Knowledge Reduction in Formal Contexts Based on Covering Rough Sets

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Abstract. Rough set theory and formal concept analysis are two approaches for data analysis related to each other, formal contexts are their common framework. In this paper, by investigating relationship between covering rough set and concept lattice, we study attribute reduction of formal context. Judgement theorems of consistent attribute sets and reduct attribute sets are given.

Keywords: Formal contexts; Concept lattices; Covering rough sets; Attribute reduction.

1 Introduction

Formal concept analysis (FCA) [1] is a kind of important mathematical tool for data analysis and knowledge processing. This theory is based on a binary relation between a set of objects and a set of attributes or properties. Consequently, formal concepts can be constructed, and all formal concepts form a concept lattice. FCA has a lot of successful applications in many domains[2,3].

The traditional rough set theory proposed by Pawlak [4] is formulated based on an equivalence relation on a universe of discourse. By means of equivalence classes of the universe of discourse, a pairs of approximations, respectively called lower and upper approximations, of any subsets of the universe of discourse are defined. Instead of the partition by a covering, covering rough sets can be constructed [5,6,7]. The notions of lower and upper approximations can also be defined based on two universes linked by a binary relation[8,9,10].

Rough set theory and formal concept analysis are two kinds of complementary tools for data analysis and knowledge processing. Many efforts have been made to combine two theories [11,12,13,14], in which object-oriented and attribute-oriented concept lattices have been proposed based on rough set theory. Attribute reduction methods for two kinds of concept lattices have been given in [15].

In this paper, the focus is on the attribute reduction of formal concepts via covering rough set theory. The objective is to find simple judgement methods of consistent or reduct attribute sets of formal contexts. Section 2 reviews basic knowledge of formal concept analysis and covering rough sets. In Section 3, we present the sufficient and necessary conditions for justifying whether an attribute subset of a formal context is a consistent or reduct attribute set. Section 4 concludes the paper.

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2 Preliminaries

In this section, some notions about concept lattices and covering rough sets are first reviewed.

2.1 Formal Concepts and Concept Lattice

Let U and A be two nonempty and finite sets, I a binary relation from U to A. Then the triplet (U, A, I) is called a *formal context*, and U is called object set, A *attribute set*, $(x, a) \in I$ means that object x possesses attribute a.

Example 1. Table 1. shows a formal context $T = (U, A, I)$, where $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{a, b, c, d, e, f\}$. In this table, for example, object 5 has properties b and f. The property b is possessed by the objects 3, 5 and 6.

Table 1. A formal context $T = (U, A, I)$

U	a	b	c	d	e	f
1	1	0	1	1	1	0
2	0	0	0	0	1	0
3	1	1	0	1	0	1
4	1	0	1	1	1	0
5	0	1	0	0	0	1
6	0	1	0	0	1	1

Let (U, A, I) be a formal context. Its formal concepts are defined by the following modal-style logical operators: $\forall X \subseteq U, B \subseteq A$,

$$X^* = \{a \in A | \forall x \in X, (x, a) \in I\}, \;\; B^* = \{x \in U | \forall a \in B, (x, a) \in I\}.$$

Specially, for $x \in U, a \in A$, $\{x\}^*$, $\{a\}^*$ are briefly denoted as x^*, a^* respectively. x^* is called the attribute set of object x in (U, A, I), a^* the object set of attribute a in (U, A, I).

In formal context (U, A, I), the following properties hold: $\forall X_1, X_2, X \subseteq U, \forall B_1, B_2, B \subseteq A$,

(1) $X_1 \subseteq X_2 \Rightarrow X_1^* \supseteq X_2^*, B_1 \subseteq B_2 \Rightarrow B_1^* \supseteq B_2^*$.

(2) $X \subseteq X^{**}, B \subseteq B^{**}$.

(3) $X^* = X^{***}, B^* = B^{***}$.

(4) $X \subseteq B^* \Leftrightarrow B \subseteq X^*$.

(5) $(X_1 \cup X_2)^* = X_1^* \cap X_2^*, \; (B_1 \cup B_2)^* = B_1^* \cap B_2^*$.

For $X \subseteq U, B \subseteq A$, (X, B) is called a *formal concept* of (U, A, I) if $X^* = B$ and $B^* = X$, and X, B are called the *extension* and the *intension* of (X, B), respectively.

We denote the set of all formal concepts of (U, A, I) as $\mathcal{L}(U, A, I)$, it forms a complete lattice under the following partial order:

$$(X_1, B_1) \le (X_2, B_2) \Longleftrightarrow X_1 \subseteq X_2 \text{ or } B_1 \supseteq B_2.$$

Thus $\mathcal{L}(U, A, I)$ is called the concept lattice of (U, A, I).

