Determining Viscoelastic and Damage Properties Based on Harmony Search Algorithm

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Abstract. This chapter documents the procedure for determining viscoelastic and damage properties using a harmony search (HS) algorithm that employs a heuristic algorithm based on an analogy with music phenomenon. To determine the viscoelastic material parameters, the steps involved in conducting the interconversion between frequency-domain and time-domain functions are outlined, based on the presmoothing of raw data using the HS algorithm. Thus, a Prony series representation of the fitted data can be obtained that includes the determination of the Prony series coefficients. To determine the damage properties of hot mix asphalt (HMA) concrete, a rate-type evolution law is applied for constructing the damage function of the HMA concrete. The damage function can be characterized by fitting experimental results using the HS algorithm. Results from laboratory tests of uniaxial specimens under axial tension at various strain rates are shown to be consistent with the rate-type model of evolution law.

1 Introduction

Linear viscoelastic (LVE) materials are rheological materials that exhibit time-temperature rate-of-loading dependence. When their response is not only a function of the current input, but also of the current and past input history, the characterization of the viscoelastic response can be expressed using the convolution (hereditary) integral. A general overview of time-dependent material properties has been presented [1]. Additionally, a detailed description of the physical response of LVE materials has been explained [2] based on ramp tests to determine the relaxation modulus which is a time-domain LVE response function.

Hot mix asphalt (HMA) concretes used in this study are composite materials consisting of aggregates and asphalt binder. Their behavior is characterized by the interaction between these two components and the LVE behavior of the HMA

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concretes, which depends on temperature, loading frequency, and strain magnitude. Studying the behavior of the HMA material requires modeling the LVE behavior through a dynamic modulus test (DMT) conducted in stress-control within the LVE range. This test is run on all previously untested specimens to obtain a LVE fingerprint (linear viscoelastic properties characteristic of HMA specimen) and to determine the shift factors for the undamaged state after constructing dynamic modulus and phase angle mastercurves. Sinusoidal loading in tension and compression sufficient to produce total strain amplitude of about 70 micro-strains was applied at different frequencies. Based on earlier work [3], the 70 micro-strain limit was found not to cause significant damage to the specimen. For mastercurve construction, several replicates are tested at four temperatures: –10, 5, 25 and 40°C.

Several methods have been proposed to convert the dynamic modulus, a LVE response function in the frequency domain, to the corresponding relaxation modulus in the time domain. For a time-domain LVE function, the Prony series is a popular representation, mainly because of its ability to describe a wide range of viscoelastic response and its relatively simple and rugged computational efficiency associated with its exponential basis functions. In this study, an approach is proposed to overcome the problem – namely, oscillations in the fitted curve – associated with the Prony series fitting. The experimental source data are smoothed through a defined log-sigmoidal function using the heuristic optimization technique of the harmony search (HS) algorithm. The coefficients of the log-sigmoidal function can be determined when a defined error norm converges into the minimum point. Thus, this procedure is required to represent the oscillated broadband data using the smoothed log-sigmoidal function. The fitted log-sigmoidal function is used to obtain the time-domain Prony series. This approach has proved very effective and stable in fitting a Prony series with positive coefficients to LVE response function data, as illustrated in an example using experimental data from a dynamic modulus test on HMA concrete.

In recent years, some success has been achieved in developing a mechanistic constitutive equation of HMA concrete for a viscoelastic continuum damage (VECD) model. Kim et al. [4] developed a uniaxial VECD model by applying the elastic-viscoelastic correspondence principle to separate the effects of viscoelasticity, and then employing internal state variables based on the work potential theory to account for the damage evolution under loading. From the verification study it was found that this constitutive model has the ability to predict the hysteretic behavior of the material under both monotonic and cyclic loading up to failure, varying loading rates, random rest durations, multiple stress/strain levels, and different modes of loading (controlled-stress versus controlled-strain). Daniel and Kim [5] discovered a unique damage characteristic curve that describes the reduction in material integrity as damage grows in the HMA specimen, regardless of the applied loading conditions (cyclic versus monotonic, amplitude/rate, and frequency). Chehab et al. [6] demonstrated that the time-temperature superposition principle is valid not only in the LVE state, but also with growing damage. This finding allows the prediction of mixture responses at various temperatures from laboratory testing from a single temperature.

To characterize the damage function with respect to an internal state variable for the uniaxial behavior of an HMA specimen, the HS algorithm can be applied to fit experimental data into a defined damage function, based on minimizing an error norm.

The outline of this chapter is as follows. Section 2 contains the details of the heuristic HS algorithm. Section 3 provides the HMA material parameters, which must be determined by the HS algorithm. The determination of material parameters and their application to the uniaxial behavior of HMA concrete with damage evolution is shown in Section 4. The concluding remarks found in Section 5 summarize the chapter.

2 Harmony Search Algorithm

Many engineering optimization problems are very complex in nature and quite difficult to be solved using gradient-based search algorithms. If there is more than one local optimum in the problem, the result may depend on the selection of an initial point, and the obtained optimal solution may not necessarily be the global optimum. To determine the LVE and damage properties of HMA concrete, the HS algorithm, which has an analogy between music and optimization, was used in this study.

The HS algorithm conceptualizes a behavioral phenomenon of musicians in the improvisation process, where each musician continues to experiment and improve his or her contribution in order to search for a better state of harmony [7, 8]. This section describes the HS algorithm based on the heuristic algorithm that searches for a globally optimized solution. First, a brief overview of the HS algorithm used to formulate solution vectors in which the optimization process is generated and the object function is evaluated, is provided. Finally, the application procedure of the HS algorithm is explained in detail.

2.1 Algorithm Procedure

The procedure for a harmony search, which consists of Steps 1 to 5, is shown in Figure 1. The algorithm parameters are specified in Step 1, as follows: the *harmony memory size* (HMS) is initialized as the number of solution vectors in *harmony memory* (HM); the *harmony memory considering rate* (HMCR, between 0 and 1) is the rate of memory consideration; the *pitch adjusting rate* (PAR, between 0 and 1) is the rate of pitch adjustment; and the maximum number of improvisations, or stopping criterion, is the termination of the HS program. In addition, the optimization problem is specified as follows:

$$\text{Minimize } f(\mathrm{X}) \text{ subject to } x_i \in X_i,\ i = 1,2,...,N, \tag{1}$$

where $f(\mathrm{X})$ is an objective function; X is the set of each decision variable, x_i; N is the number of decision variables; X_i is the set of the possible range of values for each decision variable, that is $Lx_i \leq X_i \leq Ux_i$; and Lx_i and Ux_i are the lower and upper bounds for each decision variable, respectively.

The HM is a memory location where all the solution vectors (sets of decision variables) are stored. The HM is similar to the genetic pool in the genetic algorithm (GA) [9]. Here, the HMCR and PAR are parameters that are used to improve the solution vector, as defined in Step 3.

In Step 2, the HM matrix is initially filled with as many randomly generated solution vectors as the HMS, as well as with the corresponding function value of each random vector, f(X).

$$\mathrm{HM} = \left[\begin{array}{cccc|c} x_1^1 & x_2^1 & \cdots & x_N^1 & f(\mathrm{X}^1) \\ x_1^2 & x_2^2 & \cdots & x_N^2 & f(\mathrm{X}^2) \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_N^{HMS} & f(\mathrm{X}^{HMS}) \end{array}\right]. \quad (2)$$

In Step 3, a new harmony vector, $\mathrm{X}' = (x_1', x_2', \ldots, x_N')$, is improvised based on the following three mechanisms: 1) random selection, 2) memory consideration, and 3) pitch adjustment. In the random selection, the value of each decision variable, x_i', in the new harmony vector is randomly chosen within the value range with a probability of (1 – HMCR). The HMCR, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in the HM, and (1 – HMCR) is the rate of randomly selecting one value from the possible range of values.

The value of each decision variable obtained by the memory consideration is examined to determine whether it should be pitch-adjusted. This operation uses the PAR parameter, which is the rate of pitch adjustment as it should be pitch-adjusted to neighboring pitches with a probability of HMCR × PAR, while the original pitch obtained in the memory consideration is kept with a probability of HMCR × (1 – PAR). For example, this operation uses the PAR parameter, which ranges between 0 and 1.

