Chapter 4
Induced Discounting and Risk Management

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Abstract The goal of this paper is to specify and summarize new approaches to discounting proposed in our catastrophic risk management studies. The main issue is concerned with justification of investments, which may turn into benefits over long and uncertain time horizon. For example, how can we justify mitigation efforts for expected 300-year flood that can occur also next year. The discounting is supposed to impose time preferences to resolve this issue, but this view may be dramatically misleading. We show that any discounted infinite horizon sum of values can be equivalently replaced by undiscounted sum of the same values with random finite time horizon. The expected duration of this stopping time horizon for standard discount rates obtained from capital markets does not exceed a few decades and therefore such rates may significantly underestimate the net benefits of long-term decisions. The alternative undiscounted random stopping time criterion allows to induce social discounting focusing on arrival times of the main concern (stopping time) events rather than horizons of market interests.

In general, induced discount rates are conditional on the degree of social commitment to mitigate risk. Random stopping time events affect these rates, which alter the optimal mitigation efforts that, in turn, change events. This endogeneity of the induced discounting restricts exact evaluations necessary for using traditional deterministic methods and it calls for stochastic optimisation methods. The paper provides insights in the nature of discounting that are critically important for developing robust long-term risk management strategies.

4.1 Introduction

The implication of uncertainties and risks for justifying long-term investments is a controversial issue. How can we justify investments, which may possibly turn into benefits over long and uncertain time horizons in the future? This is a key issue for catastrophic risk management. For example, how can we justify investments in

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climate change mitigations, say, in flood defense systems to cope with foreseen extreme 1,000-, 500-, 250-, and 100-floods? The lack of proper evaluations for dealing with extreme events dramatically contributes to increasing losses from human-made and natural disasters [20]. The analysis of floods that occurred in the summer of 2002 across central Europe [14] shows that the potential areas of vulnerability to extreme floods have multiplied as a consequence of failed development planning. Underestimation and ignorance of low probability/high consequence events have led to the growth of buildings and industrial land and sizable value accumulation in flood prone areas without proper attention being paid to flood mitigations. A challenge is that an endogenously created catastrophe,[1] say a 300-year flood, has never occurred before in a given region. Therefore, purely adaptive policies relying on historical observations provide no awareness of the "unknown" risk although, a 300-year flood may occur next year. For example, the 2002 floods in Austria, Germany and the Czech Republic were classified (in different regions) as 1,000-, 500-, 250-, and 100-year events [14].

A key issue is development of policies with proper long-term perspectives. The traditional discounting is supposed to impose necessary time preferences, but this view may be dramatically misleading. There are several possibilities for choosing discount rates (see, for example, the discussion in [2, 19, 24, 29]). The traditional approach is to use the rates obtained in capital markets. The geometric or exponential discount factor $d_t = (1 + r)^{-t} = e^{-1n(1+r)t} \approx e^{-rt}$ (for small r) is usually connected with a constant rate r of returns from capital markets. Since returns in capital markets are linked to assets with a lifespan of a few decades, this choice may completely reduce the impacts that investments have beyond these intervals (Sect. 4.2). Another serious problem [21, 31] arises from the use of the expected value $r = E\xi$ and the discount factor $(1 + r)^{-t}$. It implies additional significant reduction of future values in contrast to the expected discount factor $E(1 + \xi)^{-t}$, because $E(1 + \xi)^{-t} >> (1 + r)^{-t}$. These issues are discussed in Sects. 4.2 and 4.3.

An appropriate interest rate is especially difficult to define when decisions involve time horizons beyond the interests of the current generation. If future generations are not present in the market, e.g., long-term environmental damages are not included in production costs, the market interest rates do not reflect the preferences of future generations. According to Arrow et al. [2] "the observed market rates of interest refer to how individuals are willing to trade off consumption over their own life. These may or may not bear close correspondence to how a society is willing to trade off consumption across generations".

Debates on proper discount rates for long-term problems have a long-standing history [2, 29]. Ramsey [26] argued that applying a positive discount rate r to discount values across generations is unethical. Koopmans [17], contrary to Ramsey, argued that zero discount rate r would imply an unacceptably low level of current consumption. The use of so-called social discount rates produces two effects [2].

[1] As a consequence of inappropriate policies.

The "prescriptive" approach tends to generate relatively low discount rates and thus favors mitigation measures and the wellbeing of future generations. The "descriptive" approach tends to generate higher discount rates and thus favors less spending on mitigation and the wellbeing of the current generation.

The constant discount rate has only limited justification [3,12,24,29]. As a compromise between "prescriptive" and "descriptive" approaches, Cline [4] argues for a declining discount rate: 5% for the first 30 years, and 1.5% later. There have been proposals for other schedules and attempts to justify the shape of proper decline. Papers [21, 31] show that uncertainty about r produces a certainty-equivalent discount rate, which will generally be declining with time. Weitzman [31] proposed to model discount rates by a number of exogenous time dependent scenarios. He argued for rates of 3–4% for the first 25 years, 2% for the next 50 years, 1% for the period 75–300 years and 0 beyond 300 years. Newel and Pizer [21] analyzed the uncertainty of historical interest rates by using data on the US market rate for long-term government bonds. They proposed a different declining discount rate justified by a random walk model. Chichilinsky [3] proposed a new concept for long-term discounting with a declining discount rate by attaching some weight on the present and the future consumption. All these papers aim to derive an appropriate exogenous social discount rate.

