Encoding the Revision of Partially Preordered Information in Answer Set Programming

Mariette Sérayet, Pierre Drap, and Odile Papini

LSIS-CNRS 6168, Université de la Méditerranée, ESIL - Case 925, Av de Luminy, 13288 Marseille Cedex 9 France
{serayet,drap,papini}@esil.univmed.fr

Abstract. Most of belief revision operations have been proposed for totally preordrered information. However, in case of partial ignorance, pieces of information are partially preordered and few effective approaches of revision have been proposed. The paper presents a new framework for revising partially preordered information, called Partially Preordered Removed Sets Revision (PPRSR). The notion of removed set, initially defined in the context of the revision of non ordered or totally preordered information is extended to partial preorders. The removed sets are efficiently computed thanks to a suitable encoding of the revision problem into logic programming with answer set semantics. This framework captures the possibilistic revision of partially preordered information and allows for implementing it with ASP. Finally, it shows how PPRSR can be applied to a real application of the VENUS european project before concluding.

1 Introduction

Belief revision has been extensively studied in the domain of knowledge representation for artificial intelligence, mainly for totally preordered information. A characterization of belief revision has been provided by Alchourron, Gärdenfors, Makinson (AGM) with a set of postulates that any revision operation should satisfy [6]. Katsuno and Mendelzon (KM) reformulated AGM's postulates and provided a representation theorem that characterizes revision operations based on total preorders [11]. Belief revision has been discussed within different frameworks (probabillity theory, Sphon's conditional functions, Grove's system of spheres, etc $\cdots$). Some approaches have been implemented, among them, Removed Sets Revision which has been initially proposed in [15] for revising a set of propositional formulae. This approach stems from removing a minimal number of formulae, called removed set, to restore consistency. The Removed Sets Revision (RSR) and then a prioritized form of Removed Sets Revision, called Prioritized Removed Sets Revision (PRSR) [1] have been encoded into answer set programming and allowed for solving a practical revision problem coming from a real application in the framework of geographical information system.

However in some applications, an agent has not always a total preorder between situations at his disposal, but is only able to define a partial preorder

C. Sossai and G. Chemello (Eds.): ECSQARU 2009, LNAI 5590, pp. 421–433, 2009.
© Springer-Verlag Berlin Heidelberg 2009

between situations, particularly in case of partial ignorance and incomplete information. In such cases, an epistemic state can be represented by either a partial preorder on interpretations or a partially preordered belief base.

The revision of partially preordered information has been less investigated in the literature, however Lagrue and al. [4] pointed out that the KM's postulates are not appropriate for partial preorders and proposed a suitable definition of faithful assignment, called P-faithful assignment, a new set of postulates and a representation theorem. Some revision operations initially defined for total preorders, such as revision with memory and possibilistic revision have been sucessfully extended to partial preorders [2]. This paper proposes a new framework for revising partially preordered information and provides an efficient implementation thanks to Answer Set Programming. The main contributions of this paper are the following:

- It extends the Removed Sets Revision to partially preordered information, called Partially Preordered Removed Sets Revision (PPRSR). The paper shows how the notion of removed set, roughly speaking, the subsets of formulae to remove to restore consistency, initially defined in the context of non ordered [15] or totally ordered [1] information is extended to the case of the revision of partially preordered information,
- It provides an implementation of PPRSR with ASP. The revision problem is translated into a logic program with answer set semantics and a one-to-one correspondence between removed sets and preferred answer sets is shown. The computation of answer sets is performed with any ASP solver supporting the minimize statement.
- It shows that the possibilistic revision of partially preordered information can be captured within the PPRSR framework allowing for an efficient implementation with ASP.

The rest of this paper is organized as follows. Section 2 fixes the notations and gives a refresher on RSR, on answer set programming and on partial preorders. Section 3 presents the Partially Preordered Removed Set Revision (PPRSR) and shows how it captures the possibilistic revision. Section 4 details the encoding of PPRSR into logic programming with answer set semantics and the computation of answer sets thanks to ASP solvers. It then shows the one-to-one correspondence between removed sets and preferred answer sets. Section 5 illustrates how PPRSR can be applied in the context of the VENUS project before concluding.

2 Background and Notations

2.1 Notations

In this paper we use propositional calculus, denoted by $\mathcal{L}_{\mathcal{PC}}$, as knowledge representation language with usual connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$. Let X be a set of propositional formulae, we denote by $Cons(X)$ the set of logical consequences of X. We denote by $\mathcal{W}$ the set of interpretations of $\mathcal{L}_{\mathcal{PC}}$ and by $Mod(\psi)$ the set of models of a formula ψ, that is $Mod(\psi) = \{\omega \in \mathcal{W}, \omega \models \psi\}$ where $\models$ denotes the inference relation used for drawing conclusions.

2.2 Removed Sets Revision

We briefly recall the Removed Sets Revision approach. Removed Sets Revision [15] deals with the revision of a set of propositional formulae by a set of propositional formulae[1]. Let K and A be finite sets of clauses. Removed Sets Revision (RSR) focuses on the minimal subsets of clauses to remove from K, called *removed sets*, in order to restore the consistency of $K \cup A$. More formally: let K and A be two consistent sets of clauses such that $K \cup A$ is inconsistent. R a subset of clauses of K, is a removed set of $K \cup A$ iff (i) $(K \backslash R) \cup A$ is consistent; (ii) $\forall R' \subseteq K$, if $(K \backslash R') \cup A$ is consistent then $\mid R \mid \leq \mid R' \mid$[2]. Let denote by $\mathcal{R}(K \cup A)$ the collection of removed sets of $K \cup A$, RSR is defined as follows: let K and A be two consistent sets of clauses, $K \circ_{RSR} A =_{def} \bigvee_{R \in \mathcal{R}(K \cup A)} Cons((K \backslash R) \cup A)$. According to a semantic point of view, $\mid \mathcal{NS}_K(\omega) \mid$ denotes the number of clauses of K falsfied by an interpretation ω and a total preorder on interpretations is defined by: $\omega_i \leq_K \omega_j$ iff $\mid \mathcal{NS}_K(\omega_i) \mid \leq \mid \mathcal{NS}_K(\omega_j) \mid$. Removed Sets Revision can be semantically defined by $Mod(K \circ_{RSR_{sem}} A) = min(Mod(A), \leq_K)$. It minimizes the number of clauses falsified by the models of A and $Mod(K \circ_{RSR} A) = Mod(K \circ_{RSR_{sem}} A)$. In case of prioritized belief bases, RSR has been extended to Prioritized Removed Sets Revision (PRSR) [1].