If the pitch adjustment decision for x_i' is made with a probability of PAR, x_i' is replaced with $x_i' \pm rand \times bw$, where *rand* and *bw* are random numbers (e.g., a value between 0 and 1) and bandwidths between the lower and upper bounds, respectively. The value of (1 – PAR) sets the rate of performing nothing. Thus, the pitch adjustment is applied to each variable as follows:

$$x_i' \leftarrow \begin{cases} x_i' + rand \times bw, \ \ with\ a\ probability\ of\ \ HMCR \times PAR \times 0.5 \\ x_i' - rand \times bw, \ \ with\ a\ probability\ of\ \ HMCR \times PAR \times 0.5 \\ x_i', \ \ with\ a\ probability\ of\ \ HMCR \times (1 - PAR) \end{cases} \quad (3)$$

If the new harmony vector is better than the worst harmony in the HM, based on the evaluation of an objective function value, the new harmony is included in the HM, and the existing worst harmony is excluded from the HM. Finally, if the stopping criterion (or maximum number of improvisations) is satisfied, the computation is terminated. Otherwise, Steps 3 and 4 are repeated.

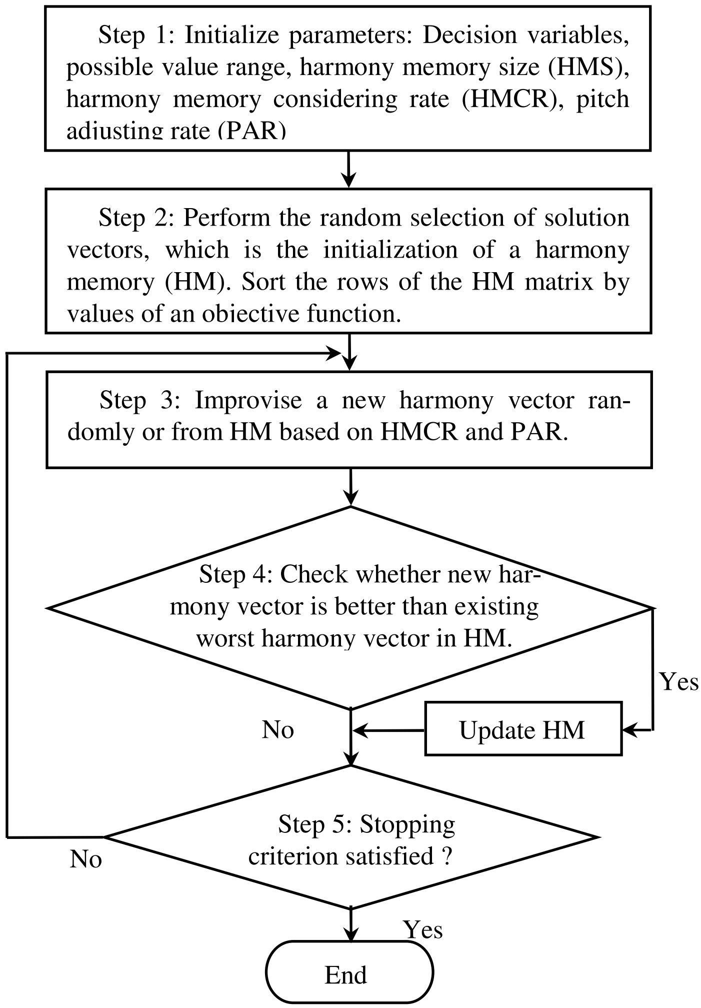


Fig. 1 The Harmony Search Minimization Algorithm

2.2 Application to the Determination of Viscoelastic and Damage Properties of HMA Concrete

In terms of applying the HS algorithm to the determination of viscoelastic and damage properties of HMA materials, a VECD model is used, which is generally applied to describe the fatigue phenomenon occurring at the HMA layers of a flexible pavement. Thus, the optimization problems, which are related to the VECD model, can be solved using the HS algorithm process with respect to a) obtaining the LVE properties from experimental data, b) determining Prony series parameters that are used to represent the LVE HMA by a generalized Maxwell model, and c) constructing the damage parameter of HMA concrete by fitting a defined function into the experimental data. Section 3 explains the detailed procedure of the proposed HS algorithm-based method to determine the viscoelastic and damage properties.

3 Material Parameters of HMA Concrete

An LVE response function defines the response of an LVE material to a unit load. As long as the loading conditions do not contribute to damage in the material, the response can be expressed using the convolution (hereditary) integral. Thus, the LVE material property of HMA concrete is represented by the generalized Maxwell model, which can be viewed as a Prony series expansion of the relaxation modulus. The Prony series coefficients are estimated from the experimental data using the following material modeling procedure:

1) Obtain the storage modulus as a function of loading frequency, based on smoothing the experimental data.
2) Convert the frequency domain data into time domain data.
3) Determine the coefficients of the Prony series representation.

In this study, an approach is proposed to overcome the problems with Prony series fitting, i.e., the oscillations in the fitted curve and the non-negative coefficients of the Prony series. First, the frequency-dependent source data are smoothed through a defined log-sigmoidal function using the HS algorithm; thus, the coefficients of the log-sigmoidal function can be determined when an error norm converges to the minimum point. This step is required to represent the oscillated broadband data using the smooth log-sigmoidal function. Secondly, the fitted log-sigmoidal function is used to obtain the discrete time-domain relaxation modulus by an analytical method. Then, the HS algorithm is also used to find the solution that represents the continuous time-domain LVE function in the form of a log-sigmoidal function. Finally, a continuous spectrum method is used to force the coefficients of the Prony series to be positive.

To take the damage of HMA materials into consideration, the constitutive model based on previous research [4, 10] can be used. The model uses the elastic-viscoelastic correspondence principle to eliminate the time dependence of the material. The work potential theory [11, 12] is then used to model the damage growth in the material as well as to include the viscoelastic effects of microcracking. The work potential theory proposes the following rate-type damage evolution law:

$$\dot{S}_m = \left(-\frac{\partial W^R}{\partial S_m} \right)^{\alpha_m}, \tag{4}$$

where the overdot represents the derivative with respect to time; W^R is the pseudo strain energy density function; α_m is a material-dependent constant; and m is not summation.

In its application to the uniaxial behavior of HMA concrete [13], the experimental stress-strain constitutive relationship is incorporated into the one-dimensional pseudo strain energy density function of the material in the following form:

$$W^R = \frac{1}{2} C(S)(\varepsilon^R)^2 , \tag{5}$$

where the damage function, $C(S)$, depends on a single damage parameter, S; and ε^R is the pseudo strain. In order to determine a state variable, S, which is used to trace a state of material damage according to the relationship with the pseudo strain energy density function in Equation (5), the HS algorithm can be applied for fitting the experimental data into a defined function of $C(S)$ based on minimizing an error norm.

3.1 Testing Systems and Methods

In this study, 12.5 mm and 9.5 mm mixtures were used for unmodified HMA and lime-modified HMA, respectively. The 12.5 mm and 9.5 mm mixtures were used based on the Superpave mix design [14]. The aggregate blend used consisted of 95.5% by mass granite aggregates, 3.5% natural sand, and 1% baghouse fines. In case of the lime-modified HMA, hydrated lime (1% by aggregate weight) is substituted for a portion of the baghouse fines. The asphalt binders used were performance grade (PG) 70-22 and PG 58-28 for unmodified and lime-modified HMA, respectively. The asphalt contents were 5.2% for the unmodified HMA; 5.3% and 5.8% for the lime-modified HMA by mass. Mixing and compaction temperatures were 166°C and 153°C, respectively. Compaction was done using the Superpave gyratory compactor.

The mixtures were compacted into gyratory plugs of 150 mm in diameter by 178 mm in height. Then, they were cut and cored to cylindrical specimens with dimensions of 100 mm in diameter and 150 mm in height for dynamic modulus tests and 75 mm in diameter and 140 mm in height for constant crosshead tests.

Regarding testing setup and methods, two different closed-loop servo-hydraulic testing machines were utilized in this study. The first is a MTS 810 loading frame equipped with either a 25 kN or 8.9kN load cell, depending on the nature of the test. For this machine, an environmental chamber, equipped with liquid nitrogen coolant and a feedback system, was used to control and maintain the test temperature. The second machine is a UTM-25 machine, manufactured by IPC Global of Australia. This machine is equipped with a 25 kN load cell. The environmental chamber for the UTM-25 is refrigerator-driven and also uses a feedback system to maintain a consistent temperature during the testing.

Measurements of axial deformations, load, and crosshead movements were taken for all tests. The data acquisition system of both machines was identical, consisting of a National Instruments 16-bit data acquisition card and Labview software. In all tests conducted in this study, axial displacement measurements were taken with linear variable differential transformers (LVDTs) from IPC Global.

