Chapter 5
Cost Effective and Environmentally Safe Emission Trading Under Uncertainty

T. Ermolieva, Y. Ermoliev, G. Fischer, M. Jonas, and M. Makowski

Abstract The aim of this paper is to analyze robust cost-effective and environmentally safe carbon emission trading schemes under uncertainties of emissions and costs, and asymmetric information of participants. The proposed model allows to control explicitly the safety of Kyoto (or other) targets by taking long-term perspectives on emission trading. The dynamics of this scheme is driven by bilateral trades with different endogenous disequilibrium prices between mutually beneficial trades, but finally the system converges to cost-effective and environmentally safe global equilibrium. The safety constraints work as a discounting mechanism that discounts the reported emissions to detectable undershooting levels. This, in turn, provides incentives for participants to reduce uncertainties. The model shows that uncertainties and short term market perspectives may easily prevent price-based trading to be environmentally safe and cost-effective scheme. The desirable equilibrium emerges only under proper price-formation mechanisms. The role of the proposed computerized multi-agent trading system is central for dealing with long-term perspectives, irreversibility and lock-in equilibriums of trades. This system can be viewed as a device for decentralized collective regulation of trades based on unified approaches to modeling of uncertainty, calculation of costs and trading rules.

5.1 Introduction

The public property of large scale pollution makes it impossible to organize complete environmental markets with private demand for and private supply of pollution control [2, 3, 7, 15, 18, 20, 22]. Yet, the idea of carbon trading markets is becoming increasingly popular for global climate change control. At the same time, the existence of various exogenous and endogenous inherent uncertainties raises serious concerns regarding the ability of carbon trading markets to

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fulfill the main purpose of the climate change control without creating world-wide irreversible socio-economic and environmental disruptions. Definitely, that interests of profit oriented markets may contradict the main concerns of the Kyoto agreements [29].

Considered in this paper is bilateral exchange of emission rights. It is assumed that parties with high emission reduction costs buy emissions from parties with low emission reduction costs within prescribed targets. In other words, parties can be engaged in a mutually beneficial bilateral emission exchange process [8] driven by cost minimizing and environmentally safe bilateral trades without the need for a market. This approach is close to important ideas on decentralized non-monetary exchange [21, 28].

In contrast, carbon trading markets, which become increasingly popular in recent years, are more similar to stock markets. Parties hold a number of permits to emit a specific amount of emissions. Parties that cannot to keep their emissions at the given level (called cap) must buy permits on the market at a prevailing market price.

Thus, in the bilateral emission trading scheme the exchange of emission rights is driven by the abatement costs and safety constraints, while in the carbon trading markets the exchange of emissions is driven by prevailing market prices. Such price signals with potential bubbles created by speculators may have no connections with minimization of abatement costs and achieving environmental safety constraints.

There are two principle approaches to control pollution: centralized cooperative command-and-control methods and decentralized market simulating schemes. If there was a social planner (central agency or regulator) fully informed about emissions and abatement cost functions of all parties, the primal problem of finding emission levels that meet given environmental standards in a cost-effective way would be a straightforward task. This could be done by dealing with nonconvex cost functions typically encountered in long-term evaluations involving new technologies with increasing returns. However, without such a planner, the primal model has to be solved in a decentralized manner.

The aim of this paper is to develop an integrated approach for designing cost-effective and environmentally safe decentralized emission trading schemes robust with respect to uncertainties of emissions, costs and asymmetric information of parties. The bilateral emission trading scheme of Sect. 5.5 corresponds to a decentralized solution of the primal model, whereas schemes of Sect. 5.7 simulate decentralized price-based market's solutions. The cost-effectiveness and environmental safety of latter solutions critically depend on proper price signals, which usually reflect instantaneous market situations rather than long-term costs and environmental constraints of the dual model. The complexity of the primal model is a vital issue for the existence of proper prices.

There is a number of uncertainties, affecting outcomes of examined model. First of all, emissions of Green-House Gases (GHGs) are not directly observable. A comprehensive discussion of related issues can be found in the volume by [1], and in [12]. In general, emissions can be estimated with information on the GHG-emitting activities by applying specific conversion factors and from atmospheric measurements using inversion models. The accuracy of the reported emissions depends on the quality of the monitoring system in each specific country and on the accuracy of the conversion factors used [26]. As emissions of GHGs cannot be observed perfectly, the uncertainty can be misused by concealing unreported emissions. A central issue becomes a trade-off between reductions of emissions and uncertainties. For example, carbon prices in the European Union crashed and caused instabilities in late April 2005 after the Czech Republic, Estonia, France, the Netherlands and Sweden reported lower than anticipated emissions [4, 27].

Apart from emissions, another essential uncertainties are those related to the emission amounts and reduction costs. Parties have incentives to keep this information private and the specific costs may remain unknown to the other parties. Besides, they may vary according to unknown market conditions. They are also subject to both industry wide and firm specific shocks.

The novelty of this paper is in integrated analysis of emission trading schemes under various types of natural and human related uncertainties. Section 5.2 illustrates the need for proper treatment of uncertainties by using available historical observations of CO_2 emissions. It shows that the use of uncertainty intervals as practiced by International Panel for Climate Change (IPCC) can leave out of consideration an essential mass of potential emissions. Therefore, Sect. 5.3 introduces a simple and realistic stochastic model allowing to represent both, human related uncertainties and uncertainties associated with the natural variability of emissions. The model focuses on proper representation of potentially controversial experts' judgments and path-dependencies of emissions. As a result, this allows us to introduce safety constraints and undershooting mechanism to control the robustness of emission targets during trading process.

Section 5.4 introduces a basic model allowing to analyze different trading schemes. In particular, the model shows that the trade equilibrium under uncertainty is significantly affected by uncertainty. This emphasizes the need for proposed integrated modeling of uncertainties, safety constraints and emission trading schemes.

The dynamic bilateral trading scheme of Sect. 5.5 can be viewed as a stochastic decomposition procedure. The trade at each step takes place towards minimization of safety-adjusted costs of meeting parties. This generates disequilibrium random prices which are endogenously driven towards the cost-effective and environmentally safe equilibrium price. This section analyses also difficulties involved in setting up such an equilibrium price in monetary trading schemes. Standard market models usually imply (e.g., by an arbitrage free type of assumption) that markets operate under equilibrium prices. Section 5.6 outlines a computerized Multi-Agent Decentralized Trading System dealing with the irreversibility of emission trades. Section 5.7 analyses path-dependencies of myopic trading schemes relying on instantaneous markets situations. It shows that short-term market perspectives preclude achieving desirable long-term emission reduction goals. Section 5.8 concludes and outlines important numerical results. The Appendix provides a proof of the convergence. It also discusses stable core solutions of bilateral emission trading scheme.

