Spatial Cognition of Geometric Figures in the Context of Proportional Analogies

Angela Schwering, Kai-Uwe Kühnberger, Ulf Krumnack, and Helmar Gust

University of Osnabrück, Germany
{aschweri,kkuehnbe,krumnack,hgust}@uos.de

Abstract. The cognition of spatial objects differs among people and is highly influenced by the context in which a spatial object is perceived. We investigated experimentally how humans perceive geometric figures in geometric proportional analogies and discovered that subjects perceive structures within the figures which are suitable for solving the analogy. Humans do not perceive the elements within a figure individually or separately, but cognize the figure as a structured whole. Furthermore, the perception of each figure in the series of analogous figures is influenced by the context of the whole analogy. A computational model which shall reflect human cognition of geometric figures must be flexible enough to adapt the representation of a geometric figure and produce a similarly structured representation as humans do while solving the analogy. Furthermore, it must be able to take into account the context, i.e. structures and transformations in other geometric figures in the analogy.

Keywords: computational model for spatial cognition, geometric proportional analogy, re-representation, adaptation, context.

1 Introduction

The cognition of spatial objects involves the construction of a consistent and meaningful overall picture of the environment. Gestalt Psychology (Wertheimer 1912; Köhler 1929; Koffka 1935) argues that human perception is holistic: instead of collecting every single element of a spatial object and afterwards composing all parts to one integrated picture, we experience things as an integral, meaningful whole. The whole contains an internal structure described by relationships between the individual elements.

Perception of the same thing can be different possibly due to differences between humans, due to changes in the context, or due to ambiguity in the figure itself. The following figures show several examples with ambiguous perceptions. The Necker cube shown in Fig. 1 is an example for a multistable perceptual experience where two alternative interpretations tend to pop back and forth unstably. The cube can be seen in two ways, because it is not possible to decide, which one of two crossing lines is in the front or in the back. Fig. 1(b) and (c) show two possible ways to perceive it.

K. Stewart Hornsby et al. (Eds.): COSIT 2009, LNCS 5756, pp. 18–35, 2009.

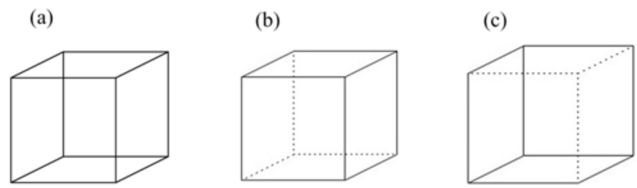


Fig. 1. The Necker Cube (a) is an ambiguous line drawing. Figure (b) and (c) show two possible ways to interpret the Necker Cube.

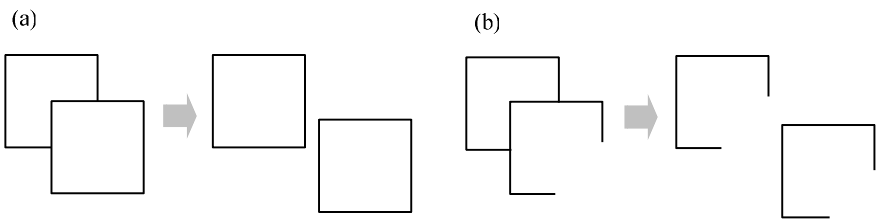


Fig. 2. The perception of a figure is influenced by its context: figure (a) is usually perceived as two complete squares one covering the other, although the covered square is only incompletely visible. In figure (b), the “covered” square is usually perceived as incomplete, because the other square (the context) is incomplete as well.

Fig. 2 is an example where the perception is influenced by the context. Figure (a) shows one complete square and an incomplete square. Most people tend to perceive one square as being covered by the other and therefore complete the non-visible part of the square in their mind to two complete squares. In figure (b), it is more likely that people perceive both squares as incomplete, because the visible square is incomplete as well.

These figures may serve as examples where identical geometric figures are perceived differently and the perception of one element is influenced by its context. A computational model of spatial cognition must be able to compute different perceptions, i.e. different representations for the same spatial object. We will introduce a language for describing geometric figures and show how Heuristic-Driven Theory Projection (HDTP) can adapt representations to reflect different perceptions.

HDTP is a computational approach for analogy making and analogical reasoning. It represents the source and the target stimulus symbolically as two logical theories. In the analogy identification process, HDTP compares both theories for common patterns and establishes a mapping of analogous formulas. The mapping of analogous formulas is captured at an abstract level: The generalized theory formally describes the common patterns of the source and the target stimulus and the analogical relation between them. The symbolic basis of HDTP allows not only the representation of the geometric figures, but also for the representation of general rules which describe how representations can be adapted to reflect different perceptions. The separation of the knowledge about the geometric figure and the abstract knowledge of human perceptions allows HDTP to compute different representations, i.e. compute different conceptualizations of the same geometric figure on-the-fly. HDTP proposes different possible analogies depending on the conceptualization of the figure.

In this paper, we investigate the spatial cognition of simple geometric figures and develop a computational model to compute different perceptions. We conducted experiments with proportional analogies, where subjects have to find a follow-up for a series of geometric figures. Subjects selected different solutions depending on the perception of the geometric figure. In section 2 we describe the experiment, present the results and analyze how subjects perceived the geometric figures in the context of an analogy. Section 3 introduces "Heuristic-Driven Theory Projection" (HDTP), a formal framework to compute analogies. A logical language is used to describe the individual elements in a simple geometric figure in an unstructured manner. From this flat representation it is possible to automatically build up different possible structures and compute "perceptions" which are reasonable to solve the analogy. This mechanism is called re-representation (section 3.3). In section 4, we sketch related work on computational models for solving geometric proportional analogies and discuss the differences to our approach. Section 5 evaluates the applicability of the approach for simple geometric figures and outlines, how HDTP could be used to model human cognition of complex spatial objects.

