An Analysis of Gabor Detection

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Abstract. An elementary function that is now commonly referred to Gabor function, Gabor filter and Gabor wavelet was derived from uncertainty relation for information by Gabor to overcome the representation limit of Fourier analysis. Analyzing a signal by a Gabor filter in terms of convolution or spatial filtering, two pieces of information—phase and magnitude—can be obtained. In the paper, Gabor filter is considered as a Gabor atom detector. This analysis demonstrates that when the *k*-value defined as $k = \|g_{ni}\|^2 / \|g_{nr}\|^2$, where g_{nr} and g_{ni} are respectively the real and imaginary parts of a Gabor filter g_n, is close to one, the target phase can be estimated by Gabor phase and the target magnitude can be estimated by Gabor magnitude. However, when the *k*-value decreases, the quality of this approximation also decreases. The corresponding error bounds are derived.

Keywords: Gabor wavelet, Gabor filter, Gabor atom, Detection.

1 Introduction

To break the representation limit of Hitherto communication theory that describes signals either in time domain or Fourier domain, Gabor proposed a method of analyzing signals in which time and frequency information can be captured simultaneously. This method was especially designed for the signals having finite duration and whose frequencies very with time (e.g., sound) [1]. It is constituted by three fundamental components, uncertainty relation for information, elementary functions that are now generally referred to Gabor functions, Gabor filters and Gabor wavelets, and an algorithm for computing decomposition coefficients. The uncertainty relation for information said that for any function, the product of its effective width in time domain and its effective width in frequency domain is limited by an infimum $1/2$ [1]. Based on the uncertainty relation for information, Gabor discovered that an elementary function,

$$g_1(t) = \exp(-0.5(t-\mu)^2 / \sigma^2)\exp(iwt), \tag{1}$$

where σ controls the resolution of the analysis and μ and w are the positions of the function in time and frequency domains, respectively, reaches the theoretical limit. Given a signal *f*, Gabor attempted to decompose *f* in terms of the elementary functions (i.e., $f \approx \sum_j c_j g_j$), where g_js are the elementary functions with different parameters and c_js are the corresponding coefficients [1]. This decomposition approach is called

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Gabor expansion. Since the elementary functions are not orthogonal, several algorithms were proposed to compute the Gabor expansion coefficients [2].

1.1 Two-Dimensional Gabor Filters

Gabor was interested in Gabor expansion for one-dimensional signals, whereas since 1980, the two-dimensional (2D) versions of the elementary functions (called 2D Gabor filters) have been extensively used as convolution filters, which was motivated by the research results in biological vision systems. In 1980, Daugman proposed 2D Gabor filters for modeling simple receptive fields in striate cortex. He completed 2D uncertainty relation for information in 1985 [3] and demonstrated that 2D Gabor filters occupy an irreducible volume in a four-dimensional information hyperspace, whose axes can be interpretable as 2D visual space, orientation and spatial frequency [4]. Even though the properties of 2D Gabor filters (e.g., orientation selectivity and trade-off between spatial and frequency resolutions) match the early psychophysical and physiological findings, they were finally confirmed for modeling simple receptive fields through a series of 2D experiments on simple cells of human beings [4] and cats [5-7]. However, not all the current psychophysical and physiological researchers agree this model [8].

Gabor filters have been regarded as an important tool for a variety of image processing and pattern recognition problems (e.g., image enhancement [9], compression [10], texture analysis [11], edge and line detection [12], biometric recognition [13], object detection [14] and segmentation [15]). To maximize systems performance in terms of accuracy, researchers used optimization algorithms to tune the Gabor parameters [16] and to increase filtering speed, they proposed steerable Gabor-type filters [17], simplified Gabor wavelet [18] and recursive Gabor filtering scheme [19].

In addition to applications, researchers have derived a wide range of theoretical results from Gabor filters — Lee extended Gabor filters to Gabor wavelets [20]; Okajima indicated that Gabor filters can be derived from mutual information maximization [21]; Daugman noticed that Gabor filters are not polar separable in spatial domain or in frequency domain but are Cartesian separable in some special cases [3] and Yu et al. discovered a skewness property of Gabor wavelets [22].

