

On the Expressiveness of Forwarding in Higher-Order Communication*

Cinzia Di Giusto, Jorge A. Pérez, and Gianluigi Zavattaro

Dipartimento di Scienze dell'Informazione, Università di Bologna, Italy

Abstract. In higher-order process calculi the values exchanged in communications may contain processes. There are only two capabilities for received processes: execution and forwarding. Here we propose a limited form of forwarding: output actions can only communicate the parallel composition of statically known closed processes and processes received through previously executed input actions. We study the expressiveness of a higher-order process calculus featuring this style of communication. Our main result shows that in this calculus termination is decidable while convergence is undecidable.

1 Introduction

Higher-order process calculi are calculi in which processes can be communicated. They have been put forward in the early 1990s, with CHOCS [1], Plain CHOCS [2], the Higher-Order π -calculus [3], and others. Higher-order (or process-passing) concurrency is often presented as an alternative paradigm to the first order (or name-passing) concurrency of the π -calculus for the description of mobile systems. These calculi are inspired by, and formally close to, the λ -calculus, whose basic computational step — β -reduction — involves term instantiation. As in the λ -calculus, a computational step in higher-order calculi results in the instantiation of a variable with a term, which is then copied as many times as there are occurrences of the variable.

HOCORE is a core calculus for higher-order concurrency, recently introduced in [4]. It is *minimal*, in that only the operators strictly necessary to obtain higher-order communications are retained. This way, continuations following output messages have been left out, so communication in HOCORE is asynchronous. More importantly, HOCORE has no restriction operator. Thus all channels are global, and dynamic creation of new channels is impossible. This makes the absence of recursion also relevant, as known encodings of fixed-point combinators in higher-order process calculi require the restriction operator. The grammar of HOCORE processes is:

$$P ::= a(x).P \mid \bar{a}\langle P \rangle \mid P \parallel P \mid x \mid \mathbf{0} \quad (*)$$

An input prefixed process $a(x).P$ can receive on name (or channel) a a process to be substituted in the place of x in the body P ; an output message $\bar{a}\langle P \rangle$ can send P (the output object) on a ; parallel composition allows processes to interact. Despite this

* Research partially funded by EU Integrated Projects HATS (contract number 231620) and SENSORIA (contract number 016004).

minimality, via a termination preserving encoding of Minsky machines [5], HOCORE was shown to be Turing complete. Therefore, in HOCORE, properties such as *termination* (i.e. non existence of divergent computations) and *convergence* (i.e. existence of a terminating computation) are both undecidable. In contrast, somewhat surprisingly, strong bisimilarity is decidable, and several sensible bisimilarities coincide with it.

In this paper, we shall aim at identifying the intrinsic source of expressive power in HOCORE. A substantial part of the expressive power of a concurrent language comes from the ability of accounting for infinite behavior. In higher-order process calculi there is no explicit operator for such a behavior, as both recursion and replication can be encoded. We then find that infinite behavior resides in the interplay of higher-order communication, in particular, in the ability of *forwarding* a received process within an *arbitrary context*. For instance, consider the process $R = a(x).\bar{b}\langle P_x \rangle$ (here P_x stands for a process P with free occurrences of a variable x). Intuitively, R receives a process on name a and forwards it on name b . It is easy to see that since objects in output actions are built following the syntax given by $(*)$, the actual structure of P_x can be fairly complex. One could even “wrap” the process to be received in x using an arbitrary number of k “output layers”, i.e., by letting $P_x \equiv \bar{b}_1\langle \bar{b}_2\langle \dots \bar{b}_k\langle x \rangle \dots \rangle \rangle$. This *nesting capability* embodies a great deal of the expressiveness of HOCORE: as a matter of fact, the encoding of Minsky machines in [4] depends critically on nesting-based counters. Therefore, investigating suitable limitations to the kind of processes that can be communicated in an output action appears as a legitimate approach to assess the expressive power of higher-order concurrency.

With the above consideration in mind, in this paper we propose HO^{-f} , a sublanguage of HOCORE in which output actions are limited so as to rule out the nesting capability (Section 2). In HO^{-f} , output actions can communicate the parallel composition of two kinds of objects: (i) statically known closed processes (i.e. that do not contain free variables), and (ii) processes received through previously executed input actions. Hence, the context in which the output action resides can only contribute to communication by “appending” pieces of code that admit no inspection, available in the form of a black-box. More formally, the grammar of HO^{-f} processes is that in $(*)$, except for the production for output actions, which is replaced by the following one:

$$\bar{a}\langle x_1 \parallel \dots \parallel x_k \parallel P \rangle$$

where $k \geq 0$ and P is a closed process. This modification directly restricts forwarding capabilities for output processes, which in turn, leads to a more limited structure of processes along reductions.

The limited style of higher-order communication enforced in HO^{-f} is relevant from a pragmatic perspective. In fact, communication in HO^{-f} is inspired by those cases in which a process P is communicated in a translated format $\llbracket P \rrbracket$, and the translation is not compositional. That is, the cases in which, for any process context C , the translation of $C[P]$ cannot be seen as a function of the translation of P , i.e. there exists no context D such that $\llbracket C[P] \rrbracket = D[\llbracket P \rrbracket]$. This setting can be related to several existing programming scenarios. The simplest example is perhaps mobility of already compiled code, on which it is not possible to apply inverse translations (such as reverse engineering). Other examples include *proof-carrying code* [6] and communication of *obfuscated code* [7]. The former features communication of executable code that comes with a certificate: a recipient can only check the certificate and decide whether to execute the code

or not. The latter consists of the communication of source code that is made difficult to understand for, e.g., security/copyright reasons, while preserving its functionality.

The main contribution of the paper is the study of the expressiveness of HO^{-f} in terms of decidability of termination and convergence. Our main results are:

1. Similarly as HOCORE, HO^{-f} is Turing complete (Section 3). The calculus thus retains a significant expressive power despite of the limited forwarding capability. This result is obtained by exhibiting an encoding of Minsky machines.
2. In sharp contrast with HOCORE, termination in HO^{-f} is *decidable* (Section 4). This result is obtained by appealing to the theory of well-structured transition systems [8], following the approach used in [9].