Sections 4.2 and 4.3 develop a different approach for social discounting. It is shown that any discounted sum, so-called net present value (NPV) criterion, $\sum_{t=0}^{\infty} d_t V_t$ of expected values $V_t = Ev_t$ for random variables (r.v.) $v_t, t = 0, 1, \ldots, d_t = (1 + r_t)^{-t}$ under constant and declining discount rates r_t equals the average undiscounted (in the agreement with Ramsey's concerns) random sum $E \sum_{t=0}^{\tau} v_t$ with a random stopping time τ defined by the given discounting d_t. Therefore, discount rates can be associated with the occurrences of "stopping time" random events determining a finite "internal" discount-related horizon $[0, \tau]$. The expected duration of τ and its standard deviation σ under modest market interest rates of 3.5% is approximately 30 years, which may have no correspondence with expected, say, 300-year extreme events and $\sigma \approx 300$. Conversely, it is shown that any stopping time random event induces a discounting. A set of mutually exclusive stopping time random events, e.g., 1,000-, 500-, 250-, and 100-year floods, induces discounting with time-declining discount rates. This case corresponds also to the discounting with uncertain discount rates r. In particular, a single stopping time random event with the standard geometric probability distribution induces the standard discounting with constant discount rate r and $d_t = (1 + r)^{-t}$.

The effects of catastrophes on the stream of values $v_t, t = 0, 1, \ldots,$ differ from the effects of market uncertainties. Section 4.4 indicates that catastrophic events pose new challenges. They often create so-called endogenous, unknown (with the lack and even absence of adequate observations) and interdependent risks, which may potentially affect large territories and communities and, on the other hand, are dramatically affected by risk management decisions. As a consequence, catastrophic risks generally make it impossible to use traditional economic and insurance models [1,3,5,8,9,16]. The concept of undiscounted random stopping time criteria allows to induce social discounting that focuses on arrivals of catastrophic events rather then the lifetime of market products. Since risk management decisions affect the occurrence of disasters in time and space, the induced discounting may depend on spatio-temporal distributions of extreme events and feasible sets of decisions, i.e., it can be viewed as a spatio-temporal discounting. The implicit dependence of the stopping time discounting on random events and decisions calls for the use of stochastic optimization methods, which allows also to address the variability (Remark 4.2) of discounted criteria by using random value $\sum_{t=0}^{\tau} v_t$ even for deterministic $v_t, t = 0, 1, \ldots$. Section 4.5 establishes connections of stopping time discounting with dynamic versions of CVaR (Conditional Value-at-Risk) risk measures. Section 4.6 illustrates how misperception of induced discounting may provoke catastrophes. Section 4.7 provides concluding remarks.

4.2 Standard and Stopping Time Induced Discounting

This section illustrated the main idea by using the standard geometric discounting. The choice of discount rate as a prevailing interest rate within a time horizon of existing financial markets is well established [18]. Uncertainties, especially related to extreme events, challenge the possibility of markets to offer proper rates for longer time horizons. The following simple Proposition 4.1 and Remark 4.2 clarify the main concerns.

The traditional financial approaches [18] often use the so-called net present value (NPV) criteria to justify investments. An investment is defined as an expected cash flow stream $V_0, V_1, \ldots, V_T, V_t = Ev_t$, over a time horizon $T \leq \infty$. Assume that r is a constant prevailing market interest rate, then alternative investments are compared by $V = V_0 + d_1 V_1 + \ldots + d_T V_T$, where $d_t = d^t, d = (1+r)^{-1}, t = 0, 1, \ldots, T$, is the discount factor and V denotes NPV.

It is usually assumed that a long-term investment activity has an infinitely long time horizon, i.e.,

$$V = \sum_{t=0}^{\infty} d_t V_t. \tag{4.1}$$

The stream of values $V_t, t = 0, 1, \ldots$, can represent an expected cash flow stream of a long-term investment activity. In economic growth models and integrated assessment models [19, 22, 29] the value V_t represents utility $U(x^t)$ of an infinitely living representative agent, or welfare $V_t = \sum_{i=1}^{n} a_i u_i(x_i^t)$ of a society with representative agents $i = \overline{1, n}$, utilities u_i, consumptions x_i^t and welfare weights α_i. Natural selection theory treats (4.1) as Darwinian fitness [28], where discount factors d_t are associated with hazard rates of an environment (Example 4.2).

The infinite time horizon in (4.1) creates an illusion of truly long-term analysis. Proposition 4.1 shows that in fact deterministic evaluation (4.1) accounts only for values V_t from a finite random horizon $[0, \tau]$ defined by a random stopping time τ with the discount-related geometric probability distribution $P[\tau \geq t] = d_t$.

Proposition 4.1. *Consider a discounted sum (4.1) with $d_t = d^t, d = (1+r)^{-1}$, $r > 0$. Let $q = d, p = 1-q$, and τ be a random variable with the geometric probability distribution $P[\tau = t] = pq^t, t = 0, 1, \ldots$. Then $d_t = P[\tau \geq t]$ and*

$$\sum_{t=0}^{\infty} d^t V_t = \sum_{t=0}^{\infty} P[\tau \geq t] V_t = E \sum_{t=0}^{\tau} V_t. \tag{4.2}$$

Conversely, for any stopping time τ with a geometric probability distribution

$$E \sum_{t=0}^{\tau} V_t = \sum_{t=0}^{\infty} d_t V_t, d_t = P[\tau \geq t].$$

Proof. We have $P[\tau \geq t] = \sum_{k=t}^{\infty} pq^k = pq^t(1-q)^{-1} = q^t = d_t$. Conversely,

$$E \sum_{t=0}^{\tau} V_t = \sum_{t=0}^{\infty} P[\tau = t] \sum_{k=0}^{t} = V_k = \sum_{t=0}^{\infty} pq^t \sum_{k=0}^{t} V_k$$
$$= \sum_{t=0}^{\infty} \left(\sum_{k=t}^{\infty} pq^k \right) V_t = \sum_{t=0}^{\infty} d_t V_t.$$

That is, any discounted deterministic sum (4.1) equals to the average undiscounted random sum $\sum_{t=0}^{\tau} V_t$ of the same values V_t. In other words, the discount factor $d_t = d^t$ induces an "internal" discount-relate time horizon $[0, \tau]$ with the geometrically distributed τ. Conversely, any geometrically distributed τ and the criterion $E \sum_{t=0}^{\tau} V_t$ induces the geometric discounting in the sum $\sum_{t=0}^{\infty} d_t V_t$.

