a value greater than the value given to b.

Recently, an extension of UTA^{GMS} has been proposed and called the $GRIP$ method [7]. The $GRIP$ method builds a set of all compatible additive value functions, taking into account not only a preorder on a set of alternatives, but also the intensities of preference among some reference alternatives. This kind of preference information is required in other well-known MCDA methods, such as $MACBETH$ [3] and AHP [24], [25]. Both UTA^{GMS} and $GRIP$ apply the robust ordinal regression to the multiple attribute additive model and, therefore, we can say that these methods apply the *additive robust ordinal regression.* In the literature, the *nonadditive robust ordinal regression* has been proposed, applying the approach of robust ordinal regression to a value function having the form of Choquet integral in order to represent positive and negative interactions between criteria [2]. The robust ordinal regression approach can be applied also to the outranking approach [10].

In this paper, we wish to consider the robust ordinal regression in a group decision context. Therefore, we consider a set of decision makers $\mathcal{D} = \{d_1, \ldots, d_p\}$ with each own preferences, and we use robust ordinal regression to investigate spaces of consensus between them. The article is organized as follows. Section 2 is devoted to presentation of the general scheme of robust ordinal regression for choice and ranking problems within MAUT, as well as basic principles of UTA^{GMS} and $GRIP$ methods. In section 3, robust ordinal regression for group choice and ranking problems is introduced within MAUT, and the UTA^{GMS}-GROUP method is presented. Section 4 presents a general scheme of robust ordinal regression for sorting problems within MAUT, as well as basic principles of $UTADIS^{GMS}$. In section 5, robust ordinal regression for group sorting problems is introduced within MAUT, and the $UTADIS^{GMS}$-GROUP method is presented. Section 6 presents a general scheme of robust ordinal regression within the outranking approach, as well as basic principles of $ELECTRE^{GMS}$. In section 7, robust ordinal regression for group decision problems is introduced within the outranking approach, and the $ELECTRE^{GMS}$-GROUP method is presented. The last section contains conclusions.

2 The Robust Ordinal Regression Approach for Choice and Ranking Problems within MAUT

MAUT provides a theoretical foundation for preference modeling using a real-valued utility function, called value function, aggregating evaluations of alternatives on multiple criteria. The value function is intended to be a preference model of a particular DM. It is also a decision model, since it gives scores to alternatives which permit to order them from the best to the worst, or to choose the best alternative with the highest score. Its most popular form is additive:

$$U(a) = \sum_{i=1}^{n} u_i(g_i(a)), \tag{1}$$

where $u_i(g_i(a)), i = 1, \ldots, n$, are real-valued marginal value functions.

Ordinal regression has been known for at least fifty years in the field of multidimensional analysis. It has been applied within MAUT, first to assess weights of an additive linear value function [27], [21], and then to assess parameters of an additive piece-wise linear value function [14]. The latter method, called UTA, initiated a stream of further developments, in both theory and applications [26].

Recently, two new methods, UTA^{GMS} [11] and $GRIP$ (Generalized Regression with Intensities of Preference) [7], have generalized the ordinal regression approach of the UTA method in several aspects, the most important of which is that they are taking into account all additive value functions (1) compatible with the preference information, while UTA is using only one such function.

2.1 The Preference Information Provided by the Decision Maker

The DM is expected to provide the following preference information:

- a partial preorder $\succeq$ on $A^R \subseteq A$ whose meaning is: for $x, y \in A^R$

$$x \succeq y \Leftrightarrow x \text{ is at least as good as } y,$$

- a partial preorder $\succeq^*$ on $A^R \times A^R$, whose meaning is: for $x, y, w, z \in A^R$,

$$(x, y) \succeq^* (w, z) \Leftrightarrow \; x \text{ is preferred to } y \text{ at least as much as } w \text{ is preferred to } z,$$

- a partial preorder $\succeq_i^*$ on $A^R \times A^R$, whose meaning is: for $x, y, w, z \in A^R$, $(x, y) \succeq_i^* (w, z) \Leftrightarrow x$ is preferred to y at least as much as w is preferred to z on criterion g_i, $i = 1, \ldots, n$.

