

Phase/Magnitude Retrieval Algorithms in Electrical Bioimpedance Spectroscopy

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Abstract— The use of an amplitude/phase retrieval algorithm in electrical impedance spectroscopy(EIS) that allows a new method to calculate the impedance spectrum in EIS systems from partial data information is described. To the authors' knowledge, this is the first time such an algorithm has been used to calculate the magnitude/phase of an impedance in EIS from one of these two available parameters. The use of magnitude or phase retrieval circuits in EIS systems may become unnecessary with the proposed technique, reducing EIS systems cost and complexity. The algorithm is theoretically validated by using a reference impedance obtained from a simulated 2R1C function that represents a model for biological tissue whose parameters are readily obtainable from literature. Experimental validation is demonstrated by using a complete EIS system to obtain the values of a known impedance across a frequency interval from 100Hz up to 500kHz, where only phase is used as input data for the algorithm to calculate the impedance magnitude. Algorithm performance is studied when noise is added to typical bioimpedance simulated data. The algorithm shows robustness with typical bioimpedance noisy data and experimental results are shown to be consistent with simulation.

Keywords— Phase/amplitude retrieval, electrical bioimpedance spectroscopy.

INTRODUCTION

In electrical bioimpedance spectroscopy (EIS) systems one measures the values of unknown electrical impedances of biological tissues at specific frequency points. In such a system, the biological tissue to be characterized is usually excited by a constant amplitude voltage or current sine-wave at determined frequencies, the current or voltage is measured between two electrodes and the transfer impedance is calculated.

The electronic circuits used in EIS systems for the acquisition of such signals have some drawbacks during its implementation, since at high frequencies acquisition electronics may become more complex and expensive. For example, at the present moment there is no commercial integrated circuit to generate waveforms for the specific purpose of biological tissue excitation in EIS. These circuits would

have to generate current or voltage sine-waves and demodulate the signals in the frequency bands commonly studied with EIS, namely, from 10Hz up to units of MHz. As a consequence, dedicated circuits operating in the radio-frequency band must be adapted.

To overcome these difficulties it would be useful to look for algorithms that may calculate the real or imaginary part of the complex valued impedances, or its magnitude and phase, instead of measuring them. A similar problem has been extensively studied in different areas in the use of Kramers-Krönig relations (KKR) to obtain the imaginary part of a complex function from its real part[1]. It has been suggested in the literature that KKR could be used in EIS[2], because the impedance value is causal and the real(imaginary) part would be calculated from each other using the Hilbert transform. There are algorithms that may be used in an optimized manner other than those calculating the Hilbert Transform[3]. Phase or magnitude retrieval algorithms solve this problem in digital signal processing theory[4]. Such algorithms use the Fast-Fourier Transform (FFT) to calculate the magnitude of a sequence from its phase or, equivalently, to obtain the phase from the FFT magnitude. The transfer impedance can be obtained from partial acquired data, either phase or magnitude, even across limited frequency intervals. Since only phase or magnitude must be acquired, the electronics can be significantly simplified. The instrumentation necessary for magnitude detection requires mostly high-speed analog-to-digital converters following high-precision instrumentation amplifiers. In the case of phase detection circuits, a high-precision analog multiplier is commonly used, providing a DC signal proportional to the phase of its input, sampled with a low acquisition rate. With this algorithm, for example, one can avoid the use of electronics to acquire magnitude, and only the phase of the impedance is acquired to calculate the magnitude.

An improvement in EIS that uses a well known phase/magnitude retrieval algorithm to calculate the impedance phase or magnitude from the available set of data, magnitude or phase, electronically obtained, is proposed. The use of the algorithm may simplify the EIS system acquisition electronics, being an alternative to the digital synchronous demodulation[5,6].

METHODOLOGY

A. Simulation methods

The proposed phase or amplitude retrieval technique in EIS systems is based on the use of an algorithm capable of reconstructing the phase or amplitude of a set of points uniformly distributed across a frequency interval starting at DC. In a practical case, the highest frequency in this interval depends on the highest frequency generated during the tissue excitation.

