Prototypical Reasoning with Low Complexity Description Logics: Preliminary Results

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Abstract. We present an extension $\mathcal{EL}^{+^\perp}\mathbf{T}$ of the description logic $\mathcal{EL}^{+^\perp}$ for reasoning about prototypical properties and inheritance with exceptions. $\mathcal{EL}^{+^\perp}\mathbf{T}$ is obtained by adding to $\mathcal{EL}^{+^\perp}$ a typicality operator $\mathbf{T}$, which is intended to select the "typical" instances of a concept. In $\mathcal{EL}^{+^\perp}\mathbf{T}$ knowledge bases may contain inclusions of the form "$\mathbf{T}(C)$ is subsumed by P", expressing that typical C-members have the property P. We show that the problem of entailment in $\mathcal{EL}^{+^\perp}\mathbf{T}$ is in co-NP.

1 Introduction

In Description Logics (DLs) the need of representing prototypical properties and of reasoning about defeasible inheritance of such properties naturally arises. The traditional approach is to handle defeasible inheritance by integrating some kind of nonmonotonic reasoning mechanism. This has led to study nonmonotonic extensions of DLs [2,3,4,5,6,12]. However, finding a suitable nonmonotonic extension for inheritance with exceptions is far from obvious.

In this work we introduce a defeasible extension of the description logic $\mathcal{EL}^{+^\perp}$ called $\mathcal{EL}^{+^\perp}\mathbf{T}$, continuing the investigation started in [7], where we extended the logic $\mathcal{ALC}$ with a typicality operator $\mathbf{T}$. The intended meaning of the operator $\mathbf{T}$ is that, for any concept C, $\mathbf{T}(C)$ singles out the instances of C that are considered as "typical" or "normal". Thus assertions as "typical football players love football" are represented by $\mathbf{T}(\mathit{FootballPlayer}) \sqsubseteq \mathit{FootballLover}$. The semantics of the typicality operator $\mathbf{T}$ turns out to be strongly related to the semantics of nonmonotonic entailment in KLM logic $\mathbf{P}$ [11].

In our setting, we assume that the TBox element of a KB comprises, in addition to the standard concept inclusions, a set of inclusions of the type $\mathbf{T}(C) \sqsubseteq D$ where D is a concept not mentioning $\mathbf{T}$. For instance, a KB may contain: $\mathbf{T}(\mathit{Dog}) \sqsubseteq \mathit{Affectionate}$; $\mathbf{T}(\mathit{Dog}) \sqsubseteq \mathit{CarriedByTrain}$; $\mathbf{T}(\mathit{Dog} \sqcap \mathit{PitBull}) \sqsubseteq \mathit{NotCarriedByTrain}$; $\mathit{CarriedByTrain} \sqcap \mathit{NotCarriedByTrain} \sqsubseteq \perp$, corresponding to the assertions: typically dogs are affectionate, normally dogs can be transported by train, whereas typically a dog belonging to the race of pitbull cannot

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(since pitbulls are considered as reactive dogs); the fourth inclusion represents the disjointness of the two concepts *CarriedByTrain* and *NotCarriedByTrain*. Notice that, in standard DLs, replacing the second and the third inclusion with $Dog \sqsubseteq CarriedByTrain$ and $Dog \sqcap PitBull \sqsubseteq NotCarriedByTrain$, respectively, we would simply get that there are not pitbull dogs, thus the KB would collapse. This collapse is avoided as we do not assume that $\mathbf{T}$ is monotonic, that is to say $C \sqsubseteq D$ does not imply $\mathbf{T}(C) \sqsubseteq \mathbf{T}(D)$.

By the properties of $\mathbf{T}$, some inclusions are entailed by the above KB, as for instance $\mathbf{T}(Dog \sqcap CarriedByTrain) \sqsubseteq Affectionate$. In our setting we can also use the $\mathbf{T}$ operator to state that some domain elements are typical instances of a given concept. For instance, an ABox may contain either $\mathbf{T}(Dog)(fido)$ or $\mathbf{T}(Dog \sqcap PitBull)(fido)$. In the two cases, the expected conclusions are entailed: $CarriedByTrain(fido)$ and $NotCarriedByTrain(fido)$, respectively.