As for (1), it is worth commenting that the encoding is not *faithful* in the sense that, unlike the encoding of Minsky machines in HOCORE, it may introduce computations which do not correspond to the expected behavior of the modeled machine. Such computations are forced to be infinite and thus regarded as non-halting computations which are therefore ignored. Only the finite computations correspond to those of the encoded Minsky machine. This way, we prove that a Minsky machine terminates if and only if its encoding in HO^{-f} converges. Consequently, convergence in HO^{-f} is *undecidable*.

As for (2), the use of the theory of well-structured transition systems is certainly not a new approach to obtain expressiveness results. However, to the best of our knowledge, this is the first time it is applied in the higher-order setting. This is significant because the adaptation to the HO^{-f} case is far from trivial. Indeed, as we shall discuss, this approach relies on approximating an upper bound on the depth of the (set of) derivatives of a process. By *depth* of a process we mean its maximal nesting of input/output actions. Notice that, even with the limitation on forwarding enforced by HO^{-f} , because of the “term copying” feature of higher-order calculi, variable instantiation might lead to a potentially larger process. Hence, finding suitable ways of bounding the set of derivatives of a process is rather challenging and needs care.

We comment further on the consequences of our results in Section 5. In this presentation we omit most proofs; these can be found in the extended version [10].

2 The Calculus

We now introduce the syntax and semantics of HO^{-f} . We use a, b, c to range over names, and x, y, z to range over variables; the sets of names and variables are disjoint.

$P, Q ::= \bar{a}(x_1 \parallel \dots \parallel x_k \parallel P)$	(with $k \geq 0$, $\text{fv}(P) = \emptyset$)	output
$a(x).P$		input prefix
$P \parallel Q$		parallel composition
x		process variable
$\mathbf{0}$		nil

An input $a(x).P$ binds the free occurrences of x in P . We write $\text{fv}(P)$ and $\text{bv}(P)$ for the set of free and bound variables in P , respectively. A process is *closed* if it does not have free variables. We abbreviate $a(x).P$, with $x \notin \text{fv}(P)$, as $a.P$, $\bar{a}(\mathbf{0})$ as \bar{a} , and

$P_1 \parallel \dots \parallel P_k$ as $\prod_{i=1}^k P_i$. Hence, an output action can be written as $\bar{a}(\prod_{k \in K} x_k \parallel P)$. We write $\prod_1^n P$ as an abbreviation for the parallel composition of n copies of P . Further, $P\{Q/x\}$ denotes the substitution of the free occurrences of x with process Q in P .

The Labeled Transition System (LTS) of HO^{-f} is defined on closed processes. There are three forms of transitions: τ transitions $P \xrightarrow{\tau} P'$; input transitions $P \xrightarrow{a(x)} P'$, meaning that P can receive at a a process that will replace x in the continuation P' ; and output transitions $P \xrightarrow{\bar{a}(P')} P''$ meaning that P emits P' at a , and in doing so it evolves to P'' . We use α to indicate a generic label of a transition.

$$\begin{array}{c} \text{INP} \quad a(x).P \xrightarrow{a(x)} P \qquad \text{OUT} \quad \bar{a}(P) \xrightarrow{\bar{a}(P)} \mathbf{0} \\ \\ \text{ACT1} \quad \frac{P_1 \xrightarrow{\alpha} P'_1}{P_1 \parallel P_2 \xrightarrow{\alpha} P'_1 \parallel P_2} \qquad \text{TAU1} \quad \frac{P_1 \xrightarrow{\bar{a}(P')} P'_1 \quad P_2 \xrightarrow{a(x)} P'_2}{P_1 \parallel P_2 \xrightarrow{\tau} P'_1 \parallel P'_2\{P/x\}} \end{array}$$

(We have omitted ACT2 and TAU2, the symmetric counterparts of the last two rules.)

Remark 1. Since we consider closed processes, in rule ACT1, P_2 has no free variables and no side conditions are necessary. As a consequence, alpha-conversion is not needed.

Definition 1. The structural congruence relation is the smallest congruence generated by the following laws:

$$P \parallel \mathbf{0} \equiv P, \quad P_1 \parallel P_2 \equiv P_2 \parallel P_1, \quad P_1 \parallel (P_2 \parallel P_3) \equiv (P_1 \parallel P_2) \parallel P_3.$$

The *alphabet* of an HO^{-f} process is defined as follows:

Definition 2 (Alphabet of a process). Let P be a HO^{-f} process. The alphabet of P , denoted $\mathcal{A}(P)$, is inductively defined as:

$$\begin{array}{l} \mathcal{A}(\mathbf{0}) = \emptyset \qquad \mathcal{A}(P \parallel Q) = \mathcal{A}(P) \cup \mathcal{A}(Q) \qquad \mathcal{A}(x) = \{x\} \\ \mathcal{A}(a(x).P) = \{a, x\} \cup \mathcal{A}(P) \qquad \mathcal{A}(\bar{a}(P)) = \{a\} \cup \mathcal{A}(P) \end{array}$$

Proposition 1. Let P be a HO^{-f} process. The set $\mathcal{A}(P)$ is finite. Also, if $P \xrightarrow{\alpha} P'$ then $\mathcal{A}(P') \subseteq \mathcal{A}(P)$.

The internal runs of a process are given by sequences of *reductions*. Given a process P , its reductions $P \longrightarrow P'$ are defined as $P \xrightarrow{\tau} P'$. We denote with \longrightarrow^* the reflexive and transitive closure of \longrightarrow ; notation \longrightarrow^j is to stand for a sequence of j reductions. We use $P \not\rightarrow$ to denote that there is no P' such that $P \longrightarrow P'$. Following [9] we now define process convergence and process termination. Observe that termination implies convergence while the opposite does not hold.

Definition 3. Let P be a HO^{-f} process. We say that P converges iff there exists P' such that $P \longrightarrow^* P'$ and $P' \not\rightarrow$. We say that P terminates iff there exist no $\{P_i\}_{i \in \mathbb{N}}$ such that $P_0 = P$ and $P_j \longrightarrow P_{j+1}$ for any j .

Termination and convergence are sometimes also referred to as *universal* and *existential* termination, respectively.

