Modeling of Operative Risk Using Fuzzy Expert Systems

Santiago Medina Hurtado

Abstract. This study is focusing on the modeling of operative loss exposure of the Allowances and Retirement Funds with the use of a Fuzzy Expert System, which evaluates the environmental and managerial factors, this allow obtain a qualification about the possibility that the company incurs in operative losses. The system can be very useful either when the quantitative information is limited due to the discrete character of the risk events or where the information about the risk factors are associated to expert's knowledge, for these reasons the modeling using formal statistical tools is difficult. Too we propose, a simple methodology to complete the knowledge matrix of the Fuzzy Expert System which combines the scoring method with the expert knowledge that allows obtain the rules of system in a simple and quick way. This extraction method saves time in the modeling of complex systems where many variable interact and where there are restrictions with the expert interactions. These systems allow having a structured vision of the sources of operational risk and where the manager should concentrate efforts to diminish the exposure. The Fuzzy Expert System can help to complement the operative risk analysis carried out with quantitative methods as Extreme value theory or Montecarlo simulation.

1   Introduction

The growing necessity to give appropriate solution to socio-economic, administrative and financial problems, starting from human perceptions and that as such we don't have enough information to apply conventional mathematical models, forcing us to search alternative models that allow obtain conclusions starting from linguistic variables. The Fuzzy Logic appears like a tool that allows us to make this transformation and provide a different answer to many real world problems where the information is incomplete, vague and subjective.

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Every day companies and institutions must to face different kind of risks, according their activities or environment. The financial literature classifies the risks in three big groups which are not independent, the market risks, the credit risks and the operative risk (Basel II 2004). However the risks are more wide than these classification (for example: regulatory risk, tributary risk, politics risk, social risk, legal risk, etc). On the other hand, the operative risk includes a great quantity of risk factors whose modeling is difficult for two aspects, first the information relative with the factor is in many cases of qualitative type and second the scarce quantity of data due to discreet character of the events that make it difficult to modeling the risk using formal statistical tools.

Most of the methodologies that address the risks have a statistical foundation and they are centered in the modeling of market and credit risks, however, for the modeling of operative risk the statistical approaches are centered in simulation processes and extreme value theory, this one fix a distribution to the tail lost. This would not have bigger difficulty if the quantitative information is enough but this is not this way. For this reason in the last years, the interest of experts and investigators has been directed toward the development of methodologies that treat the subjective characteristics. actually we are applying methodologies like Bayesian Analysis, Fuzzy Inference Systems and the Fuzzy Expert Systems to treat problems with incomplete information, in order to guide make decisions problems.

The first time that the financial community spoke of "Operative risk" was probably in 1995 when of the English Baring Bank PLC, crashed, due to the transactions carried out by an operator which consumed the capital stock of the company. This event reveled to the financial market the exist risks that could not be classified as market or credit risks and could still effect the results considerably. In the context of the financial institutions the operative risk refers to the range of possible losses associated to risks of the environment, by human fault, processes or technology.

The operative risk is a wide concept. In some cases it is difficult to make a clear distinction among the operative risk and the normal risk that institutions face in their daily operations, in some cases it is difficult to distinguish the limits. For example if a customer fails to pay their loan, it can either be due to the normal credit risk that institution face or it can also be due to the human error that occurred when the analyst studied the client's solvency. The administration program of the risk operative should clearly define that be included as operative risk to diminish the grade of ambiguity with other risk types.

Many managers of financial institutions believe that the operative risk losses could be more significant than the market risks or the credit risks losses (although this isn't clearly demonstrated). The importance of risk measure and control were considered when the Norte American banks divided and qualified its risk into 50% in credit risks, 15% for the market and liquidity risks, and 35% operative risks (Cruz 2002).

The first step toward a stricter administration of risk to financial institutions were the agreements signed at the end of 1988 by the central banks group of industrialized countries (Basel I 1988) . These agreements have led the proposal of risk management that has evolved from credit risks, to the market risk and now the operative risks. In this sense they demand the implementation of administration models with the goal of diminishing the probabilities of loss and to have an adequate technical capital to cover the potentials loss.

The BASLE is the main international agreement that regulate and propose methods that are still in evolution process to treat the risks of financial institutions, however any type of business are exposed to risks. For this reason schemes referencing to risks management have been proposed like COSO, ISO, Australian Standard, PMI, etc.

The operative risk is subdivided in two main components, first the operative risk due to internal causes and second the strategic risk due to external causes, this is indicated in the Figure 1.
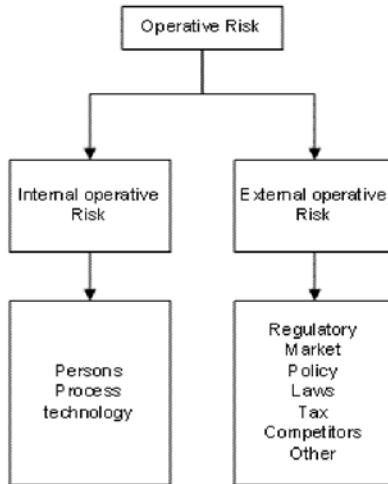

Fig. 1 Operative Risk classifications

The internal operative risks are associated to people, processes or technology used in the normal business course, anyone of these factors can experience some fails and produce financial loss. The external operative risks arise of environment factors as new competitors, political and regulatory environments, climatic factors, technology, but they also can arises of strategic plans; for example a new business lines. These factors are outside of company control. A proportion of this loss can be estimated and they should be covered, the other however can't be estimated. In both cases forecast the impact and the frequency loss is a difficult task to achieve.

In the operative risks analysis is important to quantify the potential loss expected with the purpose of covering them. The potential loss is the product of the probability that the event happens and its associated cost (severity), however, both quantities are difficult to quantify because these events don't occur frequently, that is, they are discreet events that are difficult to register and are time requires. For this reason is difficult to apply, in some cases, statistical analysis to make an objective valuation of the operative risk (Vose 1996, Cruz 2002, Moscadelli 2004, Reiss 2001, Cornalba 2004, Chernobai 2006, Chavez 2006 et all). However the difficulty in the valuation doesn't imply that the risk should be ignored, but rather, we should look for methods that allow us to model this reality.

The impact (severity) of a financial loss can be divided in two categories, a the expected, and the unexpected. In turn the unexpected loss can be classified as "severe" or "catastrophic". The expected losses should be covered with a proportion of the profits, while the severe or catastrophic losses will be covered with a proportion of the company's economic capital, insurance contracts or derivatives (Hull 2008).

The lack of historical data doesn't allow the application of statistical models, however there are people with knowledge that understand and management its processes . In this sense the qualitative information is very important because it allows, starting from a systematic process, to identify the factors that expose the company or business line to lost operational with the purpose of treating these sources of risk in an opportune way.