Remark 4.1. (Random stopping time horizon). We can consider$[0, \tau]$ being a random stopping time horizon associated with the first occurrence of a "killing", i.e., a catastrophic stopping time event. The probability that this event occurs at $t = 0, 1, \ldots$ is p and pq^t is the probability that this event occurs first time at t, i.e., τ has a geometric probability distribution. Since $p = 1 - d, d = (1+r)^{-1}$, then the expected duration of τ, $E\tau = 1/p = 1 + 1/r$. Therefore, for the interest rate of 3.5%, $r = 0.035$, the expected duration is $E\tau \approx 30$ years, i.e., this rate orients the policy analysis on an expected 30-year time horizon. The standard deviation $\sigma = \sqrt{q/p}$, i.e., it equals approximately 30 years The bias in favor of the present in discounting with the rate of 3.5% is easily illustrated [24]. For a project with long-run benefits or costs, 1 Euro of benefits or costs in years 50, 100, and 200, has a present value respectively of 0.18, 0.003, and practically 0 Euros. Definitely, this rate may have no correspondence to how society has to deal with a 300-year flood, i.e., a flood with the expected arrival time equal to 300 years. Therefore, in the risk management τ can be associated with the arrival of potential catastrophic events rather than with horizons of market interests. The induced social discounting $d_t = P[\tau \geq t]$ in this case would have proper long-term perspectives dependent on spatio-temporal patterns of catastrophes and risk management decisions (see Proposition 4.3 and Sect. 4.4). The discount rate r can be viewed also as a killing (hazard) rate [15] which makes the life expectancy of an otherwise infinitely living representative agent or society equal to $1 + 1/r$ years. Yet, depending on a concrete situation, stopping time τ can be also associated with the arrival time of a reward.

Remark 4.2. (Variability of NPV). Disadvantages of this standard criterion (4.1) are well known [18]. In particular, the NPV critically depends on the prevailing interest rate which may not be easily defined in practice. In addition, the NPV does not reveal the temporal variability of cash flow streams. Two alternative streams may easily have the same NPV despite the fact that in one of them all the cash is clustered within a few periods, but in another it is spread out evenly over time. This type of temporal heterogeneity is critically important for dealing with catastrophic losses which occur suddenly as a "spike" in time and space [9].

The criterion $E\sum_{t=0}^{\tau} V_t, V_t = Ev_t$ has visible advantages. In particular, it allows to address distributional aspects and robust strategies [6] by analyzing the random variable $\sum_{t=0}^{\tau} V_t$ (even for deterministic $v_t = V_t$), e.g., its quantiles defined as maximal $y = y_\delta$ satisfying safety constraint

$$P\left[\sum_{t=0}^{\tau} v_t \geq y\right] \geq \delta.$$

Equivalently, y_δ maximizes the concave function (see discussion in [6], p. 16)

$$y + \delta^{-1} Emin\left\{0, \sum_{t=0}^{\tau} v_t - y\right\}.$$

The optimal value of this function defines the so-called CVaR (Conditional Value-at-Risk) risk measure [27].

Therefore, if variables v_t depend on some decisions x (as in Sect. 4.4), then the maximization of function

$$F(x) = \left[y + \delta^{-l} Emin\left\{0, \sum_{t=0}^{\tau} v_t - y\right\}\right].$$

allows easy control of highly nonlinear (even for linear in x function v_t) the safety constraints (quantiles of $\sum_{t=0}^{\tau} v_t$) in an optimal manner defined by a function $F(x)$ that is adjusted to CVaR risk measure (see also Sect. 4.5).

Remark 4.3. (Shock testing). The sensitivity of models w.r.t. "shocks" (extreme scenarios, events, stresses) is often assessed by introducing them into discounted criteria [22, 29]. From Proposition 4.1 it follows that this may lead to serious miscalculations. Let us consider criterion (4.1) with discount factors, $d_t = d^t, d = (1+r)^{-1}$ and assume that a "shock" arrives at a random time moment $\theta \in \{0, 1, \ldots\}$ with probability $P[\theta = t] = \pi\gamma^t, \gamma = 1 - \pi = (1 + \rho)^{-1}$. Then the expected value, $E\sum_{t=0}^{\theta} d_t V_t = \sum_{t=0}^{\infty} d^t \gamma^t V_t = E\sum_{t=0}^{\tau} \gamma^t V_t = E\sum_{t=0}^{min(\tau,\theta)} V_t$, where $P[\tau = t] = pq^t$ with $q = d$, $p = 1 - q$. Therefore, the stopping time of the "shocked" evaluation $E\sum_{t=0}^{\theta} d^t V_t$ is defined by $min(\tau, \theta)$. The discount rate of this evaluation is $(1 + r)^{-1} \cdot (1 + p)^{-1} = (1 + r + \rho + r\rho)^{-1}$, i.e., the shocked evaluation increases the rate of the original discounting and, hence, the bias in favor of the present.

Example 4.1. (Catastrophic Risk Management). The implications of Proposition 4.1 for long-term policy analysis are rather straightforward. Let us consider some important cases. It is realistic to assume [24] that the cash flow stream, typical for investment in a new nuclear plant, has the following average time horizons. Without a disaster the first six years of the stream reflect the costs of construction and commissioning followed by 40-years of operating life when the plant is producing positive cash flows and, finally, a 70-year period of expenditure on decommissioning. The flat discount rate of 5%, as Remark 4.1 shows, orients the analysis on a 20-year time horizon. It is clear that a lower discount rate places more weight on distant costs and benefits. For example, the explicit treatment of a potential 200-year disaster would require at least the discount rate of 0.5% instead of 5%. A related example is investments in climate change mitigations to cope with potential climate change related extreme events. Definitely, a rate of 3.5%, as often used in integrated assessment models [29], can easily illustrate that climate change does not matter. A shock testing of these models reduces even further their internal stopping time horizon.