2.2 Possible and Necessary Rankings

A compatible value function is able to restore the preference information expressed by the DM on A^R and $A^R \times A^R$. Each compatible value function induces, moreover, a complete preorder on the whole set A. In particular, for any two solutions $x, y \in A$, a compatible value function orders x and y in one of the following ways: $x \succ y$, $y \succ x$, $x \sim y$. With respect to $x, y \in A$, it is thus reasonable to ask the following two questions:

- Are x and y ordered in the same way by *all* compatible value functions?
- Is there *at least one* compatible value function ordering x at least as good as y (or y at least as good as x)?

In the answer to these questions, UTA^{GMS} and $GRIP$ produce two rankings on the set of alternatives A, such that for any pair of alternatives $a, b \in A$:

- in the *necessary* ranking (partial preorder), a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for *all* value functions compatible with the preference information,
- in the *possible* ranking (strongly complete and negatively transitive relation), a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for *at least one* value function compatible with the preference information.

The necessary ranking can be considered as *robust* with respect to the preference information. Such robustness of the necessary ranking refers to the fact that any pair of alternatives compares in the same way whatever the additive value function compatible with the preference information. Indeed, when no preference information is given, the necessary ranking boils down to the dominance relation, and the possible ranking is a complete relation.

As $GRIP$ is taking into account additional preference information in form of comparisons of intensities of preference between some pairs of reference alternatives, the set of all compatible value functions restoring the whole preference information is also used to produce four types of relations on $A \times A$, such that for any four alternatives $a, b, c, d \in A$:

- the *necessary* relation $(a, b) \succeq^{*^N} (c, d)$ (partial preorder) holds (a is preferred to b *necessarily* at least as much as c is preferred to d), if and only if $U(a) - U(b) \geq U(c) - U(d)$ for *all* compatible value functions,
- the *possible* relation $(a, b) \succeq^{*^P} (c, d)$ (strongly complete and negatively transitive relation) holds (a is preferred to b *possibly* at least as much as c is preferred to d), if and only if $U(a) - U(b) \geq U(c) - U(d)$ for *at least one* compatible value functions,
- the *necessary* relation $(a, b) \succeq^{*_i^N} (c, d)$ (partial preorder) holds (on criterion i, a is preferred to b *necessarily* at least as much as c is preferred to d), if and only if $u_i(a) - u_i(b) \geq u_i(c) - u_i(d)$ for *all* compatible value functions ($i = 1, \ldots, n$),
- the *possible* relation $(a, b) \succeq^{*_i^P} (c, d)$ (strongly complete and negatively transitive relation) holds (on criterion i, a is preferred to b *possibly* at least as much as c is preferred to d), if and only if $u_i(a) - u_i(b) \geq u_i(c) - u_i(d)$ for *at least one* compatible value functions ($i = 1, \ldots, n$).

3 Robust Ordinal Regression for Group Decision about Choice and Ranking: The UTA^{GMS}-GROUP Method

The UTA^{GMS}-GROUP method applies the robust ordinal regression approach to the case of group decision, in which several DMs cooperate to make a collective decision. DMs share the same "description" of the decision problem (the same set of alternatives, evaluation criteria and performance matrix). Each DM provides his/her own preference information, composed of pairwise comparisons of some reference alternatives. The collective preference model accounts for the preference expressed by each DM. Although in the considered framework it is also possible to handle preference information about intensity of preference, we will skip this preference information for the lack of space.

Let us denote the set of DMs by $\mathcal{D}=\{d_1,\ldots,d_p\}$. For each DM $d_h \in \mathcal{D}' \subseteq \mathcal{D}$, we consider all compatible value functions. Four situations are interesting for a pair $(a,b) \in A$:

- $a \succsim_{\mathcal{D}'}^{N,N} b$: $a \succsim^N b$ for *all* $d_h \in \mathcal{D}'$,
- $a \succsim_{\mathcal{D}'}^{N,P} b$: $a \succsim^N b$ for *at least one* $d_h \in \mathcal{D}'$,
- $a \succsim_{\mathcal{D}'}^{P,N} b$: $a \succsim^P b$ for *all* $d_h \in \mathcal{D}'$,
- $a \succsim_{\mathcal{D}'}^{P,P} b$: $a \succsim^P b$ for *at least one* $d_h \in \mathcal{D}'$.