A typical 2R1C model for lung tissue impedance in EIS was simulated [7]. The utilized impedance is composed of a circuit that represents an impedance of lung tissue with values obtained from literature [8]. This circuit is a resistance, $R=99.5\Omega$, in parallel with a circuit containing a capacitor, $C=9.86nF$, in series with a resistance, $S=23.8\Omega$. Lu [7] describes how a parametric model of such an impedance is obtained and provides details of what the R, S and C parameters represent in a tissue impedance.

The simulated Fourier Transform of the impedance is used as input to the algorithm. The phase retrieval algorithm was implemented in a MATLAB script and a set of $N=128$ simulated impedance points were calculated and are shown in Figure 1. In the following algorithm, the impedance magnitude as depicted in the upper graph was used as input data. The simulated phase, shown in the graph below, is used as a reference for the iterations in the algorithm and to calculate the stopping parameter of the loop, ϵ . The frequency on the x-axis is normalized and the maximum value in rad/s, π , corresponds to 10MHz.

B. Algorithm description

The algorithm as first proposed in [4] is depicted in the flowchart in Figure 2 with a few modifications. It is capable of reconstructing a Fourier Transform magnitude or phase from the missing values, phase or magnitude. The signal sequence that produces the Fourier Transform must be real, have a finite extent and a non-factorable z-transform, if the phase is to be retrieved. For magnitude retrieval, the sequence must be real, have a finite extent and no zeros on the unit circle or in reciprocal pairs [9]. Since the use in EIS requires that the signal remains in the frequency domain and is always obtained from actual impedance values, the constraints on the input data set resides in the zero-pole location of the impedance function obtained with the 2R1C parameters. The first block in the flowchart represents the

data input uniformly spaced in the frequency domain. The size of the input sequence, N , is a power of 2 and in a practical case can be produced by a demodulation circuit. If the algorithm is set to retrieve magnitude, a phase sequence of N samples is used as input to the algorithm. In a magnitude retrieving algorithm the phase, $\theta_{est}(k)$, is initialized with random values, $\theta_{random}(k)$, and the magnitude vector with N samples is the simulated impedance, as depicted in the second block of Figure 1. The sequence formed by N samples, $Y_{OR}(k)=|Y_{OR}(k)|\exp(j\theta_{est}(k))$, is the first impedance estimate. In the following block, an M -point inverse Fast Fourier Transform (IFFT) is calculated from the estimate, $Y_{OR}(k)$, and a time-domain signal, $y_{est}(n)$, is produced and only its real part is considered. The samples with 'n' not in $\{0,1,\dots,N-1\}$ are set to zero, and causality is guaranteed in the $y_{est}(n)$ sequence. An M -point Fast Fourier Transform (FFT) is calculated for $y_{est}(n)$ producing the next phase estimate contained in the vector $Y_{est}(k)=|Y_{est}(k)|\exp(j\theta_{est+1}(k))$. $Y_{est}(k)$ is forced to have the original input $|Y_{OR}(k)|$ and the procedure is repeated until the phase does not show relevant changes in the root mean square value (RMS) of the phase estimate. Namely, if the RMS error of $\epsilon=|\theta_{est}(k)-\theta_{est+1}(k)|$ is smaller than a constant stopping parameter value set to 10^{-6} , the algorithm stops, where ϵ is a value much lower than any usual EIS value resolution. In the magnitude retrieval algorithm, phase must be exchanged for magnitude and vice-versa.

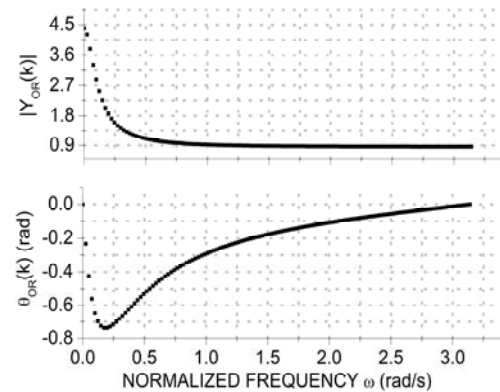


Fig. 1 Fourier Transform of the 2R1C model impedance with magnitude $|Y_{OR}(k)|$ (upper graph) and phase $\theta_{OR}(k)$ (below).; using 2R1C parameters from [7].