Actually many researches apply Fuzzy Logic (FL), to treat problems where the information is subjective and vague. The potential use of FL is extendes to the entire field of the social sciences (Kaufman 1990, Kulkarni 2001, Medina 2008, Glykas et al. 2004, Xirogiannis et al. 2008). In the case of the operative risk, the FL allows to the analysis of the risk with a smaller cost and a structured vision of the operational risk to be carried out when the quantitative information is limited. This doesn't mean that the Fuzzy Logic can replace the statistical methods that measures of the operational risks, however it can provide a rigorous theoretical scheme for the treatment of many problems and to help make decisions. FL is an administrative tools that allows measure the state of a process and help us to makes better management.

This article is organized in the following way. The numeral 2 describes the operation of a FIS. In this numeral a method to extract the base of knowledge in an easy and quick way is proposed; this is the critical part of the FIS. The numeral 3 formally defines the Fuzzy Expert Systems FES. The numeral 4 expose an application of modeling of operative risk. The last numeral presents the summations.

2   Fuzzy Logic Systems

The Classic Logic isn't appropriate when we try to describe facts that are not completely true or completely false, because it completely excludes the possibilities among these two values. The Fuzzy Logic on the other hand, allows us to use relative concepts of reality, defining the grades of membership and following the reasoning patterns similar to the human thought (Kosko, 1995).

The Fuzzy Logic formalized by Zadeh in 1965 is based on the theory of the Fuzzy Set. It indicates the grade of membership of an element to the group determined by its membership function that takes values in the interval [0, 1] (Jang, 1997, Kulkarni, 2001, Kasabov, 1998 and Kosko, 1995, Buckley 2002). This way, while in the framework of classic logic, for example, the utility of a company take only two values, it is low wiht grade zero value (0) or it is high wiht one value (1), for the Fuzzy Logic is possible also all intermediate conditions of utility like "very low", "relatively high", "middle", "lightly low", etc. whit values in [0, 1].

The extreme values assumed by the classic logic are only a particular case inside the universe of the Fuzzy Logic, because it allows being relatively imprecise to represent problems and to arrive to adequate solutions (Kosko, 1995). The

Fuzzy Logic opens the possibilities, by giving solution to problems that are expressed from the human perspective and for this simple condition, they can't have an unique solution from the "false" or "true", since it can take intermediate conditions to give satisfactory solutions to the problems.

The Mamdani Fuzzy Inference Systems FIS, (Mamdani, 1977, 1981) was the first system proven in a practical way as universal approximator of functions. Later in 1992 Kosko and Wang formally settled that any relationship among input and output variables can be approximated by means of FIS, built in linguistic terms with a high grade of accuracy (universal approximator).

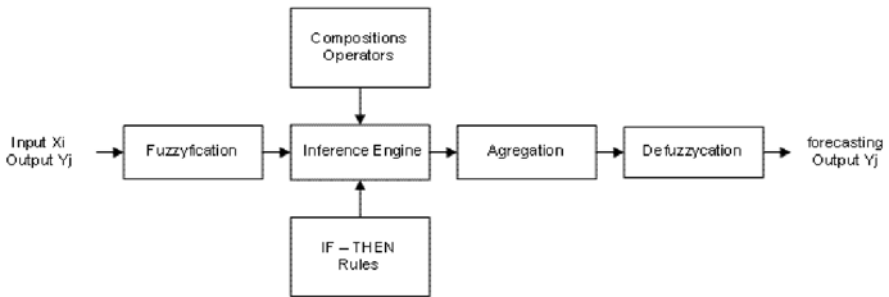To build a FIS the steps to follow are indicated in the Figure 2.

Fig. 2 Steps to building a FIS

Next, we explain the steps to build a FIS:

Fuzzyfication. The first step to build a FIS is to define the inputs and outputs of the system (linguistic variables), their linguistic terms (fuzzy sets) and their membership functions. The "linguistic variable" can take ambiguous or inexact values, for example the linguistic variable "Profitability" can take the linguistic terms "lower", "middle" and "high" and have a semantic meaning for the annalist. This terms are represented by means of fuzzy sets. The Figure 3 present three fuzzy sets lower, middle and high (linguistic terms) for the variable Operative Margin.
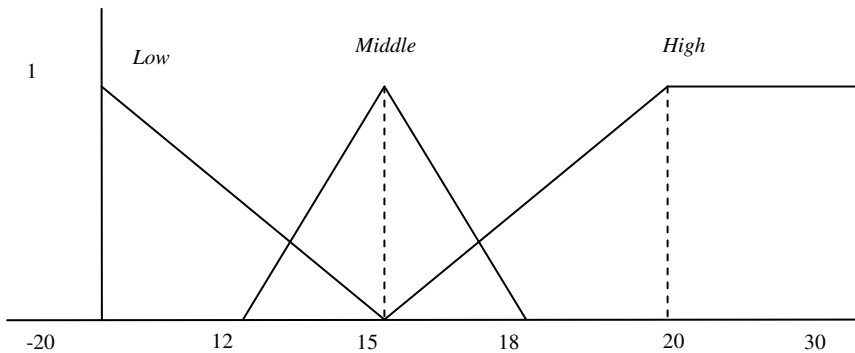

Fig. 3 Fuzzy sets assign to Operative Margin

Formally, to fuzzy Set is defined by:

Let, X the universe of discourse
    x is an element anyone of X and
    A is a collection of elements $x \in X$ and they have a defined characteristic $(A \subset X)$. A is a fuzzy set in X and define the group of orderly couples

$$A = [(x, \mu_A (x)) / x \in X] \tag{1}$$

Where $\mu_A (x)$ is a membership function of fuzzy set A.

The membership function assigns to each element of X a real number among [0,1], denominated "grade of membership" of the element to the fuzzy set A. The types of membership functions commonly utilized are: Triangular, Trapezoidal, Gaussiana, Sigmoidal and Generalized of Bell. They are chosen to reach an appropriate correspondence among the inputs and output spaces of a system.

Construction of the rule base: to build a FIS we should define the fuzzy rules Ifthen: These rules specify the relationship among input and output variables of the system. The fuzzy relationships determine the grade of association between elements of two or more sets.

The IF – THEN Rules type Mandani has the shape:

        "IF X1 is A1 and X2 is A2 and.......and Xk is Ak THEN Y is B"

Where A1, A2,…., Ak, B are linguistic terms defined by mean of fuzzy sets to each linguistic variable in the universe of discourse X1,X2,…..,Xk and Y. "Xi is Ai and…" is called the antecedent of the rule or premise and "Then Y is B" it is called the consequent or conclusion. The previous rule, defines a fuzzy relationship in the dimensional space k+1, characterized by a function of membership $\mu_{Ak \rightarrow B} (X1,X2,….., Xk,Y) \in [0; 1]$.