5.2 Uncertainties and Trends of Carbon Fluxes

This section illustrates a general need for proper representation of emission uncertainties. Next section addresses these issues in a more specific context of emission trading.

Uncertainties of emissions are often represented by means of intervals. In reality, emissions may have different likelihoods within these intervals, i.e., rather general skewed probability distributions. In this case, the use of uncertainty intervals can leave out of consideration essential patterns of emission changes as in Figs. 5.2 and 5.3.

Figures 5.1, 5.2, and 5.3 illustrate trends and natural variability of net carbon fluxes on the global scale http://lgmacweb.env.uea.ac.uk/lequere/co2/carbon.budget.htm. The global carbon budget is composed of the fossil fuel emissions, the emissions stemming from land use, the ocean uptake, and the terrestrial uptake estimated as a residual of all the sources minus the ocean uptake and atmosphere increase.

Figures 5.2 and 5.3 show the dynamics of changes in emissions and emissions uncertainties. The histogram in Fig. 5.2 is skewed to the left. In the next study period, Fig. 5.3, the situation changes: more values are concentrated on the right hand side. Between these two periods, the system turns from sink to source of CO_2. Definitely, it is impossible to characterize these changes only by uncertainty intervals.

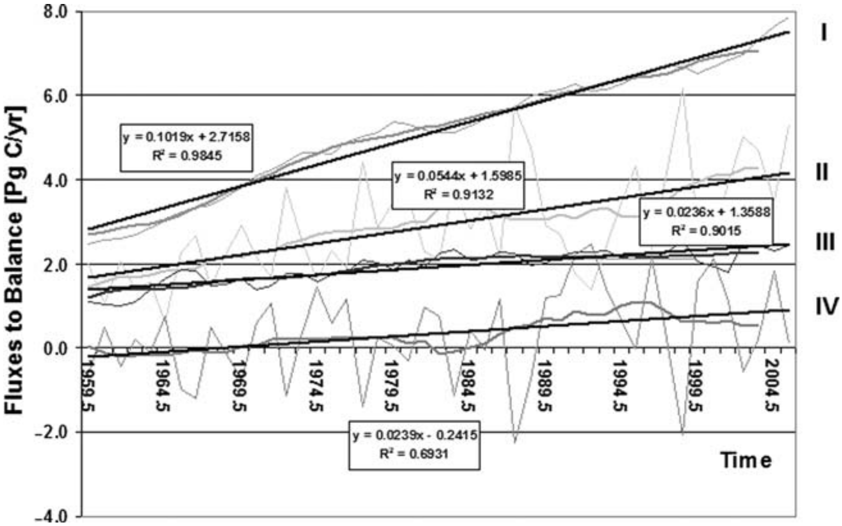

Fig. 5.1 Emission trends: fossil fuel and cement burning (I); CO_2 in the atmosphere (II); mean ocean uptake (III); net terrestrial flux (IV). *Bold lines* correspond to smoothed trajectories of respective fluxes. Regression equations in *boxes* describe linear trends

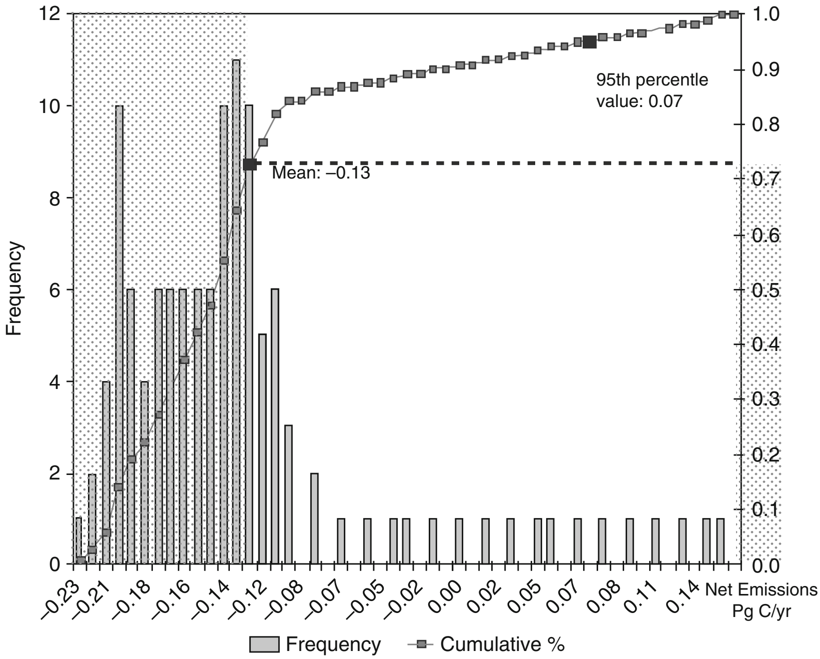

Fig. 5.2 Global CO_2 net terrestrial uptake, 1960–1970

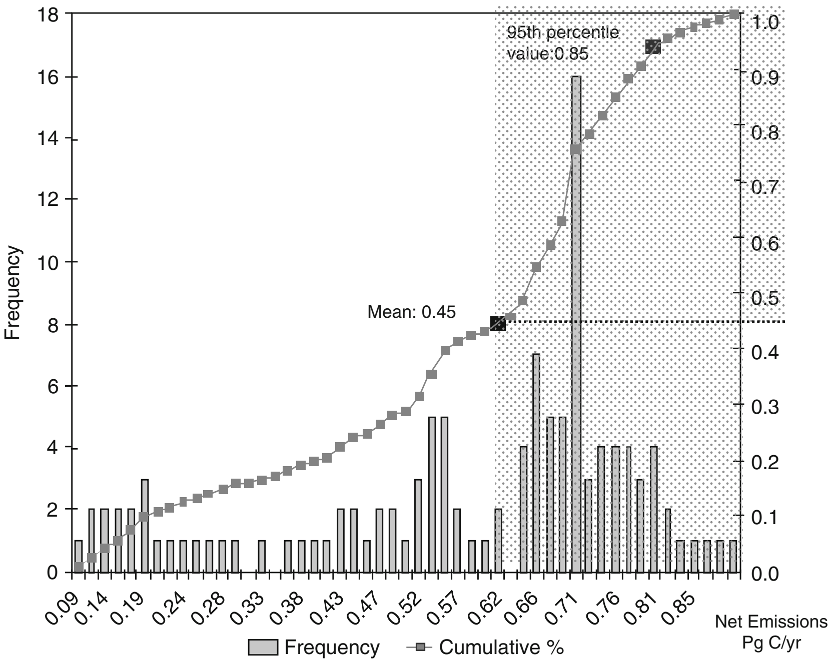

Fig. 5.3 Global CO_2 net terrestrial uptake, 1985–1995

5.3 Detectability of Emission Changes

A simple way to introduce the detectability of emission changes can be based on a straightforward representation of emission trends and uncertainties by equally-sided intervals as in Fig. 5.4 (see [1, 19]). The main idea is illustrated in Fig. 5.4. Let us assume that uncertainty of emission e_1 in the base year t_1 is characterized by equally sided interval $[e_1 - \epsilon, e_1 + \epsilon]$. The uncertainty of reported emission e_2 in the commitment year t_2 $(t_1 < t_2)$ is characterized by the same type of interval $[e_2 - \epsilon, e_2 + \epsilon]$. We assume that $e_1 > e_2$, although the case $e_1 < e_2$ is also possible, e.g., as a result of emission trading. The detectability of emission changes requires that the change in net carbon emissions $\Delta e = e_1 - e_2$ at time t_2 is greater than the uncertainty in the reported net carbon emissions at time t_2.