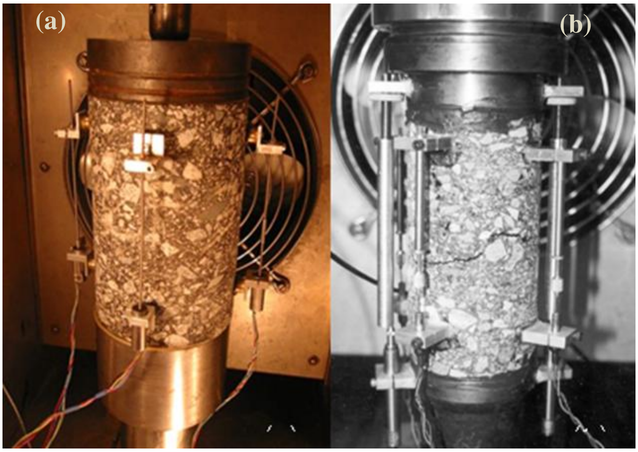


Fig. 2 LVDTs Mounting on Specimens: (a) Dynamic Modulus Test; and (b) Constant Crosshead Rate Test

Tests conducted in this study include dynamic modulus testing and constant crosshead rate testing. All tests were done in both machines. The dynamic modulus test (DMT) shown in Figure 2a is performed by applying a sinusoidal load to an asphalt concrete specimen to obtain the linear viscoelastic material properties of asphalt mixtures. The loading amplitude is adjusted based on the material stiffness, temperature, and frequency to keep the strain response within the linear viscoelastic range.

Regarding the determination of the damage function, $C(S)$, the constant crosshead rate tests were conducted in tension mode till failure of the specimen at different crosshead rates as shown in Figure 2b. Testing temperatures were 5°C and 25°C for the lime-modified HMA and the unmodified HMA, respectively, in this study.

3.2 Determination of LVE Material Parameters

In order to get the experimental data to be fitted in this study, a DMT is performed by controlling a micro-strain level of 70 that is targeted as the limit for the LVE. The loading is applied until steady-state response is achieved, at which point several cycles of data are collected. After each frequency, a five-minute rest period is allowed for specimen recovery before the next loading block is applied. The frequencies are applied from the fastest to the slowest ranging from 1 to 20 Hz.

From the DMT, the complex modulus, E^*, which includes the dynamic modulus ($|E^*|$) and the phase angle (ϕ), can be determined. The complex modulus can also be viewed as a composition of storage (E') and loss modulus (E'') as follows:

$$E^* = E' + iE'' \tag{6}$$

where i is the $\sqrt{-1}$. The dynamic modulus is the amplitude of the complex modulus and is defined as:

$$|E^*| = \sqrt{(E')^2 + (E'')^2} \;. \tag{7}$$

The values of the storage and loss modulus, which are shown in Figure 3, are related to the dynamic modulus and phase angle as follows:

$$E' = |E^*| \cos\phi \text{ and } E'' = |E^*| \sin\phi \;. \tag{8}$$

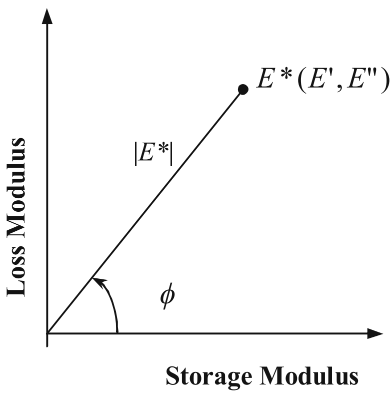


Fig. 3 Complex Modulus Schematic Diagram

As the material becomes more viscous, the phase angle increases and the loss component of the complex modulus increases. Conversely, a decreasing phase angle indicates more elastic behavior and a larger contribution from the storage modulus. The dynamic modulus at each frequency is calculated by dividing the steady state stress amplitude, σ_{amp}, by the strain amplitude, ε_{amp}:

$$|E^*| = \frac{\sigma_{amp}}{\varepsilon_{amp}} \;. \tag{9}$$

The phase angle, ϕ , is associated with the time lag, Δt , between the strain input and stress response at the corresponding frequency, f:

$$\phi = 2\pi f \Delta t \;. \tag{10}$$

In order to determine the storage modulus prior to the conversion (i.e., from frequency domain to time domain) using a Prony series function, the discrete raw data need to be fitted using a continuous function. Thus, the quality of raw data can be significantly improved using a defined log-sigmoidal function that describes the full viscoelastic range of the HMA, from the glassy state to the low frequency plateau. The log-sigmoidal function, $f(\omega)$, is defined as

$$f(\omega) = a_1 + \frac{a_2}{\left\{ a_3 + \frac{a_4}{\exp\left[a_5 + a_6 \log_{10}(\omega)\right]} \right\}} \tag{11}$$

where $a_{1,2,\ldots,6}$ are the coefficients determined by the HS algorithm, and ω is the angular frequency. For example, the optimizing solution between the storage modulus data and the log-sigmoidal function that is used to determine the coefficients of Equation (11) can be expressed by the following Equation (12):

$$\text{Minimize } g(\omega_i) = \sum_{i=1}^{N} \left| \log_{10}\left[E'(\omega_i)\right] - f(\omega_i) \right|, \tag{12}$$

where $g(\omega_i)$ is the objective function (e.g., error norm); $E'(\omega_i)$ is the storage modulus; subscript i denotes the individually selected angular frequency; N is the total number of selected angular frequencies; and the vertical bar indicates the absolute value.

The interconversion between LVE material functions of a frequency-domain, E', and a time-domain, $E(t)$, has been provided by several researchers [15, 16]. In this study, an approximate analytical solution developed by [15] is used in the following form:

$$E(t) \cong \frac{1}{\lambda'} E'(\omega) \big|_{\omega=(1/t)}, \tag{13}$$

where λ' is an adjustment factor that is defined by $\Gamma(1\text{-}n)\cos(n\pi/2)$; Γ is a gamma function; and n is the local log-log slope of the storage modulus, that is,

$$n = \frac{d \log_{10} E'(\omega)}{d \log_{10} \omega}. \tag{14}$$

Thus, the time-domain relaxation modulus can also be fitted using the defined log-sigmoidal function with respect to time, which is the same as Equation (11), based on the above function-fitting algorithm.

In order to introduce the time-domain Prony series representation, the uniaxial, non-aging, isothermal stress-strain constitutive equation for a LVE material can be considered, as follows:

$$\sigma(t) = \int_0^t E(t-\tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau, \tag{15}$$

where $\sigma(t)$, $\varepsilon(t)$, and $E(t)$ are the stress, strain, relaxation modulus components in a time domain t, respectively. The relaxation modulus of Equation (13), $E(t)$, which is based on a generalized Maxwell model consisting of a series of springs and dashpots [17] in the form of a Prony series, can be expressed as follows:

$$E(t) = E_\infty + \sum_{m=1}^{M} E_m \exp(-t/\rho_m), \tag{16}$$

where E_∞, ρ_m, and E_m are the infinite relaxation modulus, relaxation time, and Prony coefficients, respectively.

Considering the case of an infinite number of Maxwell components with continuously distributed relaxation times, and neglecting the infinite relaxation modulus, E_∞, based on the continuous spectrum method [18-20], the relaxation modulus can be defined by

$$E(t) = \int_{-\infty}^{\infty} L(\rho)\exp(-t/\rho)d\rho\ , \tag{17}$$

where L(ρ) represents a continuous distribution of the relaxation modulus, and is defined by

$$L(\rho) = \lim_{k\to\infty} \frac{(-k\rho)^k}{(k-1)!} E^{(k)}(k\rho)\ . \tag{18}$$

In order to determine the discrete Prony series coefficients, E_m, in Equation (16) from the continuous spectrum equation of Equation (17), the continuous spectrum can be approximated by subdividing lnρ into time intervals $\Delta(\ln\rho_m)$ = $\ln 10\Delta(\log_{10}\rho_m)$, as follows:

$$E(t) = \int_{-\infty}^{\infty} L(\rho)\exp(-t/\rho)d\ln\rho \approx \sum_{m=1}^{M} L(\rho_m)\exp(-t/\rho_m)\Delta(\ln\rho_m)\ . \tag{19}$$