The set of fuzzy rules (also call "base of knowledge") are obtained by knowledge experts through interviews, questionnaires or panel technical, however, in many occasions the access to experts isn't available but a numeric database of input and output variables is available. This database allows obtain the set of fuzzy rules applying some extraction algorithms. [Hammell and Sudkamp 1994, Wang et al. 2001, Wang and Mendel 1992, Tsang and Yeung 1997, Wang and Hong 1999, Männle e 1999, Herrera and Verdegay 1995, Chin et al. 2003, Espinosa and Vandewalle 2000].

To represent causal relationships among input and output variables of systems can be a very useful use of fuzzy cognitive maps which give us a complete representation of the situation and their relationships (Kosko 1994, Xirogiannis and Glykas 2004, Xirogiannis and Glykas 2007). For example suppose that an analyst tries to measure the effect of the slowness system in the level of operative risk. As a first step some causes and effects are identified and bound to the event of "slowness of the system." A slow system is caused by the reduction of the satisfaction

of the employees, which redounds in a reduction of procedural transactions; this satisfaction commits to other events that increase the operative risk of the business just as it is shown in the Figure 4 (Cruz 2002). Later this representation should be expressed in form of IF-THEN fuzzy rules. The Figure 4 also represent the effect of factors like electric risk, regulatory policy, socio-political risks over the energy spot prices and the decision of load deal for a trader company of electrical energy.
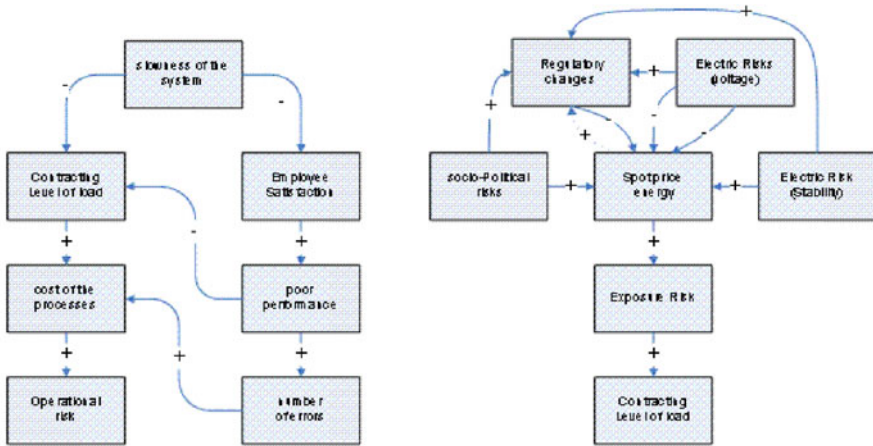
Fig. 4 Examples of Fuzzy cognitive maps

The fuzzy rules can be represented by means of matrix arrangements like in the example indicated in the figure 5. The inputs are located in the columns or row of the matrix and show the linguistic labels of their fuzzy set. The levels of the output variable (Low, Middle or High organizational Administration) are located in the central part of the matrix. This matrix represent the fuzzy rules in a concise form.

ORGANIZATIONAL MANAGEMENT				
Strategic managements	**Risk Managements**	**Exposure to process factor**		
		Low	**Middle**	**High**
Inadequate	wrong	HIGH	HIGH	HIGH
Adequate	wrong	MIDDLE	MIDDLE	MIDDLE
Inadequate	moderately good	MIDDLE	HIGH	HIGH
Adequate	moderately good	LOW	MIDDLE	MIDDLE
Inadequate	Good	MIDDLE	MIDDLE	HIGH
Adequate	Good	LOW	LOW	MIDDLE

Fig. 5 Matrix arrangements for Organizational management

The matrix should be completed following the procedures like panel sessions, interviews or surveys, etc. In this article we suggest a method to capture the expert

opinion by means of scoring method which allows us to obtain the rules quickly, as it is indicated next:

- The first step is ponder each input Xi, according to importance or effect that each Xi has on output Yj.

  Let P_i weights assign to each input. i=1,..,n. $P_i \in [0,1]$ and $\sum_{i=1}^{n} P_i = 1$

  Where n = number of input variables to the FIS.
      To higher weight (Pi), more relative importance of the input variable (is bigger the effect over the output variable)

  Example: If we are evaluating the variable: **"Exposition to External Factors"**, we can use three input variables and these are qualified with the following weight according to their importance by the expert's opinion:

Other Entities dependence	(24%)
Legal norms Knowledge	(29%)
Regulatory stability	(47%)

- Ponder the importance of each fuzzy set (linguistic label) assigned to each input "Xi". Then "Cij" is the weight assigned to each fuzzy set of the input Xi, and indicates the effect on the output Yj.
   Where:

  i=1,…,n   n is number of input variables to the FIS.
  j=1,…,m   m is each fuzzy set defined to input i.

  And $\sum_{j=1}^{m} C_{ij} = 1$ $\forall i = 1,..,n$

  Example: the input "*Regulatory stability*" has assigned three fuzzy set (three levels: very low, low and middle). The expert should assign weights to each level according to importance or effect that this one has over the output variable:

  Very low   weight (50%)
  Low   weight (39%)
  Middle   weight (11%)

- The expert must qualify any scenarios of the knowledge matrix, that is, the experts should define a set of reference rules, that allows to later, validate the FIS output. In general extreme and middle scenarios are evaluated easier for qualifying when the system is very big. The rules are expressed by:

  If X1 is A1 and X2 is A2 and...... and Xk is Ak Then Y is B

Example:
"IF *Regulatory stability* is Very Low And *Legal norms Knowledge* is Low And *Other Entities dependence* is middle Then *Exposition to External Factors* is High"

- The score for each cell of matrix knowledge is calculated by means of the relationship:

$$K_{ij} = \sum_{i=1}^{n} C_{ij}.P_i \qquad \text{For all } j=1,\ldots,m \qquad (2)$$

Where: i=1,2, n number of input variables.
j=1,..m number of fuzzy set assign to each input variables.

- The next step is correlating all scores in each cell of the score matrix with the fuzzy sets assign to output variable Yj. This procedure is carried out by obtaining the maximum and minimum value of the scores matrix, be these K_{max} and K_{min} respectively. K_{max} and K_{min} determine the scores range that can take the output Yj, therefore, the fuzzy sets must be assigned to this range. The score on the intersection of fuzzy sets determine the score from which is assigned a fuzzy set to each cell of the matrix (Compared the score of each cell with the score on the intersection).

