Levelings and Geodesic Reconstructions

Jose Crespo

GIB - LIA
DLSIIS
Facultad de Informática
Universidad Politécnica de Madrid
28660 Boadilla del Monte (Madrid), Spain
jcrespo@fi.upm.es

Abstract. This paper investigates some geodesic implementations that have appeared in the literature and that lead to connected operators. The focus is on two so-called self-dual geodesic transformations. Some fundamental aspects of these transformations are analyzed, such as whether they are actually levelings, and whether they can treat each grain or pore independently from the rest (connected-component locality). As will be shown, one of the geodesic self-dual reconstructions studied appears to be not a leveling. Nevertheless, it possesses a distinctive characteristic: it can process grains and pores in a connected-component local manner. The analysis is performed in the set or binary framework, although results and conclusions extend to (flat) gray-level operators.

Keywords: Levelings, geodesic operators, geodesic reconstructions.

1 Introduction

Levelings are a class of operators that are connected and that satisfy certain constraints. In the set or binary framework, levelings are called set or binary levelings. The analysis of this paper is performed in the set or binary framework, although results and conclusions extend to (flat) gray-level operators that commute with thresholding.

Geodesic transformations are a usual way to implement levelings. This paper investigates two so-called self-dual geodesic operations presented in the literature. In the analysis, the sequence of performing an under-reconstruction followed by an over-reconstruction (and vice-versa) is also considered.

In particular, this work investigates the following fundamental properties of them: (a) their leveling nature (i.e., if they are levelings), and (b) whether they treat each grain or pore independently from the rest (whether they are *connected-component local*). A significant finding, as will be shown, is that one self-dual geodesic reconstruction would not be a leveling. This fact does not necessarily implies that it is not useful; it has a distinctive characteristic that can make it interesting in certain applications. Researchers and users of geodesic reconstructions should know the properties of them.

M.H.F. Wilkinson and J.B.T.M. Roerdink (Eds.): ISMM 2009, LNCS 5720, pp. 82–91, 2009.

The paper is organized as follows. Section 2 provides some background, including some definitions of concepts related to connected operators and levelings. The two self-dual reconstructions (as well as the elementary geodesic transformations they are based on) are described in Section 3, and are analyzed in Section 4. Then, Section 5 concludes the paper.

2 Background

2.1 Basic Definitions

Some general references in the field of Mathematical Morphology are the following [1][2][3][4][5][6][7].

In the theoretical expressions in this paper, we will be working on the lattice $\mathcal{P}(E)$, where E is a given set of points (the space) and $\mathcal{P}(E)$ denotes the set of all subsets of E (i.e., $\mathcal{P}(E) = \{A : A \subseteq E\}$). Nevertheless, results can be extended to gray-level functions by means of the so called flat operators [2][6].

In this work we will deal later with the duality concept, which has a precise definition in Mathematical Morphology. Two morphological operators ψ_1 and ψ_2 are *dual* of each other if $\psi_1 = \complement\psi_2\complement$, where $\complement$ symbolizes the complement operator. As a particular case, a morphological operator ψ is said to be *self-dual* if $\psi = \complement\psi\complement$.

Connected operators do not introduce discontinuities and extend partitions in the sense that the partition of the output is *coarser* that that of the input [8] [9]. For binary images (or sets), they treat the connected components of the input and its complement in an all or nothing way. The operator that extracts the connected-component a point x belongs to is the opening γ_x [3]. In this work, the space connectivity is assumed to be a strong connectivity [10][11], which avoids the existence of isolated grains and pores. More particularly, connected subsets of $\mathbb{Z}^2$ with four- or eight-connectivity are used as the space E of points in this paper. Connected operations can be considered as graph operations. Image representations based on inclusion trees can be useful [12].