2.3 Partial Preorders

A partial preorder, denoted by $\preceq$ on a set A is a reflexive and transitive binary relation. Let x and y be two members of A, the equality is defined by $x = y$ iff $x \preceq y$ and $y \preceq x$. The corresponding strict partial preorder, denoted by $\prec$, is such that, $x \prec y$ iff $x \preceq y$ holds but $x = y$ does not hold. We denote by $\sim$ the incomparability relation $x \sim y$ iff $x \preceq y$ does not hold nor $y \preceq x$. The set of minimal elements of A with respect to $\prec$, denoted by $Min(A, \prec)$, is defined as: $Min(A, \prec) = \{x \in A, \nexists y \in A : y \prec x\}$.

Generally, epistemic states are represented by total preorders on interpretations, however, as mentionned in the introduction, in case of partial ignorance, the agent is unable to compare all situations between them and a partial preorder seems to be more suitable to represent epistemic states.

Let Ψ be an epistemic state and $Bel(\Psi)$ its corresponding belief set, Ψ is first represented by a partial preorder on interpretations, denoted by $\preceq_\Psi$. In [4], a suitable definition of faithful assignment is given: let $Bel(\Psi) = min(\mathcal{W}, \prec_\Psi)$, $\preceq_\Psi$ is a P-faifhful assignment if (1) if $\omega, \omega' \models Bel(\Psi)$ then $\omega \prec_\Psi \omega'$ does not hold, (2) if $\omega' \not\models Bel(\Psi)$, then there exists ω such that $\omega \models Bel(\Psi)$ and $\omega \prec_\Psi \omega'$, (3) if $\Psi = \Phi$ then $\preceq_\Psi = \preceq_\Phi$. Moreover, [4] gives a set of postulates an operation $\circ$ has to satisfy and a representation theorem such that $Mod(Bel(\Psi \circ \mu)) = min(Mod(\mu), \preceq_\Psi)$. An alternative syntactic but equivalent representation of an epistemic state, Ψ is a partially preordered belief base, denoted by $(\Sigma, \preceq_\Sigma)$, where Σ is a set of

[1] We consider propositional formulae in their equivalent conjonctive normal form (CNF).

[2] $\mid R \mid$ denotes the number of clauses of R.

propositional formulae, and $\preceq_\Sigma$ is a partial preorder on the formulae of Σ. Several ways of defining a partial preorder on subsets of formulae belonging to Σ, called comparators, from a partial preorder on a set of formulae Σ have been proposed: inclusion-based [10], possibilistic [3], lexicographic [16] comparators. They are such that the preferred formulae are kept in the belief base. In our approach, according to the Removed Sets strategy, we adopt a dual point of view in the sense that we want to prefer the subsets of formulae to remove. For example, we rephrase the possibilistic comparator (or weak comparator) used in [3], already defined in [12] and reused by [8] as follows. Y is preferred to X if for each element of Y, there exists at least one element of X which is preferred to it, more formally: let $\preceq_\Sigma$ be a partial preorder on Σ, $Y \subseteq \Sigma$ and $X \subseteq \Sigma$. Y is preferred to X, denoted by $Y \trianglelefteq_w X$ iff $\forall y \in Y$, $\exists x \in X$ such that $x \preceq_\Sigma y$.

We now briefly recall the extension of the semantic possibilistic revision to partial preorders [2]. Let π be a possibility distribution [5] and let Ψ be an epistemic state, represented by $(\mathcal{W}, \preceq_\Psi)$, such that $\forall \omega, \omega' \in \mathcal{W}$, $\omega \preceq_\Psi \omega'$ iff $\pi(\omega') \preceq \pi(\omega)$. The possibilistic revision of Ψ by a propositional formula μ leads to the epistemic state $\Psi \circ_\pi \mu$, represented by $(\mathcal{W}, \preceq_{\Psi\circ_\pi\mu})$ which considers all the counter-models of μ as impossible and preserves the relative ordering between the models of μ. More formally, $\Psi \circ_\pi \mu$ corresponds to the following partial preorder: (i) if ω, $\omega' \in Mod(\mu)$ then $\omega \preceq_{\Psi\circ_\pi\mu} \omega'$ iff $\omega \preceq_\Psi \omega'$, (ii) if ω, $\omega' \notin Mod(\mu)$ then $\omega =_{\Psi\circ_\pi\mu} \omega'$, (iii) if $\omega \in Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then $\omega \prec_{\Psi\circ_\pi\mu} \omega'$.

2.4 Answer Sets

A *normal logic program* is a set of rules of the form $c \leftarrow a_1, \ldots, a_n, not\ b_1, \ldots, not\ b_m$ where $c, a_i (1 \leq i \leq n), b_j (1 \leq j \leq m)$ are propositional atoms and the symbol *not* stands for *negation as failure*. For a rule r like above, we introduce $head(r) = c$ and $body(r) = \{a_1, \cdots, a_n,\ b_1, \cdots, b_m\}$. Furthermore, let $body^+(r) = \{a_1, \cdots, a_n\}$ denotes the set of positive body atoms and $body^-(r) = \{b_1, \cdots, b_m\}$ the set of negative body atoms, and $body(r) = body^+(r) \cup body^-(r)$. Let r be a rule, r^+ denotes the rule $head(r) \leftarrow body^+(r)$, obtained from r by deleting all negative body atoms in the body of r.