2 Spatial Cognition of Geometric Figures to Solve Analogies

Here, we give an overview of the experiment focusing only on the results relevant for the computational model. Details about the design and the results of the experiment can be found in (Schwering et al. 2008; Schwering et al. 2009a).

2.1 Setting of the Experiment

The human subject test investigated preferred solutions for proportional analogies of the form (A:B)::(C:D) - read A is to B as C is to D - where A, B and C are a given series of figures and the analogy is completed by inserting a suitable figure for D. All analogies in the test were ambiguous and allowed for different plausible solutions. The analogies were varied in such a way that different perceptive interpretations might be triggered which result in different solutions. For the experiment[1] we used the Analogy Lab, a web-based software platform especially developed for this purpose. Each subject was subsequently shown 20 different analogies randomly chosen from 30 different stimuli: for each analogy they saw the first three objects from an analogy (figure A and B from the source domain and figure C from the target domain) and had to select their preferred solution from three given possible answers (Fig. 3). In every analogy, all three possibilities were reasonable solutions of the analogy; however different solutions required different perceptions of the geometric figures A, B, and C.

[1] The experiment consisted of different parts: One part was choosing the preferred solution from three given possible answers. In a second part, participants had to construct themselves via drag&drop their solution. For this analysis we use only data from the choice-part of this experiment.

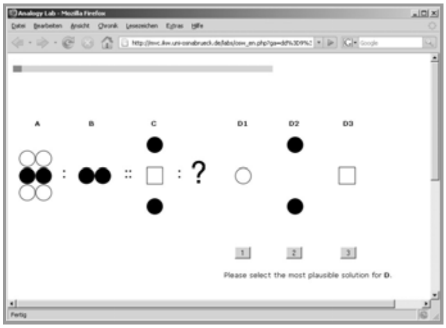


Fig. 3. The analogy lab[2] is a web-based tool to conduct experiments. This screenshot shows one analogy with three possible solutions which can be selected.

The aim of this experiment was to investigate the subjects' perception of geometric figures[3], but also to investigate how the perception changes across different variations of one analogy.

The experiment revealed that subjects applied different strategies to solve the analogies and came up with different solutions. The different solutions can be explained, when the elements in figures A, B, and C are structured differently.

2.2 Different Conceptualization of the Same Stimulus

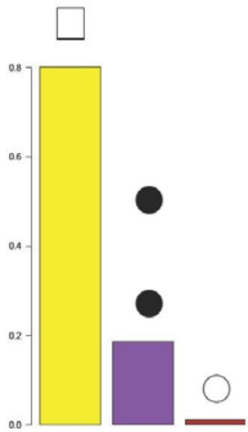
In the experiment, we investigated 30 different analogies. From this set we selected four analogies to be presented as examples in this paper. We discuss the possible perceptions of the geometric figures, present the preferences of different solutions and discuss how a conceptualization of the figure is related to one solution. We analyze how a computational model could reflect the human perception by reproducing the same groupings and same relations as the subjects did.

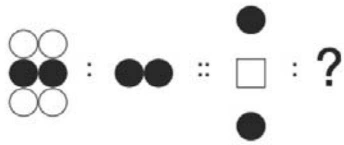
Fig. 4 shows the first analogy: the majority of the 161 subjects who solved this analogy selected the geometric figure consisting of one single white square as solution for this analogy. This solution results[4], if the elements in figure A, B and C are grouped into middle elements and outer elements. Figure B can be constructed from figure A by deleting all outer objects. The second preferred solution, the two black circles, results if the subjects group the geometric figures A, B and C according to color and delete all white objects while all black objects remain. The third solution was chosen only two times. It can be explained by keeping the middle elements with

[2] http://mvc.ikw.uos.de/labs/cc.php

[3] In a different experiment, we let subjects comment on their solution. From these comments we got evidence that subjects built up different structured representations to solve the analogy in one or the other way. Due to space limitation, we cannot include a detailed comment analysis in this paper.

[4] We would like to point out that these are our interpretations. We base these interpretations on comments that the participants of our experiments gave after solving each analogy. Although in most cases our interpretation seems to be very straight forward, there can be other interpretations that led subjects choose a solution.





Data:

- 161 subjects solved this analogy
- 129 (80%) selected the solution with one white square
- 30 (19%) selected solution with two black circles
- 2 (1%) selected the solution with one white circle

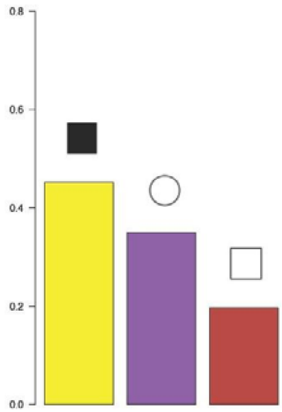
Fig. 4. The first analogy can be solved by focusing on the position of the elements or on the color. The results show that the majority of subjects preferred to keep the middle object, while several subjects chose to keep the black objects. Only two subjects selected the white circle as solution.

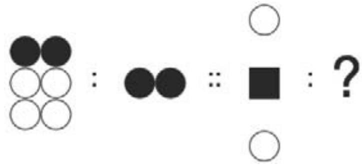
their position and color, but changing the shape to a circular shape. However, this solution is obviously not preferred.

At a more general level, we can reveal different strategies that subjects applied to solve this analogy. The majority of subjects considered the relative position of the elements and grouped elements in middle and outer elements. The second biggest group of participants focused on the color and formed one group with white elements and one group with black elements.