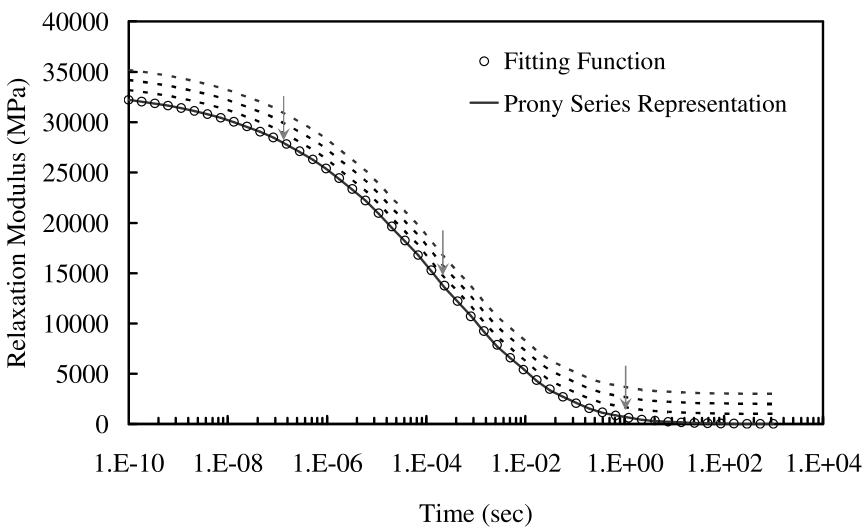


Fig. 4 Adjusting the Vertical Shift of a Relaxation Modulus Curve through the HS Algorithm to Obtain an Infinite Relaxation Modulus, E_∞

The Prony coefficients for the chosen relaxation time can be determined by

$$E_m = L(\rho_m)\ln 10\Delta(\log_{10}\rho_m)\,. \tag{20}$$

Finally, the infinite relaxation modulus, E_∞, of a Prony series representation can be found by adjusting the vertical shift of a relaxation modulus curve through the HS algorithm, as shown in Figure 4.

3.3 Determination of Damage Function, C(S)

Schapery [12] developed a theory using the method of thermodynamics of irreversible processes to describe the mechanical behavior of elastic composite materials with growing damage. Three fundamental elements comprise the work potential theory:

1) Strain energy density function, $W = W(\varepsilon_{ij}, S_m)$ (21)

2) Stress-strain relationship, $\sigma_{ij} = \dfrac{\partial W}{\partial \varepsilon_{ij}}$ (22)

3) Damage evolution law, $-\dfrac{\partial W}{\partial S_m} = \dfrac{\partial W_s}{\partial S_m}$, (23)

where σ_{ij} and ε_{ij} are stress and strain tensors, respectively; S_m are the internal state variables; and $W_s = W_s(S_m)$ is the dissipated energy due to structural changes. Using Schapery's elastic-viscoelastic correspondence principle and the rate-type damage evolution law [11-13], the physical strains, ε_{ij}, are replaced with pseudo strains, ε_{ij}^R, to include the effect of viscoelasticity. The correspondence principle proposes the extended elastic-viscoelastic correspondence principle, which is applicable to both LVE and nonLVE materials. Schapery [11, 12] suggests that constitutive equations for certain viscoelastic media are identical to those for the elastic cases, but stresses and strains are not necessarily physical quantities in the viscoelastic body. Instead, they are pseudo variables in the form of convolution integrals. According to Schapery, the pseudo strain in a uniaxial case is defined as

$$\varepsilon^R = \frac{1}{E_R}\int_0^t E(t-\tau)\frac{d\varepsilon}{d\tau}d\tau\,, \tag{24}$$

where E_R and $E(t)$ are the reference modulus and relaxation modulus, respectively. The use of pseudo strain, as defined in Equation (24), accounts for all the hereditary effects of the material through the convolution integral. Thus, the strain energy density function, $W = W(\varepsilon, S)$, transforms to the pseudo strain energy density function.

The pseudo strain energy density function of the material is formulated using Equation (22). For the uniaxial case, the stress can be determined by using the relationship of Equation (5), as follows:

$$\sigma = \frac{\partial W^R}{\partial \varepsilon^R} = C(S)\varepsilon^R . \tag{25}$$

The damage evolution law, Equation (4), is reduced to the following single equation for S:

$$\dot{S} = \left(-\frac{\partial W^R}{\partial S} \right)^{\alpha} . \tag{26}$$

To characterize the function $C(S)$ in Equation (25), the damage evolution law and experimental data are used. Using the measured stresses and calculated pseudo strains, the C values can be determined through Equations (24) and (25). For uniaxial loading conditions, a single damage variable, S, is used along with the associated power, α. Using the experimental data, the following incremental relationship can be obtained by combining Equations (5) and (26):

$$\Delta S = \left[\left\{ -\frac{1}{2} \Delta C (\varepsilon^R)^2 \right\}^{\alpha} \Delta t \right]^{1/(1+\alpha)} \quad \text{and} \tag{27}$$

$$S \cong \sum_{n=1}^{N} \left[\frac{1}{2} (C_{n-1} - C_n)(\varepsilon_n^R)^2 \right]^{\alpha/(1+\alpha)} (t_n - t_{n-1})^{1/(1+\alpha)} . \tag{28}$$

The value of α can be found by using the following relationship: $\alpha = 1 + 1/n$, in which $n = -\log E(t)/\log(t)$. Furthermore, a relationship is constructed between C and S, based on a functional form, as follows:

$$C(S) = \exp(-\beta \cdot S^{\gamma}) , \tag{29}$$

where β and γ are the parameters that are determined through the best fitting process of the HS algorithm between the functional form and the experimental data.

4 Determination of Material Parameters and Uniaxial Behaviour of HMA Concrete with Damage Evolution

A detailed application of the HS algorithm to determine the material parameters is as follows: 1) determine the coefficients of the fitting function described in Equation (11); 2) determine the Prony series coefficients based on the fitting function; 3) characterize the damage function, $C(S)$, using experimental data; and 4) predict the damage behavior of HMA concrete at various strain input rates.

4.1 Determination of the Coefficients of the Fitting Function

In order to smooth the discrete raw data that need to be fitted using the defined log-sigmoidal function of Equation (11), the HS algorithm can be used based on minimizing the objective function of Equation (12). Figure 5 shows the error norm of the objective function defined by Equation (12) in terms of the number of iterations. Based on Figure 5, it is noted that the best solution converges as the number of iterations increases. Finally, Figure 6 shows the log-sigmoidal function smoothly fitted with the raw data; therefore, the coefficients of Equation (11) in the frequency domain can be found through the HS algorithm as shown in Table 1.

At the low frequencies shown in Figure 6b, some minor loss of information may be resulted due to some local irregularities of the scattered data. However, the fitting function provides a smooth representation over all frequencies.

4.2 Time-Domain Prony Series Representation

To determine the Prony series coefficients, the analytical solution of Equation (13) is utilized to convert a frequency-domain to a time-domain modulus; also, the time-domain modulus is fitted with the defined log-sigmoidal function of Equation (11) as shown in Table 1. Based on Equation (20), the Prony coefficients for the chosen relaxation times can be found, and the infinite relaxation modulus can be found, as shown in Figure 4. Finally, Prony series representation of the relaxation modulus is shown in Figure 7. Table 2 shows the Prony series coefficients of the unmodified and lime-modified HMA concretes.

4.3 Comparison between HS Algorithm and Regression Method in Terms of Prony Series Representation

The regression method, which is used for comparing with the HS algorithm, is based on fitting raw data into a predefined function through a least squares method as shown in Figure 8. Thus, the storage modulus is expressed by the regression equation that is similar to other work [4].

In order to determine the Prony series coefficients in the regression model, a viscoelastic constitutive relationship derived from the generalized Maxwell model can be used. The mechanical model consists of a series of springs and dashpots in an arrangement shown in Figure 9. For a given applied strain, ε, the stress in the single spring, σ_∞, is given as follows:

$$\sigma_\infty = E_\infty \varepsilon \ . \tag{30}$$

The stress, σ_m, in each of the Maxwell components combining a spring with a dashpot is governed by the differential equation:

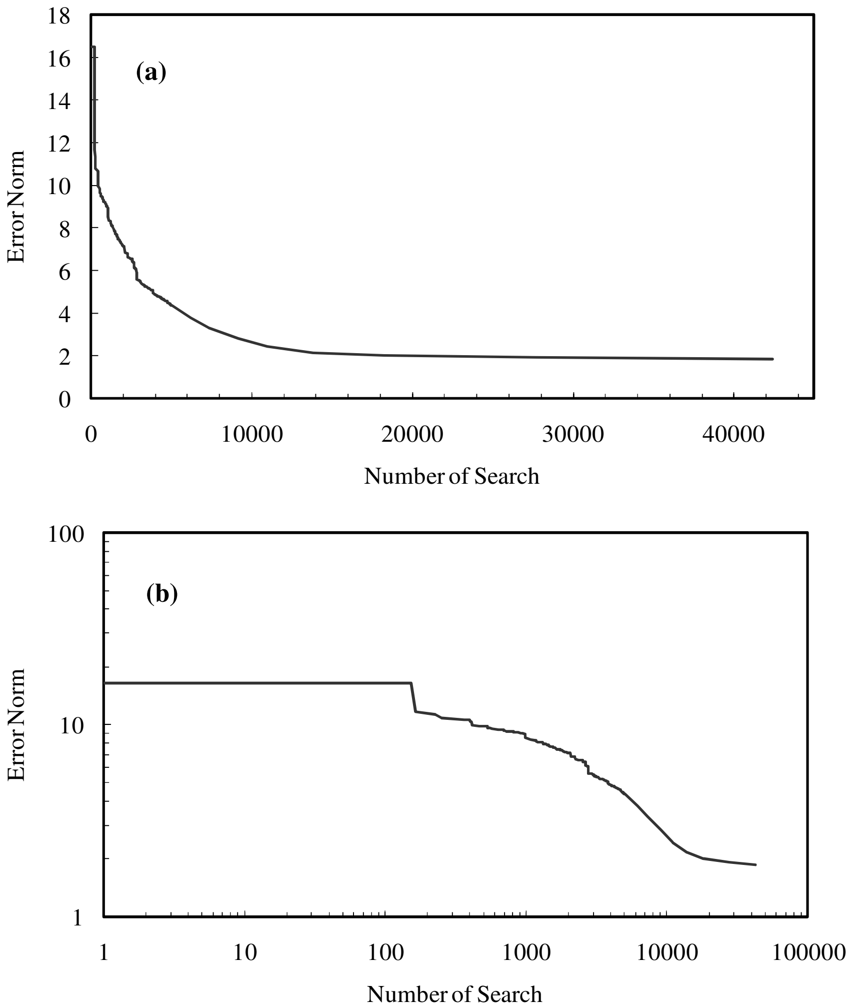


Fig. 5 Convergence of the Objective Function during the Number of Searches: (a) Semi-Log Scale; (b) Log-Log Scale

$$\frac{d\varepsilon}{dt} = \frac{1}{E_m}\frac{d\sigma_m}{dt} + \frac{\sigma_m}{\eta_m} \tag{31}$$

where η_m is the coefficient of viscosity, and E_m is the spring stiffness (e.g., Prony coefficient) in the m^{th} term or Prony coefficient. Based on the linearity of the material components, the total stress in the generalized Maxwell model is obtained by a summation as follows:

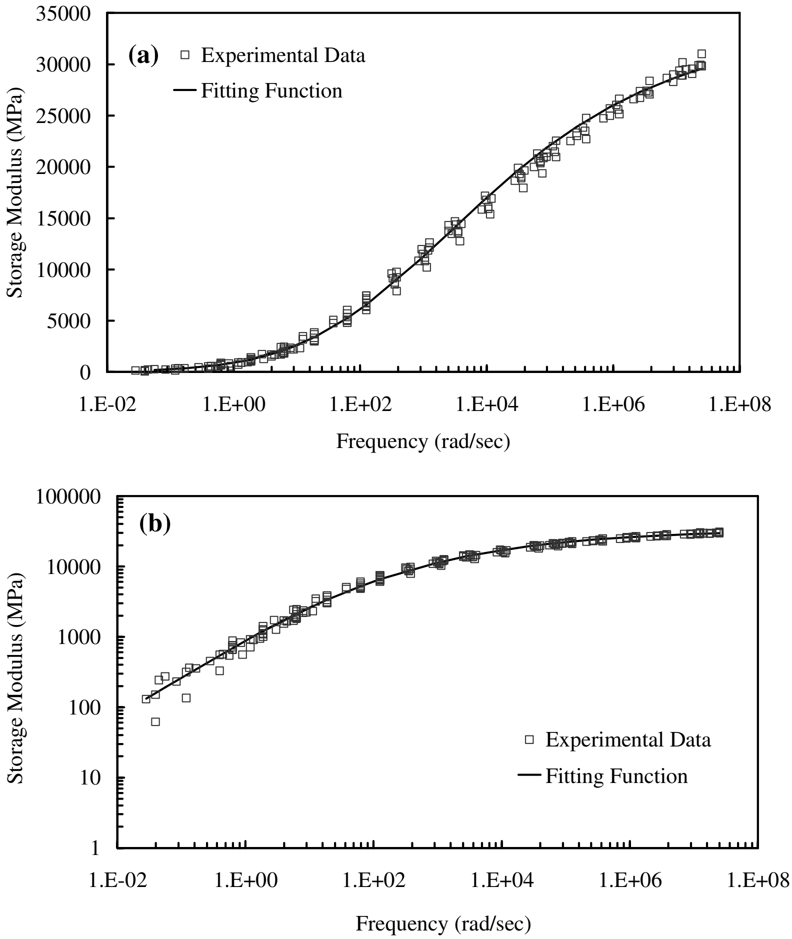


Fig. 6 Fitting the Experimental Data of the Unmodified HMA to the Log-Sigmoidal Function: (a) Semi-Log Scale; (b) Log-Log Scale

$$\sigma = \sigma_\infty + \sum_{m=1}^{M} \sigma_m \ . \tag{32}$$

Fourier transform is useful in solving the above differential equation based on the elastic-viscoelastic correspondence principle, where elastic moduli are replaced by their corresponding viscoelastic counterparts in the Fourier transform domain [21, 22]. Hence, the differential equation relating stress to strain is converted into an algebraic equation. Applying the Fourier-transform technique to Equations (30) to (32), and then eliminating the stresses σ_∞ and σ_m from the equation gives:

Table 1 Coefficients of the Log-Sigmoidal Function in the Time and Frequency Domain

		a_1	a_2	a_3	a_4	a_5	a_6
Unmodified HMA (Binder content: 5.2%)	Coefficients in frequency	4.51792	1.21033	-0.29528	-4.11821	2.17693	-0.52211
	Coefficients in time	4.52263	-2.46005	0.59640	1.28745	0.39382	0.52051
Lime-modified HMA (Binder content: 5.3%)	Coefficients in frequency	1.78212	-5.38729	-1.92668	-0.40234	-0.17790	0.48422
	Coefficients in time	1.59498	-5.32351	-1.76661	-0.69466	0.34174	-0.43839
Lime-modified HMA (Binder content: 5.6%)	Coefficients in frequency	2.02698	-1.51221	-0.59659	-5.22501	3.28673	0.49089
	Coefficients in time	1.94225	13.47153	5.11108	0.62004	-1.09828	-0.46373

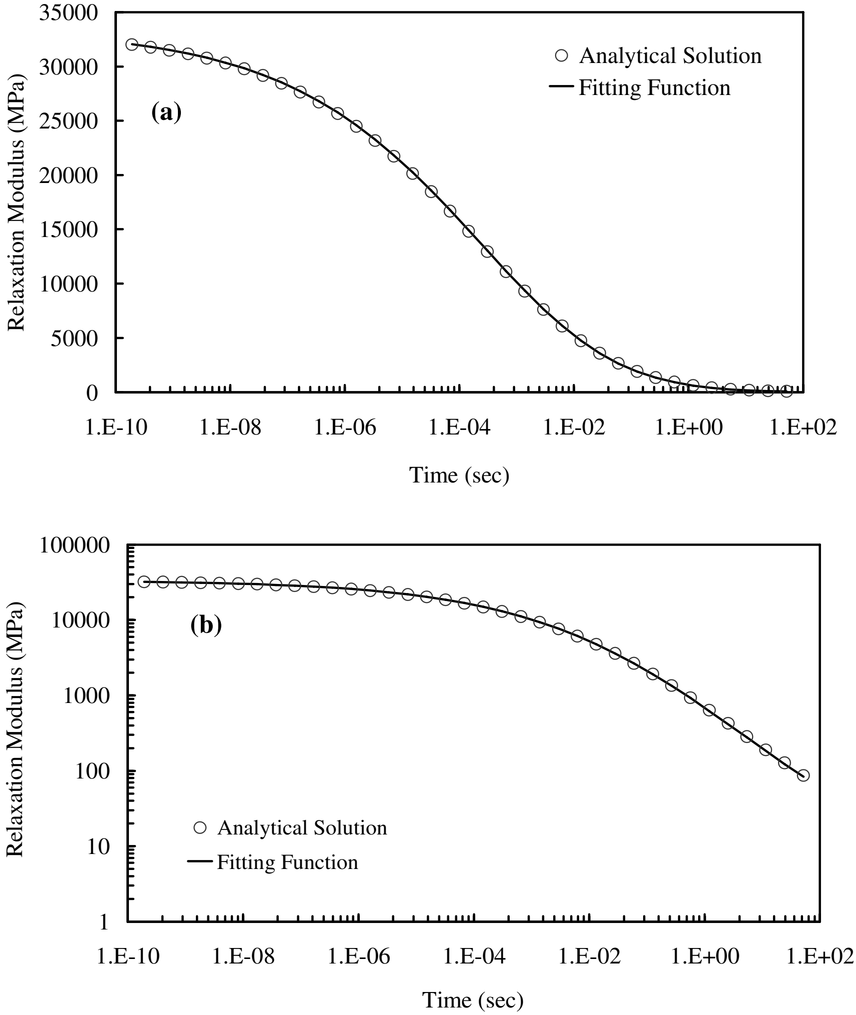


Fig. 7 Fitting the Experimental Data of the Unmodified HMA to the Log-Sigmoidal Function: (a) Semi-Log Scale; (b) Log-Log Scale