A set of atoms X is *closed under* a basic program P iff for any rule $r \in P$, $head(r) \in X$ whenever $body(r) \subseteq X$. The smallest set of atoms which is closed under a basic program P is denoted by $CN(P)$. The *reduct* or Gelfond-Lifschitz transformation [13], P^X of a program P relatively to a set X of atoms is defined by $P^X = \{r^+ \mid r \in P\ and\ body^-(r) \cap X = \emptyset\}$. A set of atoms X is an *answer set* of P iff $CN(P^X) = X$.

3 Partially Preordered Removed Sets Revision (PPRSR)

Let Ψ be an epistemic state for partially preordered information. Ψ is syntactically represented by $(\Sigma, \preceq_\Sigma)$ where Σ is a set of formulae and $\preceq_\Sigma$ is a partial preorder on Σ. Ψ can be represented from a semantic point of view as $(\mathcal{W}, \preceq_\Psi)$ where $\mathcal{W}$ is the set of interpretations and $\preceq_\Psi$ is a partial preorder on $\mathcal{W}$ such that

$Mod(Bel(\Psi)) = min(\mathcal{W}, \prec_\Psi)$. We present the Partially Preordered Removed Sets Revision (PPRSR) of an epistemic state Ψ by a formula μ. According to the syntactic point of view, we focus on the preferred subsets of formulae to remove from Σ to restore consistency. We first define the potential removed sets as follows:

Definition 1. *Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ. Let μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. R, a subset of formulae of Σ, is a potential removed set of $\Sigma \cup \{\mu\}$ iff $(\Sigma \backslash R) \cup \{\mu\}$ is consistent.*

Example 1. Let $\Sigma = \{a, b, a \vee \neg b, \neg a \vee b\}$ and $\preceq_\Sigma$ be a given partial preorder:

$$\begin{tikzcd} a \arrow[d] & a \vee \neg b \arrow[d] \\ b & \neg a \vee b \end{tikzcd}$$

$(b \leftarrow a$ means that $b \prec_\Sigma a)$. We revise Σ by $\mu = \neg a \vee \neg b$. $\Sigma \cup \{\mu\}$ is inconsistent. The potential removed sets are $R_0 = \{a, a \vee \neg b\}, R_1 = \{a, a \vee \neg b, \neg a \vee b\}, R_2 = \{a, b, a \vee \neg b\}, R_3 = \{a, b, a \vee \neg b, \neg a \vee b\}, R_4 = \{b, \neg a \vee b\}, R_5 = \{b, a \vee \neg b, \neg a \vee b\}, R_6 = \{a, b, \neg a \vee b\}, R_7 = \{a, b\}$.

Let $\mathcal{R}(\Sigma \cup \{\mu\})$ be the set of potential removed sets. Among them, we want to prefer the potential removed sets which allow us to remove the formulae that are not preferred according to $\preceq_\Sigma$. This leads to define a partial preorder on subsets of formulae of Σ, called comparator [3,16], denoted by $\trianglelefteq_C$. We now generalize the notion of Removed Sets to subsets of partially preordered formulae. We denote by $\mathcal{R}_C(\Sigma \cup \{\mu\})$ the set of removed sets of $\Sigma \cup \{\mu\}$.

Definition 2. *Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ. Let μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. $R \subseteq \Sigma$ is a removed set of $\Sigma \cup \{\mu\}$ iff*

1. *R is a potential removed set.*
2. *$\nexists R' \in \mathcal{R}(\Sigma \cup \{\mu\})$ such that $R' \subsetneq R$.*
3. *$\nexists R' \in \mathcal{R}(\Sigma \cup \{\mu\})$ such that $R' \lhd_C R$.*

Example 2. In the examples, we will use the weak comparator, denoted by $\trianglelefteq_w$ and defined in 2.3. We have $R_0 \trianglelefteq_w R_1$ because $a \preceq_\Sigma a$ and $\neg a \vee b \preceq_\Sigma a \vee \neg b$. The partial preorder on the potential removed sets is: $R_0 \trianglelefteq_w R_1$, $R_0 \trianglelefteq_w R_2$, $R_0 \lhd_w R_3$, $R_0 \lhd_w R_4$, $R_0 \sim_w R_7$, $R_1 \sim_w R_2$, $R_1 \trianglelefteq_w R_3$, $R_1 \sim_w R_7$, $R_2 \trianglelefteq_w R_3$, $R_7 \trianglelefteq_w R_2$, $R_7 \trianglelefteq_w R_3$, $R_3 =_w R_4 =_w R_5 =_w R_6$. We have $\nexists R' \in \mathcal{R}(\Sigma \cup \{\mu\})$ such that $R' \lhd_w R_0$ and $R' \lhd_w R_7$. Moreover, R_0 and R_7 are minimal according to the inclusion. So, $\mathcal{R}_w(\Sigma \cup \{\mu\}) = \{R_0, R_7\}$.

Remark: We could refine the notion of removed set with an extra preference according to a strategy P (cardinality or minimality). $\mathcal{R}_{C,P}(\Sigma \cup \{\mu\})$ denotes the set of removed sets of $\Sigma \cup \{\mu\}$ according to the strategy P. In this case, a preferred removed set according to a strategy P is a removed set R such that $\nexists R' \in \mathcal{R}_C(\Sigma \cup \{\mu\})$ such that $R' <_P R$. According to the cardinality, $R_Y \leq_{CARD} R_X$ iff $|R_Y| \leq |R_X|$ with $|X|$ the cardinality of the set X. According to the minimality, $R_Y \leq_{MIN} R_X$ iff $|R_Y \cap MIN| \leq |R_X \cap MIN|$ with $MIN = \{x | x \in \Sigma, \nexists y \in \Sigma, y \prec_\Sigma x\}$.