The Figure 6 shows an example of three triangular fuzzy sets assigned to output Yj. These are distributed on the range K_{max} and K_{min}. The scores on the intersections are L1 and L2, they determine the points from which for a specific score, a fuzzy set should be assigned. For example to scores smaller than L1 assigned *Low*, between L1 and L2 *medium* and *high* on other cases (bigger than L2).
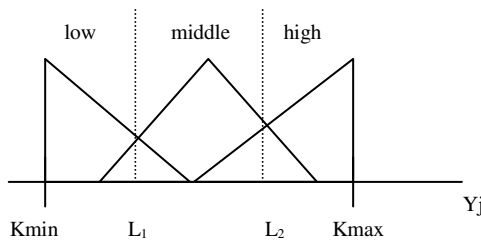

Fig. 6 Triangular fuzzy set to output Yj

The system response can be modified by changing the position of the limits L_1 and L_2 such that the system response is adjusted to the referenced fuzzy rules qualified by the experts previously. This process permits us to obtain the fuzzy rules matrix.

- Then, a validation process should begin wiht the fuzzy rules matrix suggested on the previous step to adjust the system. All cells are evaluated, first it needs to be verified that the reference rules are obtained and then it needs to be verified that all other rules are consistent, otherwise, the limits L1 and L2 must be moved to adjust the system response.

Composition operations. The basic operations carried out with fuzzy sets are Union, Intersection, Complementation, the Cartesian product and the Cartesian Coproducts. These operations are carried out by means of binary operators, classified as T-norms (for intersection operations) or S-norms (for operations of union). (Kaufman 1990, Kaufman 1982, Jang and Mizutani 1997, Kulkarni 2001, Kasabov 1998).

The fuzzy rules define a fuzzy relationship in the space dimensional k+1, characterized by a membership function $\mu_{Ak \to B}$ $(X_1, X_2, \ldots, X_k, Y) \in [0; 1]$, the basic operations with fuzzy sets are utilized like implication relationships and used to derive membership functions of fuzzy set n-dimensional.

Inference Mechanisms (Approximate Reasoning): The Approximate Reasoning is an inference procedure to derive conclusions from fuzzy rules type IF-THEN and values that take the inputs Xi of systems, applying composition relationships Max-Min or Max-product. These permits to infer a fuzzy value B´ if we have fuzzy inputs into k-dimensional space (A'_k) and we have defined the implications relationships:

$$R : A_k \to B, \quad \text{then}$$

$$B' = A'_k \circ (A_k \to B) \tag{3}$$

For example if the system is defined by the following two fuzzy rules (the rules defined fuzzy relationship):

Rule 1: IF x is A1 AND y is B1 Then z is C1, or
Rule 2: IF x is A2 AND y is B2 Then z is C2

Figure 7 represents the approximate reasoning. The systems response C´ is obtained from the inputs: "x" is A´ and "y" is B´ (A´ and B´ can be fuzzy Sets) and the previous fuzzy rules. Each rule is expressed by:

$$R1 = (A1xB1) \to C1 \text{ and } R2 = (A2xB2) \to C2. \tag{4}$$

The composition max-min can be used to infer $\mu_{C'}(z)$. The composition operator "∘" is distributed on the union of two fuzzy relations by:

$$C´ = (A´ \times B´)°(R1 \cup R2) \tag{5}$$
$$C´ = \{(A´ \times B´)°R1\} \cup \{(A´ \times B´)°R2\}$$

$$C´ = C1´ \cup C2´ \tag{6}$$

Where C1´ y C2´ are fuzzy sets derivate of fuzzy rules 1 and 2 respectively. This result can be extended for n-rules.

Aggregation. In this step the outputs of each rule ($C_1´$, $C_2´$,...,$C_n´$ activation strength of each rule) are combined to obtain only one fuzzy set C´. The inputs of the aggregation process are the truncated membership functions obtained by approximate reasoning for each n-rule. The aggregation method is commutative; it doesn't care the order in which the output of each rule is added.

This process defines a method to find C´= ($C_1´ \cup C_2´ \cup$.......$\cup C_n´$). Where $C_1´$, $C_2´$,...,$C_n´$ are fuzzy sets inferred by rules 1,2,…, n. and C´ is a fuzzy set or output system with membership function $\mu_{C´}(z)$.. The *Maximum* operator is the aggregation operator that is most utilized. The output C´ is defined by:

$$C´ = [(z, \mu_{C´}(z)/ z \in Z] \tag{7}$$

Where Z is the discourse universe of output variable and their membership function is given by:

$$\mu_{C´}(z) = Max (C_1´, C_2´,…, C_n´) \tag{8}$$

The fuzzy set C´ = C1´ $\cup$ C2´ aggregate the truncated fuzzy set of each rule. See Figure 7.

Defuzzyfication: In this last step, is obtained a crisp value (K) from the fuzzy set C´ that provides the solution to the output system (see Figure 7) and to permit to make decisions. The concretion methods mostly used are gravity (center of area), Bisector, Middle of maxima, More small of maximum and bigger of maxima (Jang and Mizutani 1997, Kasavov 1998).

The center of area is the concretion method most utilized, because it is similar to calculate the expected value of a distributions probability. The crisp value Zc´, is obtained by integrating the output membership function $\mu_{C´}$ by the means of the following relationship:

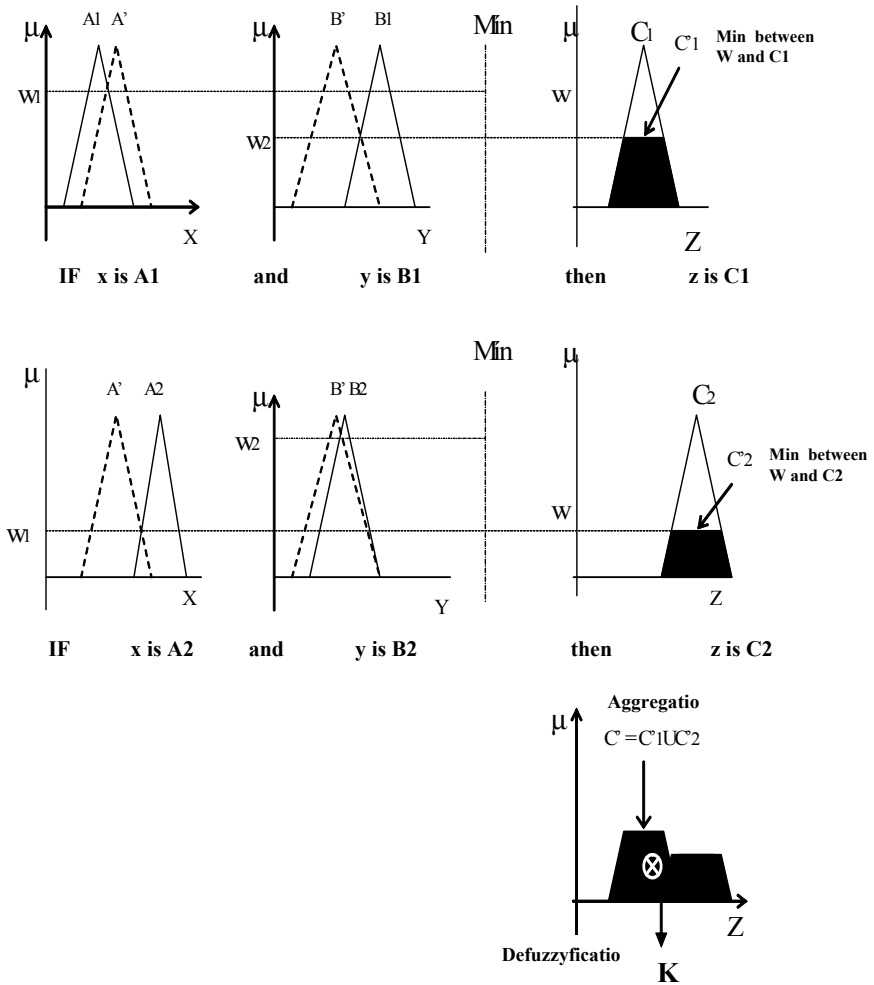$$Z_{C´} = \frac{\int_z \mu_{C´}(z)z\, dz}{\int_z \mu_{C´}(z)dz} \tag{9}$$