Table 1. Reduction of Minsky machines

$$\begin{array}{l}
\text{M-INC} \frac{i : \text{INC}(r_j) \quad m'_j = m_j + 1 \quad m'_{1-j} = m_{1-j}}{(i, m_0, m_1) \longrightarrow_{\text{M}} (i + 1, m'_0, m'_1)} \qquad \text{M-JMP} \frac{i : \text{DECJ}(r_j, s) \quad m_j = 0}{(i, m_0, m_1) \longrightarrow_{\text{M}} (s, m_0, m_1)} \\
\text{M-DEC} \frac{i : \text{DECJ}(r_j, s) \quad m_j \neq 0 \quad m'_j = m_j - 1 \quad m'_{1-j} = m_{1-j}}{(i, m_0, m_1) \longrightarrow_{\text{M}} (i + 1, m'_0, m'_1)}
\end{array}$$

3 Convergence Is Undecidable

In this section we show that $\text{HO}^{-\text{f}}$ is powerful enough to model Minsky machines [5], a Turing complete model. We present an encoding that is not *faithful*: unlike the encoding of Minsky machines in HOCORE , it may introduce computations which do not correspond to the expected behavior of the modeled machine. Such computations are forced to be infinite and thus regarded as non-halting computations which are therefore ignored. Only finite computations correspond to those of the encoded Minsky machine. More precisely, given a Minsky machine N , its encoding $\llbracket N \rrbracket$ has a terminating computation if and only if N terminates. This allows to prove that convergence is undecidable.

We begin by briefly recalling the definition of Minsky machines; we then present the encoding into $\text{HO}^{-\text{f}}$ and discuss its correctness.

Minsky machines. A Minsky machine is a Turing complete model composed of a set of sequential, labeled instructions, and two registers. Registers r_j ($j \in \{0, 1\}$) can hold arbitrarily large natural numbers. Instructions $(1 : I_1), \dots, (n : I_n)$ can be of two kinds: $\text{INC}(r_j)$ adds 1 to register r_j and proceeds to the next instruction; $\text{DECJ}(r_j, s)$ jumps to instruction s if r_j is zero, otherwise it decreases register r_j by 1 and proceeds to the next instruction. A Minsky machine includes a program counter p indicating the label of the instruction being executed. In its initial state, the machine has both registers set to 0 and the program counter p set to the first instruction. The Minsky machine stops whenever the program counter is set to a non-existent instruction, i.e. $p > n$. A *configuration* of a Minsky machine is a tuple (i, m_0, m_1) ; it consists of the current program counter and the values of the registers. Formally, the reduction relation over configurations of a Minsky machine, denoted $\longrightarrow_{\text{M}}$, is defined in Table 1.

In the encoding of a Minsky machine into $\text{HO}^{-\text{f}}$ we will find it convenient to have a simple form of guarded replication. This construct can be encoded in $\text{HO}^{-\text{f}}$ as follows.

Input-guarded replication. We follow the standard encoding of replication in higher-order process calculi, adapting it to input-guarded replication so as to make sure that diverging behaviors are not introduced. As there is no restriction in $\text{HO}^{-\text{f}}$, the encoding is not compositional and replications cannot be nested. In [4] the following encoding is shown to preserve termination.

Definition 4. *Assume a fresh name c . The encoding of input-guarded replication is as follows:*

$$\llbracket [a(z). P]_{\text{it}} \rrbracket = a(z). (Q_c \parallel P) \parallel \bar{c}(a(z)). (Q_c \parallel P)$$

Table 2. Encoding of Minsky machines

REGISTER r_j	$\llbracket r_j = m \rrbracket_M = \prod_1^m \overline{u_j}$
INSTRUCTIONS ($i : I_i$)	
$\llbracket (i : \text{INC}(r_j)) \rrbracket_M$	$= !p_i. (\overline{u_j} \parallel \text{set}_j(x). \overline{\text{set}_j} \langle x \parallel \text{INC}_j \rangle \parallel \overline{p_{i+1}})$
$\llbracket (i : \text{DECJ}(r_j, s)) \rrbracket_M$	$= !p_i. \overline{m_i}$ $\parallel !m_i. (\overline{\text{loop}} \parallel u_j. \overline{\text{loop}}. \text{set}_j(x). \overline{\text{set}_j} \langle x \parallel \text{DEC}_j \rangle \parallel \overline{p_{i+1}})$ $\parallel !m_i. \text{set}_j(x). (x \parallel \overline{\text{set}_j} \langle \mathbf{0} \rangle \parallel \overline{p_s})$
where	
$\text{INC}_j = \overline{\text{loop}} \parallel \text{check}_j. \overline{\text{loop}}$	$\text{DEC}_j = \overline{\text{check}_j}$

where $Q_c = c(x). (x \parallel \overline{c} \langle x \rangle)$, P contains no replications (nested replications are forbidden), and $\llbracket \cdot \rrbracket_{ii}$ is an homomorphism on the other process constructs in HO^{-f} .

Encoding Minsky machines into HO^{-f} . The encoding of Minsky machines into HO^{-f} is denoted by $\llbracket \cdot \rrbracket_M$ and presented in Table 2. We begin by defining the encoding of the configurations of a Minsky machine; we then discuss the encodings of registers and instructions.

Definition 5 (Encoding of Configurations). *Let N be a Minsky machine with registers r_0, r_1 and instructions $(1 : I_1), \dots, (n : I_n)$. For $j \in \{0, 1\}$, suppose fresh, pairwise different names $r_j, p_1, \dots, p_n, \text{set}_j, \text{loop}, \text{check}_j$. Also, let DIV be a divergent process (e.g. $\overline{w} \parallel !w. \overline{w}$). Given the encodings in Table 2, we have:*

1. The initial configuration $(1, 0, 0)$ of N is encoded as:

$$\llbracket (1, 0, 0) \rrbracket_M ::= \overline{p_1} \parallel \prod_{i=1}^n \llbracket (i : I_i) \rrbracket_M \parallel \text{loop}. \text{DIV} \parallel \overline{\text{set}_0} \langle \mathbf{0} \rangle \parallel \overline{\text{set}_1} \langle \mathbf{0} \rangle .$$