Example 3. We can apply strategies: $\mathcal{R}_{w,CARD}(\Sigma \cup \{\mu\}) = \{R_0, R_7\}$ and $\mathcal{R}_{w,MIN}(\Sigma \cup \{\mu\}) = \{R_0\}$.

The revision of an epistemic state represented by $(\Sigma, \preceq_\Sigma)$ by a formula μ is a new epistemic state represented by $(\Sigma \circ_{\trianglelefteq_C} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_C} \mu})$ and is defined as follows:

Definition 3. *Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ. Let μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. The Partially Preordered Removed Sets Revision (PPRSR) is defined by:*

- $\Sigma \circ_{\trianglelefteq_C} \mu = \bigvee_{R \in \mathcal{R}(\Sigma \cup \{\mu\})} Cons((\Sigma \backslash R) \cup \{\mu\})$
- $\preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}$: *(i)* $\forall \psi \in \Sigma$, $\mu \prec_{\Sigma \circ_{\trianglelefteq_C} \mu} \psi$; *(ii)* $\forall \psi, \phi \in \Sigma$, $\psi \preceq_{\Sigma \circ_{\trianglelefteq_C} \mu} \phi$ *iff* $\psi \preceq_\Sigma \phi$

Example 4. According to the example 1, Ψ is syntactically represented by $(\Sigma, \preceq_\Sigma)$ and revising by μ using the weak comparator gives $\Sigma \circ_{\trianglelefteq_w} \mu = Cons(\{b, \neg a \vee b, \neg a \vee \neg b\}) \vee Cons(\{a \vee \neg b, \neg a \vee b, \neg a \vee \neg b\})$ and $\preceq_{\Sigma \circ_{\trianglelefteq_w} \mu}$:

$$\begin{tikzcd}[column sep=small] a \arrow[d] & & a \vee \neg b \arrow[d] \\ b \arrow[dr] & & \neg a \vee b \arrow[dl] \\ & \neg a \vee \neg b & \end{tikzcd}$$

In order to establish the equivalence between the syntactic and the semantic representations of Ψ, we use the following definition where $F_\Sigma(\omega)$ denotes the set of formulae of Σ falsified by an interpretation ω.

Definition 4. $\forall \omega, \omega' \in \mathcal{W}$, $\omega \preceq_\Psi^C \omega'$ *iff* $F_\Sigma(\omega) \trianglelefteq_C F_\Sigma(\omega')$ *and* $F_\Sigma(\omega') \nsubseteq F_\Sigma(\omega)$.

Using this definition, the semantic representation of Ψ is $(\mathcal{W}, \preceq_\Psi^C)$ and is such that $Mod(\Sigma) = min(\mathcal{W}, \prec_\Psi^C)$. Moreover the following proposition holds.

Proposition 1. *Let Ψ be an epistemic state and $\preceq_\Psi^C$ be a partial preorder on $\mathcal{W}$ associated to Ψ. Then, $\preceq_\Psi^C$ is a P-faithful assignment.*

We are now able to define the semantic counterpart of PPRSR as follows:

Definition 5. *Let Ψ be an epistemic state and μ be a formula. $Mod(\Psi \circ_{\trianglelefteq_C^{sem}} \mu) = min(Mod(\mu), \prec_\Psi^C)$.*

The equivalence between the semantic and the syntactic PPRSR is given by the following proposition.

Proposition 2. *Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ and μ be a formula. $Mod(\Sigma \circ_{\trianglelefteq_C} \mu) = Mod(\Psi \circ_{\trianglelefteq_C^{sem}} \mu)$.*

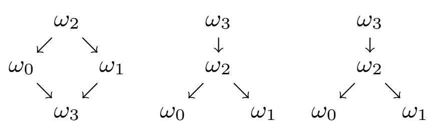
The semantic representation of the revised epistemic state is $(\mathcal{W}, \preceq^w_{\Psi \circ_{\trianglelefteq_C^{sem}} \mu})$ with $\preceq^w_{\Psi \circ_{\trianglelefteq_C^{sem}} \mu}$ defined by $\omega \preceq^w_{\Psi \circ_{\trianglelefteq_C^{sem}} \mu} \omega'$ iff $F_{\Sigma \circ_{\trianglelefteq_C} \mu}(\omega) \trianglelefteq_w F_{\Sigma \circ_{\trianglelefteq_C} \mu}(\omega')$ and $F_{\Sigma \circ_{\trianglelefteq_C} \mu}(\omega') \nsubseteq F_{\Sigma \circ_{\trianglelefteq_C} \mu}(\omega)$. When we select the weak comparator defined in 2.3, the PPRSR framework can capture the possibilistic revision recalled in 2.3 and the following proposition holds.