In this work, we present some preliminary results on *low complexity* Description Logics extended with the typicality operator $\mathbf{T}$. In particular we focus on the logic $\mathcal{EL}^{+^\perp}$ of the well known $\mathcal{EL}$ family. The logics of the $\mathcal{EL}$ family allow for conjunction ($\sqcap$) and existential restriction ($\exists R.C$). Despite their relatively low expressivity, a renewed interest has recently emerged for these logics. Indeed, theoretical results have shown that $\mathcal{EL}$ has better algorithmic properties than its counterpart $\mathcal{FL}_0$, which allows for conjunction and value restriction ($\forall R.C$). Also, it has turned out that the logics of the $\mathcal{EL}$ family are relevant for several applications, in particular in the bio-medical domain; for instance, medical terminologies, such as GALEN, SNOMED, and the Gene Ontology used in bioinformatics, can be formalized in small extensions of $\mathcal{EL}$.

We present some results about the complexity of $\mathcal{EL}^{+^\perp}\mathbf{T}$. We show that, given an $\mathcal{EL}^{+^\perp}\mathbf{T}$ KB, if it is satisfible, then there is a *small* model whose size is polynomial in the size of KB. The construction of the model exploits the facts that (1) it is possible to reuse the same domain element (instance of a concept C) to fulfill existential formulas $\exists r.C$ w.r.t. domain elements; (2) we can restrict our attention to a class of models in which the preference relation $<$ is multilinear and polynomial, that is it determines a set of disjoint chains of elements of polynomial length. The construction of the model allows us to conclude that the problem of deciding entailment in $\mathcal{EL}^{+^\perp}\mathbf{T}$ is in co-NP.

Technical details and proofs can be found in the accompanying report [10].

2 The Logic $\mathcal{EL}^{+^\perp}\mathbf{T}$

We consider an alphabet of concept names $\mathcal{C}$, of role names $\mathcal{R}$, and of individuals $\mathcal{O}$. The language $\mathcal{L}$ of the logic $\mathcal{EL}^{+^\perp}\mathbf{T}$ is defined by distinguishing *concepts* and *extended concepts* as follows: (Concepts) $A \in \mathcal{C}$, $\top$, and $\bot$ are *concepts* of $\mathcal{L}$; if $C, D \in \mathcal{L}$ and $r \in \mathcal{R}$, then $C \sqcap D$ and $\exists r.C$ are *concepts* of $\mathcal{L}$. (Extended concepts) if C is a concept, then C and $\mathbf{T}(C)$ are extended concepts of $\mathcal{L}$. A knowledge base is a pair (TBox,ABox). TBox contains (i) a finite set of GCIs $C \sqsubseteq D$, where C is an extended concept (either C' or $\mathbf{T}(C')$), and D is a concept, and (ii) a finite set of role inclusions (RIs) $r_1 \circ r_2 \circ \cdots \circ r_n \sqsubseteq r$. ABox contains expressions of the form $C(a)$ and $r(a,b)$ where C is an extended concept, $r \in \mathcal{R}$, and $a, b \in \mathcal{O}$. In order to provide a semantics to the operator $\mathbf{T}$, we extend the definition of a model used in "standard" terminological logic $\mathcal{EL}^{+^\perp}$:

Definition 1 (Semantics of T). *A model $\mathcal{M}$ is any structure $\langle \Delta, <, I \rangle$, where Δ is the domain; $<$ is an irreflexive and transitive relation over Δ, and satisfies the following* Smoothness Condition*: for all $S \subseteq \Delta$, for all $a \in S$, either $a \in Min_<(S)$ or $\exists b \in Min_<(S)$ such that $b < a$, where $Min_<(S) = \{a : a \in S$ and $\nexists b \in S$ s.t. $b < a\}$. I is the extension function that maps each extended concept C to $C^I \subseteq \Delta$, and each role r to a $r^I \subseteq \Delta^I \times \Delta^I$. For concepts of $\mathcal{EL}^{+^\perp}$, C^I is defined in the usual way. For the $\mathbf{T}$ operator: $(\mathbf{T}(C))^I = Min_<(C^I)$. A model satisfying a KB (TBox,ABox) is defined as usual. Moreover, we assume the unique name assumption.*