The following two sections define two constraints that are particularly useful for studying the connected operator class: *connected-component (c.c.) locality* and *adjacency stability.*

2.2 Connected-Component Locality

Definition 1. *[13][14][15] Let E be a space endowed with γ_x, $x \in E$. An operator $\psi : \mathcal{P}(E) \longrightarrow \mathcal{P}(E)$ is said to be* **connected-component local** *(or* **c.c.-local***) if and only if, $\forall A \in \mathcal{P}(E)$:*

- *ψ preserves (or, respectively, removes) a non-empty grain G of A in operation $\psi(A)$ if and only if ψ preserves (respectively, removes) grain G in operation $\psi(G)$.*
- *ψ preserves (or, respectively, fills) a non-empty pore P of A in operation $\psi(A)$ if and only if ψ preserves (respectively, fills) pore P in operation $\psi(E \setminus P)$.*

Where "\" denotes the set subtraction operation. (Note that connected operators just preserve or remove grains, and preserve or fill pores.)

Thus, a c.c.-local connected operator is one that (1) fills grains and/or remove pores, and (2) treats each grain or pore independently from the rest of grains and pores. The connected-component local operator concept was also later discussed in [11], where the term "grain-operator" is used.

2.3 Adjacency Stability

The adjacency stability constraint restrains in some way the behavior of adjacent flat zones, in particular the switch from grain to pore and vice-versa. The adjacency stability concept was introduced in [13][14], and was further studied in [15]. A related concept and formulation are discussed in [11].

Definition 2. *Let E be a space endowed with γ_x, $x \in E$. An operator $\psi : \mathcal{P}(E) \longrightarrow \mathcal{P}(E)$ is* **adjacency stable** *if, for all $x \in E$:*

$$\gamma_x(\mathrm{id} \bigvee \psi) = \gamma_x \bigvee \gamma_x \psi. \tag{1}$$

Property 1. Extensive and anti-extensive mappings are adjacency stable.

The next property states the composition laws of adjacency stable connected operators, and, conjointly with Property 1, provides a way to build operators of this class.

Property 2. The class of adjacency stable connected operators is closed under the sup, the inf and the sequential composition operations.

2.4 Levelings

Definition 3. *An image g is a leveling of an input image f if and only if:*

$$\forall\ (p, q) \text{ neighboring pixels} : g_p > g_q \ \Rightarrow\ f_p \geq g_p \text{ and } g_q \geq f_q \tag{2}$$

The previous definition of leveling is that in [16, Definition 4, p. 193] [17, Definition 2.2, p. 4]. A more general definition is introduced in [18, Definition 10, p. 62].

Set levelings are those defined in the set or binary framework. As discussed in [19], the leveling concept is equivalent to the adjacency stability connected operator concept that was presented in [13][14][15], which therefore constitute the origin of the leveling concept in the set or binary framework. Composition laws of set levelings (which are extended to and satisfied by flat gray-level levelings as well) can be found in [13][14][15] (see Property 1 and Property 2). Regarding some clarifications about whether levelings satisfy the strong property, see [19][20]. Levelings are useful operators for image filtering that simplify an image while imposing input-output restrictions, and that can be computed using compositions of morphological connected operators (from Properties 1 and 2).

Other works about levelings are [21][22][23][24].

3 Geodesic Reconstructions: Definitions and Formulae

The geodesic reconstructions that will be investigated in this work are defined in the following. This work will focus specially on the R_ν and $\mathrm{R}_{\nu'}$ self-dual geodesic reconstructions.

- **Reconstruction $\mathbf{R}_\nu$**
  The R_ν self-dual reconstruction is based on the elementary self-dual geodesic operator ν. (Note: the description follows the presentation in [6, Section 6.1.3], although the notation varies in minor details.)