Fig. 5 shows a variation of the first analogy: In figure A, the two top circles are black and all other circles are white and in figure C the colors are flipped compared to figure C in the previous analogy. This variation has a huge effect on the preference distribution and also on the preferred perception. The majority of the subjects chose the figure with one black square as solution for this analogy. Subjects choosing this solution presumably grouped according to colors and deleted all white elements while they kept the black ones. The second preferred solution was one white circle. These subjects focused on the relative position: The top elements form one group and the others form another group. The analogy is solved by keeping the top elements and moving them to the middle of the figure. The third preferred solution keeps the color of the top elements and the shape of the middle elements.

Although both analogies are very similar, the resulting preferences are relatively different. The majority of subjects chose either a grouping strategy based on the position or based on color, but in the first analogy the position-strategy was clearly preferred, while in the second analogy the color was more preferred. The strategy of transferring the color from elements in the source domain but keeping the same shape as in figure C was hardly applied in analogy one (only 1% of the participants), but applied by 20% of the participants in analogy two.



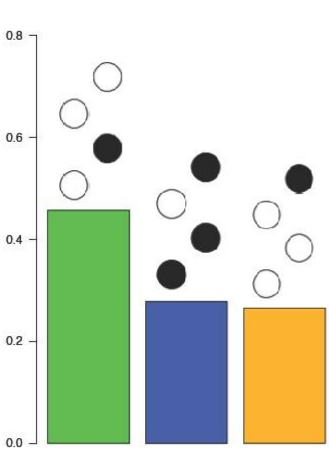


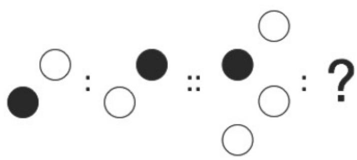
Data:

- 157 subjects solved this analogy
- 71 (45%) selected the solution with one black square
- 55 (35%) selected the solution with one white circle
- 31 (20%) selected the solution with one white square

Fig. 5. The second analogy can be solved by focusing on the color of the elements (preferred solution) or on the position (second preferred solution). It is also possible to treat shape and color differently and transfer only color while the shape remains the same (third preferred solution).

Fig. 6 shows an analogy where the geometric figure B can be perceived as a 180° rotation of figure A. In this case the figure is seen as one whole and is not divided into any subgroups. Subjects who selected the most preferred solution presumably applied this strategy.

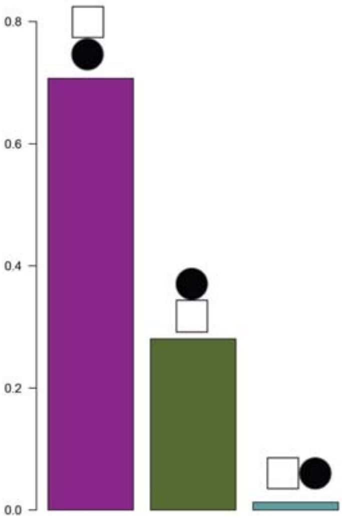




Data:

- 162 subjects solved this analogy
- 74 (46%) selected the solution where the right bottom circle is black and the other circles are white
- 45 (28%) selected the solution where the top left circle is white and the other circles are black
- 43 (27%) selected the solution where the top circle is black and the other circles are white

Fig. 6. The preferred solution of the third analogy is constructed via rotating the whole figure 180°. Participants choosing this solution presumably did not divide the figures into subgroups, but grouped all circles in figure A, all circles in figure B and all circles in figure C in three separate groups independently of their color. The second preferred solution results from a color flip. The third solution can be explained by dividing figure C in two groups: the upper two circles form one group because they repeat figure A and the lower circles from a second group.



Data:

- 157 subjects solved this analogy
- 111 (71%) selected the solution where the white square is above the black circle
- 44 (28%) selected the solution where the black circle is above the white square
- 2 (1%) selected the solution where the white square is left of the black circle

Fig. 7. The first solution of the fourth analogy has different explanations: the elements can be grouped in circles and squares and switch position. They can be perceived as one whole and rotated. They can be perceived as one whole with a mirroring axis between the circle and the square. If the mirroring axis is defined relative to the figure, the most preferred solution is correct. If the axis is defined absolute, the second preferred solution is the correct one.

The second preferred solution is constructed by flipping the colors, i.e. circles are grouped according to the color and all black circles become white and all white circles become black. In the third preferred solution, figure A is mapped on the two upper circles in figure C. Obviously, figure C is perceived as two groups: one contains the upper two circles and the second one contains the lower two. In this case, one part of figure C is an identical repetition of figure A. The solution is constructed by applying the transformation between A and B to that subgroup of C, that is identical to A. The additional subgroup of C - the two bottom white circles - remain the same.

The fourth analogy is shown in Fig. 7. The most preferred solution has different possible explanations: Each figure consists of two elements: a circle and a square. From figure A to B the circle and square change position, therefore the solution is a white square above the black circle. The same solution can be constructed with a different interpretation: figure A is perceived as a whole and is rotated 180°. A third interpretation is also possible: subjects might have perceived a vertical symmetry axis between the circle and the square. Figure B is mirrored along this axis. If the axis is perceived relative to the elements in the figure, the axis in C runs horizontally between the circle and the square. A very similar explanation exists for the second preferred solution: participants perceived as well a vertical symmetry axis between the circle and the square and mirrored figure C along a vertical axis as well. The third solution was only selected by 2 subjects and is not very preferred.

2.3 Results of the Experiment

The experiment shows that analogies have different solutions depending on how geometric figures are perceived. The preferred perception is influenced by the context, i.e. by the other figures in the analogy (cf. analogy 1 and analogy 2). Proportional analogies are a suitable framework to investigate human perception of geometric figures, because different perceptions can be easily discovered if they lead to different solutions.