Under the non-restrictive assumption that the first-order linear approximations for emissions (as in Fig. 5.1), $e(t)$ and uncertainties $\epsilon(t)$ trends are applicable for $t_1 \leq t \leq t_2$, the detection time t^* is defined as the first time moment when net emission change Δe outstrips the uncertainty interval. In a sense, this is a worst-case evaluation. As in Figs. 5.2 and 5.3, considerable mass of real emissions can be concentrated in a much smaller subinterval. For example, the uncertainty interval of random variable with a normal probability distribution is $(-\infty, \infty)$, whereas practically entire probability mass may be concentrated within $[-1, 1]$ interval. Therefore, by using stochastic uncertainty models it is possible to derive with high probability a more optimistic t^*. This is the main issue of stochastic models discussed in [16, 17]. An overview of different approaches can be found in [1]. The goal of the stochastic models is to rank the trading parties by a safety indicator showing the percentage of detectable emission changes within a given time interval.

Let us consider a rather general stochastic model for a representation of a controversial data about uncertainties. We assume that the uncertainty of emissions e_1,

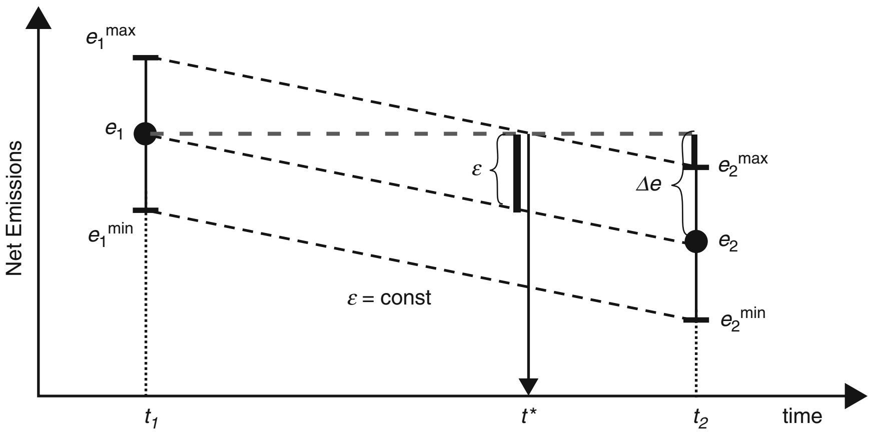

Fig. 5.4 Simplified illustration of detection time t^*

e_2 is characterized by a set of, in general, disconnected intervals. For example, Figs. 5.2 and 5.3 may suggest to represent uncertainty by a number of subintervals characterized by simple, say, uniform (conditional) probability distributions.

The following simple example illustrates the main idea of the model.

Example 5.1 (Controversial experts). Experts judgments are used in situations with the lack or even absence of real observations. Assume that two experts, Ex.1 and Ex.2, characterize the uncertainty of e_1 by overlapping intervals $[1, 8]$, $[5, 10]$. Then overall uncertainty of e_1 can be characterized by intervals $[1, 5]$, $[5, 8]$, $[8, 10]$ with likelihoods $1/4, 2/4, 1/4$ derived from the "voting" of experts: $(1, 0)$, $(1, 1)$, $(0, 1)$, i.e., for interval $[1, 5]$ votes only Ex.1, both experts vote for interval $[5, 8]$, and only Ex.2 votes for $[8, 10]$. In general, experts may characterize uncertainty by disconnected intervals. For example, Ex.1 may insist on equally probable intervals $[1, 3]$, $[5, 8]$ conditional on implementation of different technologies. Uncertainty of e_2 can be characterized in a similar manner.

Consider now a general model. For simplicity of notation we omit the index i of parties. Assume that (specific for each party i) uncertain emission e_1 is characterized by intervals $[e_1^{\min}, e_1^1]$, $[e_1^2, e_1^3]$, $[e_1^4, e_1^5], \dots, [e_1^R, e_1^{\max}]$ with probabilities p_r, $\sum_r p_r = 1$. These intervals can be derived from real observations, experts opinions, and scenarios of future developments. In addition, likelihoods of emissions within an interval can be characterized by a conditional on r distribution, say, uniform, normal or the degenerated distribution concentrated in the middle of this interval as in Fig. 5.4. In a similar manner, emissions e_2 are characterized by intervals $[e_2^{\min}, e_2^1]$, $[e_2^2, e_2^3]$, $[e_2^4, e_2^5], \dots, [e_2^L, e_2^{\max}]$ with some conditional on r and l distributions. Path-dependencies between emissions e_1 and e_2 are induced by the following stochastic model. An interval r at the base year t_1 is selected with the probability p_r, emission level e_1 is sampled from the conditional distribution in this interval; an interval l of a trend from r to l is selected with probability q_{rl}, $\sum_l q_{rl} = 1$, and finally, the end point e_2 of the random linear path (e_1, e_2) is sampled from the distribution in interval l conditional on r and l. Let us denote the obtained linear random path by $e(t, \omega)$, $t_1 \le t \le t_2$, where ω denotes the pair of points (e_1, e_2).

Linear paths $e(t, \omega)$ create the uncertainty ranges at t_2. For example, if $e(t_1, \omega)$ belongs to interval r, then the uncertainty of e_2 is defined on the basis of only feasible transitions from interval r to random intervals l with positive q_{rl}, $q_{rl} > 0$.

The proposed stochastic model allows to introduce path-dependencies of emissions subject to some essential conditions, say, the implementation of new emission reduction technologies or monitoring equipment. This may simplify the detection of emission changes. Exact detection is in general a difficult task because the resolution of all involved uncertainties may be prohibitively costly. Yet, it is possible to define likelihoods of changes. For example, it is possible to find a minimal time t_2 such that emission changes are detected during $[t_1, t_2]$ with a specified likelihood. It is also possible to find the likelihood of the changes within given interval $[t_1, t_2]$ that is used in the next section.