Example 4.2. (Darwinian fitness). Ramsey [26] had introduced discounting, first of all, as a mathematical device ensuring the convergence of infinite horizon cumulative values. Its various explanations supported by empirical studies were proposed afterwards suggesting that humans and animals place less weights on the future then on the present [28]. A reason is that future rewards run more risk of disappearing. Hence, they should be discounted, where the discount rate is the hazard rate. For example, evidence from selection experiments indicates the existence of a trade-off between short-term and long-term fertility, i.e., the existence of life-history strategy that discounts the future. In other words, natural selection puts a premium on immediate reproductivity. Accordingly, an animal can be treated as a rational optimizer maximizing its Darwinian fitness, that can be taken to be equivalent to maximizing the expected number of offsprings. In a simple case, fitness is defined [28] then as integral $F = \int_0^\infty m(t)s(t)dt$, where $m(t)$ is the expected rate of reproductive output at age t if the animal survives to that age, and $s(t)dt$ is the probability of surviving to age t. It is highly unlikely that an animal is able to learn discount factors (probability density $s(t)$) in order to maximize the Darwinian fitness. The equivalent distribution free stopping time criterion requires observations of only lifetime intervals τ, which can be easily used for adaptive adjustments of life-history strategies.

4.3 Time Declining Discount Rates

This section extends Proposition 4.1 to general time declining discount rates. It also shows that a time declining discount rate can be associated even with a set of mutually exclusive geometrically distributed extreme (stopping time) events. This rate is determined in a sense by the least probable event.

Let us consider now a stream of random variables (r.v.) $v_0, v_1, \ldots$ affected by a set of random events including potential catastrophic events. Formally, we can think of v_t as a function $v_t(\omega)$ defined on a probability space $\{\Omega, P\}$ with the set Ω of related random events and the probability measure P on Ω. We assume that v_t does not depend on the "future", i.e., we assume that $\{\Omega, P\}$ is adapted to a sequence of increasing σ -algebras $A_0 \subseteq A_1 \subseteq \ldots$ (subsets of events from Ω, which occur before $t = 0, 1, \ldots$), such that v_t is measurable (defined on) w.r.t. A_t. In what follows, all random variables are assumed to be defined on $\{\Omega, P\}$.

Let $\sigma_{k,t} = \sigma(v_k, \ldots, v_t)$ be the σ-algebra generated by $v_k, \ldots, v_t$. Consider a stopping time τ, which we define as a r. v. $\tau \in \{0, 1, \ldots\}$, such that event, $\{\tau \leq t\}, t = 0, 1, \ldots$ does not depend on values $v_{t+1}, v_{t+2}, \ldots$, i.e., $\sigma_{t+1,\infty}$.

Proposition 4.2. *Consider a discounted sum* $\sum_{t=0}^{\infty} d_t V_t$, $d_t = (1 + r_t)^{-t}$, *where* r_t *is an increasing positive sequence,* $V_t = E v_t$. *Then there is a stopping time* τ *such that* $P[\tau \geq t] = d_t$ *and*

$$\sum_{t=0}^{\infty} d_t V_t = \sum_{t=0}^{\infty} P[\tau \geq t] E v_t = E \sum_{t=0}^{\tau} v_t. \tag{4.3}$$

Conversely, let $E \mid v_t \mid$ *is uniformly bounded. Then, for any stopping time* τ

$$E \sum_{t=0}^{\tau} v_t = \sum_{t=0}^{\infty} d_t V_t, d_t = P[\tau \geq t],$$

where V_t *is conditional expectation:*

$$V_t = E[v_t \mid \tau \geq t]$$

Proof. Consider such any r.v. τ,$\tau \in \{0, 1, \ldots\}$ that $\{\tau \leq t\}$ does not dependent on values $v_0, \ldots, v_{t-1}$ and $P[\tau = t] = d_t - d_{t+1}, t = 0, 1, 2, \ldots$. Clearly, $P[\tau \geq 0] = d_0 - d_1 + d_1 - d_2 + \ldots = d_0 = 1$, $P[\tau \geq t] = d_t$ and

$$\sum_{t=0}^{\infty} d_t V_t = \sum_{t=0}^{\infty} P[\tau \geq t] V_t.$$

Let now $f_t := \sum_{k=0}^{t} v_k$. From the rearrangement known as the Kolmogorov–Prohorov's theorem it follows that

$$E f_\tau = \sum_{t=0}^{\infty} E[f_t; \tau = t] = \sum_{t=0}^{\infty} \sum_{k=0}^{t} E[v_k; \tau = t] = \sum_{k=0}^{\infty} E[v_k; \tau \geq k]$$
$$= \sum_{k=0}^{\infty} P[\tau \geq k] V_k.$$

where $V_k = E[v_k \mid \tau \geq k]$ and $E[v_t; A]$, denotes unconditional expectation $E[v_t I_A]$, I_A is the indicator function of event A. The last assertion follows from the identity $\{\tau \geq t\} = \{\tau > t-1\}$, i.e., from the independence of $\{\tau \geq t\}$ on $\sigma_{t,\infty}$. The change in the order of sums is possible due to the uniform boundness of $E \mid v_t \mid$.

Corollary 4.1. *If $v_0, v_1, \ldots$ are independent r.v. or $\{\tau \geq t\}, t = 0, 1, 2, \ldots,$ does not depend on $v_0, v_1, \ldots, v_{t-1}$, then V_t in both cases of Proposition 4.2 is unconditional expectation $V_t = Ev_t$. If $v_0, v_1, \ldots$ are independent identically distributed r.v., then the Wald's identity follows from Proposition 4.1:*

$$E \sum_{t=0}^{\tau} v_t = E v_0 E_\tau.$$

Proof. It follows from the following rearrangements:

$$\sum_{t=0}^{\infty} P[\tau \geq t] = \sum_{k=0}^{\infty} \sum_{t=k}^{\infty} P[\tau = t] = \sum_{t=0}^{\infty} t P[\tau = t] = E\tau.$$

Example 4.3. (Expected catastrophic losses). Assume that a catastrophic event may occur at $t = 0, 1, 2, \ldots$ with probability p. It is usually defined as $(1/p)$-year event, say a 100-year flood. Define τ as the arrival time of the first catastrophe and let $v_t = 0, 0 \leq t \leq \tau - 1$, $v_\tau = L_\tau$, where L_τ is conditional expected losses given that the event occurs at τ. Since $l_t \neq 0$ only for, $t = \tau$, then the expected (unconditional) losses at τ are:

$$Ev_\tau = pL_0 + qpL_1 + q^2 pL_2 + \ldots = \sum_{t=0}^{\infty} q^t V_t = \sum_{t=0}^{\infty} P[\tau \geq t] V_t,$$

where $V_t = pL_t$.