To fit the current research focus, 2D Gabor filters are analyzed. The results given in this paper are not difficult to be converted to the one-dimensional case. A 2D Gabor filter in spatial domain is defined as

$$g(x,y) = \exp\{-\pi[x'^2 a^2 + y'^2 b^2]\} \times \exp\{-2\pi i[u_0 x' + v_0 y']\} , \tag{2}$$

where $x' = (x - x_0)\cos\alpha + (y - y_0)\sin\alpha$ and $y' = -(x - x_0)\sin\alpha + (y - y_0)\cos\alpha$ [3]. There are seven degrees of freedom in 2D Gabor filters: (x_0, y_0) is the center of the filter in spatial domain, $\omega_0 = \sqrt{u_0^{\,2} + v_0^{\,2}}$ is the spatial frequency, $\tan^{-1}(v_0 / u_0)$ is the relative orientation between the complex wave and the Gaussian function, a and b control the shape of the Gaussian function and α is the orientation of the Gaussian function. Fig. 1 shows a Gabor filter. Without loss of generality, in the rest of this paper, (x_0, y_0) is set to (0, 0) and α is also set to 0. To eliminate the influence of the power of Gabor filter, a normalized Gabor filter, $g_n = g/\|g\|$ is commonly used. Its spatial filtering outputs, phase and magnitude are

$$P_1(I, g_n) = \tan^{-1}\frac{\iint I(x,y)g_{ni}(x,y)dxdy}{\iint I(x,y)g_{nr}(x,y)dxdy}\ , \tag{3}$$

$$M_1(I, g_n) = \sqrt{\left[\iint I(x,y)g_{ni}(x,y)dxdy\right]^2 + \left[\iint I(x,y)g_{nr}(x,y)dxdy\right]^2}\ , \tag{4}$$

where I is a 2D signal; g_{nr} and g_{ni} represent the real and imaginary parts of the Gabor filter g_n. For convenience, g_n is used to denote $g_n(x, y)$. The same notations are employed for other symbols.

(a) (b)

Fig. 1. (a) The real part and (b) the imaginary part of a Gabor filter

1.2 Motivation

Given a sinusoid, $f_c(x) = A_c\cos(u_c x - \phi_c)$, its magnitude, A_c and phase, ϕ_c can be detected by Fourier analysis, i.e., $A_c = \sqrt{{a_c}^2 + {b_c}^2}$ and $\phi_c = \tan^{-1}(a_c / b_c)$, where $a_c = \frac{u_c}{\pi}\int_{-\pi/u_c}^{\pi/u_c} f_c(x)\sin(u_c x)dx$ and $b_c = \frac{u_c}{\pi}\int_{-\pi/u_c}^{\pi/u_c} f_c(x)\cos(u_c x)dx$ are two Fourier coefficients. In other words, Fourier functions can be served as a detector for sinusoid signals. In this paper, Gabor filter is considered as a Gabor atom detector defined as

$$Z(x,y) = \frac{A_Z}{\|g\|}\exp(-\pi[x^2a^2 + y^2b^2]) \times \cos(-2\pi(u_0x + v_0y) - \phi)\ . \tag{5}$$

Table 1 lists five sets of parameters, target phases and the corresponding Gabor phases. In some cases (e.g., case 2), the phase differences are very significant, whereas in other cases, their differences are negligible. Gabor magnitude also suffers from the same problem that will be demonstrated in Section 2. The aim of this paper is to analyze these differences. These differences are referred to as detection errors.

The rest of this paper is organized as follows. Section 2 analyzes the detection errors. Section 3 reveals a sufficient condition for errorless detection. Section 4 gives error bounds for Gabor phase and magnitude. Section 5 offers some concluding remarks.