The following proposition can be easily proved.

Proposition 1. $\mathcal{L}(U, A, I) = \{(X^{**}, X^*) | X \subseteq U\}$.

2.2 Covering Rough Sets

Let U be a finite set called a universe of discourse. A *set system* $\mathcal{S}$ on U is a family of subsets of U. *Dual set system* of $\mathcal{S}$ is denoted as $\sim \mathcal{S}$ defined by: $\sim \mathcal{S} = \{C \subseteq U | \sim C \in \mathcal{S}\}$, where $\sim C$ denotes the complement of set C. A set system $\mathcal{C}$ is called a *covering* of U if $U = \bigcup_{C \in \mathcal{C}} C$, and $(U, \mathcal{C})$ is called a *covering approximation space* [6].

In a covering approximation space $(U, \mathcal{C})$, Yao [16] defined a pair of dual lower and upper approximation operators as follows: $\forall X \subseteq U$,

$$\underline{apr}_{\mathcal{C}}(X) = \cup\{C \in \mathcal{C} | C \subseteq X\},$$
$$\overline{apr}_{\mathcal{C}}(X) = \{x \in U | \forall C \in \mathcal{C}(x \in C \Rightarrow C \cap X \neq \emptyset)\}.$$

They satisfy the following basic properties:

(L0) $\underline{apr}_{\mathcal{C}}(X) = \sim \overline{apr}_{\mathcal{C}}(\sim X)$, (U0) $\overline{apr}_{\mathcal{C}}(X) = \sim \underline{apr}_{\mathcal{C}}(\sim X)$;
(L1) $\underline{apr}_{\mathcal{C}}(U) = U$, (U1) $\overline{apr}_{\mathcal{C}}(\emptyset) = \emptyset$;
(L2) $X \subseteq Y \Rightarrow \underline{apr}_{\mathcal{C}}(X) \subseteq \underline{apr}_{\mathcal{C}}(Y)$, (U2) $X \subseteq Y \Rightarrow \overline{apr}_{\mathcal{C}}(X) \subseteq \overline{apr}_{\mathcal{C}}(Y)$;
(L5) $\underline{apr}_{\mathcal{C}}(X) \subseteq X$, (U5) $X \subseteq \overline{apr}_{\mathcal{C}}(X)$;
(L7) $\underline{apr}_{\mathcal{C}}(X) \subseteq \underline{apr}_{\mathcal{C}}(\underline{apr}_{\mathcal{C}}(X))$, (U7) $\overline{apr}_{\mathcal{C}}(\overline{apr}_{\mathcal{C}}(X)) \subset \overline{apr}_{\mathcal{C}}(X)$.

3 Attribute Reduction of Formal Contexts

Let (U, A, I) be a formal context. It is called *regular* if for any $x \in U$ there exists $a, b \in A$ such that $(x, a) \in I$ and $(x, b) \notin I$; for any $a \in A$ there exists $x, y \in U$ such that $(x, a) \in I$ and $(y, a) \notin I$. In the sequel, we assume that the following formal contexts are regular.

A covering $\mathcal{C}_A$ of U can be induced from a regular formal context (U, A, I), and

$$\mathcal{C}_A = \{a^* | a \in A\}.$$

It should be point out that there may be many attributes with the same common objects set, that is, there may be some equal sets in covering $\mathcal{C}_A$.

Theorem 1. *Let (U, A, I) be a formal context. For any $X \subseteq U$, $X^{**} = \overline{apr}_{\sim \mathcal{C}_A}(X)$.*

Proof. For any $X \subseteq U$, we have

$$\begin{aligned} X^{**} &= \{x \in U | \forall a \in X^*, x \in a^*\} \\ &= \{x \in U | \forall a \in A(X \subseteq a^*, x \in a^*)\} \\ &= \{x \in U | \forall a \in A(x \notin a^* \Rightarrow X \not\subseteq a^*)\} \\ &= \{x \in U | \forall a \in A(x \in \sim a^* \Rightarrow (\sim a^*) \cap X \neq \emptyset)\} \\ &= \overline{apr}_{\sim \mathcal{C}_A}(X). \end{aligned}$$

Thus, the conclusion is proved. □

If we denote the set of all extensions of formal concepts of (U, A, I) as $\mathcal{L}_O(U, A, I)$, then the next theorem directly follows from Propositions 1 and Theorem 1.

Theorem 2. $\mathcal{L}_O(U, A, I) = \{\overline{apr}_{\sim\mathcal{C}_A}(X) | X \subseteq U\}$.