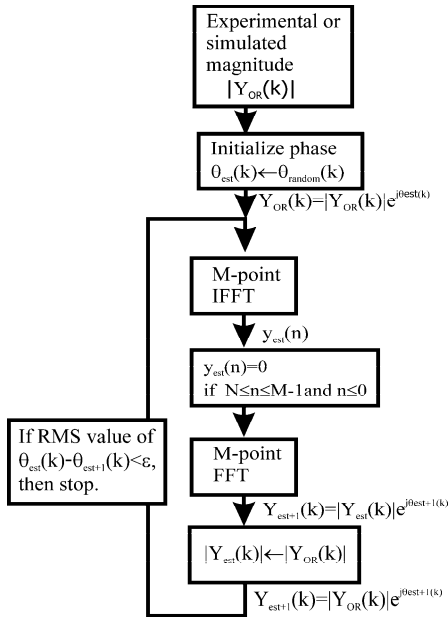


Fig. 2 Phase retrieval algorithm flowchart for EIS systems

VALIDATION PROCEDURE

The simulated data were used as reference for a typical impedance spectrum that would validate the algorithm in EIS systems. For these data, white Gaussian noise was added to the simulated magnitude spectrum of the 2R1C model, causing the magnitude-to-noise ratio (MNR) to vary. The number of iterations in the algorithm loop to reach a stopping parameter of $\epsilon=10^{-6}$ for each MNR level was also calculated and each obtained curve represents the behavior of the algorithm for a specific number of samples of the input data. The error was calculated using the mean value of the vector obtained from the absolute difference between the retrieved and simulated data normalized with maximum value in the data for different MNR. These points represent a relative error when calculated using normalized values.

A multi-channel EIS system developed at the authors' University was used to validate experimentally the technique by measuring the impedance of a resistance, $R=1000.35\Omega$, in parallel with a capacitance, $C=1nF$.

RESULTS AND DISCUSSION

In Figure 3, the error curve for the phase associated with the MNR=80dB shows the intrinsic error in the reconstruction without relevant distortion caused by noise in the process.

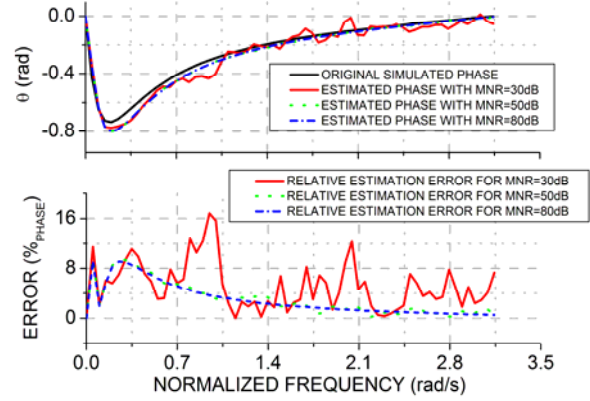


Fig. 3 In the upper graph, simulated and retrieved phase curves with $N=128$ and $MNR=30dB$, $50dB$ and $80dB$; below, the relative error in the reconstructed phase.

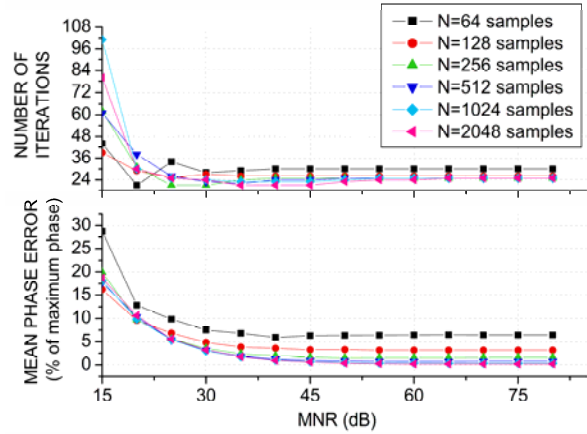


Fig. 4 The number of iterations to reach $\epsilon=10^{-6}$ for a given MNR. Curves correspond to $N= 64, 128, 256, 512$ and $1024, 2048$. Below, normalized phase error in the phase reconstruction as a function of the MNR.