2. A configuration (i, m_0, m_1) of N , after k_j increments and l_j decrements of register r_j , is encoded as:

$$\llbracket (i, m_0, m_1) \rrbracket_M = \overline{p_i} \parallel \llbracket r_0 = m_0 \rrbracket_M \parallel \llbracket r_1 = m_1 \rrbracket_M \parallel \prod_{i=1}^n \llbracket (i : I_i) \rrbracket_M \parallel$$

$$\text{loop}. \text{DIV} \parallel \overline{\text{set}_0} \langle \text{LOG}_0[k_0, l_0] \rangle \parallel \overline{\text{set}_1} \langle \text{LOG}_1[k_1, l_1] \rangle .$$

A register r_j that stores the number m is encoded as the parallel composition of m copies of the unit process $\overline{u_j}$. To implement the test for zero it is necessary to record how many increments and decrements have been performed on the register r_j . This is done by using a special process LOG_j , which is communicated back and forth on name set_j . More precisely, every time an increment instruction occurs, a new copy of the process $\overline{u_j}$ is created, and the process LOG_j is updated by adding the process INC_j in parallel. Similarly for decrements: a copy of $\overline{u_j}$ is consumed and the process DEC_j is added to LOG_j . As a result, after k increments and l decrements on register r_j , we have that $\text{LOG}_j = \prod_k \text{INC}_j \parallel \prod_l \text{DEC}_j$, which we abbreviate as $\text{LOG}_j[k, l]$.

Each instruction $(i : I_i)$ is a replicated process guarded by p_i , which represents the program counter when $p = i$. Once p_i is consumed, the instruction is active and an interaction with a register occurs. We already described the behavior of increments. Let us

now focus on decrements, the instructions that can introduce divergent —unfaithful— computations. In this case, the process can internally choose either to actually perform a decrement and proceed with the next instruction, or to jump. This can be seen as a guess the process makes on the actual number stored by the register r_j . Therefore, two situations can occur:

1. *The process chooses to decrement r_j .* In this case instruction p_{i+1} is immediately enabled, and the process launches process \overline{loop} and then tries to consume a copy of $\overline{u_j}$. If this operation succeeds (i.e. the content of r_j is greater than 0) then a synchronization with the input on $loop$ that guards the updating of LOG_j (represented as an output on name set_j) takes place. Otherwise, the unit process $\overline{u_j}$ could not be consumed (i.e. the content of r_j is zero and the process made a wrong guess). Process \overline{loop} then synchronizes with the external process $loop$. DIV, thus spawning a divergent computation.
2. *The process chooses to jump to instruction p_s .* In this case instruction p_s is immediately enabled, and it is necessary to check if the actual value stored by r_j is zero. To do so, the process receives the process LOG_j and launches it. If the number of increments is equal to the number of decrements then complementary signals on the name $check_j$ will match each other. In turn, this allows each signal \overline{loop} executed by an INC_j process to be matched by a complementary one. Otherwise, then it is the case that at least one of those \overline{loop} signals remains active (i.e. the content of the register is not zero); a synchronization with the process $loop$. DIV then takes place, and a divergent computation is spawned.

Before executing the instructions, we require both registers in the Minsky machine to be set to zero. This is to guarantee correctness: starting with values different from zero in the registers (without proper initialization of the logs) can lead to inconsistencies. For instance, the test for zero would succeed (i.e. without spawning a divergent computation) even for a register whose value is different from zero.

We now state that the encoding is correct.

Theorem 1. *Let N be a Minsky machine with registers $r_0 = m_0, r_1 = m_1$, instructions $(1 : I_1), \dots, (n : I_n)$, and configuration (i, m_0, m_1) . Then (i, m_0, m_1) terminates if and only if process $\llbracket (i, m_0, m_1) \rrbracket_M$ converges.*

As a consequence of this theorem we have that convergence is undecidable.

Corollary 1. *Convergence is undecidable in HO^{-f} .*

4 Termination Is Decidable

In this section we prove that termination is decidable for HO^{-f} processes. As hinted at in the introduction, this is in sharp contrast with the analogous result for HOCORE . The proof appeals to the theory of well-structured transition systems, whose main definitions and results we summarize next.

Well-Structured Transition Systems. The following results and definitions are from [8], unless differently specified. Recall that a *quasi-order* (or, equivalently, preorder) is a reflexive and transitive relation.

Definition 6 (Well-quasi-order). A well-quasi-order (*wqo*) is a quasi-order \leq over a set X such that, for any infinite sequence $x_0, x_1, x_2 \dots \in X$, there exist indexes $i < j$ such that $x_i \leq x_j$.

Note that if \leq is a wqo then any infinite sequence x_0, x_1, x_2, \dots contains an infinite increasing subsequence $x_{i_0}, x_{i_1}, x_{i_2}, \dots$ (with $i_0 < i_1 < i_2 < \dots$). Thus well-quasi-orders exclude the possibility of having infinite strictly decreasing sequences.

We also need a definition for (finitely branching) transition systems. This can be given as follows. Here and in the following \rightarrow^* denotes the reflexive and transitive closure of the relation \rightarrow .

Definition 7 (Transition system). A transition system is a structure $TS = (S, \rightarrow)$, where S is a set of states and $\rightarrow \subseteq S \times S$ is a set of transitions. We define $Succ(s)$ as the set $\{s' \in S \mid s \rightarrow s'\}$ of immediate successors of S . We say that TS is finitely branching if, for each $s \in S$, $Succ(s)$ is finite.

Fact 1. The LTS for HO^{-f} given in Section 2 is finitely branching.

The function $Succ$ will also be used on sets by assuming the point-wise extension of the above definitions. The key tool to decide several properties of computations is the notion of *well-structured transition system*. This is a transition system equipped with a well-quasi-order on states which is (upward) compatible with the transition relation. Here we will use a strong version of compatibility; hence the following definition.

Definition 8 (Well-structured transition system). A well-structured transition system with strong compatibility is a transition system $TS = (S, \rightarrow)$, equipped with a quasi-order \leq on S , such that the two following conditions hold:

1. \leq is a well-quasi-order;
2. \leq is strongly (upward) compatible with \rightarrow , that is, for all $s_1 \leq t_1$ and all transitions $s_1 \rightarrow s_2$, there exists a state t_2 such that $t_1 \rightarrow t_2$ and $s_2 \leq t_2$ holds.