Remark 5.1 (Modifications of model). The proposed stochastic model can be further generalized or simplified subject to available data. Straight lines of linear emission paths can be theoretically substituted by more general stochastic paths (processes) or scenario trees, although the proposed linear stochastic trends allow simple calculations. If path-dependencies of emissions are not essential, then the model directly deals only with uncertainties of e_2. In more general situations, uncertainties are also characterized by a set of probability distributions, i.e., there is a set of feasible p_r, q_{rl}. There exist different approaches to deal with arising "uncertainty-of-uncertainty" issues, in particular, the use of non-Bayesian worst-case distributions [6].

Comparative analysis of deterministic and stochastic simple detection models can be found at http://www.iiasa.ac.at/Research/FOR/unc-prep.html,FOR/vt-concept.html, and in [10].

5.4 Trade Equilibrium Under Uncertainty

GHG's control policy as other environmental policies have to be designed in such a way that they are environmentally safe and cost-effective. The models proposed in this section provide a basis for designing rather different decentralized emission trading schemes.

The models reflect the following key features. The participants (countries, companies or other emitting entities) are given a right to emit a specific amount for which they obtain an equivalent number of allowances (emission permits). Such amounts are called the "cap" (Kyoto or other targets). If participants emit more than the corresponding cap reduced by the amount of uncertainty (undershooting level) ensuring that the actual emission does not exceed the cap with a given safety (likelihood) level, they are required to reduce uncertainty or/and to buy additional credits from the parties which emit less than their cap. The transfer of permits is called "trading". Standard deterministic models belong to a specific class of the proposed models. Since they ignore uncertainty, actual emissions may considerably overshoot allowed targets.

Let us briefly consider a deterministic model with uncertainty intervals proposed in [13, 14], that will be further extended to include stochastic safety constraints. The decision problem of each party can be separated in two interdependent subproblems. Firstly, for a given amount of permits, each party solves individual problem deciding whether to spend resources on abating emissions or investing in uncertainty reduction to satisfy emission targets. This problem does not require the information from any other party. Secondly, the party needs to decide whether or not to exchange permits with other parties. This decision problem involves the cost functions of other parties. In the model this information is private and therefore the methodology of decentralized optimization [8, 10] is required.

For the individual optimization problem, we define the least costs $f_i(y_i)$ for party i to comply with imposed targets for a given amount of permits y_i and the target K_i as the minimum of emission reduction costs $c_i(x_i)$ and costs of uncertainty reduction (e.g., by investments in monitoring) $d_i(u_i)$:

$$f_i(y_i) = \min_{u_i, x_i}[c_i(x_i) + d_i(u_i)], \tag{5.1}$$

$$x_i + u_i \leq K_i + y_i, x_i \geq 0, u_i \geq 0, \tag{5.2}$$

for all i, where x_i is the reported emissions at source i, u_i is its uncertainty, and y_i is the amount of emission permits acquired by source i (y_i is negative if i is a net supplier of permits). Therefore, constraint (5.2) requires that the reported emission x_i undershoots the target K_i by the level of uncertainty u_i. Similar concept of undershooting is also used in [24]. There are also suggestions [23] to represent uncertainty by a fraction of x_i. Example 5.2 shows that this case can be reduced to the case of additive uncertainty as in (5.2). Let us also note, that the model can be formulated in terms of emission reductions that require only slight changes of terminology.

The second optimization problem with asymmetric information involves finding the permit vector $y = (y_1, \ldots, y_n)$ or distribution of permits minimizing unknown total or social cost function

$$F(y) = \sum_{i=1}^{n} f_i(y_i) \tag{5.3}$$

subject to

$$\sum_{i=1}^{n} y_i = 0. \tag{5.4}$$

Suppose that the cost functions $c_i(x_i)$ and $d_i(u_i)$ are positive, decreasing, convex in x_i and u_i respectively and continuously differentiable. Therefore, $f_i(y_i)$ is also convex, positive, decreasing and differentiable. Then, from the Lagrangian minimization a trade equilibrium can be defined as the vector $y = (y_1, \ldots, y_n)$ satisfying the following equations:

$$f_i'(y_i) = -\lambda, \qquad \sum_{i=1}^{n} y_i = 0. \tag{5.5}$$

The condition (5.5) states that the marginal value of a permit shall in equilibrium be equal to a specific unknown level (price) λ same for all parties. It is clear that at the equilibrium vector y^* the constraints (5.2) will hold with equality, i.e.,

$$f_i(y) = \max_{x_i}[c_i(x_i) + d_i(K_i + y_i - x_i)] = \max_{u_i}[c_i(K_i + y_i - u_i) + d_i(u_i)].$$

Therefore from (5.1), (5.2) it follows that at the equilibrium $y_i = y_i^*$, $\lambda = \lambda^*$, $x_i = x_i^*$, $u_i = u_i^*$:

$$c_i'(x_i) = d_i'(u_i) = -\lambda, \qquad \sum_{i=1}^{n} y_i = 0, \tag{5.6}$$

where (x_i^*, u_i^*) is the solution of the subproblem (5.1), (5.2) for (y^*, λ^*), $y^* = (y_1^*, \dots, y_n^*)$ satisfying (5.5). This equation states that in the cost-effective and environmentally safe equilibrium, the marginal cost of holding emissions down to x_i^* will be equal to the marginal cost of holding uncertainty down to u_i^*. It shows that the explicit introduction of uncertainty u_i and the safety constraints (5.2) into emission trading schemes may significantly affect the equilibrium and, hence, the design of proper emission trading schemes. In particular, it means that equilibrium market prices λ^* must satisfy (5.5), (5.6). In other words, if λ^* is known and f_i are convex functions, then λ^* decentralizes joint model (5.3), (5.4) into individual solutions of (5.5). For non-convex function $F(y)$, (5.5) are not sufficient to find an equilibrium solution of the model.

However, there is no social planner that knows the cost functions of all parties. Therefore, even in the convex case the optimal value of $F(y)$ and λ^* cannot be resolved by solving (5.5). The scheme of bilateral trade presented in Sect. 5.5 allows to compute the equilibrium x_i^*, u_i^*, λ_i^* without revealing private information on functions f_i.

Remark 5.2 (Long-term perspectives, detectability and undershooting). The basic model can be easily extended to a dynamic version. In this article we do not consider it explicitly. Instead, we introduce below long-term perspectives by explicit treatment of future uncertainties and dynamic trading processes. The environmental constraint (5.2) assumes that the known emissions plus the uncertainty of emissions undershoot the emission target. This corresponds exactly to the detectability concept in Fig. 5.4. Constraint (5.2) discounts, in a sense, the reported emissions to levels undershooting emission targets. As (5.6) show, this provides incentives for the uncertainty reduction.