4 Robust Ordinal Regression for Sorting Problems: The $UTADIS^{GMS}$ Method

Robust ordinal regression has also been proposed for sorting problems in the new $UTADIS^{GMS}$ method [12], considering an additive value function (1) as a preference model. Let us remember that sorting procedures consider a set of k predefined preference ordered classes $C_1, C_2, \ldots, C_k$, where $C_{h+1} \gg C_h$ ($\gg$ a complete order on the set of classes), $h = 1,\ldots,k-1$. The aim of a sorting procedure is to assign each alternative to one class or to a set of contiguous classes. The robust ordinal regression takes into account a *value driven sorting procedure*, that is, it uses a value function U to decide the assignments in such a way that if $U(a) > U(b)$ then a is assigned to a class not worse than b.

We suppose the DM provides preference information in form of possibly imprecise assignment examples on a reference set of alternatives $A^R \subseteq A$, i.e. for $a^R \in A^R$ the DM defines a desired assignment $a^R \rightarrow [C_{L^{DM}(a^R)}, C_{R^{DM}(a^R)}]$, where $[C_{L^{DM}(a^R)}, C_{R^{DM}(a^R)}]$ is an interval of contiguous classes $C_{L^{DM}(a^R)}$, $C_{L^{DM}(a^R)+1}, \ldots, C_{R^{DM}(a^R)}$. An assignment example is said to be precise if $L^{DM}(a^R) = R^{DM}(a^R)$, and imprecise, otherwise.

Given a value function U, a set of assignment examples is said to be *consistent with* U iff

$$\forall a^R, b^R \in A^R, U(a^R) \geq U(b^R) \; \Rightarrow \; R^{DM}(a^R) \geq L^{DM}(b^R). \qquad (2)$$

Given a set A^R of assignment examples and a corresponding set $\mathcal{U}_{A^R}$ of compatible value functions, for each $a \in A$ we define the *possible assignment* $C_P(a)$ as the set of indices of classes C_h for which there exist *at least one* value function $U \in \mathcal{U}_{A^R}$ assigning a to C_h, and the *necessary assignment* $C_N(a)$ as set of indices of classes C_h for which *all* value functions $U \in \mathcal{U}_{A^R}$ assign a to C_h.

5 Robust Ordinal Regression for Group Decision about Sorting: The $\boldsymbol{UTADIS^{GMS}}$-GROUP Method

Given a set of DMs $\mathcal{D}=\{d_1,\ldots,d_p\}$, for each DM $d_r\in\mathcal{D}'\subseteq\mathcal{D}$ we consider the set of all compatible value functions $\mathcal{U}^{d_r}_{A^R}$. Given a set A^R of assignment examples, for each $a\in A$ and for each DMs $d_r\in\mathcal{D}'$ we define his/her possible and necessary assignments as

$$C_P^{d_r}(a)=\{h\in H \text{ such that } \exists U\in\mathcal{U}^{d_r}_{A^R} \text{ assigning } a \text{ to } C_h\} \quad (3)$$

$$C_N^{d_r}(a)=\{h\in H \text{ such that } \forall U\in\mathcal{U}^{d_r}_{A^R}, \mathcal{U} \text{ is assigning } a \text{ to } C_h\} \quad (4)$$