The next proposition shows that a set of even geometrically distributed events can induce discounting with time declining discount rates. Let us assume that there is a set of mutually exclusive events (see also Sect. 4.4) of "magnitude" $i = 1, \ldots, n$. The probability of scenario i is $\theta_i, \sum_{i=1}^{n} \theta_i = 1$ and, conditional on this scenario, the event i occurs for the first time at τ_i with the probability $P[\tau_i = t] = p_i q_i^t, q_i = 1 - p_i, t = 0, 1, \ldots$. Thus, the occurrence of events at t is characterized by a mixed geometric distribution $\sum_{i=1}^{n} \theta_i p_i q_i^t$. Let τ be the arrival time of a first event. Then $d_t = P[\tau \geq t] = \sum_{i=1}^{n} \theta_i P[\tau_i \geq t]$. Since $P(\tau_i \geq t) = p_i q_i^t + p_i q_i^{t+1} + \ldots = q_i^t$, then evaluation (4.1) takes the form

$$V = \sum_{t=0}^{\infty} d_t V_t, d_t = \sum_{i=1}^{n} \theta_i q_i^t. \tag{4.4}$$

This equation essentially modifies the standard geometric discounting. Nevertheless, the induced discount factors d_t for large t tend to be defined by the smallest discount rate of the least probable event. The following proposition is similar to the conclusion in [31].

Proposition 4.3. *Discount factor $d_t = \sum_{i=1}^{n} \theta_i q_i^t$ in (4.4) is determined for $t \to \infty$ by the standard geometric discount factor $q_i^t *$ associated with the least probable event $i^\bullet$,*

$$p_{i*} = \min_i \ p_i \ : d_t / q_{i*}^t \to \theta_{i*} \text{ for } t \to \infty.$$

Proof. $d_t = q_{i*}^t \sum_{i=1}^{n} \theta_i \chi_i(t)$, where $\chi_i(t) = (q_i / q_i *)^t$. From $p_{i*} < p_i$, $p_i = 1 - q_i$, it follows that $\chi_i(t) \to 0, t \to \infty$, for $i \neq i^\bullet$ and $\chi_{i^\bullet}(t) = 1$. Hence, $d_t / q_{i*}^t \to \theta_{i*}$ for $t \to \infty$.

Remark 4.4. (Finite time horizon T). Propositions 4.1, 4.2, 4.3 hold true also for a finite time horizon $T < \infty$ after substituting probabilities $P[\tau = t]$, $P[\tau \geq t]$ by conditional probabilities $P[\tau = t \mid \tau \leq T]$ and $P[\tau \geq t \mid \tau \leq T]$.

Remark 4.5. (Distribution free approach). Propositions 4.1, 4.2 provide two alternative approaches for discounting: standard discounted criterion of the left-hand side of (4.1), (4.2) with an exogenous discounting, or undiscounted criterion of the right hand side with τ defined by random arrival time of stopping time events. Proposition 4.3 shows that the corresponding induced discounting $d_t = P[\tau \geq t]$ can be a complex implicit function of spatio-temporal patterns of events. The next section illustrates, that τ may depend also on various decisions. All these make it rather difficult to evaluate exact risk profiles $P[\tau \geq t]$ and exogenous discount factors d_t. Therefore, this would require the use of the distribution-free random stopping time criterion and STO methods rather then the standard distribution-based discounted criterion and deterministic optimization methods.

4.4 Endogenous Discounting

This section summarizes typical motivations for developing spatio-temporal catastrophic risk management models with rather natural versions of the stopping time concepts. A typical model may include often the following loop and the potential for positive feedbacks, branching and disequilibrium:

1. Stopping time induces discounting in the form of dynamic risk profiles $d_t = P[\tau \geq t]$.
2. The discounting affects optimal mitigation efforts.
3. Mitigation efforts affect the stopping time τ, risk profiles $P[\tau \geq t]$ and the discounting d_t (return to point 1).

This means that the stopping time criterion induces endogenous spatio-temporal endogenous discounting.

Example 4.4. (Evaluation of a Flood Management Program). Consider a simple version of the catastrophic flood management model developed for the Upper Tisza river region [9]. The spatio-temporal structure of this model was motivated by the following reasons.

Throughout the world, the losses from floods and other natural disasters are mainly absorbed by the immediate victims and their governments [13]. The insurance industry and its premium payers also absorb a portion of catastrophic losses, but even in the wealthy countries this share is relatively small. With increasing losses from floods, governments are concerned with escalating costs for flood prevention, flood response, compensation to victims, and public infrastructure repair. As a new policy, many officials would like to increase the responsibility of individuals and local governments for flood risks and losses [25], but this is possible only through location-specific analysis of risk exposures and potential losses, the mutual interdependencies of these losses, and the sensitivities of the losses to new risk management strategies.

This is a methodologically challenging task requiring at least the development of spatio-temporal catastrophe models [5, 8, 9, 30]. Although rich data usually exist on aggregate levels, the sufficient location specific data are not available, especially data relevant to new policies. Moreover, catastrophes affect large territories and communities producing mutually dependent losses with analytically intractable multidimensional probability distributions dependent also on various decisions. This critically distinguishes the arising problems from a standard risk management situations, e.g., the well-known asset-liability management. The standard methods, in particular, the existing extreme event theory, are not applicable to rational management of catastrophic risks. The new GIS-based catastrophe models [9,30] are needed to simulate the occurrence of potential extreme events and the samples of mutually dependent catastrophic losses for which no or very few historic observations exist.