In the stochastic model of Sect. 5.3 uncertainty of emissions by party i at the commitment year t_2 is characterized by a random variable $e(t_2, \omega_i)$. A reported emission x_i provides additional information that modifies $e(t_2, \omega)$. For example, if it is known for sure that x_i belongs to an interval l_i, then distribution of $e_i(t_2, \omega_i)$ is induced only by feasible transitions from initial intervals r_i to l_i with corresponding probability distributions. We can also say that reported emission x_i transforms $e_i(t_2, \omega_i)$ into a random variable $\epsilon_i(x_i, \omega)$. Therefore, (5.2) of the deterministic model has to be understood now in a probabilistic sense as the following safety constraint. Let us define the uncertainty of reported emission x_i as $\xi_i(x_i, \omega) = \epsilon_i(t_2, x_i, \omega) - x_i$. Then the safety constraint can be written as probabilistic version of the deterministic constraint (5.2):

$$P[x_i + \xi_i(x_i, \omega) \le K_i + y_i] \ge Q_i, \tag{5.7}$$

for all parties i, where Q_i is a safety level ensuring that the probability of all potential emission paths to x_i satisfying the emission target K_i exceeds Q_i. Thus the interval uncertainty u_i is substituted by a random variable $\xi_i(x_i, \omega)$ dependent, in general, on x_i. In reality, the uncertainty characterized by ξ_i can be reduced by improvements of monitoring systems. Let us introduce the variable u_i to control ξ_i

within the desirable safety level Q_i. If $z_i(x_i)$ is the minimal z such that

$$P[\xi_i(x_i, \omega_i) \leq z] \geq Q_i,$$

then the safety constraint (5.7) can be substituted by the following equivalent constraint

$$x_i + u_i \leq K_i + y_i, u_i \leq z_i(x_i), \tag{5.8}$$

Remark 5.3 (Risk-based undershooting). Equation (5.8) shows that the stochastic model induces risk-based upper bounds on uncertainty intervals. Therefore, it allows, e.g., in cases illustrated by Figs. 5.3 and 5.4 to introduce but risk-based undershooting of emission targets defined by "critical" quantile $z_i(x_i)$, which is less conservative than standard interval uncertainty.

For the simplicity of notation, let us denote now by ω the vector of all uncertain parameters affecting cost functions and emissions, i.e., some components of ω such as market prices affect only cost functions, whereas other components affect emissions. In other words, suppose that all uncertain variables are defined on a probability space with a set of scenarios (events) ω. For random cost functions $c_i(x_i, \omega)$, $d_i(u_i, \omega)$, we can redefine functions $f_i(y_i)$ in (5.1) as

$$f_i(y_i) = \min_{x_i u_i} E[c_i(x_i, \omega) + d_i(u_i, \omega)], \tag{5.9}$$

where the minimization in (5.9) is subject to constraint (5.8). In this model ex-ante decisions x_i, u_i take a long-term perspective: they have to be optimal against all potential future scenarios ω and threats regulated by safety constraints. Uncertainties of cost functions c_i, d_i may be due to unknown in advance market performance, production shocks, and technological uncertainties.

Example 5.2 (Linear equivalent). Often, $\xi_i(x_i, \omega)$ is represented as $\xi_i(x_i, \omega) = \gamma_i x_i + \epsilon_i$, where $0 \leq \gamma_i \leq 1$, and ϵ_i is a random variable. In particular, uncertainty u_i in (5.2) can be given as $u_i = \gamma_i x_i$. The uncertainty in these cases can be controlled by γ_i in the following manner. Let $\epsilon_i(Q)$ be the minimal z such that $P[\epsilon_i \leq z] \geq Q_i$, e.g., $\epsilon_i(Q) = 0$ for constraints (5.2). Then constraint (5.8) is reduced to linear constraint

$$x_i + u_i \leq K_i - \epsilon_i(Q_i) + y_i, u_i \leq x_i.$$

After solving individual subproblem subject to this constraint, the optimal γ_i can be found as $\gamma = u_i / x_i$.

Remark 5.4 (Nonconvexity). Safety constraints (5.7) are well known in financial applications as the Value-at-Risk indicator. Similar constraints are typical for safety regulation of insurance companies, nuclear power plants, and catastrophic risk management [9]. Unfortunately, due to these constraints $f_i(y_i)$ and $F(x)$ may not be a convex function. In order to ensure convexity and/or robustness of decisions x_i, u_i

under rare extreme events it is possible to modify slightly cost functions c_i, d_i based on the "Conditional Value at Risk" [25] indicators as it is used in [9] and in [6] for general stochastic optimization problems.

Additional nonconvexities of functions $f_i(y_i)$ may be due to increasing returns of cost functions $c_i(\cdot)$, $d_i(\cdot)$, with respect to new emission abatement technologies with increasing returns. Nonconvexities generate the so-called duality gap between solutions of the basic model (5.3), (5.4) and its dual model (see Sect. 5.6) precluding the price-based dual (market) schemes to achieve cost-effective and environmentally safe solutions.

5.5 Dynamic Bilateral Trading Processes

The overall goal of the parties participating in the emission trading is to jointly achieve emission targets by redistributing the emission permits y_i, i.e., to find a vector y that would minimize social costs of all parties (5.3) under safety constraints (5.8), where cost functions $f_i(y_i)$ are defined according to (5.9). It is assumed that a party i knows its expected cost function $f_i(y_i)$, but the expected cost function $F(y)$ is unknown.

The basic feature of the trading scheme is similar to the procedure in [8] for convex function $F(y)$: two parties meet (e.g., picked at random) and, if possible, exchange emission permits in a mutually beneficial way. A new pair is picked and the procedure is repeated. The Appendix provides the proof that this dynamic process will lead the parties to an equilibrium despite the information of each party's cost is private and $F(y)$ is not necessarily a convex function.

The following simple equations illustrate that the bilateral exchange of emissions is beneficial for both parties. Let $y^k = (y_1^k, \dots, y_n^k)$ be the vector of emission permits after k trades. Consider two parties i and j at step k with permits y_i^k and y_j^k. An exchange of permits between them leads to a new distribution of permits $y^{k+1} = (y_1^{k+1}, \dots, y_n^{k+1})$, $y_l^{k+1} = y_l^k$ for $l \neq i, j$. According to (5.5), if there exist any two parties i and j having different marginal costs on emission reduction $f_i'(y_i^k) \neq f_j'(y_j^k)$, then the permit vector $y^k = (y_1^k, \dots, y_n^k)$ is not cost efficient. Without loss of generality, assume that $f_i'(y_i^k) - f_j'(y_j^k) < 0$. Constraint (5.4) requires that the feasible exchange in permits has to be such that $y_i^{k+1} + y_j^{k+1} = y_i^k + y_j^k$.