The following theorem is a special case of Theorem 4.6 in [8] and will be used to obtain our decidability result.

Theorem 2. Let $TS = (S, \rightarrow, \leq)$ be a finitely branching, well-structured transition system with strong compatibility, decidable \leq , and computable $Succ$. Then the existence of an infinite computation starting from a state $s \in S$ is decidable.

We will also need a result due to Higman [11] which allows to extend a well-quasi-order from a set S to the set of the finite sequences on S . More precisely, given a set S let us denote by S^* the set of finite sequences built by using elements in S . We can define a quasi-order on S^* as follows.

Definition 9. Let S be a set and \leq a quasi-order over S . The relation \leq_* over S^* is defined as follows. Let $t, u \in S^*$, with $t = t_1 t_2 \dots t_m$ and $u = u_1 u_2 \dots u_n$. We have

that $t \leq_* u$ if and only if there exists an injection f from $\{1, 2, \dots, m\}$ to $\{1, 2, \dots, n\}$ such that $t_i \leq u_{f(i)}$ and $i \leq f(i)$ for $i = 1, \dots, m$.

The relation \leq_* is clearly a quasi-order over S^* . It is also a wqo, since we have the following result.

Lemma 1 ([11]). *Let S be a set and \leq a wqo over S . Then \leq_* is a wqo over S^* .*

Finally we will use also the following proposition, whose proof is immediate.

Proposition 2. *Let S be a finite set. Then the equality is a wqo over S .*

Termination is Decidable in HO^{-f} . Here we prove that termination is decidable in HO^{-f} . The crux of the proof consists in finding an upper bound for a process and its derivatives. This is possible in HO^{-f} because of the limited structure allowed in output actions. We proceed as follows. First we define a notion of normal form for HO^{-f} processes. We then characterize an upper bound for the derivatives of a given process, and define an ordering over them. This ordering is then shown to be a wqo that is strongly compatible with respect to the LTS of HO^{-f} given in Section 2. The decidability result is then obtained by resorting to the results from [8] reported before.

Definition 10 (Normal Form). *Let $P \in \text{HO}^{-f}$. P is in normal form iff*

$$P = \prod_{k=1}^l x_k \parallel \prod_{i=1}^m a_i(y_i). P_i \parallel \prod_{j=1}^n \overline{b_j} \langle P'_j \rangle$$

where each P_i and P'_j are in normal form.

Lemma 2. *Every process $P \in \text{HO}^{-f}$ is structurally congruent to a normal form.*

We now define an ordering over normal forms. Intuitively, a process is larger than another if it has more parallel components.

Definition 11 (Relation \preceq). *Let $P, Q \in \text{HO}^{-f}$. We write $P \preceq Q$ iff there exist $x_1 \dots x_l, P_1 \dots P_m, P'_1 \dots P'_m, Q_1 \dots Q_m, Q'_1 \dots Q'_m$, and R such that*

$$\begin{aligned} P &\equiv \prod_{k=1}^l x_k \parallel \prod_{i=1}^m a_i(y_i). P_i \parallel \prod_{j=1}^n \overline{b_j} \langle P'_j \rangle \\ Q &\equiv \prod_{k=1}^l x_k \parallel \prod_{i=1}^m a_i(y_i). Q_i \parallel \prod_{j=1}^n \overline{b_j} \langle Q'_j \rangle \parallel R \end{aligned}$$

with $P_i \preceq Q_i$ and $P'_j \preceq Q'_j$, for $i \in [1..m]$ and $j \in [1..n]$.

The normal form of a process can be intuitively represented in a tree-like manner. More precisely, given the process in normal form

$$P = \prod_{k=1}^l x_k \parallel \prod_{i=1}^m a_i(y_i). P_i \parallel \prod_{j=1}^n \overline{b_j} \langle P'_j \rangle$$

we shall decree its associated tree to have a root node labeled x_1, \dots, x_k . This root node has $m + n$ children, corresponding to the trees associated to processes P_1, \dots, P_m and P'_1, \dots, P'_m ; the outgoing edges connecting the root node and the children are labeled $a_1(y_1), \dots, a_m(y_m)$ and $\overline{b_1}, \dots, \overline{b_n}$.

This intuitive representation of processes in normal form as trees will be useful to reason about the structure of HO^{-f} terms. We begin by defining the *depth* of a process. Notice that such a depth corresponds to the maximum depth of its tree representation.

Definition 12 (Depth). Let $P = \prod_{k=1}^l x_k \parallel \prod_{i=1}^m a_i(y_i) \cdot P_i \parallel \prod_{j=1}^n \overline{b_j} \langle P'_j \rangle$ be a HO^{-f} process in normal form. The depth of P is given by

$$\text{depth}(P) = \max\{1 + \text{depth}(P_i), 1 + \text{depth}(P'_j) \mid i \in [1..m] \wedge j \in [1..n]\}.$$

Given a natural number n and a process P , the set $\mathcal{P}_{P,n}$ contains all those processes in normal form that can be built using the alphabet of P and whose depth is at most n .

Definition 13. Let n be a natural number and $P \in \text{HO}^{-f}$. We define the set $\mathcal{P}_{P,n}$ as follows:

$$\begin{aligned} \mathcal{P}_{P,n} = \{Q \mid & Q \equiv \prod_{k \in K} x_k \parallel \prod_{i \in I} a_i(y_i) \cdot Q_i \parallel \prod_{j \in J} \overline{b_j} \langle Q'_j \rangle \\ & \wedge \mathcal{A}(Q) \subseteq \mathcal{A}(P) \\ & \wedge Q_i, Q'_j \in \mathcal{P}_{P,n-1} \forall i \in I, j \in J\} \end{aligned}$$

where $\mathcal{P}_{P,0}$ contains processes that are built out only of variables in $\mathcal{A}(P)$.

As it will be shown later, the set of all derivatives of P is a subset of $\mathcal{P}_{P,2 \cdot \text{depth}(P)}$.