In general, a catastrophe model represents the study region by grids, e.g., a relatively small pilot Upper Tisza region is represented by $1{,}500 \times 1{,}500$ grids [9]. Depending on the purpose of the study, these grids are aggregated into a much smaller number of cells (locations, compartments) $j = 1, 2, \ldots, m$. These cells may correspond to a collection of households at a certain site, a collection of grids with similar land-use characteristics, or an administrative district or grid with a segment of gas pipeline. The choice of cells provides a desirable representation of losses. Accordingly, cells are characterized by their content, in general, not necessarily in monetary units. Values can be measured in real terms, without using an aggregate dollar value. The content of cells is characterized by the vulnerability curves calculating random damages to crops, buildings, infrastructure, etc., under a simulated catastrophic scenario.

Catastrophic floods which are simulated by the catastrophe model, affect at random different cells and produce mutually dependent random losses L_j^t, $j = 1, \ldots, m$, from a catastrophic event at time t. These losses can be modified by various decisions. Some of the decisions reduce losses, say a dike, whereas others spread them on a regional, national, and international level, e.g., insurance contracts. If $x = (x_1, x_2, \ldots, x_n)$ is the vector of the decision variables, then L_j^t is a random function $L_j^t(x)$.

Flood occurrences in the region are modeled according to specified probabilistic scenarios of catastrophic rainfalls and the reliability of dikes. There are three dikes allocated along the region's river branch. Each of them may break after the occurrence at a random time of a 100-, 150-, 500-, and 1,000-year rainfall characterized by the so-called up-stream discharge curves calculating the amount of discharged water to the river branch per unit of time. In fact, the discharge curves upscale the information about complex rainfall and run-off processes affected by land-use and land-transformation policies. This brings considerable uncertainty in the definition of a $1/p$-year flood, $p = 1/100$, 1/150, 1/500,1/1,000. Therefore, a 100-year discharge curve may represent, in fact, a set of floods with different frequencies p, say, $1/150 \leq p \leq 1/100$. In addition to the interval, the uncertainty about p can be given by a prior distribution. Therefore, a single discharge curve, in general, corresponds to a set of $1/p$-year floods, where p is characterized by a prior probability distribution. For example, it can be characterized by a finite number of probabilistic scenarios $p_1, \ldots, p_n$ with prior probabilities $\theta_1, \ldots, \theta_n$ as in Proposition 4.3.

The stopping time can be defined differently, depending on the purpose of the policy analysis. A catastrophic flood in our example occurs due to the break of one of the three dikes. These events are considered as mutually exclusive events, since the break of a dike in the pilot region releases the "pressure" on other dikes. Therefore, the stopping time τ can be defined as the first time moment of a dike break. In this case, the probability or induced discount factor $d_t = P[\tau \geq t]$ is an implicit function of t, probabilities $\theta_i, p_i, i = 1, \ldots, n$, and the probability of a dike break. The situation is complicated further by the deterioration of dikes in time and/or by inappropriate maintenance of the flood protection system (see also Sect. 4.6), e.g., modifications to the dikes, the removal of some of them, and building new retention areas and reservoirs. Besides these structural decisions, the stopping time τ can be affected by other decisions, e.g., land use policies. Accordingly, depending on goals, the definition of stopping time τ can be further modified. For example, let us assume that the region [10] participates in the flood management program through payments to a mutual catastrophe fund, which has to support a flood protection system and compensates losses to victims. To enforce the participation in the program, the government provides only partial coverages of losses. The stability of this program critically depends on the insolvency of the fund that may require a new definition of τ. Let β be a fixed investment rate enabling the support of the system of dikes on a certain safety level and ξ be a random time of a first catastrophic flood. Denote by L_j^{ξ} random losses at location j, $j = \overline{1, m}$, at time $t = \xi$ and by π_j the premium rate paid by location j to the mutual catastrophe fund. Then, its accumulated risk reserve at time ξ together with a fixed partial compensation of losses $\chi \sum_j L_j^{\xi}$ by the government is $R_{\xi} = \xi \sum_j \pi_j + \chi \sum_j L_j^{\xi} - \sum_j \varphi_j L_j^{\xi} - \beta\xi$, where $0 \leq \varphi_j \leq 1$, is the portion of losses compensated by the fund at location j. Let us also assume that the functioning of the flood management program is considered as a long-term activity assuming that growth and aging processes compensate each other. Then, the insolvency of the fund is associated with the event:

$$\xi \sum_j \pi_j + \chi \sum_j L_j^{\xi} - \sum_j \varphi_j L_j^{\xi} - \beta\xi < 0. \tag{4.5}$$

Inequality (4.5) defines extreme random events affected by various feasible decisions x including components $(\pi_j, \varphi_j, \chi, b_j, \beta, j = \overline{1,m}$. The likelihood of event (4.5) determines the vulnerability of the program. It is more natural now to define the stopping time τ as the first time when event (4.5) occurs. In this case τ would depend on all components of vector x and the induced discounting would focus on time horizons associated with the occurrence of the event (4.5).

4.5 Dynamic Risk Profiles and CVaR Risk Measure

Example 4.5 illustrates that the probability distributions $P[\tau \geq t]$, $t = 0, 1, \ldots,$ itself represent key safety characteristics of catastrophic risk management programs. Induced discounting $d_t = P[\tau \geq t]$ then "controls" these risk profiles implicitly through their contributions to discounted goals of programs. Another possibility as this section shows is to impose explicitly safety constraints of the type $P[\tau \geq t] \geq \gamma_t$, for some safety levels $\gamma_t, t = 0, 1, \ldots$. In this case resulting robust strategies would directly control the safety constraints.

Example 4.5. (Safety constraints). The occurrence of disasters is often associated with the likelihood of some processes abruptly passing "vital" thresholds. This is a typical situation for insurance, where the risk process is defined similar to (4.5) by flows of premiums and claims whereas thresholds are defined by insolvency constraints. A similar situation arises in the control of environmental targets and in the design of disaster management programs [5,8,9]. Assume that there is a random process R_t and the threshold is defined by a random ρ_t. In spatial modeling, R_t and ρ_t can be large-dimensional vectors reflecting the overall situation in different locations of a region. Let us define the stopping time τ as the first time moment t when R_t is below ρ_t. By introducing appropriate risk management decisions x it is often possible to affect R_t and ρ_t in order to ensure the safety constraints $P[R_t \geq \rho_t] \geq \gamma$, for some safety level γ, or $\gamma_t, t = 0, 1, 2, \ldots$.