$$\breve{\sigma} = \left(E_\infty + \sum_{m=1}^{M} \frac{i\omega_n \rho_m E_m}{i\omega_n \rho_m + 1} \right) \breve{\varepsilon} \text{ , } n=1,\ldots,N \tag{33}$$

where $\breve{\sigma}$ and $\breve{\varepsilon}$ are defined as stress and strain in the Fourier-transform domain, with the relaxation time of the m^{th} Maxwell element expressed as follows:

$$\rho_m \equiv \frac{\eta_m}{E_m} . \tag{34}$$

Table 2 Prony Series Coefficients of the Unmodified and Lime-Modified HMA Concretes

Unmodified HMA (Binder content: 5.2%)		Lime-modified HMA (Binder content: 5.3%)		Lime-modified HMA (Binder content: 5.6%)	
E_∞: 22.40 MPa		E_∞: 879.58 MPa		E_∞: 800.46 MPa	
ρ_m	E_m (MPa)	ρ_m	E_m (MPa)	ρ_m	E_m (MPa)
1.E-10	507.22	1.E-10	366.30	1.E-10	317.03
1.E-09	834.93	1.E-09	560.07	1.E-09	497.47
1.E-08	1353.67	1.E-08	849.91	1.E-08	774.59
1.E-07	2139.18	1.E-07	1274.78	1.E-07	1191.33
1.E-06	3237.37	1.E-06	1877.92	1.E-06	1796.96
1.E-05	4555.73	1.E-05	2690.98	1.E-05	2628.59
1.E-04	5683.89	1.E-04	3696.78	1.E-04	3665.22
1.E-03	5855.13	1.E-03	4766.10	1.E-03	4748.17
1.E-02	4555.50	1.E-02	5597.54	1.E-02	5513.79
1.E-01	2485.68	1.E-01	5765.33	1.E-01	5492.53
1.E+00	949.99	1.E+00	4999.45	1.E+00	4501.30
1.E+01	282.39	1.E+01	3541.64	1.E+01	2976.67
1.E+02	77.29	1.E+02	2047.75	1.E+02	1624.62
1.E+03	22.60	1.E+03	1005.01	1.E+03	780.13

The complex modulus can be obtained from the constitutive equation shown in Equation (33) according to the following equation:

$$E^* = E_\infty + \sum_{m=1}^{M} \frac{i\omega_n \rho_m E_m}{i\omega_n \rho_m + 1}, \; n=1,\ldots,N. \tag{35}$$

From Equation (35), the storage modulus in frequency-domain can be determined by taking the real parts of the complex modulus:

$$E'(\omega_n) = E_\infty + \sum_{m=1}^{M} \frac{\omega_n{}^2 \rho_m^2 E_m}{\omega_n{}^2 \rho_m^2 + 1}, \; n=1,\ldots,N. \tag{36}$$

In order to obtain the time-domain relaxation modulus, the Prony series function in Equation (16) is determined by using the equivalent E_∞, ρ_m, and E_m shown in Equation (36); E_∞ can be found by the limit of $E'(\omega)|_{0<\omega<<1}$. The Prony-series coefficients, E_m, are obtained based on the selected relaxation times and reduced frequencies, ρ_m and ω_n, and the following linear algebraic equation:

$$\vec{f} = \mathbf{E}^{-1}\vec{d} \tag{37}$$

where the column vectors, $\vec{f}$ and $\vec{d}$, are E_m and $E'(\omega_n)$-E_∞ respectively; the superscript –1 denotes an inversion; the matrix, $\mathbf{E}^{-1}$, is as follows:

$$\mathbf{E} = E_{n,m} = \sum_{m=1}^{M} \frac{\omega_n{}^2 \rho_m^2}{\omega_n{}^2 \rho_m^2 + 1}, \; n=1,\ldots,N. \tag{38}$$

Based on the HS algorithm and regression method, two Prony series representations of relaxation modulus are shown in Figure 10 using the unmodified HMA. In case of the regression method, some oscillation can be observed between 0.01 and 1000 sec. Furthermore, some Prony coefficients shown in Table 3 resulted in negative values, which do not physically make sense because spring stiffnesses should be positive. In order to obtain the positive values of moduli in case of the regression approach, a non-linear optimization process [23] is necessary to approach a better solution.

However, the HS algorithm used in this study addresses the problem of negative Prony series coefficients or oscillations based on fitting raw data with the sigmoidal function when the source data exhibit significant variability.

4.4 Determination of the Parameters of Damage Function C(S)

In order to calculate the damage parameter, S, in terms of the damage function, $C(S)$, the value of α is found to be equal to 2.9 using the log-scale slope, n, in case of the unmodified HMA concrete. Using the HS algorithm as well as the raw data obtained from the calculation of Equation (28), the parameters β and γ of Equation

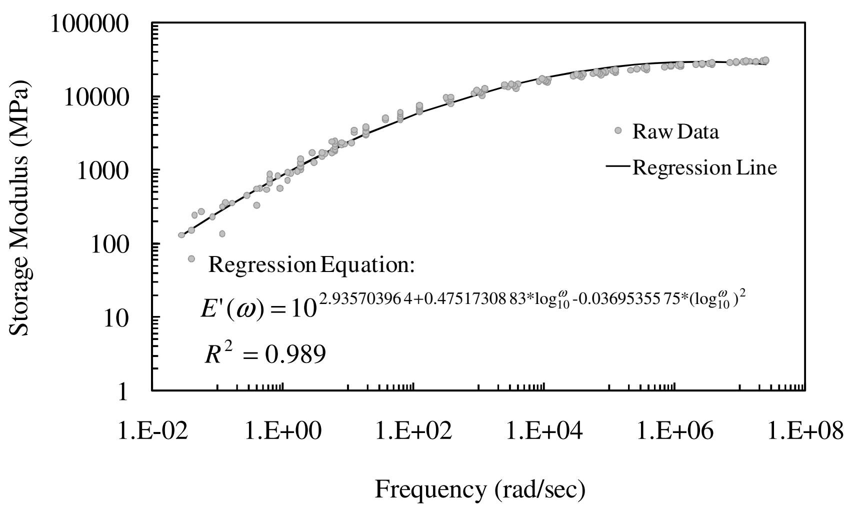


Fig. 8 Fitting Raw Data into a Predefined Function through a Least Squares Method

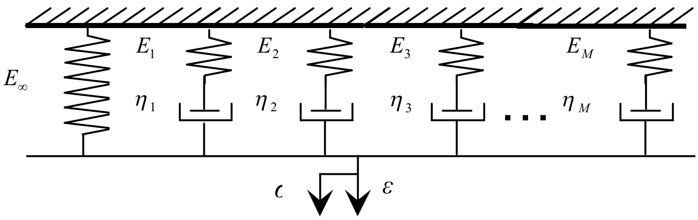


Fig. 9 Generalized Maxwell Model Used for the LVE Modeling

(29) are determined as 0.002757 and 0.543455, respectively, in case of the unmodified HMA concrete. Figure 11 shows the defined function that is fitted using the experimental data of the unmodified HMA. As the same of the determination procedure of the unmodified HMA material parameters, the material parameters of the lime-modified HMA were found as shown in Table 4.