Grouping is a common strategy to establish the required structure to solve the analogy. In the examples above, grouping based on similarity (such as grouping of elements with common color or common shape) and grouping based on position play important roles. The position is often defined relative, e.g. middle and outer elements seem to be more prominent than other positions. Spatial proximity or continuous movement are other criteria for structuring geometric figures. In analogy three, figure C is an extended version of figure A. In such cases, the extended figure can be divided into two groups: One group comprising the original figure and the second group comprising the additional elements.

3 A Computational Model for Geometric Analogies

The holistic Gestalt perception contradicts the atomistic way computers process information. A computational model for spatial cognition must be able to compute an overall, holistic representation from a list of single elements. We developed a language to describe geometric figures. The analogy model HDTP[5] computes differently structured representations of a geometric figure based on a flat list of single elements.

3.1 Heuristic-Driven Theory Projection (HDTP)

HDTP is a symbolic analogy model with a mathematically sound basis: The source and the target domain are formalized as theories based on first-order logic. HDTP distinguishes between domain knowledge—facts and laws holding for the source or the target domain—and background knowledge, which is assumed to be generally true. Knowledge about a geometric figure is captured by domain knowledge, while general principles of perception are captured in the background knowledge (Fig. 9).

An analogy is established by aligning elements of the source with analogous elements of the target domain. In the mapping phase, source and target are compared for structural commonalities. HDTP (Gust et al. 2006; Schwering et al. 2009c) uses anti-unification to identify common patterns in the source and target domain. Anti-Unification (Plotkin 1970; Krumnack et al. 2007) is the process of comparing two formulae and identifying the most specific generalization subsuming both formulae.

---

[5] This paper shall present the idea of the computational model and sketch the overall process. A detailed description of the syntactic and semantic properties of HDTP can be found here (Gust et al. 2006; Krumnack et al. 2007; Schwering et al. 2009c).

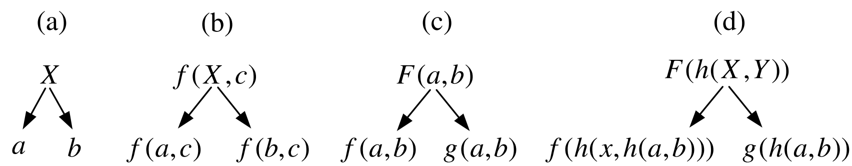


Fig. 8. Anti-unification compares two formulae and creates the least general generalization. While (a) and (b) are first-order anti-unification, (c) and (d) require second-order anti-unification to capture the common structure of the formulae.

We use anti-unification to compare the source theory with the target theory and construct a common, general theory which possibly subsumes many common structures of the source and the target domain. Fig. 8 gives several examples for anti-unification. Formulae are generalized to an anti-instance where differing constants are replaced by a variable. In (a) and (b), first-order anti-unification is sufficient. The formulae in (c) and (d) differ also w.r.t. the function symbols. While first-order anti-unification fails to detect commonalities when function symbols differ, higher-order anti-unification generalizes function symbols to a variable and retains the structural commonality. In example (d), *F* is substituted by *f/g*, *X* is substituted by *x/a* and *Y* is substituted by *h(a, b)/b*. A detailed description of anti-unification in HDTP can be found in (Krumnack et al. 2007). An example for anti-unification of formulas describing geometric figures is shown below in Fig. 13.

Fig. 9 sketches the HDTP architecture to solve geometric proportional analogies. Figure A and figure B of the analogy are part of the source domain, while figure C

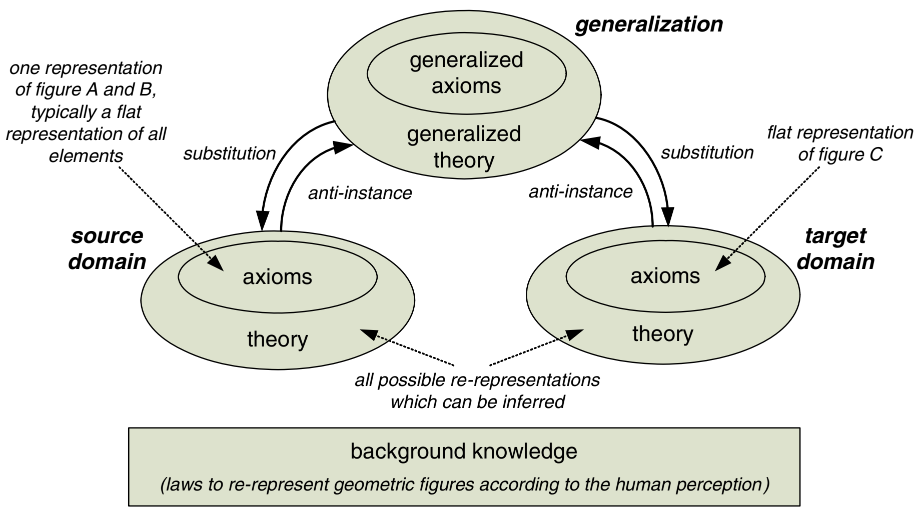


Fig. 9. Overview of the HDTP architecture

(and the still missing figure D) are part of the target domain. All elements in a geometric figure are described by a set of axioms in a formal language (cf. section 3.2). The background knowledge contains laws how to compute structured representations of a geometric figure. Our experiment revealed that one possible strategy is grouping elements with a common color; therefore, the background knowledge contains a law for filtering elements with a common color out of all elements belonging to one figure. Applying these laws to the axiomatic description of a figure leads to a structure (re-)representation of this figure.

To solve the analogy, HDTP compares figure A and figure C for structural commonalities and establishes a mapping between analogous elements in figure A and C. HDTP uses anti-unification for the mapping process and computes a generalization of the commonalities. The generalized theory with its substitutions specifies formally the analogical relation between source and target. Additional information about the source domain - in proportional geometric analogies this is information how to construct figure B from figure A - is transferred to the target domain and applied to figure C to construct figure D (Schwering et al. 2009b).