The use of this type safety constraints is a rather standard approach for coping with risks in the insurance, finance, and nuclear industries. For example, the safety regulations of nuclear plants assume that the violation of safety constraints may occur only once in 10^7 years, i.e., $\gamma = 1 - 10^{-7}$. It is remarkable that the use of stopping time criterion as in the right-hand side of (4.2) has strong connections with the dynamic safety constraints and dynamic versions of static CVaR risk measures [27]. Let us illustrate this by using the simplest version of climate change stabilization models discussed in [23].

Assume that $R_t = \sum_{k=0}^{t} x_k$, where decision variables

$$x_k \geq 0, k = 0, 1, \ldots, t, \qquad t \leq T < \infty.$$

We can consider x_k to be a CO_2 emission reduction at the beginning of period k. At time t the target value on total emission reduction R_t in period t is given as a random variable ρ_t. It is assumed that the exact value of ρ_t may be revealed at a random period τ, $P[\tau \geq t] = d_t$. The decision path $x = (x_0, x_1, \ldots, x_T)$ has to be chosen ex-ante in period $t = 0$ to mitigate climate change impacts associated with the case $R_\tau < \rho_\tau$. Consider the loss function associated with emission mitigation strategy x and given τ:

$$V(x) = E \sum_{t=0}^{\tau} [c_t x_t + b_t max\,\{0, \rho_t - R_t\}\, I_{t=\tau}], \tag{4.6}$$

where deterministic coefficients c_t can be viewed as marginal costs, and b_t as risk factors.

This can be written (Example 4.3) as

$$V(x) = \sum_{t=0}^{T} d_t \left[c_t x_t + b_t Emax \left\{ 0, \rho_t - \sum_{k=0}^{t} x_k \right\} \right].$$

Assume that $V(x)$ is a continuously differentiable function, e.g., a component of random vector $\rho = (\rho_0, \rho_1, \ldots, \rho_T)$ has a continuous density function. Also, assume for now that there exists a positive optimal solution $x^* = (x_0^*, x_1^*, \ldots, x_T^*)$, $x_t^* > 0$, minimizing $V(x)$ subject to $x_t \geq 0, t = 0, 1, \ldots, T$. Then, from the optimality condition for stochastic minimax problems (see discussions in [6], p. 16) it follows that for $x = x^*$,

$$V_{x_t} = c_t - \sum_{k=t}^{T} b_k P \left[\sum_{s=0}^{k} x_s \leq \rho_k \right] = 0, t = 0, 1, \ldots, T.$$

From this it follows sequentially for $t = T, T-1, \ldots, 0$,

$$P\left[\sum_{k=0}^{T} x_k \leq \rho T\right] = c_T/b_T,\; P\left[\sum_{k=0}^{t} x_k \leq \rho_t\right] = (c_t - c_{t+1})/b_t, t = 0, 1, \ldots, T-1. \tag{4.7}$$

Since $Emax\,\{0, \rho_t - R_t\} = E\rho_t I_{\rho_t \geq R_t} - R_t P[\rho_t \geq R_t]$, then from (4.7) it follows that $V(x^*) = Epb_\tau I_{\rho_\tau \geq R_t}$, which can be viewed as a dynamic CVaR (Conditional-Value-at-Risk) risk measure. Equations (4.7) can be used to control dynamic risk profiles, say, profiles with a given safety level γ as in Example 4.5:

$$1 - \gamma = c_T/b_T = (c_t - c_{t+1})/b_t t = 0, 1, \ldots, T-1,$$

by appropriate choice of risk factors b_t similar to stationary CVaR risk measures. In this case the minimization of (4.6) controls safety constraints (4.7) with given safety level γ, i.e.,

$$P\left[\sum_{k=0}^{t} x_k \leq \rho_k\right] = 1 - \gamma, \qquad t = 0, 1, \ldots, T. \tag{4.8}$$

This is a remarkable result, since the safety constraints, as a rule, are non-convex and even discontinuous, whereas the minimization of function (4.6) is often a convex problem for important practical cases.

Equations (4.7) are derived so far from the existence of the positive optimal solution x^*. The following proposition clarifies this assumption.

Proposition 4.4. *The existence of positive optimal solution follows from* $c_T/d_T < 1, (c_t - c_{t+1})/d_t < 1, t = 0, 1, \ldots, T-1$, *and the monotonicity of quantiles* $\beta_t, \beta_0 < \beta_1 < \ldots < \beta_T$ *defined by equations*

$$P[\beta_T \leq \rho_T] = c_T/d_T, P[\beta_t \leq \rho_t] = (c_t - c_{t+1})/d_t, t = 0, 1, \ldots, T-1$$

Proof. Indeed, the first requirement guarantees that $x_0^* > 0, \sum_{k=0}^{t} x_0^* > 0$, $t = 1, 2, \ldots, T$. From the second requirement it follows that $x_0^* + x_1^* > x_0^*$, $i.e., x_1^* > 0$, and so on.

Let us note that in general cases outlined in Example 4.5, process R_t is given by stochastic equations $R_{t+1} - R_t = g(t, x_t), t = 0, 1, \ldots, T-1$, where $g(t, x_t)$ is a random function. In this case (4.7), (4.8) would have a form of conditional expectation rather then quantiles. It is even easy to see for $g(t, x_t) = a_t x_t$, where a_t are random variables. In rather general cases a minimization problem (4.6) can be solved by distribution-free stochastic optimization methods proposed in [5,7–9], i.e., methods which don't use (in general) exact probability distributions.

Remark 4.6. (Robust decision). The stopping time τ in model (4.6) is not associated with the violation of safety constraint (4.8). In catastrophic risk management the model (4.6) is usually considered as an auxiliary submodel. For example, if random ρ_t are affected by a set of decisions y with a cost function $F(y)$, then the minimization of function $V(x) + F(y)$ yields robust decision minimizing total costs under safety constraints (4.8) and a dynamic version of the CVaR risk measure.