Notice that the meaning of $\mathbf{T}$ can be split into two parts: for any a of the domain Δ, $a \in (\mathbf{T}(C))^I$ just in case (i) $a \in C^I$, and (ii) there is no $b \in C^I$ such that $b < a$. In order to isolate the second part of the meaning of $\mathbf{T}$, we introduce a new modality $\Box$. The basic idea is simply to interpret the preference relation $<$ as an accessibility relation. By the Smoothness Condition, it turns out that $\Box$ has the properties as in Gödel-Löb modal logic of provability G. The interpretation of $\Box$ in $\mathcal{M}$ is as follows: $(\Box C)^I = \{a \in \Delta \mid$ for every $b \in \Delta$, if $b < a$ then $b \in C^I\}$. We have that a is a typical instance of C ($a \in (\mathbf{T}(C))^I$) iff $a \in C^I$ and, for all $b < a$, $b \notin C^I$, namely we have that $a \in (\mathbf{T}(C))^I$ iff $a \in (C \sqcap \Box\neg C)^I$. From now on, we consider $\mathbf{T}(C)$ as an abbreviation for $C \sqcap \Box\neg C$. The Smoothness Condition ensures that typical elements of C^I exist whenever $C^I \neq \emptyset$, by preventing infinitely descending chains of elements.

3 Complexity of $\mathcal{EL}^{+^\perp}\mathbf{T}$

In order to give a complexity upper bound for the logic $\mathcal{EL}^{+^\perp}\mathbf{T}$, we show that, given a model $\mathcal{M} = \langle \Delta, <, I \rangle$ of a KB, we can build a *small* model of KB whose size is polynomial in the size of the KB.

Theorem 1 (Small model theorem). *Let KB=(TBox,ABox) be an $\mathcal{EL}^{+^\perp}\mathbf{T}$ knowledge base. For all models $\mathcal{M} = \langle \Delta, <, I \rangle$ of KB and all $x \in \Delta$, there exists a model $\mathcal{N} = \langle \Delta^\circ, <^\circ, I^\circ \rangle$ of KB such that (i) $x \in \Delta^\circ$, (ii) for all $\mathcal{EL}^{+^\perp}\mathbf{T}$ concepts C, $x \in C^I$ iff $x \in C^{I^\circ}$, and (iii) $\mid \Delta^\circ \mid$ is polynomial in the size of KB.*

Due to space limitations, here we only give a sketch of the proof, whose details can be found in [10]. The construction comprises three steps.

(step A) First of all, in order to reduce the size of the model, we cut a portion of it that includes x. We build a model $\mathcal{M}'$ by means of the following construction. For each atomic concept $C \in \mathcal{C}$ and for each role $r \in \mathcal{R}$ we let $S(C)$ and $R(r)$ be the mappings computed by the algorithm defined in [1] to compute subsumption by means of completion rules. As usual, for a given individual a in the ABox, we write a^I to denote the element of Δ corresponding to the extension of a in $\mathcal{M}$. We make use of three sets of elements: Δ_0 will be part of the domain of the model being constructed, and it contains a portion of the domain Δ of the initial model. All elements introduced in the domain must be processed in order to satisfy the existential formulas. *Unres* is used to keep track of not yet processed elements. Finally, Δ_1 is a set of elements that will belong to the domain of the constructed model. Each element w_C of Δ_1 is created for a corresponding atomic concept C and is used to satisfy any existential formula $\exists r.C$ throughout the model. In the following by w_C we mean the domain element of Δ_1 which is added for the atomic concept C. We provide an algorithmic description of the construction of model $\mathcal{M}'$ from the given model $\mathcal{M}$. Observe that $\mathcal{M}$ can be an infinite model.

$\Delta_0 := \{x\} \cup \{a^I \in \Delta \mid a$ occurs in the ABox $\}$
Unres$:=\{x\} \cup \{a^I \in \Delta \mid a$ occurs in the ABox $\}$
$\Delta_1 := \emptyset$
while *Unres* $\neq \emptyset$ **do**
    extract one y from *Unres*
    for each $\exists r.C$ occurring in KB s.t. $y \in (\exists r.C)^I$ **do**
        if $\nexists w_C \in \Delta_1$ **then**
            choose $w \in \Delta$ s.t. $(y, w) \in r^I$ and $w \in C^I$
            $\Delta_0 := \Delta_0 \cup \{w\}$
            Unres$:=$*Unres* $\cup \{w\}$
            create a new element w_C associated with C
            $\Delta_1 := \Delta_1 \cup \{w_C\}$
            add $w <' w_C$
            add (y, w_C) to $r^{I'}$
        else
            add (y, w_C) to $r^{I'}$
    for each $y_i \in \Delta$ such that $y_i < y$ **do**
        $\Delta_0 := \Delta_0 \cup \{y_i\}$
        Unres$:=$*Unres* $\cup\{y_i\}$
for each $w_C, w_D \in \Delta_1$ with $C \neq D$ **do**
    if $(C, D) \in R(r)$ **then** add (w_C, w_D) to $r^{I'}$