When compared to processes in languages such as Milner's CCS, higher-order processes have a more complex structure. This is because, by virtue of reductions, an arbitrary process can take the place of possibly several occurrences of a single variable. As a consequence, the depth of (the syntax tree of) a process cannot be determined (or even approximated) before its execution: it can vary arbitrarily along reductions. Crucially, in HO^{-f} it is possible to bound such a depth. Our approach is the following: rather than solely depending on the depth of a process, we define measures on the relative position of variables within a process. Informally speaking, such a position will be determined by the number of prefixes guarding a variable. Since variables are allowed only at the top level of the output objects, their relative distance will remain invariant during reductions. This allows to obtain a bound on the structure of HO^{-f} processes. Finally, it is worth stressing that even if the same notions of normal form, depth, and distance can be defined for HOCORE, a finite upper bound for such a language does not exist. We first define the maximum distance between a variable and its binder.

Definition 14. Let $P = \prod_{k \in K} x_k \parallel \prod_{i \in I} a_i(y_i) \cdot P_i \parallel \prod_{j \in J} \overline{b_j} \langle P'_j \rangle$ be a HO^{-f} process in normal form. We define the maximum distance of P as:

$$\begin{aligned} \text{maxDistance}(P) = \max\{ & \text{maxDist}_{y_i}(P_i), \\ & \text{maxDistance}(P_i), \text{maxDistance}(P'_j) \mid i \in I, j \in J\} \end{aligned}$$

where

$$\text{maxDist}_x(P) = \begin{cases} 1 & \text{if } P = x, \\ 1 + \text{maxDist}_x(P_z) & \text{if } P = a(z) \cdot P_z \wedge x \neq z, \\ 1 + \text{maxDist}_x(P') & \text{if } P = \overline{a} \langle P' \rangle, \\ \max\{\text{maxDist}_x(R), \text{maxDist}_x(Q)\} & \text{if } P = R \parallel Q, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 3 (Properties of maxDistance). Let P be a HO^{-f} process. It holds that:

1. $\text{maxDistance}(P) \leq \text{depth}(P)$
2. For every Q such that $P \xrightarrow{\alpha} Q$, $\text{maxDistance}(Q) \leq \text{maxDistance}(P)$.

We now define the maximum depth of processes that can be communicated. Notice that the continuations of inputs are considered as they could become communication objects themselves along reductions:

Definition 15. Let $P = \prod_{k \in K} x_k \parallel \prod_{i \in I} a_i(y_i) \cdot P_i \parallel \prod_{j \in J} \overline{b_j}(P'_j)$ be a HO^{-f} process in normal form. We define the maximum depth of a process that can be communicated ($\text{maxDepCom}(P)$) in P as:

$$\text{maxDepCom}(P) = \max\{\text{maxDepCom}(P_i), \text{depth}(P'_j) \mid i \in I, j \in J\}.$$

Lemma 4 (Properties of maxDepCom). Let P be a HO^{-f} process. It holds that:

1. $\text{maxDepCom}(P) \leq \text{depth}(P)$
2. For every Q such that $P \xrightarrow{\alpha} Q$, $\text{maxDepCom}(Q) \leq \text{maxDepCom}(P)$.

Notation 1. We use $P \xrightarrow{\tilde{\alpha}} P'$ if, for some $n \geq 0$, there exist $\alpha_1, \dots, \alpha_n$ such that $P \xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_n} P'$.

Generalizing Lemmata 3 and 4 we obtain:

Corollary 2. Let P be a HO^{-f} process. For every Q such that $P \xrightarrow{\tilde{\alpha}} Q$, it holds that:

1. $\text{maxDistance}(Q) \leq \text{depth}(P)$
2. $\text{maxDepCom}(Q) \leq \text{depth}(P)$.

We are interested in characterizing the derivatives of a given process P . We shall show that they are over-approximated by means of the set $\mathcal{P}_{P, 2 \cdot \text{depth}(P)}$. We will investigate the properties of the relation \preceq on such an approximation; such properties will also hold for the set of derivatives.

Definition 16. Let $P \in \text{HO}^{-f}$. Then we define $\text{Deriv}(P) = \{Q \mid P \longrightarrow^* Q\}$

The following results hold because of the limitations we have imposed on the output actions for HO^{-f} processes. Any process that can be communicated in P is in $\mathcal{P}_{P, n-1}$ and its maximum depth is also bounded by $\text{depth}(P)$. The deepest position for a variable is when it is a leaf in the tree associated to the normal form of P . That is, when its depth is exactly $\text{depth}(P)$. Hence the following:

Proposition 3. Let P be a HO^{-f} process. Suppose, for some n , that $P \in \mathcal{P}_{P, n}$. For every Q such that $P \xrightarrow{\alpha} Q$, it holds that $Q \in \mathcal{P}_{P, 2 \cdot n}$.

The lemma below generalizes Proposition 3 to a sequence of transitions.

Lemma 5. Let P be a HO^{-f} process. Suppose, for some n , that $P \in \mathcal{P}_{P, n}$. For every Q such that $P \xrightarrow{\tilde{\alpha}} Q$, it holds that $Q \in \mathcal{P}_{P, 2 \cdot n}$.

Corollary 3. Let $P \in \text{HO}^{-f}$. Then $\text{Deriv}(P) \subseteq \mathcal{P}_{P, 2 \cdot \text{depth}(P)}$.

To prove that \preceq is a wqo, we first show that it is a quasi order.

Proposition 4. The relation \preceq is a quasi-order.

We are now in place to state that \preceq is a wqo.

Theorem 3 (Well-quasi-order). *Let $P \in \text{HO}^{-f}$ and $n \geq 0$. The relation \preceq is a well-quasi-order over $\mathcal{P}_{P,n}$.*

Proof. The proof is by induction on n .

(–) Let $n = 0$. Then $\mathcal{P}_{P,0}$ contains processes containing only variables taken from $\mathcal{A}(P)$. The equality on finite sets is a well-quasi-ordering; by Lemma 1 (Higman's Lemma) also $=_*$ is a well quasi-ordering: it corresponds to the ordering \preceq on processes containing only variables.