Fig. 7 Approximate reasoning

3 Fuzzy Expert Systems (FES)

The interrelation of FIS allows the configuration of an FES and can be used to model complex decision making systems. A FES is a system that uses Fuzzy Logic instead of Boolean Logic to deduce conclusions from input variables. FES is different from conventional expert systems which are applied to domains well defined, where the knowledge phenomenon is wide however, problems where inaccuracy, ambiguity and bad definition exists, the development of a conventional expert system will be more difficult. The treatment of uncertainty is an important aspect in the evolution of expert systems.

In the domain of a expert system there are different sources of uncertainty and this implies that there are not exact solutions, there are approximate solutions (making decisions about management and economics, psychological, medicine etc.). For example, if a problem can be solved in different ways, the expert system should be able to recognize the possible solutions, whose answers should be over-lapped; therefore the solution is also inaccurate and uncertain. The expert system should be able to accept questions that incorporate uncertainty and provide an interpretation or reasoning appropriate to this uncertainty.

The following characteristics of decisioning problems are inherent to the process of human reasoning:

- The learning process or acquisition of knowledge is imprecise
- The knowledge of the expert contains uncertainty.
- The process of the expert's reasoning is imprecise.
- The knowledge represented in natural languages introduces uncertainty.
- The information isn't totally defined.

The previous characteristics limited the use of conventional expert systems. The incorporation of fuzzy logic in the expert systems allows their potential use in many areas of knowledge due to the treatment of characteristics associated to learning processes and human reasoning.

Let $x_i = (x_1, x_2, x_3, \ldots, x_n)$ the inputs to FES, these will feed j-Fuzzy Inferences Systems (FIS_{1j}), that is, each FISj is integrated by n+1-tupla ($x_1, x_2, x_3, \ldots, x_n, y_j$) and they have a fuzzy sets by output variable (Y_j), which allow to obtain a crisp values $Z(C_j^`)$ after apply a defuzzyfication method. See Figure 8.
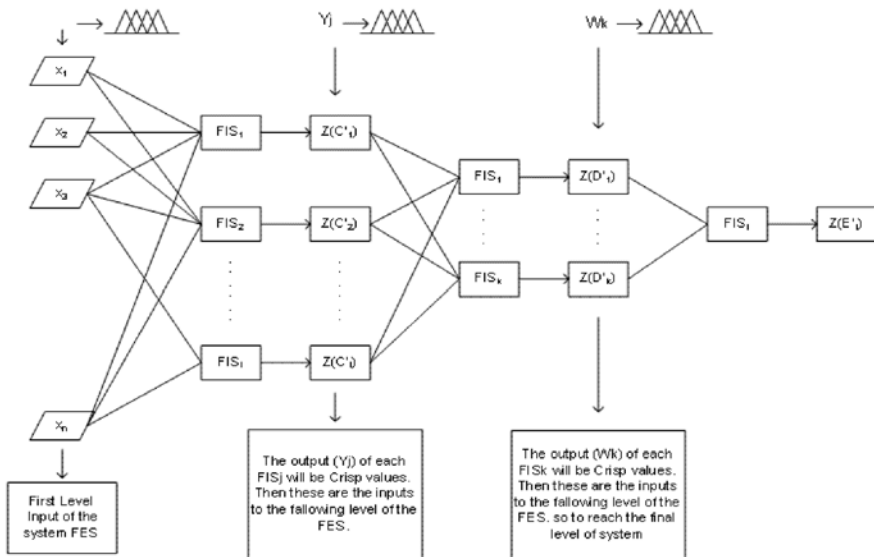

Fig. 8 FES structure

The output Y_j will be the input for the following level of the FES with crisp values $Z(C_j`)$, that is, the variable $Y_j$ will feed again k-new $FIS_k$, each one will be formed for j+1-tuplas $[Z(C_1`), Z(C_2`),.....,Z(C_j`), W_k]$, and they have a fuzzy sets by output (W_k), which allow obtained crisp values $Z(D_k`)$ after apply a defuzzyfication method. Repeating this process successively can reach the output $Z(Et`)$ of the fuzzy expert systems. (Medina 2007)

A formal expression for this process is shown below:

Level	Fuzzy Set	Crisp Value (Defuzzyfication)
	Y_1	$Z(C_1)=f_1(x_1 , x_2 , x_3 ,....., x_n)$
Level 1	Y_2	$Z(C_2)=f_2(x_1 , x_2 , x_3 ,....., x_n)$
	…………..	…………..
	Y_j	$Z(C_j)=f_j(x_1 , x_2 , x_3 ,....., x_n)$
	W_1	$Z(D_1)= f'_1 (Z(C_1), Z(C_2),.....,Z(C_j))$
Level 2	W_2	$Z(D_2)= f'_2 (Z(C_1), Z(C_2),.....,Z(C_j))$
	…………..	…………..
	W_k	$Z(D_k)= f'_k (Z(C_1), Z(C_2),.....,Z(C_j))$
Level 3		$Z(E_1)= f''_1 (Z(D_1), Z(D_2),.....,Z(D_k))$
Outputs		$Z(E_2)= f''_2 (Z(D_1), Z(D_2),.....,Z(D_k))$
		…………..
		$Z(E_t)= f''_t (Z(D_1), Z(D_2),.....,Z(D_k))$

The output $Z(E_t)$ can be written as:

$$Z(E_t)= f''_t (Z(D_1), Z(D_2),.....,Z(D_k))$$

$$Z(E_t)= f''_t \{f'_1 [Z(C_1), Z(C_2),.....,Z(C_j)],....., f'_k [Z(C_1), Z(C_2),.....,Z(C_j)]\}$$

$$Z(E_t)= f''_t \{f'_1 [f_1(x_1,..,x_n),...., f_j(x_1,..,x_n)],....., f'_k [f_1(x_1,..,x_n),...., f_j(x_1,..,x_n)]\} \qquad (10)$$

The functions $f(\cdot)$ are FIS that transform the initial input $(x_1 , x_2 , x_3 ,....., x_n)$. The equation 7.10 is a highly not lineal function.