$$[\nu^g_{(1)}(f)](x) = \begin{cases} (\delta^g_{(1)}(f))(x), & \text{if } f(x) \le g(x) \\ (\varepsilon^g_{(1)}(f))(x), & \text{otherwise} \end{cases} \tag{3}$$

  where $\delta^g_{(1)}(f) = \delta_1(f) \wedge g$, and $\varepsilon^g_{(1)}(f) = \varepsilon_1(f) \vee g$. The mask image is denoted by g, and the marker image is symbolized by f.
  The $\nu^g_{(1)}(f)$ operator can be equivalently expressed [6] as $\varepsilon_1(f) \vee \delta^g_{(1)}(f)$ or $\delta_1(f) \wedge \varepsilon^g_{(1)}(f)$ (which would follow from leveling expressions presented in [16][25]).
  The corresponding transformation of size n is $\nu^g_{(n)}(f) = \nu^g_{(1)}(\nu^g_{(n-1)}(f))$. Reconstruction R_ν denotes the iteration of ν until idempotence:

$$\mathrm{R}_\nu(g; f) = \nu^g_{(i)}(f) \tag{4}$$

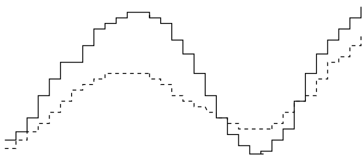
  where i is such that $\nu^g_{(i+1)}(f) = \nu^g_{(i)}(f)$.
- **Reconstruction $\mathbf{R}_{\nu'}$**
  In [6, Section 6.1.3], the next self-dual transformation variant is also presented:

$$[\nu'^g_{(1)}(f)](x) = \begin{cases} (\delta^g_{(1)}(f \wedge g))(x), & \text{if } f(x) \le g(x) \\ (\varepsilon^g_{(1)}(f \vee g))(x), & \text{otherwise} \end{cases} \tag{5}$$

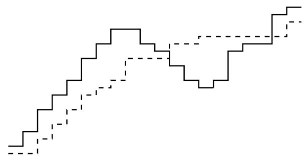
  $\nu'^g_{(n)}(f) = \nu'^g_{(1)}(\nu'^g_{(n-1)}(f))$, and the reconstruction based on $\nu'^g_{(1)}$ until idempotence is:

$$\mathrm{R}_{\nu'}(g; f) = \nu'^g_{(i)}(f) \tag{6}$$

  where i is such that $\nu'^g_{(i+1)}(f) = \nu'^g_{(i)}(f)$.
- **Under-reconstruction $\underline{\mathbf{R}}$ and over-reconstruction $\overline{\mathbf{R}}$**
  Let $\underline{\mathrm{R}}(g; f)$ denote the normal under-reconstruction or reconstruction by dilation (i.e., the iteration of $\delta^g_{(1)}(f)$ until idempotence), where $f \le g$. Let $\overline{\mathrm{R}}(g; f)$ symbolize the normal over-reconstruction or reconstruction by erosion (i.e., the iteration of $\varepsilon^g_{(1)}(f)$ until idempotence), where $f \ge g$.
  In the work of the paper, we will also consider the sequential compositions of $\underline{\mathrm{R}}$ and $\overline{\mathrm{R}}$ (i.e., $\overline{\mathrm{R}} \circ \underline{\mathrm{R}}$ and $\overline{\mathrm{R}} \circ \underline{\mathrm{R}}$) to complete the analysis and comparisons.



(a) Case study 1



(a) Case study 2

Fig. 1. Case studies. Input functions (continuous line) and marker functions (dotted line) used in the analysis. Note: the space of points is discrete.

4 Analysis of Geodesic Reconstructions $\mathbf{R}_\nu$ and $\mathbf{R}_{\nu'}$

In this section, we will analyze the geodesic reconstructions defined in the previous section on two different cases: one that can be referred to as "normal or non-problematic", and another one that has some adjacency issues. The two cases are displayed in Fig. 1, where the input functions are displayed as continuous lines, and the marker function as dotted lines:

4.1 On the Leveling Nature

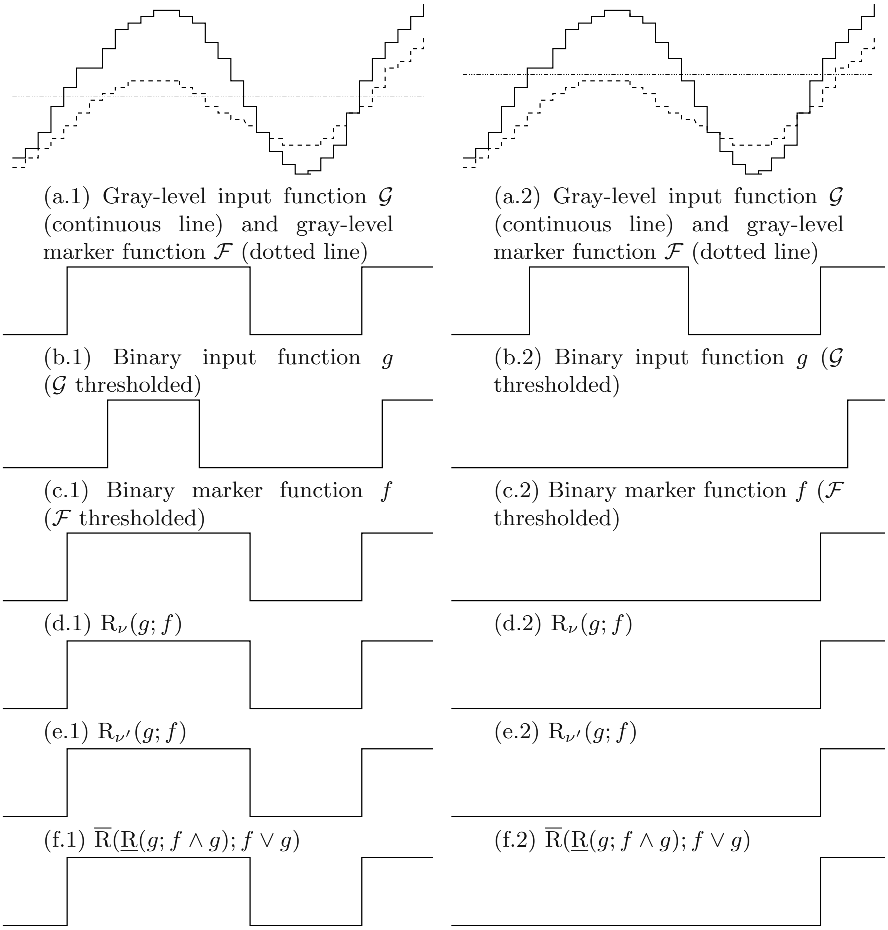
We will first use a simple example of the application of the geodesic reconstructions defined in Section 3 to a 1-D gray-level function and an associated marker, as displayed in Fig. 2. In fact, we will perform a thresholding operation to operate on a section to better illustrate the behaviors of the geodesic reconstructions regarding adjacency matters.

We can observe that all geodesic reconstructions considered compute the same result in the example of Fig. 2 at each level. In the case at the left of Fig. 2, a grain is marked and reconstructed (see Fig. 2(d.1)-(g.1)); in the right part, that same grain is removed (see Fig. 2(d.2)-(g.2)). No differences are observed.

We apply next the geodesic reconstructions considered in this paper to a case that shows some adjacency issues in Fig. 3. Two levels are considered: the first one, at the left part of Fig. 3, does not present any problem and all geodesic reconstructions compute the same result. However, the level at the right part of the figure poses some adjacency issues, and, as can be observed in Fig. 3(d.2)-(g.2), *not all geodesic reconstructions considered behave the same.*

In fact, the result shown in Fig. 3(c.2) computed by $\mathrm{R}_{\nu'}$ shows the behavior that is in fact excluded by the leveling nature: a grain has been removed and an adjacent pore has been filled. The adjacency stability equation (1) (or expression (2)) is not satisfied. Thus, based on expression (5) by itself[1], **the $\mathbf{R}_{\nu'}$ reconstruction is**

[1] We mean without imposing restrictions on the marker. As a matter of fact, in relation to this issue, it can be mentioned that the marker-based formulation (which does not get into the details of concrete geodesic implementations) of a set leveling in [22] poses some constraints on the marker (more exactly, on the markers, since it considers two) to take into account the adjacency constraints derived from the adjacency stability equation (1) (or expression (2)).