Definition 1. *Let $T = (U, A, I)$ be a formal context. For $B \subseteq A$ we call the formal context (U, B, I_B) a sub-context of T, where $I_B = I \cap (U \times B)$.*

In the following, $\forall X \subseteq U$, X^* with respect to (w.r.t.) (U, B, I_B) is denoted as X^{*_B}, and X^* w.r.t. (U, A, I) is unchangeable. Then $X^{*_B} = X^* \cap B$.

Definition 2. *Let $T = (U, A, I)$ be a formal context. An attribute subset $B \subseteq A$ is referred to as a consistent attribute set of T if $X^{**} = X^{*_B*}$ for all $X \subseteq U$. If $C \subseteq A$ is a consistent attribute set of T and there is no proper subset $D \subset C$ such that D is a consistent attribute set of T, then C is referred to as a reduct attribute set of T. The intersection of all reduct attribute sets is called the core attribute set of T.*

Theorem 3. *Let $T = (U, A, I)$ be a formal context and $B \subseteq A$. B is consistent attribute set of T iff $\mathcal{L}_O(U, B, I_B) = \mathcal{L}_O(U, A, I)$.*

Proof. It directly follows from Definition 2, Proposition 1, Theorems 1 and 2. □

From Theorem 3 we can see that the notion of consistent attribute set in Definition 2 coincides with that proposed in [17].

Theorem 4. *Let $T = (U, A, I)$ be a formal context and $B \subseteq A$. B is consistent attribute set of T iff $X \subseteq U$,*

$$\underline{apr}_{\sim\mathcal{C}_A}(X) = \underline{apr}_{\sim\mathcal{C}_B}(X) \text{ or } \overline{apr}_{\sim\mathcal{C}_A}(X) = \overline{apr}_{\sim\mathcal{C}_B}(X).$$

Proof. It directly follows from Theorems 2 and 3. □

It follows from Theorem 4 that attribute reduction of formal context coincide with reduction of covering rough set[6]. Therefore, attribute reduction of formal context can be done based on the theory of covering rough sets.

In a formal context (U, A, I), an equivalence relation E_A on A can be introduced as follows:

$$E_A = \{(a, b) \in A \times A | a^* = b^*\}.$$

The equivalence class of attribute $a \in A$ is denoted by $[a]_{E_A}$, or briefly by $[a]$, and the partition on the attribute set A w.r.t. E_A is denoted by A/E_A.

Example 2. For the formal context (U, A, I) of Example 1, the equivalence relation E_A can be figured out:

$$E_A = \{(a, a), (a, d), (b, b), (b, f), (c, c), (d, d), (d, a), (e, e), (f, b), (f, f)\}.$$

Thus

$$[a] = [d] = \{a, d\}, [b] = [f] = \{b, f\}, [c] = \{c\}, [e] = \{e\},$$
$$A/E_A = \{\{a, d\}, \{b, f\}, \{c\}, \{e\}\}.$$

Definition 3. *Let $T = (U, A, I)$ be a formal context. An attribute $a \in A$ is referred to as a class-reducible attribute of T if there are some sets in $\mathcal{C}_A - \{b^*|b \in [a]\}$ such that a^* is equal to the intersection of them; Otherwise, a is referred to as a class-irreducible attribute of T.*

When a is a class-reducible attribute of T, we call the attribute class $[a]$ as a *reducible attribute class* of T. Similarly, if a is a class-irreducible attribute of T then $[a]$ is called a *irreducible attribute class* of T.

Example 3. As for the formal context (U, A, I) of Example 1, by Definition 3 we can check that c is a class-reducible attribute, a, b, d, e, f are class-irreducible attributes. Thus $\{c\}$ is a reducible attribute class, and $\{a, d\}$, $\{b, f\}$ and $\{e\}$ are irreducible attribute classes.

Proposition 2. *Let $T = (U, A, I)$ be a formal context and $a \in A$. a is class-reducible attribute of T, $b \in B = A - [a]$, b is a class-reducible attribute of (U, B, I_B) iff b is a class-reducible attribute of T.*

Proof. If b is a class-reducible attribute of (U, B, I_B), b^* can be expressed as an intersection of some sets in $\mathcal{C}_B - \{c^*|c \in [b]\} = \mathcal{C}_A - \{c^*|c \in [a] \cup [b]\} \subseteq \mathcal{C}_A - \{c^*|c \in [a]\}$, So it certainly is a class-reducible attribute of T.