FES also allows the incorporation of information that come from models of the financial economy (such as a valuation model of companies) or forecasting models (such as a neuronal network or ARIMA model). These structures combine the entire information that comes from the expert knowledge and the mathematical or statistical models that are necessary in the analysis of complex problems. Figure 9.
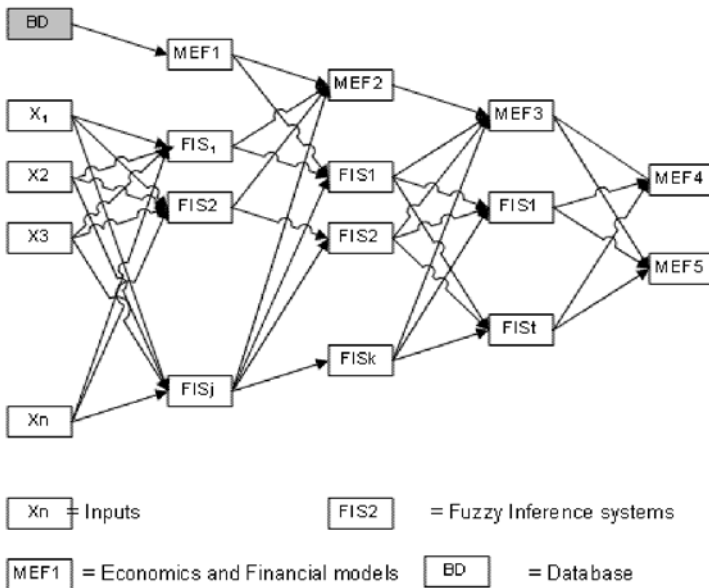
Fig. 9 FES that incorporate financial economic models

The following section shows an application of the operational risk analysis.

4 Modeling Operational Risk Factors

In this paragraph apply the methods described above to the operational risk modeling of a financial institution associated with the pension Funds and layoffs

4.1 Fuzzyfication

The first stage that should be carried out is the fuzzyfication process of system. We should identify and define clearly the input and output variables, their relationships, the number and type of fuzzy sets assigned to all variables. This generally is done by the entity's executive staff at all levels. After carrying out the consultation regarding the factors that expose the entity to operative loss or factors that can reduce their exposure, we proceeded to analyze each ideas expressed. The result of the identification and grouping of variables is shown in Figure 10.

The first level is composed by the input variables of the systems that will be qualified by experts when the systems is operating. These are:

- Regulatory stability
- knowledge of regulations
- Dependency of other entities
- Knowledge Management
- Management of human talent
- Problems of the technological platform

- Planning Design and Development
- Standardization of processes
- Supply Management
- Information security
- Risk Management
- Strategic Management

The variables of the second to fifth correspond to clusters of the inputs variables which have a clear meaning to the experts. These higher level variables define the number of FIS that contains the expert system; in this case seven FIS should be completed.

- Exposure to External Factors
- Exposure to Human factor
- Exposure to process factor
- Risk Management
- Organizational Management
- Exposure to internal factors
- Exposure to Operational Loss

This model allows the evaluation of the level of exposition to operative loss to the company. Each variable in each level was clearly defined for experts to avoid difficulties with the interpretation. This model can be integrated with the other process of risks management.
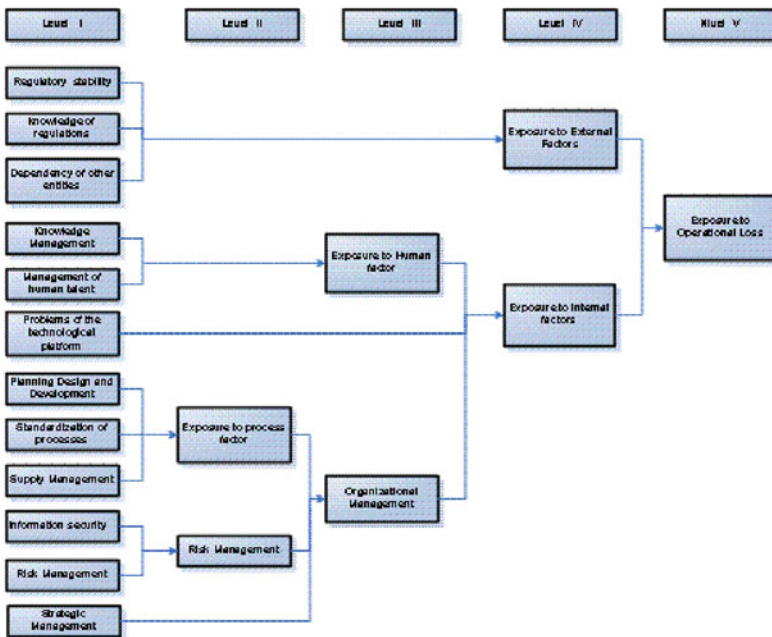

Fig. 10 FES for Operative Risk measurement

Then the fuzzy sets must be assign to each input and output variables of the system. These are defined by a range [0,1]. For example the "Exposure to Operational Loss" variable, has assigned three fuzzy sets wiht linguistic label of low, medium and high.

The Figure 11 shows triangular fuzzy sets used for the FES. All fuzzy sets are defined in the universe of discourse [0,1]. The Figure 12, shows all fuzzy variables and their fuzzy sets defined by the fuzzy Expert System.
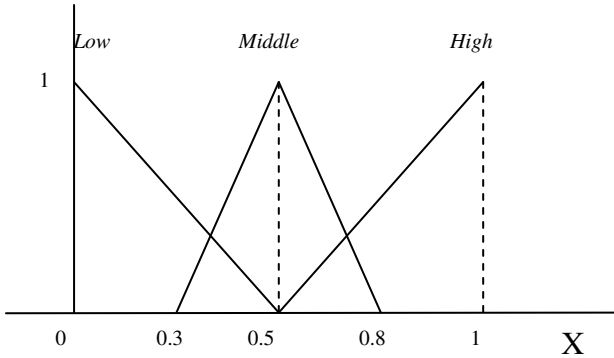

Fig. 11 Triangular Fuzzy sets

INPUT AND OUTPUT VARIABLE	LINGUISTIC LABELS		
Regulatory stability	very low	low	middle
Knowledge of regulations	very low	low	middle
Dependency of other entities	low	middle	high
Knowledge Management	inadequate	adequate	-
Management of human talent	inadequate	adequate	-
technological platform Problems	low	moderate	high
Planning Design and Development	inadequate	adequate	-
Standardization of processes	low	middle	high
Supply Management	inadequate	adequate	-
Information security	regular	good	-
Risk Administrations	regular	good	-
Strategic Management	inadequate	adequate	-
Exposure to External Factors	low	middle	high
Exposure to Human factor	low	middle	high
Exposure to process factor	low	middle	high
Risk Management	wrong	regular	good
Organizational Management	low	middle	high
Exposure to internal factors	low	middle	high
Exposure to Operational Loss.	low	middle	high

Fig. 12 Fuzzy variables and fuzzy sets of systems

4.2   Knowledge Matrix

In this stage we should defined the fuzzy rules that allows the prediction of the behavior of the systems (output variable) with different levels of input variables. It basically corresponds to the definition of the underlying relationships among the input and output variables which are expressed in IF-THEN statements.