(a.1) Gray-level input function $\mathcal{G}$ (continuous line) and gray-level marker function $\mathcal{F}$ (dotted line)

(a.2) Gray-level input function $\mathcal{G}$ (continuous line) and gray-level marker function $\mathcal{F}$ (dotted line)

(b.1) Binary input function g ($\mathcal{G}$ thresholded)

(b.2) Binary input function g ($\mathcal{G}$ thresholded)

(c.1) Binary marker function f ($\mathcal{F}$ thresholded)

(c.2) Binary marker function f ($\mathcal{F}$ thresholded)

(d.1) $\mathrm{R}_{\nu}(g; f)$

(d.2) $\mathrm{R}_{\nu}(g; f)$

(e.1) $\mathrm{R}_{\nu'}(g; f)$

(e.2) $\mathrm{R}_{\nu'}(g; f)$

(f.1) $\overline{\mathrm{R}}(\underline{\mathrm{R}}(g; f \wedge g); f \vee g)$

(f.2) $\overline{\mathrm{R}}(\underline{\mathrm{R}}(g; f \wedge g); f \vee g)$

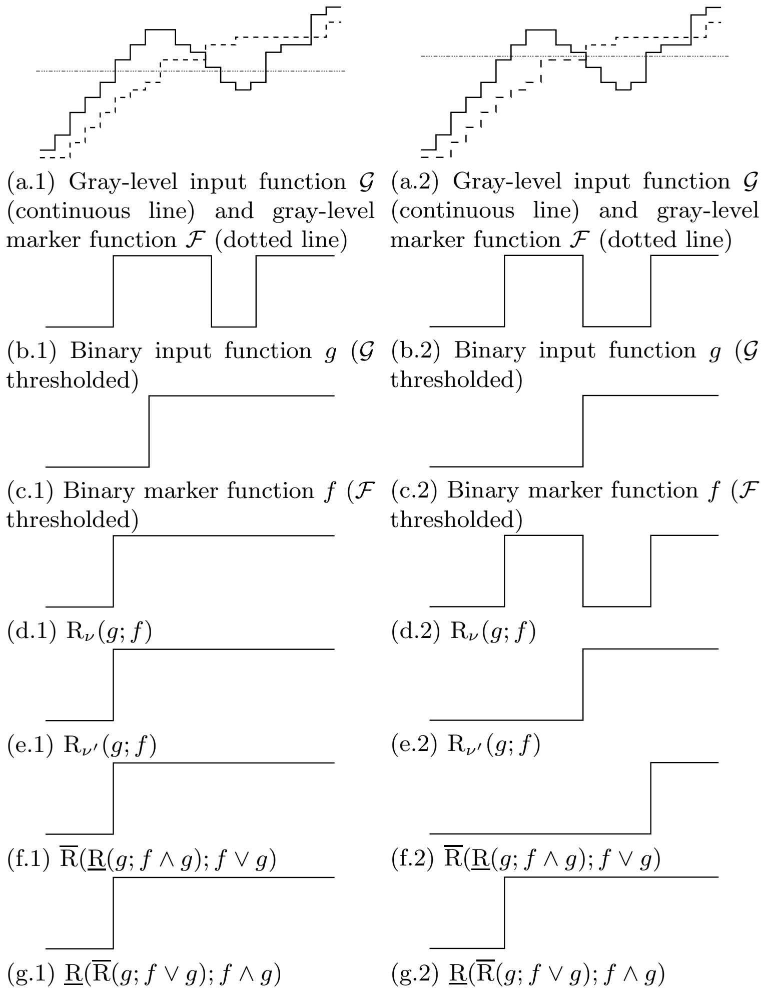
(g.1) $\underline{\mathrm{R}}(\overline{\mathrm{R}}(g; f \vee g); f \wedge g)$