4.5 *Application to the Prediction of the Damage Behaviour of HMA Concrete*

The material parameters, which are determined by the HS algorithm presented in Section 3, are used for the verification using experimental results. A series of uniaxial extension tests was performed on the cylindrical specimens at 25°C at different constant strain rates, 0.0063/sec and 0.0294/sec, for the unmodified HMA; at 5°C at different constant strain rates, 0.00003/sec and 0.000055/sec, for the lime-modified HMA. Each specimen was glued to the end plates, which were then

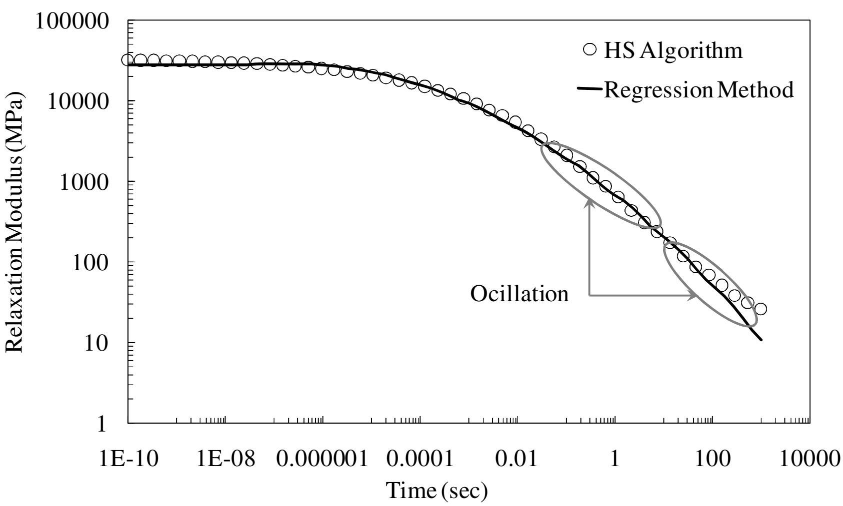


Fig. 10 Prony Series Representations Based on the HS Algorithm and Regression Method

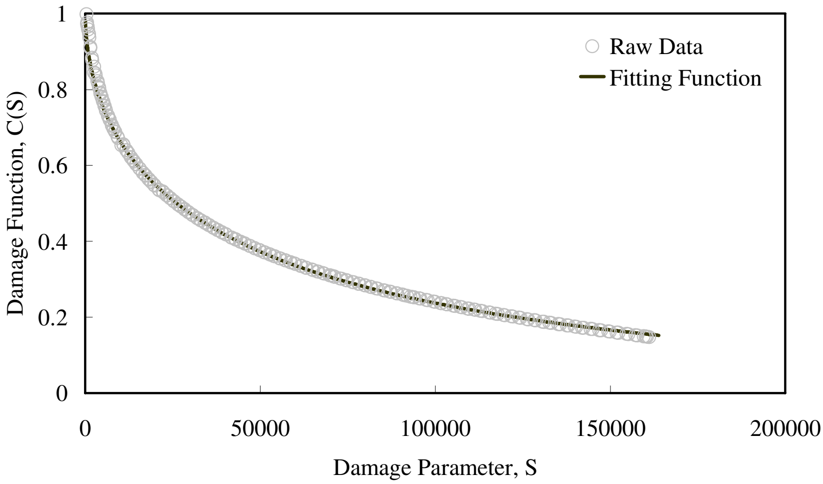


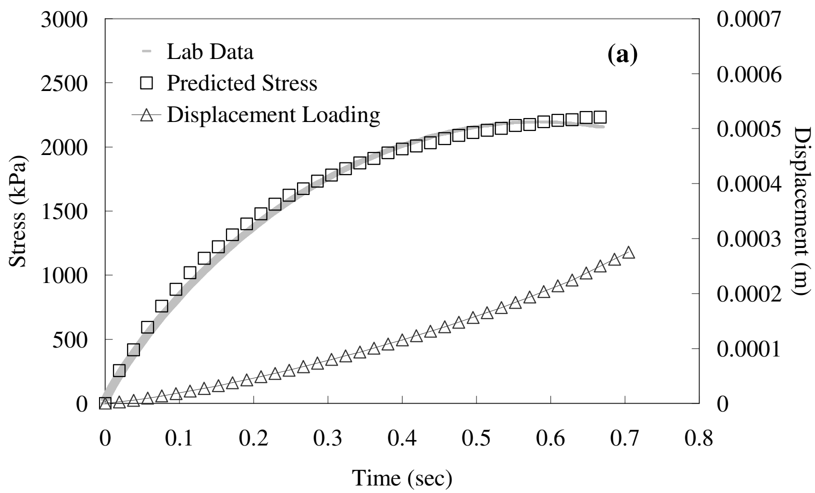
Fig. 11 Damage Function/Parameter Curve of the Unmodified HMA for Raw Data and Fitting Function

Table 3 Prony Coefficients Obtained by Using the Regression Model

Unmodified HMA (Binder content: 5.2%)	
E_∞ : 34.40 MPa	
ρ_m	E_m (MPa)
2.E-07	-1445.86
2.E-06	3756.61
2.E-05	6886.95
2.E-04	7256.92
2.E-03	5593.55
2.E-02	3371.45
2.E-01	1634.09
2.E+00	646.07
2.E+01	210.06
2.E+02	56.50
2.E+03	12.40
2.E+04	3.043
2.E+05	-2.50
2.E+06	11.48
2.E+07	-43.54

Table 4 Material Parameter of the Lime-Modified HMA

Material parameters	Lime-modified HMA (Binder content: 5.3%)	Lime-modified HMA (Binder content: 5.6%)
α	2.79	3.06
β	0.001247	0.539442
γ	0.002001	0.503317



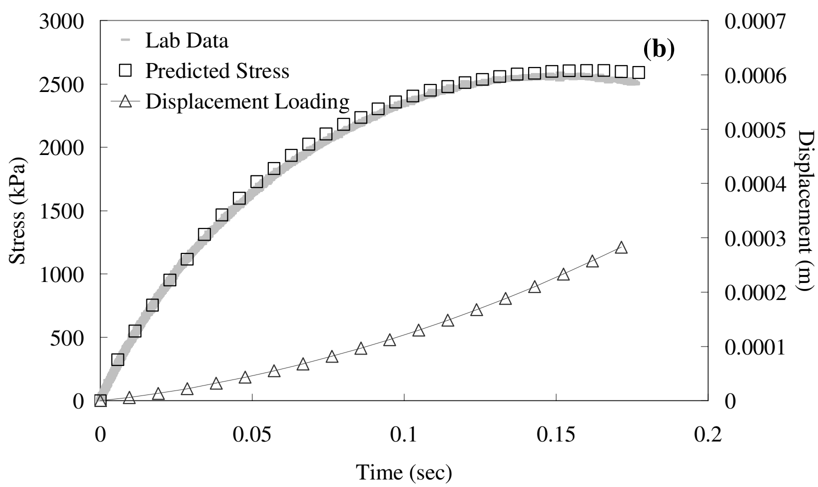


Fig. 12 Stress Prediction of the Unmodified HMA: (a) 0.0063/sec Strain Rate; (b) 0.0294/sec Strain Rate