Table 1. The differences between target phase and Gabor phase under different parameters

Case	Parameters of Gabor filter and Gabor atom				Target phase, (ϕ) (degree)	Gabor phase, $P_1(Z, g_n)$ (degree)	$\lvert P_1(Z, g_n) - \phi \rvert$ (degree)
	A	B	u_0	v_0			
1	0.05	0.05	0.1	0	50	50.00	6.83×10^{-10}
2	0.1	0.05	0.01	0	80	10.10	69.90
3	0.1	0.05	0.05	0.05	80	79.99	0.01
4	0.05	0.05	0.02	0.02	45	37.37	7.63
5	0.03	0.1	0	0.015	45	4.04	40.96

2 Analysis of the Detection Errors

2.1 Assumptions and Notations

For clear presentation, a set of notations and assumptions is essential. Gabor filters are functions in L^2 space, i.e., $\int\int g_n \times g_n^* dxdy < \infty$, where * represents a complex conjugate and the signals considered in this paper are assumed to be real-valued functions in L^2 space. Their norms and inner product are respectively defined as $\|g_n\| = \sqrt{\int\int g_n \times g_n^* dxdy}$, $\|I\| = \sqrt{\int\int I \times I^* dxdy}$ and $< g_n, I > = \int\int g_n \times I^* dxdy$, where I is a signal. This inner product is in fact a continuous version of spatial filtering. Since I is a real-valued function in L^2 space, $< g_n, I > = \int\int g_n \times I dxdy$. Using these notations, $M_1 = |< g_n, I >|$ and $P_1 = \arg(< g_n, I >)$. Convolution of I and g_n at the point (0, 0) is equivalent to $< I, g_n >$ and $< g_{n(-u_0,-v_0)}, I >$, where $g_{n(-u_0,-v_0)}$ is a Gabor filter with central frequencies at $-u_o$ and $-v_0$. Since both convolution and spatial filtering can be represented by the inner product, it is used as a basic operator in this paper.

2.2 Information Mix-Up

Using Gabor atom as a target function, this subsection shows that different information is mixed up in Gabor phase and magnitude. Using compound angle formulas, the target signal in Eq. 5 can be rewritten as

$$Z(x, y) = \frac{A_Z}{\|g\|}\exp(-\pi[x^2a^2 + y^2b^2]) \times \{\cos(-2\pi(u_0x + v_0y)) \times \cos(\phi) + \sin(-2\pi(u_0x + v_0y)) \times \sin(\phi)\} \cdot \quad (6)$$

Eq. 6 can be simplified as $Z = A_Z(\cos(\phi)g_{nr} + \sin(\phi)g_{ni})$. Let $g_{nr} = \|g_{nr}\|v_r$ and $g_{ni} = \|g_{ni}\|v_i$, where v_r and v_i are two unit vectors in L^2 space. Therefore,

$$Z = A_Z(\cos(\phi)\|g_{nr}\|v_r + \sin(\phi)\|g_{ni}\|v_i) \ . \quad (7)$$

The inner product of g_n and Z is

$$< g_n, Z >= A_Z(\cos(\phi)\|g_{nr}\| < g_n, v_r > + \sin(\phi)\|g_{ni}\| < g_n, v_i >) \tag{8}$$

$$= A_Z(\cos(\phi)\|g_{nr}\| < \|g_{nr}\|v_r + i\|g_{ni}\|v_i, v_r > + \sin(\phi)\|g_{ni}\| < \|g_{nr}\|v_r + i\|g_{ni}\|v_i, v_i >) \tag{9}$$

$$= A_Z(\cos(\phi)\|g_{nr}\|^2 < v_r, v_r > + i\sin(\phi)\|g_{ni}\|^2 < v_i, v_i >) \tag{10}$$

$$= A_Z(\cos(\phi)\|g_{nr}\|^2 + i\sin(\phi)\|g_{ni}\|^2) \tag{11}$$

The property that g_r and g_i are orthogonal is applied to Eq. 10. Using Eqs. 3 and 4, we can obtain the Gabor phase and magnitude of Z,

$$P_1(Z, g_n) = \tan^{-1}\frac{\sin(\phi)\|g_{ni}\|^2}{\cos(\phi)\|g_{nr}\|^2} \tag{12}$$

$$M_1(Z, g_n) = A_Z\sqrt{\cos^2(\phi)\|g_{nr}\|^4 + \sin^2(\phi)\|g_{ni}\|^4}\ . \tag{13}$$

Eqs. 12 and 13 clearly uncover that the phase and magnitude of the target signal and the norms of g_{nr} and g_{ni} are mixed up in the Gabor output.

