

Tractable Cases of Facility Location on a Network with a Linear Reliability Order of Links

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Abstract. In this paper we study the problem of locating k facilities to maximize the expected demand serviced in a network with unreliable links. Given a linear ordering of links, which models the dependencies among link failures, we assume that when a strong link fails, all weaker links with lower reliability also fail. This model is due to Gunneç and Salman [1], and for the single ordering case, an exact algorithm for maximizing expected serviced demand was provided in our earlier work [2] via a greedy method and dynamic programming.

Our main result in this paper is to identify the boundary of hardness of the problem as we extend the model to have more than one disaster scenario and there is a different linear order in each scenario (defining the strength of the links in the scenario). We show that in the case with two disaster scenarios, the resulting facility location problem is polynomial time solvable by proving total unimodularity of a linear programming formulation. We also supply an alternate proof of this fact by the iterative relaxation method. In addition, for the two scenario case, we show that a version maximizing the expected demand served minus the sum of facility opening costs reduces to a bipartite matching problem. We then prove NP-hardness for the case with three orderings, even when all reliability values are one or zero (i.e., every scenario is deterministic, and we have three scenarios). Following the idea of the reduction, we show that the problem with an arbitrary number of orderings generalizes the maximum coverage problem, and hence affords a greedy approximation algorithm with performance ratio $(1 - \frac{1}{e})$. We also consider the distance-bounded version of the problem where a demand point can be covered only if a facility exists within a distance limit, and show that the problem is NP-hard even for a single ordering and is equivalent to the maximum k -facility location problem. Our methods represent the first attempt at finding interesting tractable models of link failure for facility location

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planning for disasters. Our earlier results for the case of a single ordering [2] already showed the versatility of various techniques such as the Greedy method and a Dynamic Programming algorithm for addressing the many variants of the problem. This paper reveals an even richer structure for the polynomially solvable two-scenario case by showing non-trivial applications of total unimodularity, the iterative relaxation technique and weighted bipartite perfect matchings.

References

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