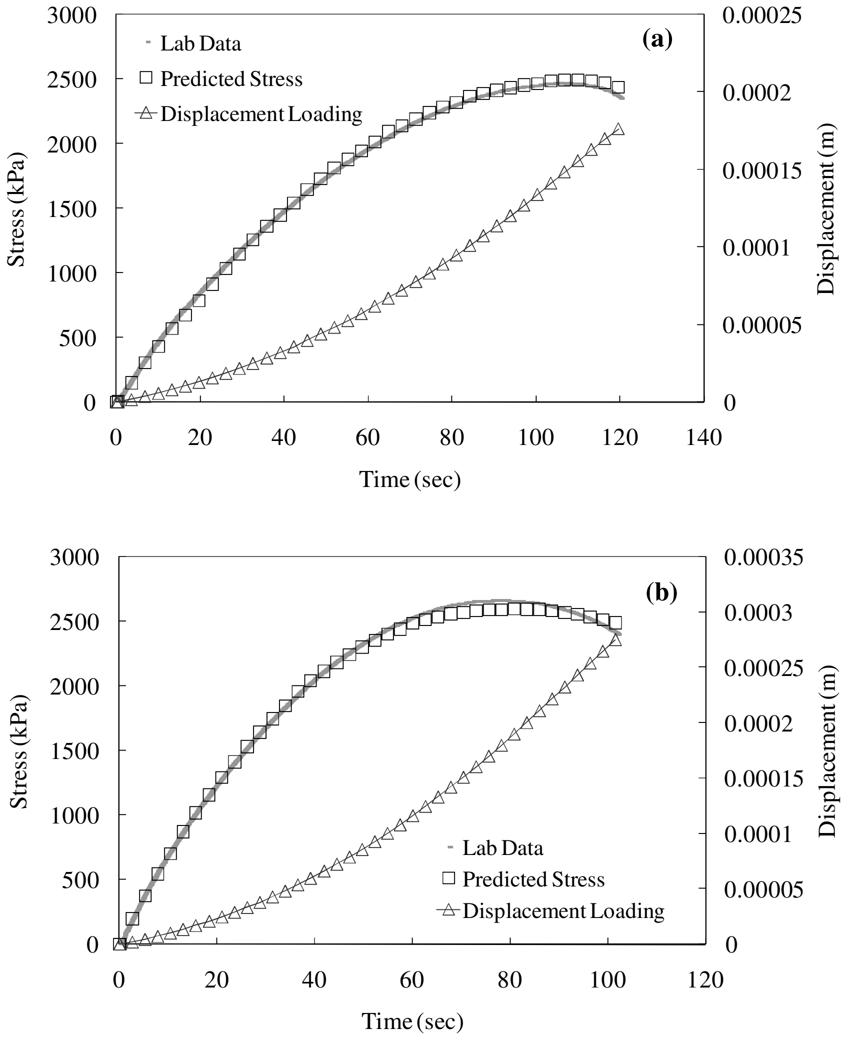


Fig. 13 Stress Prediction of the Lime-Modified HMA: (a) 0.00003/sec Strain Rate and 5.3% Binder Content; (b) 0.000055/sec Strain Rate and 5.8% Binder Content

connected to the loading frame through a load cell. Axial elongation was obtained by measuring the linear variable differential transformer (LVDT) attached to the specimen. Test results are shown in Figures 12 and 13 at various strain rates and different temperatures. The stress predictions for the given displacement load measured from the LVDT are also presented in Figures 12 and 13. Because of machine compliance (e.g., deformation of certain machine components along the loading train under load), the displacements were measured from the LVDT attached to the tested specimens. As shown in these figures, the stress predictions are accurately matched using the experimental data. Furthermore, the stress response for a given displacement load is sensitive to the strain rate. For example, a faster strain rate results in a larger stress level. The purely viscoelastic responses of the unmodified and lime-modified HMA concretes are shown in Figures 14 and 15. It can be observed that the LVE responses depart from the experimental data at the early stage.

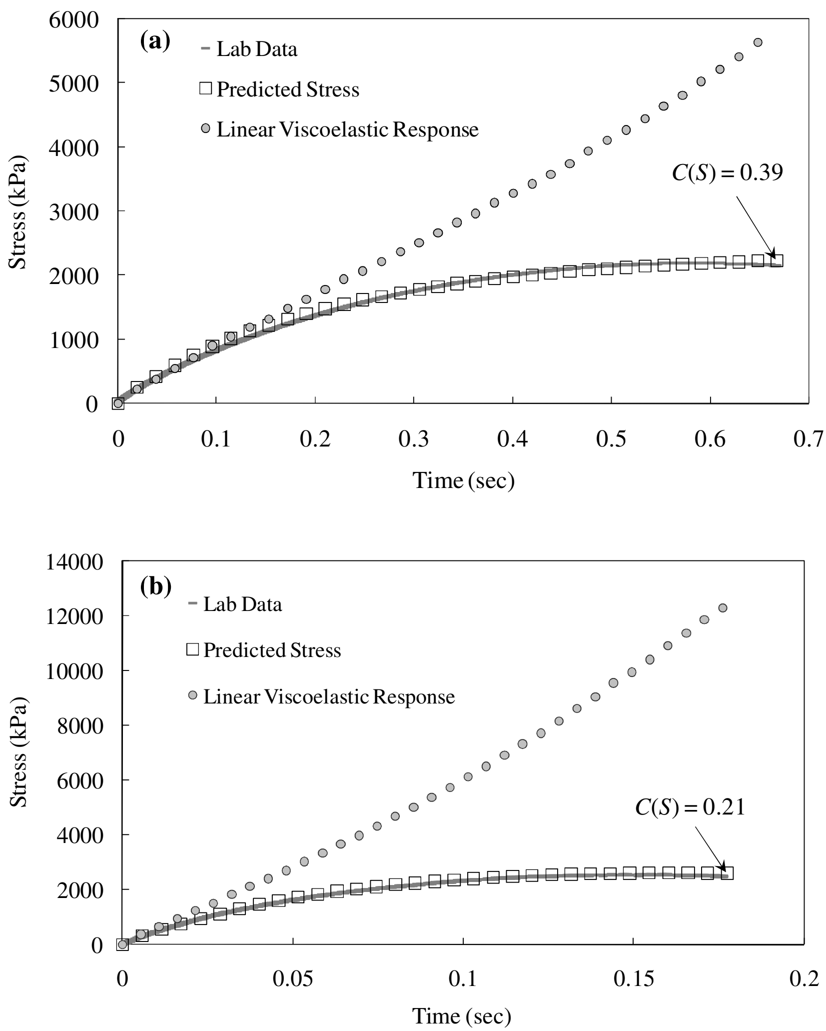


Fig. 14 LVE Response of the Unmodified HMA: (a) 0.0063/sec Strain Rate; (b) 0.0294/sec Strain Rate

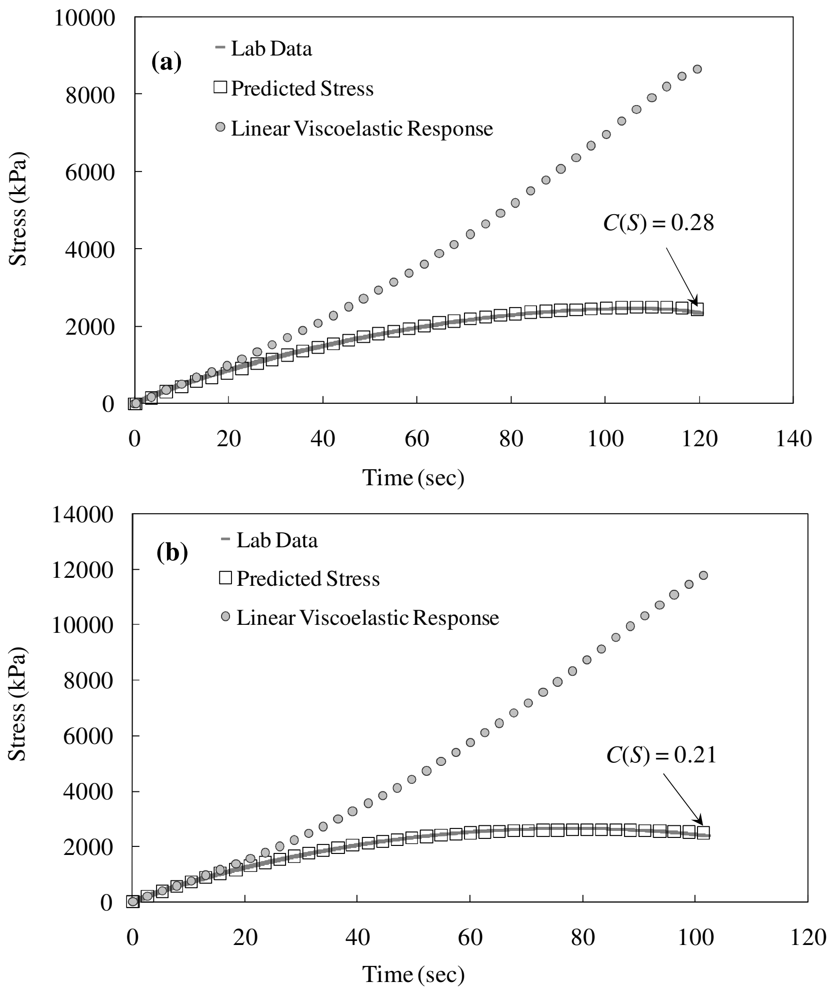


Fig. 15 LVE response of the Lime-Modified HMA: (a) 0.00003/sec Strain Rate and 5.3% Binder Content; (b) 0.000055/sec Strain Rate and 5.8% Binder Content

5 Conclusions

In this chapter, the HS algorithm has been implemented in the context of determining viscoelastic and damage properties. An interconversion method between time- and frequency- domain LVE responses for HMA concrete is presented based on a methodology of the HS algorithm to pre-smooth the experimental data using the log-sigmoidal function before fitting to a Prony series. Also, in order to model the material response of HMA concrete to different strain rate loadings, which induce damage growth, the damage function can be characterized by fitting experimental results using the HS algorithm. Through the application to the prediction of the damage behavior of HMA concrete, the stress predictions result in accurate matches using the experimental data.

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