The group of statements IF-THEN, conforms the set of fuzzy rules and are the explicit representation of the experts knowledge about the specific process which we wants to evaluate, therefore, the participation of experts in the building of the knowledge matrix is an important issue for both the validity and acceptability of the model.

The fuzzy rules take the form:

"IF *X1 is A1* and *X2 is A2* and.......and *Xk is Ak* THEN *Y is B*"

From the operative point of view, the process begins with the identification and definition of input and output variables from expert opinion. Then the knowledge matrix should be completed which incorporates the information from the input variables, fuzzy sets and the output variable. The matrixes define the relationships between variables and are completed by using the algorithm presented in Section 2.

Seven FIS should be the developer to measure the "operational loss exposure". For example, to complete the FIS related with "*Organizational Exposure Factor*" the experts defined the six rules of reference shown in the following matrix (Figure 13). The missing fields must be completed with the proposed procedure:

ORGANIZATIONAL MANAGEMENT				
Strategic managements	**Risk Managements**	**Exposure to process factor**		
		Low	**Middle**	**High**
Inadequate	wrong	HIGH	HIGH	HIGH
Adequate	wrong	-	-	-
Inadequate	moderately good	-	-	-
Adequate	moderately good	-	-	-
Inadequate	Good	-	-	-
Adequate	Good	LOW	LOW	MIDDLE

Fig. 13 Organizational management matrix knowledge

Could be thought that it's an easy job to complete the system matrixes from the experts directly, however when the number of input variables as well as the number of fuzzy sets is very large (modeling a complex problem with many chained FIS), the number of cells in the matrix increases exponentially, therefore, it takes a long time for the experts analysis to complete the matrixes.

To complete the scoring matrix (see figure 14), we used the previously proposed procedure and calculate the scores for each cell from weights given to each input variable and the weight given to the fuzzy sets of each variable. For example, the scores in the first cells are calculated as follows (see :

$$0.4549 = 0.32 \times 0.81 + 0.29 \times 0.5 + 0.39 \times 0.13$$
$$0.5212 = 0.32 \times 0.81 + 0.29 \times 0.5 + 0.39 \times 0.30$$
$$0.2565 = 0.32 \times 0.19 + 0.29 \times 0.5 + 0.39 \times 0.13$$
$$0.3228 = 0.32 \times 0.19 + 0.29 \times 0.5 + 0.39 \times 0.30$$

Output Variable			Label Linguistic					
ORGANIZATIONAL MANAGEMENT			LOW	MIDDLE	HIGH			
Strategic managements 0,3200		Risk Management 0,2900	0,3900 Baja 0,130	Exposure to Media 0,3	Alta 0,6			
0,8 Inadequate	0,5	wrong	0,4549	0,5212	0,6265	HIGH	HIGH	HIGH
0,2 Adequate	0,5	wrong	0,2565	0,3228	0,4281	LOW	MIDDLE	MIDDLE
0,8 Inadequate	0,4	moderately good	0,4259	0,4922	0,5975	MIDDLE	HIGH	HIGH
0,2 Adequate	0,4	moderately good	0,2275	0,2938	0,3991	LOW	MIDDLE	MIDDLE
0,8 Inadequate	0,1	Good	0,3389	0,4052	0,5105	MIDDLE	MIDDLE	HIGH
0,2 Adequate	0,1	Good	0,1405	0,2068	0,3121	LOW	LOW	MIDDLE

Fig. 14 Scoring matrix for organizational management exposition

The next step is to relate the scores for each cell in the matrix with the fuzzy sets assigned to the output variable of the FIS. To make this we must first determine K_{max} and K_{min} and then assign the fuzzy sets defined for the output variable in this range.

In our case $K_{max} = 0,59750$ and $K_{min} = 0,14050$. We have assign three fuzzy sets to the output variable "*Exposition to organizational factor*" (low, medium, high), then we must define the position of these fuzzy sets into the interval [0,14050 ; 0,59750]. We can make a symmetric partition of interval, in this case we must only determine the amplitude of each interval dividing the rank by three, that is,

$$\text{Rank} = 0,5975 - 0,1405 = 0,4570$$
$$\text{Amplitude} = 0,4570/3 = 0,15233$$

Then, the limits L_1 and L_2 are determined by adding these amplitude starting from K_{min}. For our case $L_1 = 0,29283$ and $L_2 = 0,44517$, this is shown in Figure 15.
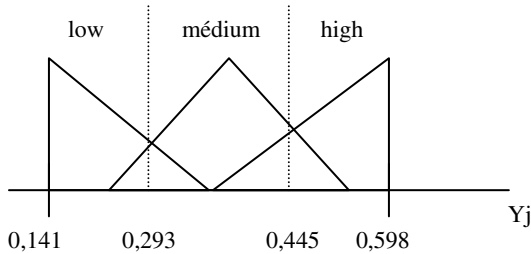

Fig. 15 Fuzzy sets for Exposition to organizational factors

The system response can be modified by varying the position of limits L1 and L2 in a way that the assignment of fuzzy sets is in accordance with all rules described by experts. Through this procedure will be determined all the rules of system.

4.3   FES Validation Process

The inference system must be validated through tests, that is to say, we must give values to the input variables of FES and then we must compare the results of system with the expert opinions. The   results must be consistent.