Moreover, for each subset of DMs $D'\subseteq\mathcal{D}$, we define the following assignments:

$$C_{P,P}^{D'}(a)=\bigcup\nolimits_{d_r\in\mathcal{D}'}C_P^{d_r}(a) \quad (5)$$

$$C_{N,P}^{D'}(a)=\bigcup\nolimits_{d_r\in\mathcal{D}'}C_N^{d_r}(a) \quad (6)$$

$$C_{P,N}^{D'}(a)=\bigcap\nolimits_{d_r\in\mathcal{D}'}C_P^{d_r}(a) \quad (7)$$

$$C_{N,N}^{D'}(a)=\bigcap\nolimits_{d_r\in\mathcal{D}'}C_N^{d_r}(a). \quad (8)$$

Possible and necessary assignments $C_P^{d_r}(a)$ and $C_N^{d_r}(a)$ are calculated for each decision maker $d_r\in\mathcal{D}$ using $\mathrm{UTADIS}^{\mathrm{GMS}}$, and then the four assignments $C_{P,P}^{D'}(a)$, $C_{N,P}^{D'}(a)$, $C_{P,N}^{D'}(a)$ and $C_{P,P}^{D'}(a)$ can be calculated for all subsets of decision makers $\mathcal{D}'\subseteq\mathcal{D}$.

6 Robust Ordinal Regression for Outranking Methods

Outranking relation is a non-compensatory preference model used in the $ELECTRE$ family of multiple criteria decision aiding methods [22]. Its construction involves two concepts known as concordance and discordance. Outranking relation, usually denoted by S, is a binary relation on a set A of actions. For an ordered pair of actions $(a,b)\in A$, aSb means "a is at least as good as b". The assertion aSb is considered to be true if the coalition of criteria being in favor of this statement is strong enough comparing to the rest of criteria, and if among the criteria opposing to this statement, there is no one for which a is significantly worse than b. The first condition is called concordance test, and the second, non-discordance test.

Let us denote by k_j the weight assigned to criterion g_j, $j=1,\ldots,n$; it represents a relative importance of criterion g_j within family G of n criteria. The indifference, preference and veto thresholds on criterion g_j are denoted by q_j, p_j and v_j, respectively. For consistency, $v_j>p_j>q_j\geq 0$, $j=1,\ldots,n$. In all formulae that follow, we suppose, without loss of generality, that all these thresholds are constant, that preferences are increasing with evaluations on particular criteria, and that criteria are identified by their indices.

The concordance test involves calculation of concordance index $C(a,b)$. It represents the strength of the coalition of criteria being in favor of aSb. This coalition is composed of two subsets of criteria:

- subset of criteria being clearly in favor of aSb, i.e., such that $g_j(a) \geq g_j(b) - q_j$,
- subset of criteria that do not oppose to aSb, while being in an ambiguous position with respect to this assertion; these are those criteria for which a weak preference relation bQa holds; i.e., such that $g_j(b) - p_j \leq g_j(a) < g_j(b) - q_j$.

Consequently, the concordance index is defined as

$$C(a,b) = \frac{\sum_{j=1}^{n} \phi_j(a,b) \times k_j}{\sum_{j=1}^{n} k_j}, \tag{9}$$

where, for $j = 1, \ldots, n$,

$$\phi_j(a,b) = \begin{cases} 1, & \text{if } \; g_j(a) \geq g_j(b) - q_j, \\ \frac{g_j(a) - [g_j(b) - p_j]}{p_j - q_j}, & \text{if } \; g_j(b) - p_j \leq g_j(a) < g_j(b) - q_j, \\ 0, & \text{if } \; g_j(a) < g_j(b) - p_j. \end{cases} \tag{10}$$

$\phi_j(a,b)$ is a marginal concordance index, indicating to what extend criterion g_j contributes to the concordance index $C(a,b)$. As defined by (10), $\phi_j(a,b)$ is a piecewise-linear function, non-decreasing with respect to $g_j(a) - g_j(b)$.

Remark that $C(a,b) \in [0,1]$, where $C(a,b) = 0$ if $g_j(a) \leq g_j(b) - p_j$, $j = 1, \ldots, n$ (b is strictly preferred to a on all criteria), and $C(a,b) = 1$ if $g_j(a) \geq g_j(b) - q_j$, $j = 1, \ldots, n$ (a outranks b on all criteria).

The result of the concordance test for a pair $(a,b) \in A$ is positive if $C(a,b) \geq \lambda$, where $\lambda \in [0.5, 1]$ is a cutting level, which has to be fixed by the DM.