On the other hand, assume that b is a class-reducible attribute of T. Then b^* can be expressed as an intersection of some sets in $\mathcal{C}_A - \{c^*|c \in [b]\}$, say $c_1^*, c_2^*, \ldots, c_k^*$. It is evident that $b^* \subset c_i^*$ for every i. If $c_i \notin [a]$ for all i, then $c_i^* \in \mathcal{C}_A - \{c^*|c \in [a] \cup [b]\}$ such that $b^* = \bigcap_{i=1}^{k} c_i^*$, which means that b is a class-reducible attribute of (U, B, I_B). If there are some $c_i \in [a]$, say $c_1^* = c_2^* = a^*$, from that a is a class-reducible attribute of T, there are attributes $d_1, d_2, \ldots, d_l \in A - [a]$ such that $a^* = \bigcap_{j=1}^{l} d_j^*$. By $a^* \subset d_j^*$ for all j and $b^* \subset a^*$, we know $d_j \notin [b]$ for all j. So $d_1, d_2, \ldots, d_l, c_3, \ldots, c_k$ belong to $A - [a] - [b]$, equivalently $d_1^*, d_2^*, \ldots, d_l^*, c_3^*, \ldots, c_k^* \in \mathcal{C}_A - \{c^*|c \in [a]\} - \{c^*|c \in [b]\}$, and $b^* = (\bigcap_{j=1}^{l} d_j^*) \cap (\bigcap_{i=3}^{k} c_i^*)$ holds. Thus b is a class-reducible attribute of (U, B, I_B). □

Proposition 2 shows that deleting reducible attribute classes from attribute set do not change reducibility of other attribute classes. Thus the following conclusion is evident.

Proposition 3. *Let a be a class-reducible attribute of (U, A, I). Then there are class-irreducible attributes $b_1, b_2, \ldots, b_n$ of (U, A, I) such that $a^* = \bigcap_{i=1}^{n} b_i^*$.*

Theorem 5. *Let $T = (U, A, I)$ be a formal context and $B \subseteq A$. B is a consistent attribute set of T iff $B \cap [a] \neq \emptyset$ for every irreducible attribute class $[a]$ of T.*

Proof. ($\Rightarrow$) Suppose that B is a consistent attribute set of T, and there exists an irreducible attribute class $[a]$ of T such that $B \cap [a] = \emptyset$. By the definition of consistent attribute set we have $a^* = (a^*)^{**} = (a^*)^{*_B*}$, that is, $a^* = (a^{**_B})^* = \bigcap_{b \in a^{**_B}} b^*$. From $B \cap [a] = \emptyset$ we have $B \subseteq A - [a]$. Noticing $a^* = \bigcap_{b \in a^{**_B}} b^*$ and $a^{**_B} \subseteq B$, we know that a is a class-reducible attribute of T, a contradiction! Therefore $B \cap [a] \neq \emptyset$ for every irreducible attribute class $[a]$ of T.

($\Leftarrow$) Assume that $B \cap [a] \neq \emptyset$ for every irreducible attribute classes $[a]$ of T. For any $X \subseteq U$, $X^{**} \subseteq X^{*_B*}$ follows from $X^{*_B} = X^* \cap B$ and the properties of operation *.

Let $x \in X^{*_B*}$. If $a \in X^{*_B}$ then $x \in a^*$. Equivalently, $\forall a \in B$, $X \subseteq a^*$ implies $x \in a^*$. Let $a \in A - B$ and suppose $X \subseteq a^*$. If a is a class-irreducible attribute of T, then $B \cap [a] \neq \emptyset$ follows from the assumption. Thus there exists $b \in B \cap [a]$, that is, $b \in B$, $a^* = b^*$. By $X \subseteq a^*$ we have $x \in b^* = a^*$. If a is a class-reducible attribute of T, from Proposition 3 we knew that there are class-irreducible attributes $b_1, b_2, \ldots, b_n$ of (U, A, I) such that $a^* = \bigcap_{i=1}^{n} b_i^*$. For every i, by $B \cap [b_i] \neq \emptyset$ we can assume $b_i \in B \cap [b_i]$, then $X \subseteq b_i^*$ follows from $X \subseteq a^* \subset b_i$. So $x \in b_i^*$ for every i. From $a^* = \bigcap_{i=1}^{n} b_i^*$ we have $x \in a^*$. We can conclude that for any $a \in A$, $X \subseteq a^*$ implies $x \in a^*$. By the definition of X^{**}, we get $x \in X^{**}$. Therefore, $X^{*_B*} \subseteq X^{**}$.