3.2 Language to Formalize Different Conceptualizations of Geometric Figures

We developed a formal language based on the "Languages of Perception" by (Dastani 1998). Basic elements of a geometric figure can be described by its (absolute) position, shape and color. We can detect groups of elements following the criteria mentioned in section 2.3. For the following example, grouping based on common shape and color is important. The language also supports other structures such as iteration of elements or groups. Since we focus on the basic principle of re-represen-tation and on the changing of flat representations to structured ones, we describe this process exemplary for grouping elements according to their shape and do not elaborate all other possible structures that could be expressed with this language.

The analogy shown in Fig. 10 was solved by grouping all circles in figure A into one group and all remaining elements (in this case a white square) into a second group. Grouping all remaining elements into one group was a common strategy in our experiment. All circles become black, while the remaining elements stay the same. With this strategy the solution to this analogy is keeping the grey square of figure C and changing the color of the circles to black.

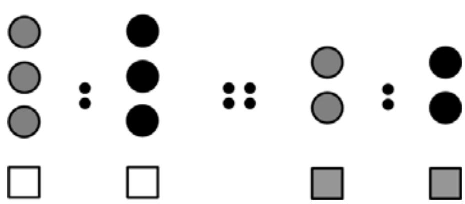


Fig. 10. In this analogy, all circles become black and the squares remain as they are

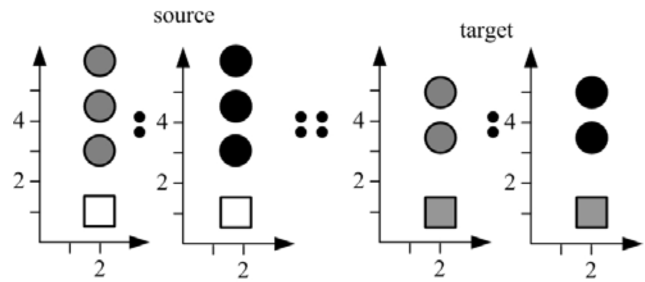


Fig. 11. In HDTP, the analogy is separated into a source and a target domain and a coordinate system determines the absolute position of elements

Fig. 11 shows the same analogy as it would be described in HDTP: figure A and B belong to the source domain and figure C and D belong to the target domain. A coordinate system is used to determine the absolute position of the elements.

HDTP starts with a flat representation of all elements. The elements of figure A are described as follows[6]:

```
% flat representation of figure A
o1 := [shape:square, color:white, position:p(2,1)]
o2 := [shape:circle, color:grey, position:p(2,3)]
o3 := [shape:circle, color:grey, position:p(2,4.5)]
o4 := [shape:circle, color:grey, position:p(2,6)]
```

Based on the flat representation, HDTP has to compute a structured representation which reflects human cognition. First, we show how a structured representation looks like for the running example and how the language supports the re-representation. In the next section, we sketch the process how HDTP automatically detects the correct re-representation steps and computes such structured representations. As we already mentioned, the source domain is perceived as two figures (figure A and figure B) and figure A is divided into a group of circles and the remaining objects (the square):

```
% representation of figure A with structure
group figA := [o1,o2,o3,o4]
group g1 := filter(figA,(shape:circle),+)
group g2 := filter(figA,(shape:circle),-)
```

Groups can be expressed extensionally or intensionally. Extensional groups are defined by listing all members of the group. This is typically the case for the group of elements belonging to one figure such as the group `figA`. Intensional groups are specified by the defining criteria such as groups `g1` and `g2`. Group `g1` is constructed by selecting those elements of group `figA` which have a circular shape. The plus and the minus sign indicate the polarity: a minus stands for the complement of a group and is used to group the remaining elements in figure A. It is also possible to combine different filters by concatenating different filtering criteria: A group containing all grey circles would be defined as follows:

```
group g1 := filter(figA,(shape:circle, color:grey),+)
```

[6] The elements of figure B are constructed from figure A by changing the color of all circles to black and keeping the square. Therefore, they are not described explicitly here.

HDTP uses its background knowledge to transform flat representations into structured ones. The background knowledge contains rules to filter a group for certain elements, i.e. filter group `figA` for all elements which have a circular shape. All circular elements are extracted, added to a list of elements which is used to construct the new group. Analogously, groups can be filtered for a certain color, absolute position, or relative position such as "middle elements".

```
group g1 := filter(figA,(position:top),+)
```

Additional rules are required for groupings based on the relative position. HDTP background knowledge contains rules to compute spatial relations "above", "below", "right", and "left" based on a single cross calculus. For example, top elements are computed by selecting those elements from a group which are not below another element. A single cross calculus is sufficient for the simple geometric analogies used in our experiment. For more complex stimuli one can choose to implement a different calculus to compute spatial relations.

Like figure A, figure C is first represented as a flat list of elements. To establish a mapping, figure C must be regrouped in a way analogous to figure A. If the same subgroups can be constructed, the same transformation can be applied. The following code shows the flat representation and the division into a group of circular elements and a second group of remaining elements.

```
% formalization of figure C as list of flat elements
o5 := [shape:square, color:white, position:p(2,1)]
o6 := [shape:circle, color:grey, position:p(2,3.5)]
o7 := [shape:circle, color:grey, position:p(2,5)]

% representation of figure C in two groups
group figC := [o5,o6,o7]
group g3 := filter(figC,(shape:circle),+)
group g4 := filter(figC,(shape:circle),-)
```

3.3 Solving the Analogy: Re-representation and Anti-unification

The previous section presented the language that is used to describe geometric figures and rules to compute higher structures. Finding the correct conceptualization of a geometric figure within a proportional analogy is an iterative process (Fig. 12): First, HDTP computes different possible conceptualization of figure A using prolog laws in the background knowledge (Schwering et al. 2009b). There are numerous ways in which figure A of the running example could be represented (Schwering et al. 2009a): it could be grouped based on shape, based on color (grey elements, versus white elements), it could be considered as one whole group or any other way of grouping. The re-representation is heuristic-driven:

- It is influenced by Gestalt principles, e.g. according to the law of similarity it makes sense to group grey elements and white elements or circles and squares.
- It is influenced by possible transformations to figure B. If several elements are repeated in B, it is likely that a transformation must exist between the elements in A and the repeated elements in B.