(–) Let $n > 0$. Take an infinite sequence of processes $s = P_1, P_2, \dots, P_l, \dots$ with $P_l \in \mathcal{P}_{P,n}$. We shall show that the thesis holds by means of successive filterings of the normal forms of the processes in s . By Lemma 2 there exist K_l, I_l and J_l such that

$$P_l \equiv \prod_{k \in K_l} x_k \parallel \prod_{i \in I_l} a_i(y_i) \cdot P_i^l \parallel \prod_{j \in J_l} \overline{b_j} \langle P_j^l \rangle$$

with P_i^l and $P_j^l \in \mathcal{P}_{P,n-1}$. Hence each P_l can be seen as composed of 3 finite sequences: (i) $x_1 \dots x_k$, (ii) $a_1(y_1) \cdot P_1^l \dots a_i(y_i) \cdot P_i^l$, and (iii) $\overline{b_1} \langle P_1^l \rangle \dots \overline{b_j} \langle P_j^l \rangle$. We note that the first sequence is composed of variables from the finite set $\mathcal{A}(P)$ whereas the other two sequences are composed by elements in $\mathcal{A}(P)$ and $\mathcal{P}_{P,n-1}$. Since we have an infinite sequence of $\mathcal{A}(P)^*$, as $\mathcal{A}(P)$ is finite, by Proposition 2 and Lemma 1 we have that $=_*$ is a wqo over $\mathcal{A}(P)^*$. By inductive hypothesis, we have that \preceq is a wqo on $\mathcal{P}_{P,n-1}$, hence by Lemma 1 relation \preceq_* is a wqo on $\mathcal{P}_{P,n-1}^*$. We start filtering out s by making the finite sequences $x_1 \dots x_k$ increasing with respect to $=_*$; let us call this subsequence t . Then we filter out t , by making the finite sequence $a_1(y_1) \cdot P_1^l \dots a_i(y_i) \cdot P_i^l$ increasing with respect to both \preceq_* and $=_*$. This is done in two steps: first, by considering the relation $=_*$ on the subject of the actions (recalling that $a_i, y_i \in \mathcal{A}(P)$), and then by applying another filtering to the continuation using the inductive hypothesis. For the first step, it is worth remarking that we do not consider symbols of the alphabet but pairs of symbols. Since the set of pairs on a finite set is still finite, we know by Higman's Lemma that $=_*$ is a wqo on the set of sequences of pairs (a_i, y_i) . For the sequence of outputs $\overline{b_1} \langle P_1^l \rangle \dots \overline{b_j} \langle P_j^l \rangle$ this is also done in two steps: the subject of the outputs are ordered with respect to $=_*$ and the objects of the output action are ordered with respect to \preceq_* using the inductive hypothesis. At the end of the process we obtain an infinite subsequence of s that is ordered with respect to \preceq . \square

The last thing to show is that the well-quasi-ordering \preceq is strongly compatible with respect to the LTS associated to HO^{-f} . We need the following auxiliary lemma:

Lemma 6. *Let P, P', Q , and Q' be HO^{-f} processes in normal form such that $P \preceq P'$ and $Q \preceq Q'$. Then it holds that $P\{Q/x\} \preceq P'\{Q'/x\}$.*

Theorem 4 (Strong Compatibility). *Let $P, Q, P' \in \text{HO}^{-f}$. If $P \preceq Q$ and $P \xrightarrow{\alpha} P'$ then there exists Q' such that $Q \xrightarrow{\alpha} Q'$ and $P' \preceq Q'$.*

Theorem 5. *Let $P \in \text{HO}^{-f}$. The transition system $(\text{Deriv}(P), \longrightarrow, \preceq)$ is a finitely branching well-structured transition system with strong compatibility, decidable \preceq , and computable Succ.*

Proof. The transition system of HO^{-f} is finitely branching (Fact 1). The fact that \preceq is a well-quasi-order on $\text{Deriv}(P)$ follows from Corollary 3 and Theorem 3. Strong compatibility follows from Theorem 4. \square

We can now state the main result of the section. It follows from Theorems 2 and 5.

Corollary 4. *Let $P \in \text{HO}^{-f}$. Termination of P is decidable.*

5 Concluding Remarks

We have studied HO^{-f} , a higher-order process calculus featuring a limited form of higher-order communication. In HO^{-f} , output actions can only include previously received processes in composition with closed ones. This is reminiscent of programming scenarios with forms of code mobility in which the recipient is not authorized or capable of accessing/modifying the structure of the received code. We have shown that such a weakening of the forward capabilities of higher-order processes has consequences both on the expressiveness of the language and on the decidability of termination.

As for the expressiveness issues, by exhibiting an encoding of Minsky machines into HO^{-f} , we have shown that convergence is undecidable. Hence, from an *absolute expressiveness* standpoint, HO^{-f} is Turing complete. Now, given the analogous result for HOCORE [4], a *relative expressiveness* issue also arises. Indeed, our encoding of Minsky machines into HO^{-f} is not faithful, which reveals a difference on the criteria each encoding satisfies. This reminds us of the situation in [12], where faithful and unfaithful encodings of Turing complete formalisms into calculi with interruption and compensation are compared. Using the terminology in [12], we can say that the presented encoding satisfies a *weakly Turing completeness* criterion, as opposed to the (stronger) *Turing completeness* criterion that is satisfied by the encoding of Minsky machines into HOCORE in [4]. The discrepancy on the criteria satisfied by each encoding might be interpreted as an expressiveness gap between HO^{-f} and HOCORE; nevertheless, it seems clear that the loss of expressiveness resulting from limiting the forwarding capabilities in HOCORE is much less dramatic than what one would have expected.

We have shown that the communication style of HO^{-f} causes a separation result with respect to HOCORE. In fact, because of the limitation on output actions, it was possible to prove that termination in HO^{-f} is decidable. This is in sharp contrast with the situation in HOCORE, for which termination is undecidable. In HO^{-f} , it is possible to provide an upper bound on the depth (i.e. the level of nesting of actions) of the (set of) derivatives of a process. In HOCORE such an upper bound does not exist. This was essential for obtaining the decidability result; for this, we appealed to the approach developed in [9], which relies on the theory of well-structured transition systems [8]. As far as we are aware, this approach to studying expressiveness issues has not previously been used in the higher-order setting. The decidability of termination might shed light on the development of verification techniques for higher-order processes.