For a number of input data samples and a spectrum profile, such errors are unavoidable and can be improved only by increasing the N . In practical signals, the error will increase with worst MNR, as illustrated by the other curves for lower MNR. Additional noisy simulated magnitude spectra were used to evaluate how many iterations would be necessary to provide the required estimate. The number of iterations and the RMS error between the original simulated signal and the estimated phase are depicted in two graphs in Figure 4 as a function of the MNR, for sequences of lengths $N= 64, 128, 256, 512, 1024, 2048$. Those curves show that after the MNR reaches a certain level, similar to values obtained for typical EIS electronic circuits larger than 30 dB, the algorithm requires more than 20 and less than 25 iterations to reach the stopping parameter $\epsilon=10^{-6}$. This parameter

can be used to establish N to be used in the signal reconstruction of an EIS system. This is a critical point, since the tissue excitation circuit in the EIS system will have to produce signals with the same number of frequencies.

The error in the reconstruction decreases and stabilizes with an increase in the number of samples, including the intrinsic error which is not produced by noise. This indicates that with a proper N , the algorithm can be sufficiently robust to replace a circuit that would measure the calculated parameter, in this case, phase. The same analysis may be applied to the magnitude retrieval algorithm, however, the signal must satisfy the constraints as previously described.

In the experiment, an error caused by interpolation in the enlargement of the sequence size appears. The RC load is a first order system, and linear interpolation may be used to obtain the other spectrum samples without much deterioration in the error curves.

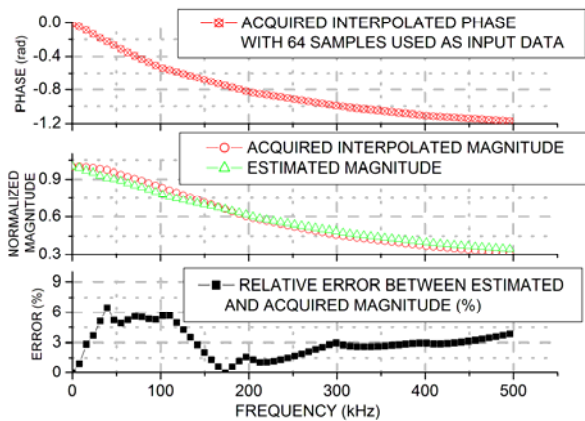


Fig. 5 In the upper graph, the acquired interpolated phase used as input to the algorithm with $N=64$; In the second graph, the normalized acquired and estimated magnitude; below, the absolute value of the error between estimated and acquired magnitude.

In the validation experiment conducted, 28 samples were acquired with the system and linear interpolation was used to enlarge the sequence to $N = 64$ points. The experimental interval was in a range from 500 Hz up to 500 kHz. In the analysis with $N = 64$ samples, an error bounded to values around 6.5% was expected. The phase data for the algorithm is shown in the upper graph in Figure 5. Since the algorithm reconstructs the magnitude to within a scale factor[9], the estimated and original spectra are depicted normalized in Figure 5 and the absolute error is shown in the same graph with magnitudes. A peak error of approximately 6.5% occurred at around 39kHz. This error occurred also due to the interpolation procedure, but remained below the

errors with worst MNR with the simulated impedance. Its standard deviation, which will be considered here as the technique uncertainty, is 1.5%. And the mean error is 2.95%. Based on the graph in Figure 5, the errors are consistent with the simulated results, since the levels remain around the bounding value obtained in the simulation for 64 samples, even considering that interpolation caused deterioration in the reconstruction. For the 2R1C model no discrepancy between theoretical and experimental results has been observed.

CONCLUSIONS

Validations of a technique to reconstruct the transfer impedance spectrum from acquired information were demonstrated. To the authors' knowledge it is the first time this well known algorithm is proposed to be used in EIS systems. Performance analysis of the algorithm with 2R1C models and using an RC circuit was implemented. The algorithm performance is shown in terms of number of iterations for convergence with typical impedance data and such data can be used as parameters to develop practical systems in DSPs.

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