For any $X \subseteq U$, combining $X^{**} \subseteq X^{*_B*}$ and $X^{*_B*} \subseteq X^{**}$ we get $X^{**} = X^{*_B*}$. By virtue of Definition 2, we can conclude that B is a consistent attribute set of T. □

Theorem 6. *Let $T = (U, A, I)$ be a formal context and $B \subseteq A$ a consistent attribute set of T. If $a \in B$ is a class-reducible attribute of T then $B - \{a\}$ is also a consistent attribute set of T.*

Proof. Because B is a consistent attribute set of T, we have $X^{**} = X^{*_B*}$, $\forall X \subseteq U$. It is evident that $X^{*_B*} \subseteq X^{*_{B-\{a\}}*}$ by $B - \{a\} \subseteq B$. If $x \in X^{*_{B-\{a\}}*}$ then $X \subseteq b^*$ with $b \in B - \{a\}$ implies $x \in b^*$. As a is a class-reducible attribute of T, from Proposition 3 it follows that there exists class-irreducible attributes $b_1, b_2, \ldots, b_n$ of T such that $a^* = \bigcap_{i=1}^{n} b_i^*$. In virtue of Theorem 5 we have $B \cap [b_i] \neq \emptyset$ for all i. So we can assume that $b_i \in B - \{a\}$ for all i. If $X \subseteq a^*$ then $X \subseteq b_i^*$ for all i. Thus $x \in b_i^*$ for all i, from which $x \in a^*$ follows. We can conclude consequently that $X \subseteq b^*$ implies $x \in b^*$ for all $b \in B$, which means $x \in X^{*_B*}$. So $X^{*_{B-\{a\}}*} \subseteq X^{*_B*}$. Therefore $X^{*_{B-\{a\}}*} = X^{*_B*}$ for any $X \subseteq U$. By Definition 2 we can conclude that $B - \{a\}$ is a consistent attribute set of T. □

Theorem 7. *Let $T = (U, A, I)$ be a formal context and $B \subseteq A$. B is a reduct attribute set of T iff $|B \cap [a]| = 1$ for any irreducible attribute class $[a]$ of T, and $B \cap [a] = \emptyset$ for any reducible attribute class $[a]$ of T. where $|\bullet|$ denotes the cardinality of set $\bullet$.*

Proof. ($\Rightarrow$) Assume that B is a reduct attribute set of T. If $[a]$ is an irreducible attribute class of T, then $B \cap [a] \neq \emptyset$ follows from Theorem 5. Suppose $|B \cap [a]| > 1$, say $b_1, b_2 \in B \cap [a]$. By Theorem 5 we know that $B - \{b_1\}$ is also a consistent attribute set of T, which is in contradiction to that B is a reduct attribute set of T. Therefore, For any irreducible attribute class $[a]$ of T, $|B \cap [a]| = 1$ holds.

On the other hand, if $[a]$ is a reducible attribute class of T, then by Theorem 6 we have that $B - [a]$ is also a consistent attribute set of T. So $B \cap [a] = \emptyset$ follows from that B is a reduct attribute set of T.

($\Leftarrow$) Suppose $|B \cap [a]| = 1$ for any irreducible attribute class $[a]$ of T, and $B \cap [a] = \emptyset$ for any reducible attribute class $[a]$ of T. Then it is apparent that $B \cap [a] \neq \emptyset$ for any irreducible attribute class $[a]$ of T, so it follows from Theorem 5 that B is a consistent attribute set of T. On account of $B \cap [a] = \emptyset$ for any reducible attribute class $[a]$ of T, we know that B consists of class-irreducible attributes of T. For any $a \in B$, we have that a is a class-irreducible attribute of T. $(B - \{a\}) \cap [a] = \emptyset$ follows from the supposition. By Theorem 5, $B - \{a\}$ is not a consistent attribute set of T. Therefore we can conclude that B is a reduct attribute set of T. □

Example 4. For the formal context (U, A, I) in Example 1, using Theorem 7 we can check that the following attribute sets are all reduct attribute sets of T:

$$\{a, b, e\}, \{a, e, f\}, \{b, d, e\}, \{d, e, f\}.$$

And $\{e\}$ is the core attribute set of (U, A, I).

If we consider attribute reduction of formal context according to Theorem 7, the main work of attribute reduction is on judging the reducibility of all attributes.

4 Conclusions

Formal contexts are a common framework of formal concept analysis and the theory of rough sets by which two theories are related tightly. This paper study attribute reduction of formal contexts for concept lattice via revealed relations between formal concepts and covering rough sets. The obtained judgement theorems for consistent attribute sets and reduct attribute sets form the basis of establishing simple methods for attribute reduction of formal contexts, it just be our next work.

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