If we take $y_i^{k+1} = y_i^k + \Delta_k$ and $y_j^{k+1} = y_j^k - \Delta_k$, $\Delta_k > 0$, then the new feasible distribution of permits reduces the total costs of parties $f_i(y_i^k) + f_j(y_j^k)$ and hence the total cost $F(y^k)$:

$$\begin{aligned} F(y^{k+1}) - F(y^k) &= f_i(y_i^{k+1}) + f_j(y_j^{k+1}) - f_i(y_i^k) - f_j(y_j^k) \\ &= \Delta_k(f_i'(y_i^k) - f_j'(y_j^k)) + o(\Delta_k) < 0, \end{aligned}$$

for a small Δ_k. We also have

$$f_i(y_i^{k+1}) - f_i(y_i^k) < f_j(y_j^k) - f_j(y_j^{k+1}). \tag{5.10}$$

i.e., the new distribution of permits reduces costs of j more than increases cost of i. Hence j is able to compensate i for the increased costs in a mutually beneficial way.

Let us summarize the trade scheme more precisely. We assume that after picking up (say at random) a pair of parties i, j these parties are able to find y_i^{k+1}, y_j^{k+1} minimizing

$$f_i(y_i) + f_j(y_j) \tag{5.11}$$

subject to constraints $y_i + y_j = y_i^k + y_j^k$, $y_i \geq 0$, $y_j \geq 0$.

This problem is solved by parties i and j only. For continuously differentiable functions $f_i(y_i)$, $f_j(y_j)$, a party j that decreases emission permit by $\Delta_k > 0$ may negotiate with i such a level Δ_k that equalizes marginal costs, i.e., $f_i^{'}(y_i^k - \Delta_k) = f_j^{'}(y_j^k + \Delta_k) = \lambda_k$, where λ_k can be viewed as an equilibrium price (in general stochastic) at step k. Let us note that price process λ_k is driven endogenously by cost-minimizing decisions of meeting parties, what is fundamentally different from standard models of financial markets with exogenously given price processes.

The sequential bilateral trades can go on as long as there are two parties with different marginal costs. The bilateral exchange of emissions equalizes marginal costs which define an intermediate "local" equilibrium price λ_k. During the process, marginal costs and prices will differ between the sequential trades, but finally the trading system converges to an equilibrium with marginal costs of all parties equal to equilibrium price as in (5.5).

It is important to compare the outlined bilateral trading scheme with a basic price-based scheme. A cost-effective and environmentally safe price signal is a solution of the dual model to the basic primal model (5.3)–(5.4). It involves finding the price λ maximizing the following concave and, in general, non-differentiable function

$$\phi(\lambda) = \min_y \sum_{i=1}^{n} (f_i(y_i) + \lambda y_i).$$

A price signal λ decentralizes the solution of internal minimization problem into individual subproblems: find solutions $y_i(\lambda)$ minimizing functions $f_i(y_i) + \lambda y_i$. In general, solutions $y_i(\lambda)$ do not satisfy the balance equation, i.e., $\sum_{i=1}^n y_i(\lambda) \neq 0$, therefore the price λ has to be adjusted towards the desirable balance. The common idea is to change current λ_k at time $k = 0, 1, \ldots$ proportionally to the imbalance, i.e., $\phi^{'}(\lambda)$ for continuously differentiable $\phi(\lambda)$:

$$\lambda_{k+1} = \lambda_k + \rho_k \sum_{i=1}^{n} y_i(\lambda_k)$$

with a small step-size ρ_k. From the convergence results of quasi-gradient methods (see, e.g., discussion in [5]) it follows that with $\rho_k = const/k$, the sequence λ_k

converges to a price $\overline{\lambda}$ maximizing $\phi(\lambda)$. If $y_i(\lambda)$ are unique solutions (for any λ and $i = \overline{1,n}$)), then $\phi(\lambda)$ is continuously differentiable function and $\overline{\lambda}$ fulfills the balance equation ($\sum_{i=1}^{n} y_i(\overline{\lambda}) = 0$) independently of the convexity $f_i(y_i)$. Otherwise, additional coordinated search is required to select $\overline{y_i}$ from the set of solutions $y_i(\overline{\lambda})$ in order to guarantee (achieve) the balance $\sum_{i=1}^{n} y_i(\overline{\lambda}) = 0$. The coordination of parties is also required for tracking values of imbalances $\sum_{i=1}^{n} y_i(\lambda_k)$ for adjusting prices λ_k, $k = 0, 1, \ldots$. Fundamental difficulties arise in the case of markets uncertainties (see Sect. 5.7) and duality gap, i.e., when $\max \phi(\lambda) < \min_y \{F(y), \sum_{i=1}^{n} y_i = 0\}$ for nonconvex function $F(y)$.

5.6 Computerized Multi-agent Decentralized Trading System

The proposed perfect market system implies that trades being bilateral, sequential (dynamic) and random do not impair the cost savings even if parties only have information on their own cost. However, there are essential obstacles that can inhibit real markets from perfect functioning according to proposed procedure. In a perfect market, a party that has sold permits in an early stage of the trading process would be able to cancel its earlier transaction. In the real emission trading market, this type of counter-actions may be impossible due to irreversibility of decisions: investments may already have been made, and these investment costs are largely sunk costs. This is the fundamental obstacle involved in the design of cost-minimizing and environmentally safe emission trading markets.

Price-based trading schemes have additional inherent obstacles. Designing environmentally safe and cost-effective price-based emission trading markets is equivalent to solving of the dual model asking for the same full information as the solution of the primary model. Additional critical limitation is the duality gap which occurs in nonconvex cases and uncertainty of market prices. The available computer technology and numerically stable optimization procedures allow to organize computerized (say, web-based) multi-agent decentralized trading system to resolve these issues.

One can imagine a distributed computer network that connects computers of parties with the computer of a central agency. The party in an anonymous manner stores information on its specific cost functions, and other characteristics of the underlying optimization model (5.8), (5.9) including specific probability distributions. The central agency stores information on the emission detection model. The computer of the central agency generates a pair of parties i, j and in an anonymous manner negotiates with computers of these partners a proper Δ_k that solves the subproblem (5.11). This can be easily organized without revealing private information of the parties. The process is repeated until equilibrium levels have been reached. This procedure allows to discover equilibrium solution that can then be implemented in reality. The information about the equilibrium price λ^* allows also to identify so-called core solution defining stable coalition of parties (see the Appendix). A network of interconnected computers is essential for a rapid, smooth and robust functioning of emission trading market. There would also be a clear separation between a first stage, in which provisional bids are made between computers of parties and reconstructing is allowed, and a second stage, when contracts have been concluded and investments in emission control are implemented.

It is well known [2] that generally the market does not generate desirable outcomes if market prices fail to reflect socio-economic and environmental impacts. In this case it is typically necessary to establish negotiation processes between involved parties to determine desirable collective solutions. From this perspective, the proposed trading system can be viewed as a device for collective negotiations and decision-making in the presence of inherent uncertainties and irreversibilities.