   The following validations can be carried out:

- Validation each knowledge matrix (each FIS) that compose the expert system. The fuzzy surfaces allows the validation of the relationships among input and output variables and their consistency. An example of a fuzzy surface is shown in the Figure 16. If the behaviors are not consistent we must review the rules, suggest the use of another fuzzy set to the input or output variables or move the fuzzy sets in the universe of discourse of each variable.
- Validation of the shape and position in the universe of discourse of the fuzzy sets defined to the input and output variables in order to improve the system response.
- Platform Validation, that is to identifying the programming problems that make the system not respond adequately. This process is done through the use of reference cases for which the analyst knows the answer. If the system does not reproduce these predefined answers it must be revised.
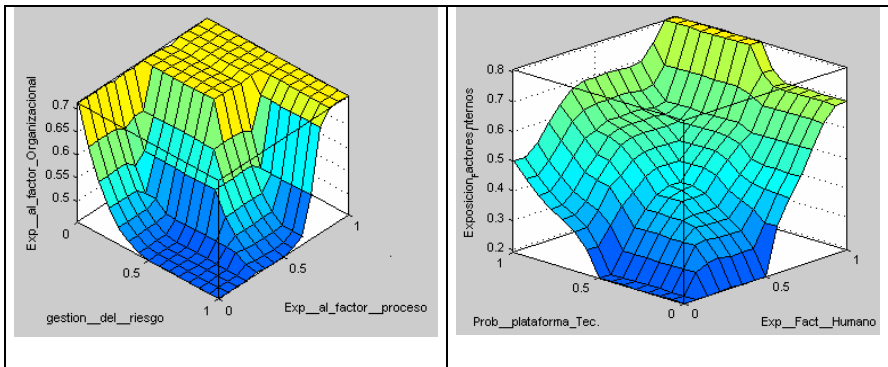
Fig. 16 Fuzzy Surface

4.4 Platform for Measurement Exposure to Operating Losses

The proposed model was programmed with the fuzzy logic functions available in MATLAB. The platform developed to measure the exposure of the operational risk as a result of measuring the factors identified from experts, this in turn gives us the evaluation of higher level variables of system, using color coding to give warnings about the state of these variables, as shown in Figure 17.
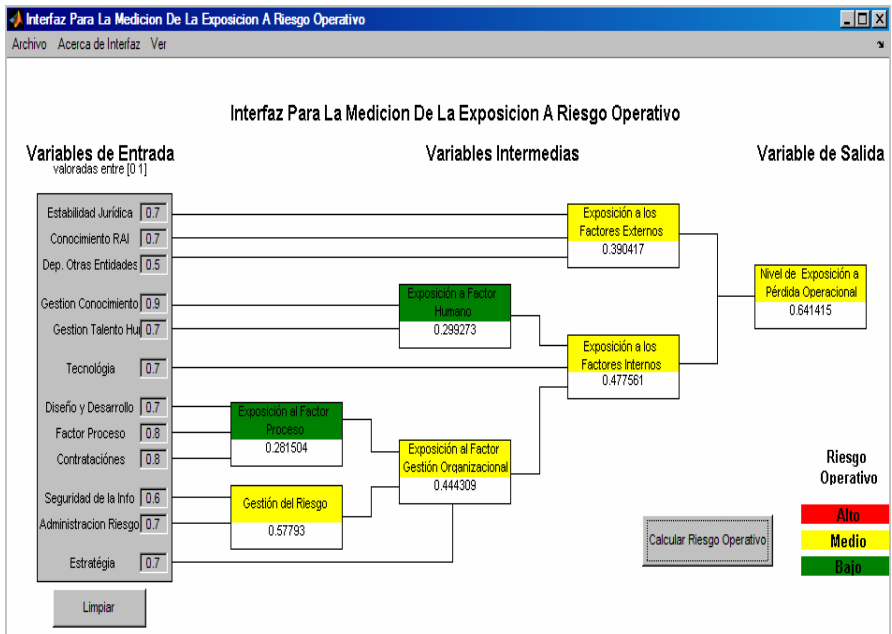

Fig. 17 FES Platform for measure the exposition to operative risk

With the model developed executive staff was asked to qualify the defined as-
pects related with the exposition of the operative risk and characterized by the
inputs previously identified. The survey yielded the following results (figure 18):

Input Variables	1	2	3	4	5	6	prom.
Regulatory stability	0.7	0.3	0.5	0.3	0.5	0.5	0.47
Knowledge of regulations	0.7	0.5	0.5	0.3	0.2	0.7	0.48
Dependency of other entities	0.5	0.5	0.5	0.3	0.4	0.6	0.47
Knowledge Management	0.9	0.95	0.7	0.65	0.2	0.9	0.72
Management of human talent	0.7	0.95	0.8	0.3	0.2	0.9	0.64
technologic platform Problems	0.7	0.5	0.7	0.6	0.4	0.5	0.57
Planning Design and Development	0.7	0.9	0.8	0.8	0.4	0.7	0.72
Standardization of processes	0.8	0.95	0.7	0.7	0.4	0.8	0.73
Supply Management	0.8	0.8	0.9	0.8	0.2	0.8	0.72
Information security	0.6	0.99	0.6	0.6	0.3	0.9	0.67
Risk Administrations	0.7	0.99	0.8	0.7	0.3	0.9	0.73
Strategic Management	0.7	0.95	0.9	0.7	0.1	0.9	0.71

Fig. 18 Input Qualification by Executive staff

5   Conclusions

The previous model can be applied to measure the executive staff's perception
about factors that expose the company to operative loss such as human factors,
technologic factors, and processes factors, etc. That is, FES allows us to evaluate
the activities of corporate structure in all aspects. FES allows the desegregations
of detailed factors associated with activities of processes or sub processes in such
a way that lets the manager improve the process of management control.

Qualitative variables associated with operative risk can be identifying by expert
opinions that manage and understand their process in all ranges of organizational
structure. This information is modeling by FES. Therefore, the development of
support systems to make decisions have more acceptances of the people, since the
developed models are participative and intuitive. (The experts are building).

All companies are exposed to risks, some of these are easy to avoid and meas-
ure, for other however, their quantification is more difficult; for this reason, the
appropriate management should be search new methodologies to improve the
management toward competitive results. FES presented in this article, will allow
identifying and measuring factors that generate a bigger or lower exposition to
operational risk and prioritize the necessary resources to improve the manage-
ment. The manager can develop strategic activities that allow getting accepted
levels of operative loss.

With the purpose of lowers the possibility of operative loss, the companies can
improve the risk management systems to promote the continuous improvement
that allow to detect and to prevent the potential risks appropriately, also can be
used insurance policy to cover the operative losses that can be materialized, the

third alternative is transfer the risk through outsourcing agreement (transfer activities that bigger vulnerability and don't have a high added value), lastly the company can avoid carry out risks activities and that they don't affect the normal operation. An appropriate combination of previous alternatives can be employees with the purpose of minimize the exposition levels to operative losses and guarantee the survival and growth of company.

Future works that can be explored is the relationship between the statistical models used to measure operative value at risk VaR (POT Peak over threshold or Monte Carlo Simulation) and the FES model as proposed in this article. ¿How these models can be integrated? ¿how arrive to economic losses starting from the FES system?, is yet a question to be solved.

Other subject that can be explored is the relationship between the Net Bayesian and a FES, both models capture the expert's knowledge by inputs but the Net Bayesian obtain as output a probability and the FES obtain a possibility, which is the relationship between both in a real problem, could be investigated.

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