(g.2) $\underline{\mathrm{R}}(\overline{\mathrm{R}}(g; f \vee g); f \wedge g)$

Fig. 2. Geodesic reconstructions for case study 1 at two levels. Part (a) shows the input gray-level function $\mathcal{G}$ (continuous line) and marker $\mathcal{F}$ (dotted line), along with the thresholding level (horizontal discontinuous line) used for parts (b) and (c), which show, respectively, the binary input function g and marker f employed for the reconstructions displayed in parts (d) and (e). Note: the left and right parts of the figure refer to different thresholding levels; the space of points is discrete.

not generally a leveling. The previous statement can depend on the particular space: it could be the case that a certain self-dual grain removing and pore filling operation is not a leveling in a certain space but it is in a subset or a superset of it (see further discussion about some of these aspects in [15]).

The other reconstructions considered, $\mathrm{R}_{\nu}(g; f)$, $\overline{\mathrm{R}}(\underline{\mathrm{R}}(g; f \wedge g); f \vee g)$ and $\underline{\mathrm{R}}(\overline{\mathrm{R}}(g; f \vee g); f \wedge g)$ do not show those adjacency issues. Nevertheless, in the situation at the right of Fig. 3 $\overline{\mathrm{R}}(\underline{\mathrm{R}}(g; f \wedge g); f \vee g)$ is not equal to $\underline{\mathrm{R}}(\overline{\mathrm{R}}(g; f \vee g); f \wedge g)$


(a.1) Gray-level input function $\mathcal{G}$ (continuous line) and gray-level marker function $\mathcal{F}$ (dotted line)

(a.2) Gray-level input function $\mathcal{G}$ (continuous line) and gray-level marker function $\mathcal{F}$ (dotted line)

(b.1) Binary input function g ($\mathcal{G}$ thresholded)

(b.2) Binary input function g ($\mathcal{G}$ thresholded)

(c.1) Binary marker function f ($\mathcal{F}$ thresholded)

(c.2) Binary marker function f ($\mathcal{F}$ thresholded)

(d.1) $\mathrm{R}_{\nu}(g; f)$

(d.2) $\mathrm{R}_{\nu}(g; f)$

(e.1) $\mathrm{R}_{\nu'}(g; f)$

(e.2) $\mathrm{R}_{\nu'}(g; f)$

(f.1) $\overline{\mathrm{R}}(\underline{\mathrm{R}}(g; f \wedge g); f \vee g)$

(f.2) $\overline{\mathrm{R}}(\underline{\mathrm{R}}(g; f \wedge g); f \vee g)$

(g.1) $\underline{\mathrm{R}}(\overline{\mathrm{R}}(g; f \vee g); f \wedge g)$

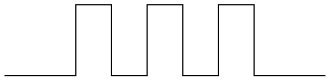
(g.2) $\underline{\mathrm{R}}(\overline{\mathrm{R}}(g; f \vee g); f \wedge g)$

Fig. 3. Geodesic reconstructions for case study 2 at two levels. Part (a) shows the input gray-level function $\mathcal{G}$ (continuous line) and marker $\mathcal{F}$ (dotted line), along with the thresholding level (horizontal discontinuous line) used for parts (b) and (c), which show, respectively, the binary input function g and marker f employed for the reconstructions displayed in parts (d) and (e). Note: the left and right parts of the figure refer to different thresholding levels; the space of points is discrete.

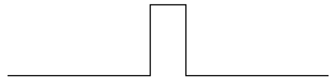
(see Fig. 3(f.2) and Fig. 3(g.2)), and they should generally be avoided if self-dual processing is desired.