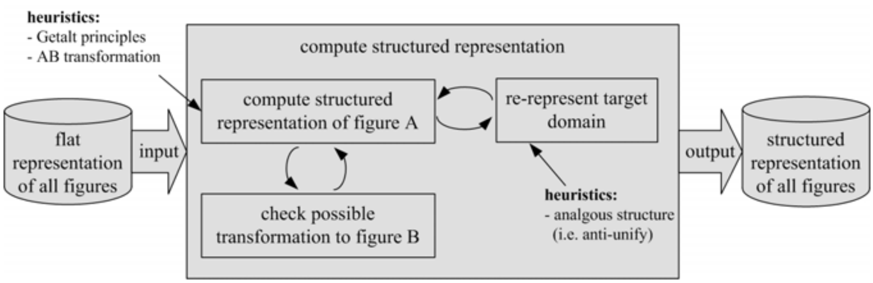


Fig. 12. Iterative process of computing the correct structured representation of the analogy

However, the structure of figure A is not independently created from the overall analogy: Once one (or several) preferred conceptualization of A exist, HDTP tries to re-represent figure C in an analogous way, i.e. it tries to establish the same groupings as in figure A. If this is not possible, the structure of figure A must be revised.

Once figures A and C have a structured representation and the transformation between A and B is known, an analogical mapping can be established via anti-unification and figure D is constructed via analogical transfer.

Fig. 13 shows the anti-unification for an object description and a group definition. The upper part shows an example of a comparison between object `o1` and object `o5` . Both objects are squares at the position (2,1), but object `o1` is white and object `o5` is grey. Both formulas differ only with respect to the identifier and with respect to their color. Therefore identifier and color are replaced by a variable `X` respectively `Y` in the generalization. The same holds for the group definitions in Fig 13(b): one group is defined on elements in figure A and the other is defined on the elements in figure C of the target domain. The generalization replaces the differing group identifier (`g1/g2`) with a variable `G` and `figA/figC` with the variable `F`.

(a)

```
X := [shape:square, color:Y, position:p(2,1)]

X -> o1                                   X -> o5
Y -> white                                Y -> grey

o1 := [shape:square,                      o5 := [shape:square,
       color:white,                              color:grey,
       position:p(2,1)]                          position:p(2,1)]
```

(b)

```
                    group G :=
              filter(F,(shape:circle),+)

G -> g1,                                  G -> g3,
F -> figA                                 F -> figC

group g1 :=                               group g3 :=
filter(figA,(shape:circle),+)             filter(figC,(shape:circle),+)
```

Fig. 13. Anti-unification of two object descriptions and two intentional group definitions

4 Related Work

Proportional analogies were studied in various domains such as the natural-language domain (Indurkhya 1989; Indurkhya 1992), the string domain (Hofstadter and Mitchell 1995), analogical spatial arrangement at a table top scale (French 2002), and in the domain of geometric figures.

In (1962; 1969), Evans developed a heuristic program to solve GPAs. Before the actual mapping process, the program computes meaningful components consisting of several line segments in each figure. Evan's analogy machine determined the relation between A-B, computed a mapping between A-C based on rotation, scaling, or mirroring, and selected an appropriate solution from a list of possible solutions. In contrast to our approach, the representation and the mapping phase are sequentially separated from each other. While we use structural criteria, Evans uses mathematical transformation to detect a suitable mapping between figure A and C.

O'Hara & Indurkhya (1992; 1993) worked on an algebraic analogy model which is able to adapt the representation of line drawing figures during the analogy-making process. Dastani et al. developed a formal language for this algebraic model to describe elements in geometric figures and compute automatically a structural, Gestalt-based representation (Dastani and Scha 2003). This approach accounts also for context effects, i.e. figure C has an effect on the conceptualization of figure A (Dastani and Indurkhya 2001). Both ideas strongly influenced our work. We reuse many ideas developed for this algebraic model and apply them to our logic-based framework.

Mullally, O'Donoghue et al. (2005; 2006) investigated GPAs in the context of maps. They used structural commonalities to detect similar configurations in maps and to automatically classify geographic features. Due to the limitation to maps, they do not support the complex spatial analysis required for our GPAs.

Several other approaches deal with the perception of visual analogies in general. Davies and Goel investigate the role of visual analogies in problem solving (Davies et al. 2008). Forbus et al. (2004) developed an approach to compare sketch drawings. Since GPAs are not the focus of these approaches, we do not discuss them here.

5 Conclusions, Discussion and Future Work

We presented HDTP, a formal framework to automatically compute different conceptualizations of the same figure. We discuss the presented approach and afterwards argue how this approach could be used in a more general context of recognition and classification of spatial objects.

5.1 Summary and Conclusions

Human spatial cognition is a holistic process: we tend to see whole patterns of stimuli when we perceive a spatial object in an environment. According to Gestalt theory, parts of the spatial object derive their meaning from the membership in the entire configuration. Computers, on the other hand, process visual information in an atomistic way. To receive similar patterns as the ones humans perceive, we need a computational model which can generate in a bottom-up manner the structure which is necessary to interpret the stimulus correctly.