4.6 Intertemporal Inconsistency

The time consistency of discounting means that the evaluation of an investment project today ($t = 0$), will have the same discount factor as the evaluation of the same project after any time interval $[0, T]$ in the future. In other words, despite delayed implementation of the project we always found ourselves in the same environment. Only geometric or exponential discounting, $d_t = d^t = e^{ln(d)t} = e^{-\lambda t}$, where $\lambda = -ln(d)$, defines a homogeneous time consistent preference:

$$\sum_{t=0}^{\infty} d^t V_t = V_0 + dV_1 + \ldots + d^{T-1} V_{T-1} + d^T [V_T + dV_{T+1} + \ldots]$$

This is also connected with the geometric probability distribution of the discount-related stopping time τ in (4.2): if $P[\tau \geq t] = d^t, 0 < d < 1$, then

$$P[\tau = t] = d^t - d^{t+1} = (1-d)d^t, t = 0, 1, \ldots.$$

In other words, the consistency is the direct consequence of the well-known "memoryless" feature of geometric and exponential probability distributions: for any $t \geq 0, s \geq 0$,

$$P[\tau = t + s \mid \tau \geq t] = d^{t+s}(1-d)/d^t = d^s(1-d)$$

Hence, independently of waiting time t, the probability of the stopping time occurrence at $t + s$ is the same as at the initial time moment $t = 0$.

For other discount factors with time-dependent rates, their time inconsistency requires appropriate adjustments of discount factors for projects undertaken later rather than earlier. The misperception of this inconsistency may provoke increasing vulnerability and catastrophic losses. Let us consider typical scenarios of such developments. Section 4.4 shows that the adequate perception of proper discounting is a challenging task requiring models that allow the explicit evaluation of related risk profiles. This section, in fact, illustrates that the design of such models has to be considered as a key mitigation measure to cope with increasing vulnerability.

A number of authors distinguish between various types of so-called "imperfect altruism" resulting in the lack of social commitment to mitigate risks. For example, there were alluded definitions of a naive, a sophisticated and a committed (ideal) society. The main differences between these three societies and how they provoke catastrophes are summarized in [11] by using a simplified flood management model outlined in Sect. 4.4. This model has the fixed 100-year horizon T in which three societies, the naive, the sophisticated, and the committed, live and plan for coping with the catastrophic losses that may occur due to break of a dyke from 150-year flood with time consistent geometric probability distribution. They are able to mitigate the reliability of dikes and losses by paying fair premiums to the catastrophe fund. But, depending on their perception of risk profiles or induced discounting, the results are dramatically different.

The current generation of *The Naive Society* is aware of a possible catastrophe. It maximizes the (identical for all generations) value function taking into account the potential need to save for paying premiums. Unfortunately, it has a misleading view on the catastrophe, namely, if the catastrophe has not occurred in the later generation the society believes that it will not occur within the current generation with the same probability. Thus, it relies on geometric probability distribution and fails to take into account the time inconsistency induced by increasing the probability of a dike break due to aging processes. Therefore, the first generation of the society postpones the implementation of decisions, i.e., the naive society puts also its preferences on consumption as the first priority consuming at a higher rate than it actually plans.

For the next generation the time is shifted forward by 20 years, and the second generation, similar to the first, plans but does not implement saving actions essential for the catastrophe fund to function. The risk profiles, time preferences, premiums, and the actions are not adjusted towards the real escalating risks. In a similar way, behave the next generations. The plans are never implemented and the view on a catastrophe remains time invariant despite dramatic increase of risk.

The Sophisticated Society implies a correct understanding of the time-inconsistent discounting induced by the deteriorating system of dikes. But this society, similar to the naive planners, also evaluates present consumption to be much higher than the future one. This leads to postponing the decisions. Due to these delays, the risk burden is increasingly shifted to the next generation, calculated premiums become higher and higher. If a catastrophe occurs, this society will also be not prepared to cope with losses as catastrophe management is not functioning.

The "pathologies" of these societies can be explained by their misperception of risks, and, the lack of committed actions.

The Committed Society is similar to that of the sophisticated society. In contrast though, this society is able to implement decisions because its calculations demonstrate that the delays in actions may dramatically affect individuals and the growth of societies as a whole. Individuals could be better off if their consumption options were limited and their choices constrained by anticipating risks. As a direct consequence of the committed actions, the premiums that the society pays for coping with catastrophes in 100 years time are much lower than those of the sophisticated society.

4.7 Concluding Remarks

The proposed new approach to discounting is based on undiscounted stopping-time criterion which is equivalent to the standard discounted criterion in the case of market-related discount factors. In general, the stopping time criterion induces the discounting that depends on spatio-temporal patterns of catastrophes and various relevant decisions. More formally, this paper demonstrates that discount factors $d_t, t = 0, 1, \ldots$ can be associated with the occurrence of an extreme ("killing") "stopping time" event at random time τ with probability $P[\tau \geq t] = d_t$. Consequently, the infinite discounted sum $\sum_{t=0}^{\infty} d_t V_t$, $V_t = E v_t$, is replaced by the undiscounted expectation $E \sum_{t=0}^{\tau} v_t$ within the finite interval $[0, \tau]$. The use of the stopping time criterion $E \sum_{t=0}^{\tau} V_t$ induces the standard discounting in the case when τ is associated with the lifetime of market products. In dealing with catastrophic risks, the stopping time τ can be associated with the arrival time of potential catastrophic events. The use of random criterion $\sum_{t=0}^{\tau} v_t$ allows to address the variability of valuations even in the case of deterministic flows $V_0, V_1, \ldots$. In this case, it is often important to substitute the expected value of random sum $\sum_{t=0}^{\tau} v_t$ by its quantiles. Mitigation efforts affect the occurrence of extreme events and, thus, they affect discounting, which in turn affects mitigations. This endogeneity of discounting restricts exact evaluations of d_t and the consequent use of deterministic methods and it calls for specific stochastic optimization methods.

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