4.2 On Connected-Component Locality

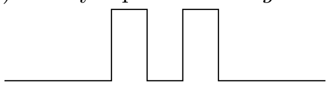
A complete study of the c.c.-locality of a marker-based connected operator implemented using reconstruction transformations (self-dual or not) would obviously



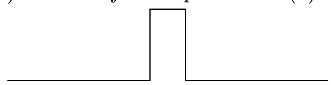
(a) Binary input function g



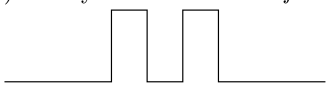
(d) Marker f' computed on (c)



(b) Binary marker function f



(e) $\mathrm{R}_{\nu'}(\mathrm{R}_{\nu'}(g; f); f')$



(c) $\mathrm{R}_{\nu'}(g; f)$

Fig. 4. Iteration of $\mathrm{R}_{\nu'}$. Note that part (c) is different from part (e) (i.e., $\mathrm{R}_{\nu'}(g; f) \neq \mathrm{R}_{\nu'}(\mathrm{R}_{\nu'}(g; f); f')$).

need to take into account the marker computation. In the following, to simplify the treatment, we will focus on the reconstruction transformations themselves (expressions (3) and (5)), except when a second sequential application is commented where the marker computation stage is also considered.

By examining the behaviors of the R_{ν} and $\mathrm{R}_{\nu'}$ self-dual reconstructions in the example of the right part of Fig. 3, we can see that the treatment of a grain (or respectively, a pore) in Fig. 3(d.2) has been influenced by an adjacent pore (respectively, a grain). Thus, reconstruction R_{ν} is not c.c.-local. Note that the geodesic operations themselves of R_{ν} make it non c.c.-local (even when the marker is c.c.-local).

Regarding the alternative reconstruction $\mathrm{R}_{\nu'}$, those issues do not arise: a grain that has not been marked would be removed by $\mathrm{R}_{\nu'}$ disregarding what happens at the adjacent pores. Analogously for pores. Thus, **$\mathbf{R}_{\nu'}$ can be used as a basis for c.c.-local connected processing**.

We will briefly comment that the switching of adjacent grains and pores that happens in non levelings (such as $\mathrm{R}_{\nu'}$) is normally linked to non-idempotent behavior when c.c.-local processing is desired. This is illustrated in Fig. 4, where two iterations of $\mathrm{R}_{\nu'}$ are applied to the input image Fig. 4(a) using Fig. 4(b) as marker, where three grains of Fig. 4(a) are signaled to be removed and two pores are signaled to be filled. Let us assume that the marker criterion is c.c.-local. Fig. 4(c) displays $\mathrm{R}_{\nu'}(g; f)$. If we apply the previous operation again, the marker computed on Fig. 4(c) is Fig. 4(d). We can see that $\mathrm{R}_{\nu'}(\mathrm{R}_{\nu'}(g; f); f')$ (Fig. 4(e)) is different from $\mathrm{R}_{\nu'}(g; f)$ (Fig. 4(c)), in other words, $\mathrm{R}_{\nu'}$ does not show idempotence. As was the case when considering the leveling nature, this can depend on the particular space.

5 Conclusion

This paper has investigated some fundamental issues that exist with geodesic reconstructions, and, particularly, the emphasis of the analysis has been on a pair of self-dual reconstructions that have appeared in the literature. It is important that researchers and users of geodesic reconstructions know the distinctive properties and characteristics of them.

The focus of the analysis has been on: (a) whether the geodesic transformations are levelings; and (b) whether they can be used for building connected-component local operators.

As has been found out, one of them is not generally a leveling. This operator possesses a characteristic that makes it interesting for certain situations, as discussed in the paper: it can be used as basis for processing grains and pores independently from the rest of grains and pores, i.e., for building connected-component local connected operators.

Acknowledgements

This work has been supported in part by "Ministerio de Ciencia e Innovación" of Spain (Ref.: TIN2007-61768).