Experiments on geometric proportional analogies have shown that subjects perceive the same geometric figure in different ways and that the preferred perception changes, if the context in the analogy is varied. Subjects apply different strategies to solve the analogies: the elements in the geometric figures are often regrouped according to shape, color or position to establish a common structure in source and target domain.

HDTP is a heuristic-driven computational framework for analogy-making and can be used to simulate the human way of solving geometric proportional analogies. We developed a logic-based language to describe geometric figures. HDTP takes such formal descriptions of the figures in the source and the target domain and tries to detect common structures. Usually, source and target are not available in an analogously structured representation at first. HDTP re-represents the descriptions to transform the flat representation into a structured representation of the geometric figure. Different structured representations reflect different conceptualizations of a geometric figure. The process of re-representation is essential to model spatial cognition of geometric figures in the context of proportional analogies: finding the analogous structural patterns in figure A and figure C can be considered as the main task in analogy-making. In the mapping process, HDTP uses the theory of anti-unification to compare a source and a target formula and computes a generalization. The analogical relation between the source and the target is established by creating a generalized theory subsuming all formulae in the source theory and all formulae in the target theory. The proportional analogy is solved by transferring the relation between figure A and B and apply it to figure C to construct figure D.

5.2 Opportunities and Drawbacks of the Approach

In our experiments, we investigated only simple, artificial stimuli so far. The stimuli had different number of elements which varied across three different shapes, three different colors and different positions. The artificial stimuli are simple enough to control variations, to emphasize different aspects and different Gestalt principles and to trigger different perceptions. With systematic variations it is possible to detect how certain variations change the perception. The number of possible re-representations and transformations is limited. The language we are using at the moment supports only simple elements, but it could be extended. Future development shall support complex forms and line drawings like the ones in Fig. 14. It shall become possible to compute an area from a given set of lines and check whether it is a familiar form such as a square or a triangle. The detection of complex structures and forms requires a spatial reasoner which can detect spatial relations.

Analogy making provides a good framework to test spatial cognition, because variations in context may lead to different perceptions which result in different solutions. The different solutions serve as indicator for different conceptualizations.

In section 3.3 we describe a heuristic-driven framework to compute different conceptualizations. We assume the relatively difficult situation where none of the figures is pre-structured. Real-world tasks are often easier. In a recognition task for example, a new stimulus (the target domain) is compared to known stimuli (the source domain). The new stimulus must be restructured to fit to the given structure of the source domain. A pre-defined structure of the source domain reduces drastically the complexity of the underlying framework.

5.3 Spatial Object Recognition as Future Application

In this paper, we discussed the computational framework only in the context of analogies. However, we think that HDTP could serve as a general framework for visual recognition and concept formation. Visual recognition of spatial stimuli is based on matching new stimuli to familiar ones. Often, things are best characterized by their structural (and functional) features, but superficial features do not reveal much about the nature of an object. Therefore, we argue that analogical comparison is very suitable to model the human cognitive process of recognition.

So far, HDTP was tested only with artificial stimuli. The language used at the moment can describe simple elements and express very limited spatial relations. However, the basic principle of the computational model presented in this paper is flexible enough to support complex spatial objects as well. First experiments have shown, that structural commonalities play an important role in object recognition (Stollinski et al. 2009). Future work will investigate HDTP in analogy making between complex stimuli like sketches of real world spatial objects.

Fig. 14 shows different sketches of an oven. Although they differ from each other, they share a lot of structural commonalities: all of them have four hotplates which are inside a polygon representing the top surface of the oven. Five temperature regulators and a spy window (with or without handle to open the door) are inside a polygon representing the front surface of the oven. Similarly to geometric figures, each sketch is represented by its primitive elements (lines and ovals). Background knowledge contains laws how to analyze geometric forms, detect polygons from lines or compute even more complex structures such as a cube.

An effective model for spatial cognition requires a spatial reasoner to compute spatial relations or different 3D perspectives on the same object in space. Already our experiments with simple geometric figures revealed such requirements: Several participants applied three dimensional transformations between figure A and B, which cannot be represented in the two-dimensional model of HDTP. Future work will investigate how existing models for spatial reasoning can be integrated. Also the representation language as well as the re-representation rules must be extended.

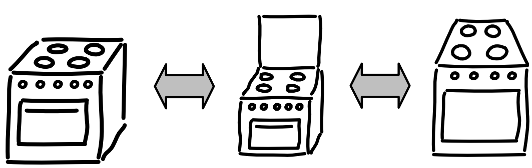


Fig. 14. Analogy-based sketch recognition compares different sketches of spatial objects and detects common structures

5.4 Sketch Map Comparison as Future Application

A second application area for analogical reasoning is the comparison of sketch maps to metric maps. While metric maps such as street maps are constructed from exact measurements, sketch maps are drawn by humans based on their cognitive map. Fig. 15 shows a sketch map and a metric map of the same area. A qualitative comparison of both maps reveals many structural commonalities: the spatial objects lie along streets forming the same street network. The sketch map is a simplified and schematized representation of the metric map.

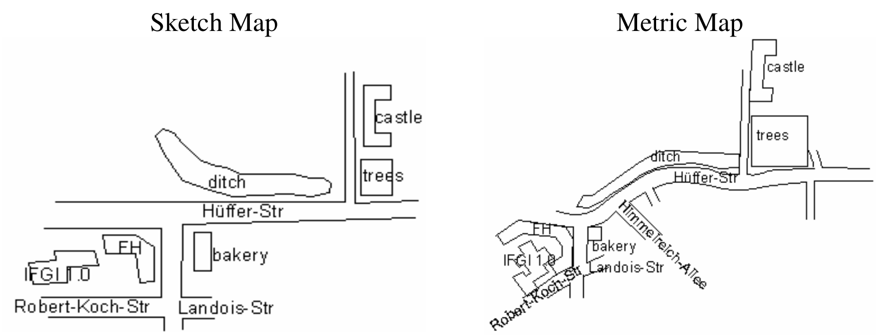


Fig. 15. Analogy-based comparison of a sketch map and a metric map of the same area reveals structural commonalities between spatial objects such as houses, streets, water-bodies and trees

Analogical comparison focuses only on structural commonalities such as the relation of geographic features to streets and streets being connected to other streets. It abstracts from metric details. Therefore, we argue that analogical comparisons are a useful tool for sketch map comparisons.