Proposition 3. *Let $\circ_\pi$ be the possibilistic revision operator. $\forall \omega, \omega' \in \mathcal{W}$, $\omega \preceq^w_{\Psi \circ_{\trianglelefteq_w^{sem}} \mu} \omega'$ iff $\omega \preceq_{\Psi \circ_\pi \mu} \omega'$*

Example 5. Let $(\Sigma, \preceq_\Sigma)$ be the syntactic representation of Ψ from the example 1 with $\Sigma = \{a, b, a \vee \neg b, \neg a \vee b\}$. The interpretations are: $\omega_0 = \{\neg a, \neg b\}$, $\omega_1 = \{\neg a, b\}$, $\omega_2 = \{a, \neg b\}$ and $\omega_3 = \{a, b\}$. Using the definition 4 with the weak comparator, we construct a partial preorder on the interpretations. The sets of formulae of Σ falsified by the interpretations are $F_\Sigma(\omega_0) = \{a, b\}$, $F_\Sigma(\omega_1) = \{a, a \vee \neg b\}$, $F_\Sigma(\omega_2) = \{b, \neg a \vee b\}$ and $F_\Sigma(\omega_3) = \emptyset$ and the partial preorder $\preceq_\Psi^w$ is given by the Fig. 1 (a). Therefore $(\mathcal{W}, \preceq_\Psi^w)$ is the semantic representation of Ψ and is such that $Mod(\Sigma) = min(\mathcal{W}, \prec_\Psi^w)$.

Let $(\Sigma \circ_{\trianglelefteq_w} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_w} \mu})$ be the syntactic representation of the epistemic state Ψ revised by μ. Using the definition 4 with the weak comparator, we construct a new partial preorder on the interpretations. The sets of formulae of $\Sigma \circ_{\trianglelefteq_w} \mu$ falsified by the interpretations are $F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega_0) = \{a, b\}$, $F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega_1) = \{a, a \vee \neg b\}$, $F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega_2) = \{b, \neg a \vee b\}$ and $F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega_3) = \{\neg a \vee \neg b\}$ and the partial preorder $\preceq_{\Psi \circ_{\trianglelefteq_w^{sem}}}$ is given by the Fig. 1 (b). Therefore $(\mathcal{W}, \preceq^w_{\Psi \circ_{\trianglelefteq_w^{sem}} \mu})$ is the semantic representation of Ψ revised by μ and with the proposition 2 is such that $Mod(\Sigma \circ_{\trianglelefteq_w} \mu) = min(Mod(\mu), \prec_\Psi^w)$.

If we apply, the semantic possibilistic revision of $(\mathcal{W}, \preceq_\Psi^w)$ by μ which preserves the relative ordering between the models of μ and considers all the counter-models of μ as impossible, we obtain the partial preorder $\preceq_{\Psi \circ_\pi \mu}$ illustrated in Fig. 1 (c). Therefore $(\mathcal{W}, \preceq^w_{\Psi \circ_{\trianglelefteq_w^{sem}} \mu}) = (\mathcal{W}, \preceq_{\Psi \circ_\pi \mu})$.



(a) $\preceq_\Psi^w$ (b) $\preceq^w_{\Psi \circ_{\trianglelefteq_w^{sem}} \mu}$ (c) $\preceq_{\Psi \circ_\pi \mu}$

Fig. 1. Partial preorders between interpretations

4 Encoding PPRSR in Answer Set Programming

In order to compute the removed sets, we extend the methods proposed by [9] and [1] to the revision of partially preordered information. We first translate our revision problem into a logic program with answer sets semantics, denoted by $\Pi_{\Sigma \cup \{\mu\}}$. The set of answer sets is denoted by $S(\Pi_{\Sigma \cup \{\mu\}})$. We then define a partial preorder between answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ and we show a one-to-one correspondence between removed sets of $\Sigma \cup \{\mu\}$ and preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$.

Let Σ be a set of partially preordered formulae and μ a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. The set of all positive literals of $\Pi_{\Sigma \cup \{\mu\}}$ is denoted by V^+ and the set of all negative literals of $\Pi_{\Sigma \cup \{\mu\}}$ is denoted by V^-. The set of all rule atoms representing formulae is defined by $R^+ = \{r_f | f \in \Sigma\}$ and $F_O(r_f)$ represents the formula of Σ corresponding to r_f in $\Pi_{\Sigma \cup \{\mu\}}$, namely $\forall r_f \in R^+, F_O(r_f) = f$. This translation requires the introduction of intermediary atoms representing subformulae of f. We denote by ρ_f^j the intermediary atom representing f^j which is a subformula of $f \in \Sigma$. To each answer set S of $\Pi_{\Sigma\cup\{\mu\}}$, an interpretation of $\Sigma \cup \{\mu\}$ is associated. Each interpretation of $\Sigma \cup \{\mu\}$ corresponds to several potential removed sets denoted by $F_O(R^+ \cap S)$.

1. In the first step, we introduce rules in order to build a one-to-one correspondence between answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and interpretations of V^+. For each atom, $a \in V^+$ two rules are introduced: $a \leftarrow not\ a'$ and $a' \leftarrow not\ a$ where $a' \in V^-$ is the negative atom corresponding to a.
2. In the second step, we introduce rules in order to exclude the answer sets S corresponding to interpretations which are not models of $(\Sigma \backslash F) \cup \{\mu\}$ with $F = \{f | r_f \in S\}$. According to the syntax of f, the following rules are introduced:
   - If $f \equiv a$, the rule $r_f \leftarrow not\ a$ is introduced;
   - If $f \equiv \neg f^1$, the rule $r_f \leftarrow not\ \rho_{f^1}$ is introduced;
   - If $f \equiv f^1 \vee \ldots \vee f^m$, the rule $r_f \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced;
   - If $f \equiv f^1 \wedge \ldots \wedge f^m$, it is though necessary to introduce several rules to the program. These rules are introduced: $\forall 1 \leq j \leq m$, $r_f \leftarrow \rho_{f^j}$.
3. The third step rules out answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ which correspond to interpretations which are not models of μ. According to the syntax of μ, the following rules are introduced:
   - If $\mu \equiv a$, the rule $false \leftarrow not\ a$ is introduced;
   - If $\mu \equiv \neg f^1$, the rule $false \leftarrow not\ \rho_{f^1}$ is introduced;
   - If $\mu \equiv f^1 \vee \ldots \vee f^m$, the rule $false \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced;
   - If $\mu \equiv f^1 \wedge \ldots \wedge f^m$, the rules $\forall 1 \leq j \leq m$, $false \leftarrow \rho_{f^j}$ are introduced.