Once the result of the concordance test has been positive, one can pass to the non-discordance test. Its result is positive for the pair $(a,b) \in A$ unless "a is significantly worse than b" on at least one criterion, i.e., if $g_j(b) g_j(a) < v_j$ for $j = 1, \ldots, n$.

It follows from above that the outranking relation for a pair $(a,b) \in A$ is true, and denoted by aSb if both the concordance test and the non-discordance test are positive. On the other hand, the outranking relation for a pair $(a,b) \in A$ is false, and denoted by $aS^c b$, either if the concordance test or the non-discordance test is negative. Knowing S or S^c for all ordered pairs $(a,b) \in A$, one can proceed to exploitation of the outranking relation in set A, which is specific for the choice, or sorting or ranking problem, as described in [8].

Experience indicates that elicitation of preference information necessary for construction of the outranking relation is not an easy task for a DM. In particular, the inter-criteria preference information concerning the weights of criteria and the veto thresholds are difficult to be expressed directly.

For this reason, some aggregation-disaggregation procedures have been proposed in the past to assist the elicitation of the weights of criteria and all the thresholds required to construct the outranking relation [19], [20], [18]. A robust ordinal regression approach to outranking methods has been presented in [10]. Below, we briefly sketch this proposal.

We assume that the preference information provided by the DM is a set of pairwise comparisons of some reference actions. The set of reference actions is denoted by A^R, and it is usually, although not necessarily, a subset of set A. The comparison of a pair of actions $(a,b) \in A^R$ states the truth or falsity of the outranking relation, denoted by aSb or aS^cb, respectively. It is worth stressing that the DM does not need to provide all pairwise comparisons of reference actions, so this comparison can be confined to a small subset of pairs.

We also assume that the intra-criterion preference information concerning indifference and preference thresholds $p_j > q_j \geq 0$, $j = 1, \ldots, n$, is given. The last assumption is not unrealistic because these thresholds are relatively easy to provide by an analyst who is usually aware what is the precision of criteria, and how much difference is non-significant or relevant.

In order to simplify calculations of the ordinal regression, we assume that the weights of criteria sum up to one, i.e. $\sum_{j=1}^{n} k_j = 1$. Thus, (9) becomes

$$C(a,b) = \sum_{j=1}^{n} \phi_j(a,b) \times k_j = \sum_{j=1}^{n} \psi_j(a,b), \tag{11}$$

where the marginal concordance index $\psi_j(a,b) = \phi_j(a,b) \times k_j$ is a monotone non-decreasing function with respect to $g_j(a) - g_j(b)$, such that $\psi_j(a,b) \geq 0$ for $g_j(a) - g_j(b) \geq -q_j$, $j = 1, \ldots, n$, and $\sum_{j=1}^{n} \psi_j(a,b) = 1$ in case $g_j(a) - g_j(b) = \beta_j - \alpha_j$ for all $j = 1, \ldots, n$, α_j and β_j being the worst and the best evaluation on criterion g_j, respectively.

The ordinal regression constraints defining the set of concordance indices $C(a,b)$, cutting levels λ and veto thresholds v_j, $j = 1, \ldots, n$, compatible with the pairwise comparisons provided by the DM have the following form:

$$\left.\begin{array}{l}
C(a,b) = \sum_{j=1}^{n} \psi_j(a,b) \geq \lambda \;\text{ and }\; g_j(b) - g_j(a) \leq v_j - \epsilon, j = 1, \ldots, n,\\
\quad \text{if } \; aSb, \;\text{ for }\; (a,b) \in A^R,\\
C(a,b) = \sum_{j=1}^{n} \psi_j(a,b) \leq \lambda + \epsilon + M_0(a,b) \;\text{ and }\; g_j(b) - g_j(a) \leq v_j - \delta M_j(a,b),\\
M_j(a,b) \in \{0,1\},\; \sum_{j=0}^{n} M_j(a,b) \leq n,\; j = 1, \ldots, n,\\
\quad \text{if } \; aS^cb, \;\text{ for }\; (a,b) \in A^R,\\
1 \geq \lambda \geq 0.5, \quad v_j \geq p_j,\; j = 1, \ldots, n,\\
\psi_j(a,b) \geq 0 \;\text{ if }\; g_j(a) - g_j(b) \geq -q_j, \;\text{ for all } (a,b) \in A^R,\; j = 1, \ldots, n,\\
\sum_{j=1}^{n} \psi_j(a,b) = 1 \;\text{ if }\; g_j(a) - g_j(b) = \beta_j - \alpha_j \;\text{ for all } (a,b) \in A^R,\; j = 1, \ldots, n,\\
\psi_j(a,b) \geq \psi_j(c,d) \;\text{ if }\; g_j(a) - g_j(b) \geq g_j(c) - g_j(d),\\
\quad \text{for all } a,b,c,d \in A^R,\; j = 1, \ldots, n,
\end{array}\right\} E(A^R)$$

where ϵ is a small positive value and δ is a big positive value. Remark that $E(A^R)$ are constraints of a 0-1 mixed linear program.

Given a pair of actions $(x,y) \in A$, the following values are useful to build necessary and possible outranking relations:

$$d(x,y) = Min\left\{\sum_{j=1}^{n} \psi_j(x,y) - \lambda\right\}, D(x,y) = Max\left\{\sum_{j=1}^{n} \psi_j(x,y) - \lambda\right\}.$$

subject to constraints $E(A^R)$, where $\psi_j(a,b) \geq \psi_j(c,d)$ if $g_j(a) - g_j(b) \geq g_j(c) - g_j(d)$, for all $a,b,c,d \in A^R \cup \{x,y\}$, $j = 1, \ldots, n$, and $g_j(y) - g_j(x) \geq v_j$, $j = 1, \ldots, n$.

Given a pair of actions $(x, y) \in A$, x *necessarily* outranks y, which is denoted by $xS^N y$, if and only if $d(x, y) \geq 0$. $d(x, y) \geq 0$ means that for all compatible outranking models x outranks y. Analogously, given a pair of actions $(x, y) \in A$, x *possibly* outranks y, which is denoted by $xS^P y$, if and only if $D(x, y) \geq 0$. $D(x, y) \geq 0$ means that for at least one compatible outranking model x outranks y. The necessary and the possible outranking relations are to be exploited as usual outranking relations in the context of choice, sorting and ranking problems.

7 Robust Ordinal Regression for Outranking Methods in Group Decision Problems

The above approach can be adapted to the case of group decision. In this case, several DMs cooperate in a decision problem to make a collective decision. DMs share the same "description" of the decision problem (the same set of actions, evaluation criteria and performance matrix). Each DM provides his/her own preference information, composed of pairwise comparisons of some reference actions. The collective preference model accounts for the preference expressed by each DM.

Let us denote the set of DMs by $\mathcal{D}=\{d_1, \ldots, d_p\}$. For each DM $d_r \in D' \subseteq D$, we consider all compatible outranking models. Four situations are interesting for a pair $(x, y) \in A$:

- $x\ S^{N,N}(D')\ y$: $xS^N y$ for all $d_r \in D'$,
- $x\ S^{N,P}(D')\ y$: $xS^N y$ for at least one $d_r \in D'$,
- $x\ S^{P,N}(D')\ y$: $xS^P y$ for all $d_r \in D'$,
- $x\ S^{P,P}(D')\ y$: $xS^P y$ for at least one $d_r \in D'$.

8 Conclusions

In this article we presented basic principles of robust ordinal regression for group decision. After recalling the robust ordinal regression methods within MAUT for choice and ranking problems (UTA^{GMS} and $GRIP$), for sorting problems (($UTADIS^{GMS}$), as well as ordinal regression methods within the outranking approach ($ELECTRE^{GMS}$), we extended all these methods to group decision introducing UTA^{GMS}-Group, $UTADIS^{GMS}$-GROUP and $ELECTRE^{GMS}$ -GROUP.

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