The model $\mathcal{M}' = \langle \Delta', <', I' \rangle$ is defined as follows:

- $\Delta' = \Delta_0 \cup \Delta_1$
- we extend $<'$ computed by the algorithm by adding $u <' v$ if $u < v$, for each $u, v \in \Delta'$;
- the extension function I' is defined as follows: • for all atomic concepts $C \in \mathcal{C}$, for all domain elements in Δ', we define: for each $u \in \Delta_0$, we let $u \in C^{I'}$ if $u \in C^I$; for each $w_D \in \Delta_1$, we let $w_D \in C^{I'}$ if $C \in S(D)$. • for all roles r, we extend $r^{I'}$ constructed by the algorithm by means of the following role closure rules: for all inclusions $r \sqsubseteq s \in$ TBox, if $(u, v) \in r^{I'}$ then add (u, v) to $s^{I'}$; for all inclusions $r_1 \circ r_2 \sqsubseteq s \in$ TBox, if $(u, v) \in r_1^{I'}$ and $(v, w) \in r_2^{I'}$ then add (u, w) to $s^{I'}$. • I' is extended so that it assigns a^I to each individual a in the ABox.

$\mathcal{M}'$ is not guaranteed to have polynomial size in the KB because in line 18 we add an element y_i for each $y_i < y$, then the size of Δ_0 may be arbitrarily large.

(step B) We refine our construction in order to obtain from $\mathcal{M}'$ a multi-linear model with a polynomial number of chains. Intuitively, a model is *multi-linear* if the relation $<$ forms a set of chains of domain elements, that is, for every u, v, z of the domain, we have that: (i) if $u < z$ and $v < z$ and $u \neq v$, then $u < v$ or $v < u$; (ii) if $z < u$ and $z < v$ and $u \neq v$, then $u < v$ or $v < u$. From $\mathcal{M}'$ we can obtain a multilinear model $\mathcal{M}''$ that preserves the interpretation of atomic concepts with respect to common elements of the domain and has a polynomial number of chains.

(step C) We finally construct a model $\mathcal{N}$ from $\mathcal{M}''$ whose domain has polynomial size in the size of KB. The idea is as follows. Let us consider a chain $w_0, w_1, w_2, \ldots$ in the multi-linear model. We can observe that, given w_i and w_j in the chain such that $w_i < w_j$, the set of negated box formulas $\neg\Box\neg C$ of which w_i is an instance is a subset of the set of negated box formulas of which w_j is an instance. We can thus shrink each chain by retaining only the elements w_i, w_j such that $w_i < w_j$ implies there exists a formula $\neg\Box\neg C$ such that w_j is an instance of $\neg\Box\neg C$ and w_i is not an instance of $\neg\Box\neg C$. As there is only a polynomial number of such box formulas $\neg\Box\neg C$, each chain will contain only a polynomial number of elements. Since the number of chains is polynomial in itself (by step B), the resulting model $\mathcal{N}$ has a polynomial size.

Given Theorem 1 above, when evaluating the entailment, we can restrict our consideration to small models, namely, to polynomial multi-linear models of the KB. We write KB $\models \alpha$ to say that a query α holds in all the models of the KB. A query α is either a formula of the form $C(a)$ or a subsumption relation $C \sqsubseteq D$. We write KB $\models_s \alpha$ to say that α holds in all polynomial multi-linear models of the KB. It holds that KB $\models \alpha$ if and only if KB $\models_s \alpha$. As a consequence, we can give an upper bound on the complexity of $\mathcal{EL}^{+^\perp}\mathbf{T}$:

Theorem 2. *In $\mathcal{EL}^{+^\perp}\mathbf{T}$, the problem of deciding whether KB $\models \alpha$ is in* co-NP*. The problems of satisfiability of a KB and of concept satisfiability are in* NP*. The problems of subsumption and of instance checking are in* co-NP*.*

4 Conclusions and Future Issues

We have presented the description logic $\mathcal{EL}^{+^\perp}\mathbf{T}$, that is $\mathcal{EL}^{+^\perp}$ extended by a tipicality operator $\mathbf{T}$ intended to select the "most normal" instances of a concept. Whereas for $\mathcal{ALC} + \mathbf{T}$ deciding satisfiability (subsumption) is EXPTIME complete (see [9]), we have shown here that for $\mathcal{EL}^{+^\perp}\mathbf{T}$ the complexity is significantly smaller, namely it reduces to NP for satisfiability (and co-NP for subsumption). This result is obtained by a "small" model property (of a particular kind: multi-linear) that fails for the whole $\mathcal{ALC} + \mathbf{T}$ as well as for $\mathcal{ALC}$. We believe that this bound is also a lower bound, but we have not proved it so far. Although validity/satisfiability for KLM logic $\mathbf{P}$ is known to be (co)NP hard, in