References

Dastani, M.: Languages of Perception. Institute of Logic, Language, and Computation (ILLC), Amsterdam, Universiteit van Amsterdam (1998)

Dastani, M.M., Indurkhya, B.: Modeling context effect in perceptual domains. In: Akman, V., Bouquet, P., Thomason, R.H., Young, R.A. (eds.) CONTEXT 2001. LNCS, vol. 2116, p. 129. Springer, Heidelberg (2001)

Dastani, M., Scha, R.: Languages for Gestalts of line patterns. Journal of Mathematical Psychology 47, 429–449 (2003)

Davies, C., Goel, A.K., Yaner, P.W.: Proteus: Visual analogy in problem solving. In: Knowledge-Based Systems (2008)

Evans, T.G.: A heuristic program to solve geometric analogy problems, Cambridge, MA, USA, Massachusetts Institute of Technology (1962)

Evans, T.G.: A program for the solution of a class of geometric-analogy intelligence-test questions. In: Minsky, M. (ed.) Semantic information processing, pp. 271–353. MIT Press, Cambridge (1969)

Forbus, K.D., Lockwood, K., Klenk, M., Tomai, E., Usher, J.: Open-domain sketch understanding: The nuSketch approach. In: AAAI Fall Symposium on Making Pen-based Interaction Intelligent and Natural, Washington, DC (2004)

French, R.M.: The computational model of analogy-making. Trends in Cognitive Sciences 6(5), 200–205 (2002)

Gust, H., Kühnberger, K.-U., Schmid, U.: Metaphors and heuristic-driven theory projection (HDTP). Theoretical Computer Science 354(1), 98–117 (2006)

Hofstadter, D.R., Mitchell, J.C.: The copycat project: A model of mental fluidity and analogy-making. In: Hofstadter, D.R., group, F.A.R. (eds.) Fluid Concepts and Creative Analogies, pp. 205–267. Basic Books, New York (1995)

Indurkhya, B.: Modes of analogy. In: Jantke, K.P. (ed.) AII 1989. LNCS, vol. 397. Springer, Heidelberg (1989)

Indurkhya, B.: Metaphor and cognition. Kluwer, Dordrecht (1992)

Koffka, K.: Principles of Gestalt Psychology. Harcourt, New York (1935)

Köhler, W.: Gestalt Psychology. Liveright, New York (1929)

Krumnack, U., Schwering, A., Gust, H., Kühnberger, K.-U.: Restricted higher-order anti-unification for analogy making. In: Orgun, M.A., Thornton, J. (eds.) AI 2007. LNCS, vol. 4830, pp. 273–282. Springer, Heidelberg (2007)

Mullally, E.-C., O'Donoghue, D., Bohan, A.J., Keane, M.T.: Geometric proportional analogies in topographic maps: Theory and application. In: 25th SGAI International Conference on Innovative Techniques and Applications of Artificial Intelligence, Cambridge, UK (2005)

Mullally, E.-C., Donoghue, D.P.: Spatial inference with geometric proportional analogies. Artificial Intelligence Review 26(1-2), 129–140 (2006)

O'Hara, S.: A model of the "redescription" process in the context of geometric proportional analogy problems. In: Jantke, K.P. (ed.) AII 1992. LNCS, vol. 642. Springer, Heidelberg (1992)

O'Hara, S., Indurkhya, B.: Incorporating (re)-interpretation in case-based reasoning. In: Wess, S., Richter, M., Althoff, K.-D. (eds.) EWCBR 1993. LNCS, vol. 837. Springer, Heidelberg (1994)

Plotkin, G.D.: A note on inductive generalization. Machine Intelligence 5, 153–163 (1970)

Schwering, A., Krumnack, U., Kühnberger, K.-U., Gust, H.: Investigating Experimentally Problem Solving Strategies in Geometric Proportional Analogies, Osnabrueck, Germany, University of Osnabrueck, p. 233 (2008)

Schwering, A., Bauer, C., Dorceva, I., Gust, H., Krumnack, U., Kühnberger, K.-U.: The Impact of Gestalt Principles on Solving Geometric Analogies. In: 2nd International Analogy Conference (ANALOGY 2009). New Bulgarian University Press, Sofia (2009a)

Schwering, A., Gust, H., Kühnberger, K.-U., Krumnack, U.: Solving geometric proportional analogies with the analogy model HDTP. In: Annual Meeting of the Cognitive Science Society (CogSci 2009), Amsterdam, The Netherlands (2009b)

Schwering, A., Krumnack, U., Kühnberger, K.-U., Gust, H.: Syntactic principles of Heuristic-Driven Theory Projection. Special Issue on Analogies - Integrating Cognitive Abilities. Journal of Cognitive Systems Research 10(3), 251–269 (2009c)

Stollinski, R., Schwering, A., Kühnberger, K.-U., Krumnack, U.: Structural Alignment of Visual Stimuli Influences Human Object Categorization. In: 2nd International Analogy Conference (ANALOGY 2009). New Bulgarian University Press, Sofia (2009)

Wertheimer, M.: Experimentelle Studien über das Sehen von Bewegung. Zeitschrift für Psychologie 61, 161–265 (1912)