5.7 Myopic Market Processes

The basic model (5.3), (5.4), (5.8), (5.9) takes long-term perspectives on emission permit trading. Parties use expectations and safety constraints in order to achieve cost-effective and environmentally safe outcomes robust against future developments. The resulting trading scheme is similar to non-monetary exchange economy [28] important for environmental control. There are no demand and supply functions. Instead, the safety constraints enforce parties to invest in emission and uncertainty reductions and consequently act as supplier of mutually beneficial emission permits until a global equilibrium emerges.

The situation becomes dramatically different in the case of price-based schemes under markets uncertainties affecting cost functions c_i, d_i of parties. The short term market perspectives orient parties on instantaneous information about prices and costs. At time interval k parties observe market-related components of uncertainty ω_k and thus know their instantaneous cost functions $c_i(x_i, \omega_k)$, $d_i(x_i, \omega_k)$. Based on this information, parties calculate cost functions

$$f_i(y_i, \omega_k) = \min_{u_i, x_i}[c_i(x_i, \omega_k) + d_i(u_i, \omega_k)], \tag{5.12}$$

subject to the safety constraints (5.8) conditional on observable uncertainties. They minimize then

$$\sum_{i=1}^{n} f_i(y_i, \omega_k) \tag{5.13}$$

subject to

$$\sum_{i=1}^{n} y_i = 0$$

by using observed price signals π_k, $k = 1, 2, \ldots,$ which separates joint model (5.13) into independent individual minimization of cost functions

$$f_i(y_i, \omega_k) + \pi_k y_i$$

by parties $i = 1, \ldots, n$, where $\pi_k y_i$ is the cost of buying ($y_i > 0$) or selling ($y_i < 0$) emission permits y_i. If π_k coincides with an equilibrium spot price, then solutions $y_i(\omega_k)$, $i = 1, \ldots, n$, of these individual models may coincide (Sect. 5.5) with the solution of joint model (5.13). In particular, they may satisfy the balance equation $\sum_{i=1}^{n} y_i = 0$. Otherwise, market prices π_k may cause disruptions of the balance and crashes of prices similar to the European carbon prices in April 2005. The following example illustrates a typical situation.

Example 5.3 (Market's uncertainty). Suppose that $f_i(y_i)$ are known deterministic functions, i.e., only prices are random. At time interval k there is a favorable situation for the exchange of emission permits for some parties i and j, e.g., $f_i'(y_i^k) \neq f_j'(y_j^k)$. Instead, high market price π_k, $\pi_k > \left|f_j'(y_j^k)\right| > \left|f_i'(y_i^k)\right|$, forces both parties to reduce emissions in the excess of targets in order to sell surpluses on the market. Disequilibrium price π_k creates an oversupply of emission permits that pushes the market price π_{k+1} towards 0. This may prevent to sell reduced emissions which turned to be of higher marginal costs with respect to new prices.

The myopic model (5.12)–(5.13) yields decisions $x_i(k, \omega_k)$, $u_i(k, \omega_k)$, $y_i(k, \omega_k)$, $k = 1, 2, \ldots$. Such decisions depend on case-specific realizations of the random variable ω_k, therefore are not robust. At time interval $k + 1$ new observation ω_{k+1} may contradict ω_k requiring significant revisions of these decisions, which may be impossible due to their irreversibility. In order to achieve a convergence, the parties must adopt a precautionary incremental behavior with respect to arriving new information. Let us consider first this type of trading scheme for the basic model (5.3)–(5.4), (5.9).

The bilateral dynamic trading process of Sect. 5.5 has deep roots in the structure of so-called stochastic gradients in the linear subspace defined by (5.4). Namely, it is easy to prove that the vector

$$g(y) = \left(f_1'(y_1) - \frac{1}{n}\sum_{j=1}^{n} f_j'(y_j), \ldots, f_n'(y_n) - \frac{1}{n}\sum_{j=1}^{n} f_j'(y_j)\right)$$

is the projection of $gradF(y) = (f_1'(y_1), \ldots, f_n'(y_n))$ in this subspace. A stochastic gradient then can be defined as the following. Pick up at random a pair (i, j) and define stochastic vector

$$\xi(y) = \frac{(n-1)}{2}(0, \ldots, 0, f_i'(y_i) - f_j'(y_j), 0, \ldots, 0, f_j'(y_j) - f_i'(y_i), 0, \ldots, 0),$$

i.e., $\xi(y) = (0, \ldots, 0)$ for $i = j$. This vector is a stochastic gradient of $F(y)$ [5], i.e., the conditional expectation $E[\xi(y)|y] = g(y)$, assuming that pairs (i, j) of distinct parties are chosen with equal probability $1/n(n-1)$. Therefore, instead of complete minimization of function (5.11) at step k, parties can move from y^k in the random direction $\xi(y^k)$ with a small step size α_k. This type of stochastic decentralized optimization processes are important in cases when functions $f_i(y_i)$

are not calculated exactly, e.g., they are affected by unknown random variables ω as in (5.12). It is easy to check that

$$F(y^k - \alpha_k \xi(y^k)) = F(y^k) - \alpha_k [f'_{i_k}(y^k_{i_k}) - f'_{j_k}(y^k_{j_k})]^2 + o(\alpha_k), \tag{5.14}$$

where (i_k, j_k) is picked up at step k random pair (i, j). Therefore, sequential change of emissions defined by equations

$$y^{k+1} = y^k - \alpha_k \xi(y^k) \tag{5.15}$$

produces monotonically decreasing for small α_k (contrary to standard stochastic gradient methods [5] random sequence $\{y^k\}$. The convergence analysis of this scheme is similar to the proof of the Theorem in the Appendix. It is also possible to derive from (5.14), that the scheme of Sect. 5.5 is in fact equivalent to the procedure (5.15) with the full step-size α_k equalizing marginal costs,

$$f'_i(y^{k+1}_i) = f'_j(y^{k+1}_j).$$

The basic adjustments in (5.15) are again pair-wise, but random encounters. Other encounters are also possible assuming that each party meets every other party.

This type procedure is also applicable in the case when parties use only observable random functions $c_i(x, \omega)$, $d_i(u, \omega)$. Suppose that instead of myopic decisions $y_i(k, \omega_k)$ parties make precautionary incremental and adaptive adjustments of vector y. We can define stochastic vector $\xi(y, \omega)$ similar to vector $\xi(y)$ as

$$\xi(y, \omega) = \frac{(n-1)}{2}(0, \ldots, 0, f'_i(y_i, \omega) - f'_j(y_j, \omega), \ldots, f'_j(y_j, \omega) \\ - f'_i(y_i, \omega), 0, \ldots, 0)$$

and proceed with changes of emission permits y^k according to procedure (5.15) with $\xi(y^k$ substituted by $\xi(y^k, \omega_k)$. Under standard assumptions the conditional expectation $E[\xi(y, \omega)|y] = g(y)$, where $g(y)$ is the projection of $grad G(y)$,

$$G(y) = \sum_{i=1}^{n} E f_i(y, \omega). \tag{5.16}$$

The convergence of the trading scheme (5.15) with the vector $\xi(y^k, \omega_k)$ to a solution minimizing $G(y)$ subject to $\sum_{i=1}^{n} y_i = 0$ can be derived from general results on the convergence with probability 1 of stochastic quasigradient methods [5]. In particular, it requires the proper step-size multipliers α_k, e.g., $\alpha_k = const/k$ is applicable. This requirement presumes no knowledge of underlying data. Yet, it suffices to stabilize exchanges y^k.