The HO^{-f} calculus is a sublanguage of HOCORE. As such, HO^{-f} inherits the many results and properties of HOCORE [4]; most notably, a notion of (strong) bisimilarity which is decidable and coincides with a number of sensible equivalences in the higher-order context. Our results thus complement those in [4] and deepen our understanding

of the expressiveness of core higher-order calculi as a whole. Furthermore, by recalling that CCS without restriction is not Turing complete and has decidable convergence, the present results shape an interesting expressiveness hierarchy, namely one in which HOCORE is strictly more expressive than HO^{-f} (because of the discussion above), and in which HO^{-f} is strictly more expressive than CCS without restriction.

Remarkably, our undecidability result can be used to prove that (weak) barbed bisimilarity is undecidable in the calculus obtained by extending HO^{-f} with restriction. Consider the encoding of Minsky machines used in Section 3 to prove the undecidability of convergence in HO^{-f} . Consider now the restriction operator $(\nu \tilde{x})$ used as a binder for the names in the tuple \tilde{x} . Take a Minsky machine N (it is not restrictive to assume that it executes at least one increment instruction) and its encoding P , as defined in Definition 5. Let \tilde{x} be the tuple of the names used by P , excluding the name w . We have that N terminates if and only if $(\nu \tilde{x})P$ is (weakly) barbed equivalent to the process $(\nu d)(\bar{d} \mid d \mid d. (\bar{w} \mid !w. \bar{w}))$.

Related Work. The most closely related work is [4], which was already discussed along the paper. We do not know of other works that study the expressiveness of higher-order calculi by restricting higher-order outputs. The recent work [13] studies finite-control fragments of Homer [14], a higher-order process calculus with locations. While we have focused on decidability of termination and convergence, in [13] the interest is in decidability of barbed bisimilarity. One of the approaches explored in [13] is based on a type system that bounds the size of processes in terms of their syntactic components (e.g. number of parallel components, location nesting). Although the restrictions such a type system imposes might be considered as similar in spirit to the limitation on outputs in HO^{-f} (in particular, location nesting resembles the output nesting HO^{-f} forbids), the fact that the synchronization discipline in Homer depends heavily on the structure of locations makes it difficult to establish a more detailed comparison with HO^{-f} .

Also similar in spirit to our work, but in a slightly different context, are some studies on the expressiveness (of fragments) of the Ambient calculus [15]. Ambient and higher-order calculi are related in that both allow the communication of objects with complex structure. Some works on the expressiveness of fragments of Ambient calculi are similar to ours. In particular, [16] shows that termination is decidable for the fragment without both restriction (as HO^{-f} and HOCORE) and movement capabilities, and featuring replication; in contrast, the same property turns out to be undecidable for the fragment with recursion. Hence, the separation between fragments comes from the source of infinite behavior, and not from the structures allowed in output action, as in our case. However, we find that the connections between Ambient-like and higher-order calculi are rather loose, so a proper comparison is difficult also in this case.

Future Work. As already mentioned, a great deal of the expressive power in higher-order calculi resides in the interplay of input and output actions. Here we have studied an alternative for limiting output capabilities; it would be interesting to investigate if suitable limitations on input actions are possible, and whether they have influence on expressiveness. Another interesting direction would be to compare higher-order and Ambient calculi from the expressiveness point of view.

Acknowledgments. We are grateful to Julian Gutierrez for his helpful comments on an earlier version of this paper.

References

1. Thomsen, B.: A calculus of higher order communicating systems. In: Proc. of POPL 1989, pp. 143–154. ACM Press, New York (1989)
2. Thomsen, B.: Plain CHOCS: A second generation calculus for higher order processes. *Acta Inf.* 30(1), 1–59 (1993)
3. Sangiorgi, D.: Expressing Mobility in Process Algebras: First-Order and Higher-Order Paradigms. PhD thesis CST-99-93, University of Edinburgh, Dept. of Comp. Sci. (1992)
4. Lanese, I., Pérez, J.A., Sangiorgi, D., Schmitt, A.: On the expressiveness and decidability of higher-order process calculi. In: Proc. of LICS 1998, pp. 145–155. IEEE Computer Society Press, Los Alamitos (2008)
5. Minsky, M.: *Computation: Finite and Infinite Machines*. Prentice-Hall, Englewood Cliffs (1967)
6. Necula, G.C., Lee, P.: Safe, untrusted agents using proof-carrying code. In: Vigna, G. (ed.) *Mobile Agents and Security*. LNCS, vol. 1419, pp. 61–91. Springer, Heidelberg (1998)
7. Collberg, C.S., Thomborson, C.D., Low, D.: Manufacturing cheap, resilient, and stealthy opaque constructs. In: Proc. of POPL 1998, pp. 184–196. ACM Press, New York (1998)
8. Finkel, A., Schnoebelen, P.: Well-structured transition systems everywhere! *Theor. Comput. Sci.* 256(1-2), 63–92 (2001)
9. Busi, N., Gabbriellini, M., Zavattaro, G.: On the expressive power of recursion, replication, and iteration in process calculi. *Math. Struct. Comp. Sci.* (to appear, 2009)
10. Di Giusto, C., Pérez, J.A., Zavattaro, G.: On the Expressiveness of Forwarding in Higher-Order Communication (Extended Version) (2009), <http://www.cs.unibo.it/~perez/hocore>
11. Higman, G.: Ordering by divisibility in abstract algebras. *Proceedings of the London Mathematical Society* (3) 2(7), 326–336 (1952)
12. Bravetti, M., Zavattaro, G.: On the expressive power of process interruption and compensation. *Math. Struct. Comp. Sci.* (to appear, 2009)
13. Bundgaard, M., Godskesen, J.C., Haagsen, B., Huttel, H.: Decidable fragments of a higher order calculus with locations. In: Proc. of EXPRESS 2008. *Electronic Notes in Theoretical Computer Science*. Elsevier, Amsterdam (2008) (to appear)
14. Bundgaard, M., Godskesen, J.C., Hildebrandt, T.: Bisimulation congruences for homer — a calculus of higher order mobile embedded resources. Technical Report TR-2004-52, IT University of Copenhagen (2004)
15. Cardelli, L., Gordon, A.D.: Mobile ambients. *Theor. Comput. Sci.* 240(1), 177–213 (2000)
16. Busi, N., Zavattaro, G.: On the expressive power of movement and restriction in pure mobile ambients. *Theor. Comput. Sci.* 322(3), 477–515 (2004)