   In order to rule out $false$ from the models of μ, the following rule is introduced: $contradiction \leftarrow false, not\ contradiction$.

Example 6. For the previous example, the logic program $\Pi_{\Sigma\cup\{\mu\}}$ is the following:

$$\begin{array}{llll} a \leftarrow not\ a' & b \leftarrow not\ b' & r_a \leftarrow a' & r_{a\vee\neg b} \leftarrow a', b \\ a' \leftarrow not\ a & b' \leftarrow not\ b & r_b \leftarrow b' & r_{\neg a\vee b} \leftarrow a, b' \\ false \leftarrow not\ a', not\ b' & & contradiction \leftarrow false, not\ contradiction & \end{array}$$

If $f = \neg a \vee b$ belongs to a removed set, then $r_{\neg a\vee b}$ should belong to an answer set. f has to be falsified and so $\neg f$, i.e. $a \wedge \neg b$, has to be satisfied that is why the rule $r_{\neg a\vee b} \leftarrow a, b'$ is introduced to $\Pi_{\Sigma\cup\{\mu\}}$.

From the logic program, we show how we obtain a one-to-one correspondence between the preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and the removed sets of $\Sigma \cup \{\mu\}$. Let S be a set of atoms, we define the interpretation over the atoms of $S \cap V^+$ as $I_S = \{a | a \in S\} \cup \{\neg a | a' \in S\}$ and the following result holds.

Proposition 4. *Let ρ a rule atom or an intermediary atom. $\rho \in CN(\Pi^S_{\Sigma\cup\{\mu\}})$ iff $I_S \not\models F_O(R^+ \cap S)$.*

The correspondence between answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and interpretations of $(\Sigma \backslash F_O(R^+ \cap S)) \cup \{\mu\}$ is given in the following proposition:

Proposition 5. *Let Σ be a set of partially preordered formulae. Let $S \subseteq V$ be a set of atoms. S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ iff S corresponds to an interpretation I_S of V^+ which satisfies $(\Sigma \backslash F_O(R^+ \cap S)) \cup \{\mu\}$.*

The proof of the proposition 5 is based on the rules construction.

Example 7. The answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ are: $S_0 = \{a', b, r_{a \vee \neg b}, r_a\}$, $S_1 = \{a, b', r_{\neg a \vee b}, r_b\}$ and $S_2 = \{a', b', r_a, r_b\}$.

In order to compute the answer sets corresponding to the removed sets, we introduce new preference relations between answer sets according to a partial preorder. We define the notion of preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ according to the weak comparator denoted by $S_w(\Pi_{\Sigma \cup \{\mu\}})$.

Definition 6. *Let $\preceq_\Sigma$ be a partial preorder on Σ, μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent, $S \in S(\Pi_{\Sigma \cup \{\mu\}})$. S is a preferred answer set of $\Pi_{\Sigma \cup \{\mu\}}$ iff $\nexists S' \in S(\Pi_{\Sigma \cup \{\mu\}})$ such that $F_O(S' \cap R^+) \lhd_w F_O(S \cap R^+)$.*

Example 8. We have $F_O(S_0 \cap R^+) \unlhd_w F_O(S_1 \cap R^+)$ and $F_O(S_2 \cap R^+) \unlhd_w F_O(S_1 \cap R^+)$. So, $S_w(\Pi_{\Sigma \cup \{\mu\}}) = \{S_0, S_2\}$.

Remark: As previously, it is possible to refine the notion of preferred answer set with an extra preference according to a strategy P. Let S_X, $S_Y \in S_w(\Pi_{\Sigma \cup \{\mu\}})$. S_Y is preferred to S_X according to $CARD$ (resp. MIN) iff $|F_O(S_Y \cap R^+)| \leq |F_O(S_X \cap R^+)|$ (resp. $|F_O(S_Y \cap R^+) \cap MIN| \leq |F_O(S_X \cap R^+) \cap MIN|$).

Example 9. We have S_0 is as preferred as S_2 according to $CARD$ and S_0 is preferred to S_2 according to MIN.

The one-to-one correspondence between preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ and the removed sets is given by the following proposition:

Proposition 6. *Let Σ be a finite set of partially preordered formulae and μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. X is a removed set of $\Sigma \cup \{\mu\}$ iff there exists a preferred answer set S of $\Pi_{\Sigma \cup \{\mu\}}$ such that $F_O(R^+ \cap S) = X$.*

Sketch of the proof: we show that the set of removed sets of $\Sigma \cup \{\mu\}$ equals the set of preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$.

Example 10. We have $F_O(S_0 \cap R^+) = \{a, a \vee \neg b\}$ and $F_O(S_2 \cap R^+) = \{a, b\}$ which correspond to the removed sets R_0 and R_7 found in the previous section.

Performing PPRSR. Regarding the implementation, CLASP [7] gives us the answer sets of $\Pi_{\Sigma \cup \{\mu\}}$. But our method requires to partially preorder the answer sets with the comparator $\unlhd_w$ to obtain the preferred answer sets corresponding to removed sets. This step is not yet implemented in ASP. We used a java program to partially preorder the answer sets to obtain the preferred answer sets. We denote by N the number of answer sets given by CLASP. The computation of the partial preorder between them can be realized in less than $\frac{N(N-1)}{2}$ comparisons. Indeed, it is sufficient to compare the minimal formulae according to $\preceq_\Sigma$ of each answer set and so using the following proposition, we reduce the cost of the computation.