$\mathcal{EL}^{+^\perp}\mathbf{T}$, we can only directly encode nonmonotonic assertions $A \mid\!\sim B$ where A is a conjunction of atoms and B is either an atom or $\perp$. As far as we know, the complexity of this fragment of $\mathbf{P}$ is unknown. Thus a lower bound for $\mathcal{EL}^{+^\perp}\mathbf{T}$ cannot be obtained from known results about KLM logic $\mathbf{P}$.

The logic $\mathcal{EL}^{+^\perp}\mathbf{T}$ in itself is not sufficient for prototypical reasoning and inheritance with exceptions, in particular we need a stronger (nonmonotonic) mechanism to cope with the problem known as *irrelevance*. Concerning the example of the Introduction, we would like to conclude that typical red dogs are affectionate, since the color of a dog is irrelevant with respect to the property of being affectionate. However, as the property of being red is not a property neither of all dogs, nor of typical dogs, in $\mathcal{EL}^{+^\perp}\mathbf{T}$ we are not able to conclude $\mathbf{T}(Dog \sqcap Red) \sqsubseteq Affectionate$. One possibility is to consider a stronger (nonmonotonic) entailment relation $\mathcal{EL}^{+^\perp}\mathbf{T}_{min}$ determined by restricting the entailment of $\mathcal{EL}^{+^\perp}\mathbf{T}$ to "minimal models", as defined in [8] for $\mathcal{ALC} + \mathbf{T}$. Intuitively, minimal models are those that maximise "typical instances" of a concept. As shown in [8], for $\mathcal{ALC} + \mathbf{T}_{min}$, minimal entailment can be decided in co-NExp$^{\mathrm{NP}}$. We believe that for $\mathcal{EL}^{+^\perp}\mathbf{T}_{min}$ we can obtain a smaller complexity upper bound on the base of the results presented here.

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References

1. Baader, F., Brandt, S., Lutz, C.: Pushing the $\mathcal{EL}$ envelope. In: Proc. of IJCAI 2005, Professional Book Center, pp. 364–369 (2005)
2. Baader, F., Hollunder, B.: Embedding defaults into terminological knowledge representation formalisms. J. Autom. Reasoning 14(1), 149–180 (1995)
3. Baader, F., Hollunder, B.: Priorities on defaults with prerequisites, and their application in treating specificity in terminological default logic. J. of Automated Reasoning (JAR) 15(1), 41–68 (1995)
4. Bonatti, P.A., Lutz, C., Wolter, F.: Description logics with circumscription. In: Proc. of KR, pp. 400–410 (2006)
5. Donini, F.M., Nardi, D., Rosati, R.: Description logics of minimal knowledge and negation as failure. ACM Trans. Comput. Log. 3(2), 177–225 (2002)
6. Eiter, T., Lukasiewicz, T., Schindlauer, R., Tompits, H.: Combining answer set programming with description logics for the semantic web. In: KR 2004, pp. 141–151 (2004)
7. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: Preferential Description Logics. In: Dershowitz, N., Voronkov, A. (eds.) LPAR 2007. LNCS (LNAI), vol. 4790, pp. 257–272. Springer, Heidelberg (2007)
8. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: Reasoning About Typicality in Preferential Description Logics. In: Hölldobler, S., Lutz, C., Wansing, H. (eds.) JELIA 2008. LNCS (LNAI), vol. 5293, pp. 192–205. Springer, Heidelberg (2008)

9. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: On Extending Description Logics for Reasoning About Typicality: a First Step. Technical Report 116/09, Dip. di Informatica, Univ. di Torino (2009)
10. Giordano, L., Gliozzi, V., Olivetti, N., Pozzato, G.L.: Reasoning About Typicality in Low Complexity Description Logics: Preliminary Results. Technical Report 121/09, Dip. di Informatica, Univ. di Torino (2009)
11. Kraus, S., Lehmann, D., Magidor, M.: Nonmonotonic reasoning, preferential models and cumulative logics. Artificial Intelligence 44(1-2), 167–207 (1990)
12. Straccia, U.: Default inheritance reasoning in hybrid kl-one-style logics. In: Proc. of IJCAI, pp. 676–681 (1993)