Remark 5.5 (Short-term market decisions). Method (5.15) with vector $\xi(y^k, \omega_k)$ leads to an array of bilateral trading schemes with a variety of trading rules. Yet, it is important to note a significant difference between functions $G(y)$ and $F(y)$ defined by (5.16) and (5.9), respectively; function $G(y)$ focuses on the variability of short-term market decisions $x_i(k,\omega)$, $u_i(k,\omega)$, whereas $F(y)$ focuses on forward-looking decisions x_i, u_i, y_i robust against all future eventualities ω. Thus, in contrast to trading scheme of Sect. 5.6, the minimization of function (5.16) takes short-term market perspectives on emission and uncertainty reductions driven by random observations ω_k of market situations. It also treats the safety constraints conditionally on myopic decisions $x_i(k,\omega)$, $u_i(k,\omega)$, that prevents to achieve robust cost-effective and environmentally safe outcomes specified by (5.3), (5.4), (5.7), (5.9). Such outcomes are guaranteed only through replacement of myopic overacting decisions $x_i(k,\omega)$, $u_i(k,\omega)$ by incremental adjustments of x_i, u_i similar to adjustments of y^k.

5.8 Concluding Remarks

The paper analyzes cost effective and environmentally safe carbon trading schemes explicitly incorporating different types of exogenous and endogenous uncertainties on emissions and the abatement costs. The feasibility of decentralized market pollution control mechanisms is usually discussed under strong assumptions that all actions are made simultaneously at known equilibrium prices, what implies existence of perfectly informed social planner. The examined dynamic bilateral trading schemes are not based on price signals, and the emerging emission prices implicitly depend on the costs functions and the safety constraints on environmental targets. With the safety constraints, the parties set the level of their exposure toward uncertainties and risks. The safety constraints discount the reported emissions to undershooting detectable levels. This type of undershooting concept or discounting should become a key element in a robust regulation of emission trades together with unified approaches to modeling emission uncertainties and cost functions. The paper shows that myopic price-based trading schemes are not able to achieve cost-effective and environmentally safe solutions. The irreversibility of trades calls for the use of the proposed computerized emission trading system providing, in a sense, collective decentralized regulation of trades. The procedures provide a constructive and easy approach for designing robust emission trading schemes. All decisions are fully decentralized, individual contrary to cost effective and environmentally safe price mechanisms requiring additional coordination to stabilize leading otherwise to nowhere trading processes. Our approach is close to important ideas on decentralized non-monetary exchange. Bilateral trading scheme with deterministic interval uncertainty has been applied [14] for the fossil fuel related carbon emissions of the major Parties of the Kyoto Protocol. Numerical findings indicate that the compliance costs increase significantly for USA, Japan and the European Union, if uncertainty of the emission levels is considered. However, although the Central and Eastern Europe, Russia, and Ukraine have larger uncertainties in emission levels, their net costs may decrease as they can sell emission reductions at a higher price. Additional simple calculations according to Remark 5.3 show that stochastic uncertainty in the emission levels reduces the compliance costs of parties.

Appendix

The convergence of trading scheme in Sect. 5.5 was proved in [8] for convex functions $f_i(y_i)$, $i = \overline{1, n}$. The following proof covers the case of nonconvex functions.

Proof of Convergence

Theorem (Convergence to an equilibrium). Let $f_i(y_i) \geq 0$ be continuously differentiable functions and let Y^* be the set of equilibriums equalizing marginal values $f_i^{'}$ as in (5.5), $F(Y^*) = F(y)$, $y \in Y^*$, y^k is defined as in Sect. 5.5. Then either

1. $y^k \in Y^*$ after a finite number of steps, or
2. The sequence $\{F(y^k)\}$ converges to its equilibrium value from the set $F(y^*)$ and all cluster points of $\{y^k\}$ belong to Y^*, or
3. If Y^* contains only a single point y^*, then $\{y^k\}$ converges to this point.

Proof. The sequence $\{F(y^k)\}$, $k = 1, \ldots$, is monotonically decreasing, $F(y^k) \geq 0$. Therefore, there exist a limit $\overline{F} = \lim_k F(y^k)$. Let us prove that $\overline{F} \in F(Y^*)$. Suppose there exists a convergent subsequence y^{k_s}, $y^{k_s} \to \overline{y}$, $s \to \infty$ and $\overline{y} \notin Y^*$. Therefore, there exist i, j such that $f_i^{'}(\overline{y}_i) \; /= f_j^{'}(\overline{y}_j)$. It means that $\lim F(y^{k_s+1}) < F(\overline{y})$, what contradicts the convergence of $F(y^k)$, i.e., $\overline{F} \in F(Y^*)$, and all cluster points of the bounded sequence y^k belong to Y^*. Hence, if Y^* is a singleton, then y^k converges to y^*.

A Core Solution

From (5.10) it follows that at each step k cooperating parties i, j can redistribute joint cost $f_i(y_i^{k+1}) + f_j(y_j^{k+1}) = \phi_i^{k+1} + \phi_j^{k+1}$, $\phi_i^{k+1} < f_i(y_i^k)$, $\phi_j^{k+1} < f_j(y_j^k)$. Therefore at the equilibrium $y^* = (y_1^*, \ldots, y_n^*)$ parties will deal actually with payments $\phi_i^* < f_i(y_i^0)$ such that $\sum_{i=1}^n \phi_i^* = \sum_{i=1}^n f_i(y_i^*) \; : \; = F_I$ where $I = 1, \ldots, n$. From this equation follows the Pareto efficiency of $\phi^* = (\phi_i^*)_{i=1,\ldots,n}$. An important question is whether the grand coalition I of parties is stable, i.e., $\sum_{I \in C} \phi_i^* \leq F_c$ for any other coalition $C \subseteq I$. Accordingly, a distribution of payments ϕ^* is a core solution if it satisfies these two equation. The bilateral trading procedure allows to find the equilibrium price λ^*. If function $F(y)$ is convex, then the payment distribution $\phi_i^* = f_i(y_i^*) + \lambda^* y_i^*$ is a core solution. If the function $F(y)$ is globally Lipschitz continuous, then the core solution remains the same (see discussion in [11]).

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