Proposition 7. *Let $\preceq_\Sigma$ be a partial preorder on Σ, μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent and $S, S' \in S(\Pi_{\Sigma \cup \{\mu\}})$. $F_O(S \cap R^+) \unlhd_w F_O(S' \cap R^+)$ iff $\forall y \in Min(F_O(S \cap R^+), \prec_\Sigma), \exists x \in Min(F_O(S' \cap R^+), \prec_\Sigma)$ such that $x \preceq_\Sigma y$ where $Min(F_O(S \cap R^+), \prec_\Sigma) = \{x | x \in F_O(S \cap R^+), \nexists y \in F_O(S \cap R^+), y \prec_\Sigma x\}$.*

Moreover, the determination of the minimal answer sets according to this partial preorder does not increase the cost since the complexity of CLASP is similar to the complexity of the SAT problem.

5 VENUS Application

The european VENUS project (Virtual ExploratioN of Underwater Sites) no (IST-034924)[3] aims at providing scientific methodologies and technological tools for the virtual exploration of deep underwater archaeology sites. In this context, technologies like photogrammetry are used for data acquisition and the knowledge about the studied objects is provided by both archaeology and photogrammetry. We constructed an application ontology in [14] from a domain ontology which describes the vocabulary on the amphorae (the studied artefacts) and from a task ontology describing the data acquisition process. This ontology consists of a set of concepts, relations, attributes and constraints like "If the typology of the amphora is Dressel 20 then the total length of the amphora should be included between 0,368 and 0.552 m." Our knowledge base contains our ontology and observations. The ontology represents the generic knowlegde which is preferred to observations. The observations on the same amphora can be preordered according to the reliability of the experts who provide them. In this context, we revise the generic knowledge and the observations by new observations. We only consider a small part of the ontology (Fig. 2) and some observations in order to provide an example where the knowledge base is expressed in propositional logic.

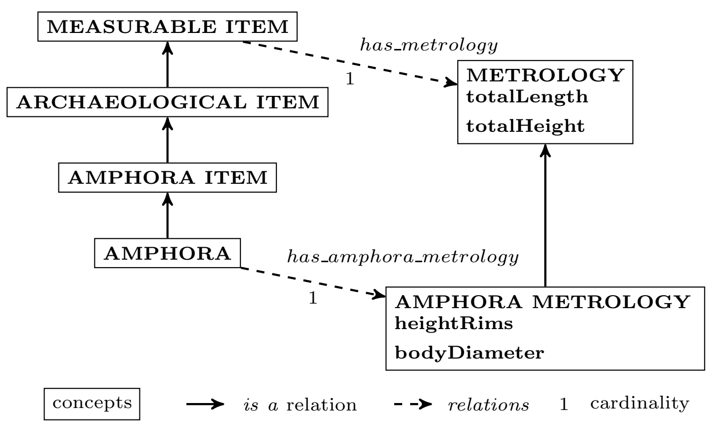


Fig. 2. Extract of the application ontology

We use the following propositional variables: a for the amphora, t for the typology, b for Beltran 2B, h for the total height, l for the total length, c_h (resp. c_l) for the constraint of compatibility between the height (resp. length) and the typology. The propositional translation of the extract of the ontology can be

[3] http://www.venus-project.eu

resumed by the set of formulae: $G = \{(\neg a \vee h) \wedge (\neg a \vee t), (\neg t \vee \neg b \vee h) \wedge (\neg t \vee \neg b \vee c_h), (\neg t \vee \neg b \vee l) \wedge (\neg t \vee \neg b \vee c_l)\}$. We then add the formulae provided by the observations of the first expert denoted by $O_1 = \{a, b, c_l, c_h, l, h, t\}$. We obtain $\Sigma = G \cup O_1$ and $\preceq_\Sigma$ is represented by the figure 3 (a). We revised by the observations given by the second expert who is more reliable than the first one, denoted by $O_2 = \{\neg c_l, \neg c_h\}$ and such that $\neg c_l \sim_{O_2} \neg c_h$, the revised preorder is represented by Fig. 3 (b). The revision presented in the section 3 is the first

$$
\begin{array}{c}
a =_\Sigma t =_\Sigma b \\
\swarrow \quad \searrow \\
c_h \qquad c_l \\
\downarrow \qquad \downarrow \\
h \qquad l \\
\searrow \quad \swarrow \\
(\neg a \vee h) \wedge (\neg a \vee t) \\
=_\Sigma \\
(\neg t \vee \neg b \vee h) \wedge (\neg t \vee \neg b \vee c_h) \\
=_\Sigma \\
(\neg t \vee \neg b \vee l) \wedge (\neg t \vee \neg b \vee c_l)
\end{array}
$$

(a) $\preceq_\Sigma$

$$
\begin{array}{c}
a =_{\Sigma \circ_{\trianglelefteq_C} \mu} t =_{\Sigma \circ_{\trianglelefteq_C} \mu} b \\
\swarrow \quad \searrow \\
c_h \qquad c_l \\
\downarrow \qquad \downarrow \\
h \qquad l \\
\downarrow \qquad \downarrow \\
\neg c_h \qquad \neg c_l \\
\searrow \quad \swarrow \\
(\neg a \vee h) \wedge (\neg a \vee t) \\
=_{\Sigma \circ_{\trianglelefteq_C} \mu} \\
(\neg t \vee \neg b \vee h) \wedge (\neg t \vee \neg b \vee c_h) \\
=_{\Sigma \circ_{\trianglelefteq_C} \mu} \\
(\neg t \vee \neg b \vee l) \wedge (\neg t \vee \neg b \vee c_l)
\end{array}
$$

(b) $\preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}$

Fig. 3. $\preceq_\Sigma$ and $\preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}$

step of the revision to apply in the VENUS context. Indeed, the revision could be defined as follows:

- $\Sigma \circ_{\trianglelefteq_C} O_2 = \bigvee_{R \in \mathcal{R}_C(\Sigma \cup O_2)} Cons((O_1 \backslash R) \cup G \cup O_2)$ with a modified definition of the potential removed sets of the definition 1. R is a potential removed set of $\Sigma \cup \{\mu\}$ iff $(O_1 \backslash R) \cup G \cup O_2$ is consistent.
- $\preceq_{\Sigma \circ_{\trianglelefteq_C} O_2}$: (i) $\forall \psi, \phi \in O_1$, $\psi \prec_{\Sigma \circ_{\trianglelefteq_C} O_2} \phi$ iff $\psi \preceq_\Sigma \phi$, (ii) $\forall \psi, \phi \in G$, $\psi \prec_{\Sigma \circ_{\trianglelefteq_C} O_2} \phi$ iff $\psi \preceq_\Sigma \phi$, (iii) $\forall \psi \in G$, $\mu \in O_2$, $\psi \prec_{\Sigma \circ_{\trianglelefteq_C} O_2} \mu$, (iv) $\forall \psi \in G$, $\phi \in O_1$, $\psi \prec_{\Sigma \circ_{\trianglelefteq_C} O_2} \phi$ and (v) $\forall \psi \in O_1$, $\phi \in O_2$ such that ψ and ϕ refers to the measures of the same attribute[4], $\phi \prec_{\Sigma \circ_{\trianglelefteq_C} O_2} \psi$.

6 Conclusion

This paper presents a new framework for revising partially preordered information called Partially Preordered Removed Sets Revision (PPRSR) which extends the Removed Sets approach to partial preorders. The paper shows that PPRSR can be successfully encoded into answer set programming and proposes an implementation stemming from ASP solvers. It shows that the extension of the possibilistic revision to partial preorders can be captured within the PPRSR

[4] It is obvious that measures of different attributes are incomparable.

framework allowing for an efficient implementation with ASP. It illustrates how PPRSR can be applied within the context of the VENUS european project dealing with archaeological information. An experimental study has now to be conducted in the context of the VENUS project in order to provide a more accurate evaluation of the performance of PPRSR. We have to deeper investigate the use of ASP solver statements in order to directly define a partial preorder between answer sets. A future work will investigate the use of the lexicogtraphic comparator for defining revision operations within the framework of PPRSR.

Acknowledgements

Work partially supported by the European Community under project VENUS (Contract IST-034924) of the "Information Society Technologies (IST) program of the 6th FP for RTD". The authors are solely responsible for the content of this paper. It does not represent the opinion of the European Community, and the European Community is not responsible for any use that might be made of data appearing therein.

References

1. Benferhat, S., Ben-Naim, J., Papini, O., Würbel, E.: Answer set programming encoding of prioritized removed sets revision: Application to gis. In: Applied Intelligence. Springer, Heidelberg
2. Benferhat, S., Lagrue, S., Papini, O.: Revising partially ordered belief. In: Proc. of NMR 2002, Toulouse, France (2002)
3. Benferhat, S., Lagrue, S., Papini, O.: Revision with partially ordered information in a possibilistic framework. Fuzzy Sets and Systems 144(1), 25–41 (2004)
4. Benferhat, S., Lagrue, S., Papini, O.: Revision of partially ordered information. In: Proc. of IJCAI 2005, Edinburgh, pp. 376–381 (2005)
5. Dubois, D., Prade, H.: Belief change and possibility theory. In: Gärdenfors, P. (ed.) Belief Revision, pp. 142–182. Cambridge University Press, U. K. (1992)
6. Gärdenfors, P.: Knowledge in Flux: Modeling the Dynamics of Epistemic States. Bradford Books/ MIT Press, Cambridge (1988)
7. Gebser, M., Kaufmann, B., Neumann, A., Schaub, T.: Clasp: A conflict-driven answer set solver. In: Baral, C., Brewka, G., Schlipf, J. (eds.) LPNMR 2007. LNCS, vol. 4483, pp. 260–265. Springer, Heidelberg (2007)
8. Halpern, J.Y.: Defining relative likelihood in partially-ordered structures. In: Proc. of UAI 1996, pp. 299–306 (1996)
9. Hué, J., Würbel, E., Papini, O.: Removed sets fusion: Performing off the shelf. In: Ghallab, M., Spyropoulos, C.D., Fakotakis, N., Avouris, N. (eds.) Proc. of ECAI 2008 (2008)
10. Junker, U., Brewka, G.: Handling partially ordered defaults in tms. In: Proceedings of IJCAI 1989, pp. 1043–1048 (1989)
11. Katsuno, H., Mendelzon, A.: Propositional Knowledge Base Revision and Minimal Change. Artificial Intelligence 52, 263–294 (1991)
12. Lewis, D.K.: Counterfactuals. Harvard University Press, Cambridge (1973)

13. Gelfond, M., Lifschitz, V.: The stable model semantics for logic programming. In: Kowalski, R.A., Bowen, K. (eds.) Proc. of ICLP 1988, pp. 1070–1080. MIT Press, Cambridge (1988)
14. Papini, O., Würbel, E., Jeansoulin, R., Curé, O., Drap, P., Sérayet, M., Hué, J., Seinturier, J., Long, L.: D3.4 representation of archaeological ontologies 1. Technical report, Projet VENUS (2008)
15. Würbel, E., Jeansoulin, R., Papini, O.: Revision: An application in the framework of gis. In: Cohn, A.G., Giunchiglia, F., Selman, B. (eds.) Proc. of KR 2000, pp. 505–516 (2000)
16. Yahi, S., Benferhat, S., Lagrue, S., Sérayet, M., Papini, O.: A lexicographic inference for partially preordered belief bases. In: Brewka, G., Lang, J. (eds.) Proc. of KR 2008, pp. 507–516 (2008)