References

1. Matheron, G.: Random Sets and Integral Geometry. Wiley, New York (1975)
2. Serra, J.: Mathematical Morphology, vol. I. Academic Press, London (1982)
3. Serra, J. (ed.): Mathematical Morphology. Theoretical Advances, vol. II. Academic Press, London (1988)
4. Heijmans, H.: Morphological Image Operators. In: Hawkes, P. (ed.) Advances in Electronics and Electron Physics. Academic Press, Boston (1994)
5. Banon, G.: Formal introduction to digital image processing. INPE, São José dos Campos (2000)
6. Soille, P.: Morphological Image Analysis, 2nd edn. Springer, Heidelberg (2003)
7. Dougherty, E., Lotufo, R.: Hands-on Morphological Image Processing. SPIE Press, Bellingham (2003)
8. Serra, J., Salembier, P.: Connected operators and pyramids. In: Proceedings of SPIE, Non-Linear Algebra and Morphological Image Processing, San Diego, July 1993, vol. 2030, pp. 65–76 (1993)
9. Salembier, P., Serra, J.: Flat zones filtering, connected operators, and filters by reconstruction. IEEE Transactions on Image Processing 4(8), 1153–1160 (1995)
10. Ronse, C.: Set-theoretical algebraic approaches to connectivity in continuous or digital spaces. Journal of Mathematical Imaging and Vision 8(1), 41–58 (1998)
11. Heijmans, H.: Connected morphological operators for binary images. Computer Vision and Image Understanding 73, 99–120 (1999)
12. Monasse, P., Guichard, F.: Fast computation of a contrast-invariant image representation. IEEE Trans. on Image Proc. 9(5), 860–872 (2000)
13. Crespo, J., Serra, J., Schafer, R.W.: Image segmentation using connected filters. In: Serra, J., Salembier, P. (eds.) Workshop on Mathematical Morphology, Barcelona, May 1993, pp. 52–57 (1993)
14. Crespo, J.: Morphological Connected Filters and Intra-Region Smoothing for Image Segmentation. PhD thesis, School of Electrical and Computer Engineering, Georgia Institute of Technology (December 1993)

15. Crespo, J., Schafer, R.W.: Locality and adjacency stability constraints for morphological connected operators. Journal of Mathematical Imaging and Vision 7(1), 85–102 (1997)
16. Meyer, F.: From connected operators to levelings. In: Heijmans, H.J.A.M., Roerdink, J.B.T.M. (eds.) Mathematical morphology and its applications to image and signal processing, pp. 191–198. Kluwer Academic Publishers, Dordrecht (1998)
17. Meyer, F., Maragos, P.: Nonlinear scale-space representation with morphological levelings. Journal of Visual Communication and Image Representation 11(3), 245–265 (2000)
18. Meyer, F.: Levelings, image simplification filters for segmentation. Journal of Mathematical Imaging and Vision 20(1-2), 59–72 (2004)
19. Crespo, J.: Adjacency stable connected operators and set levelings. In: Banon, G.J.F., Barrera, J., Braga-Neto, U.d.M., Hirata, N.S.T. (eds.) Proceedings of the 8th International Symposium on Mathematical Morphology 2007 - ISMM 2007, October 2007. São José dos Campos, Universidade de São Paulo (USP), Instituto Nacional de Pesquisas Espaciais (INPE), vol. 1, pp. 215–226 (2007)
20. Crespo, J., Maojo, V.: The strong property of morphological connected alternated filters. Journal of Mathematical Imaging and Vision 32(3), 251–263 (2008)
21. Matheron, G.: Les nivellements. Technical Report N-54/99/MM, Report Centre de Morphologie Mathmatique, E.N.S. des Mines de Paris (February 1997)
22. Serra, J.: Connections for sets and functions. Fundamenta Informaticae 41(1-2), 147–186 (2000)
23. Maragos, P.: Algebraic and PDE approaches for lattice scale-spaces with global constraints. International Journal of Computer Vision 52(2/3), 121–137 (2003)
24. Serra, J., Vachier-Mammar, C., Meyer, F.: Nivellements. In: Najman, L., Talbot, H. (eds.) Morphologie mathématique 1: approches déterministes, pp. 173–200. Lavoisier, Paris (2008)
25. Meyer, F.: The levelings. In: Heijmans, H.J.A.M., Roerdink, J.B.T.M. (eds.) Mathematical morphology and its applications to image and signal processing, pp. 199–206. Kluwer Academic Publishers, Dordrecht (1998)