3 A Sufficient Condition for Errorless Detection

Eqs. 12 and 13 not only explain the detection errors but also imply that when $\|g_{nr}\| = \|g_{ni}\|$, P_1 is equal to ϕ and $M_1 = A_Z/2$. In other words, when $\|g_{nr}\| = \|g_{ni}\|$, current Gabor filtering scheme can perform errorless phase and magnitude detection. Although Table 1 demonstrates that $\|g_{nr}\| \neq \|g_{ni}\|$ in general, it is interesting to know under what condition $\|g_{nr}\|$ is equal to $\|g_{ni}\|$. Let us consider $\|g_r\|^2 - \|g_i\|^2$.

$$\|g_r\|^2 - \|g_i\|^2 = \int\int\left(\exp(-\pi(x^2a^2 + y^2b^2))\cos(-2\pi(u_0x + v_0y))\right)^2 dxdy$$

$$-\int\int\left(\exp(-\pi(x^2a^2 + y^2b^2))\sin(-2\pi(u_0x + v_0y))\right)^2 dxdy \tag{14}$$

$$= \int\int\left(\exp(-2\pi(x^2a^2 + y^2b^2))(\cos^2(-2\pi(u_0x + v_0y)) - \sin^2(-2\pi(u_0x + v_0y)))\right)dxdy \tag{15}$$

$$= \int\int\left(\exp(-2\pi(x^2a^2 + y^2b^2))\cos(-4\pi(u_0x + v_0y))\right)dxdy \tag{16}$$

$$= \int\int\left(\exp(-2\pi(x^2a^2 + y^2b^2))\left(\cos(-4\pi u_0x)\cos(-4\pi v_0y) - \sin(-4\pi u_0x)\sin(-4\pi v_0y)\right)\right)dxdy \tag{17}$$

$$= \int\exp(-2\pi x^2a^2)\cos(-4\pi u_0x)dx\int\exp(-2\pi y^2b^2)\cos(-4\pi v_0y)dy$$

$$-\int\exp(-2\pi x^2a^2)\sin(-4\pi u_0x)dx\int\exp(-2\pi y^2b^2)\sin(-4\pi v_0y)dy \tag{18}$$

$$= \int\exp(-2\pi x^2a^2)\cos(-4\pi u_0x)dx\int\exp(-2\pi y^2b^2)\cos(-4\pi v_0y)dy \tag{19}$$

$$= \exp\left(-\frac{(4\pi u_0)^2}{4(2\pi a^2)}\right)\sqrt{\frac{\pi}{2\pi a^2}}\exp\left(-\frac{(4\pi v_0)^2}{4(2\pi b^2)}\right)\sqrt{\frac{\pi}{2\pi b^2}} \tag{20}$$

$$= \frac{1}{2ab}\exp\left(-2\pi\left(\left(\frac{u_0}{a}\right)^2 + \left(\frac{v_0}{b}\right)^2\right)\right) \tag{21}$$

Eqs. 16 and 17 apply compound angle formulas; Eq. 19 uses the property that $\int f(x)dx = 0$ if *f(x)* is an odd function and Eq. 20 utilizes the formula, $\int \exp(-px^2)\cos(-qx)dx = \exp\left(-\frac{q^2}{4p}\right)\sqrt{\frac{\pi}{p}}$, where $p \neq 0$ and $q \neq 0$. To compute $\|g_{nr}\|^2 - \|g_{ni}\|^2$, $\|g\|^2$ is essential.

$$\|g\|^2 = < g, g > \tag{22}$$

$$= \int\int \exp\{-2\pi[x^2a^2 + y^2b^2]\}dxdy \tag{23}$$

$$= \left(\frac{1}{2ab}\right) \tag{24}$$

Combining Eqs. 21 and 24, the equality,

$$\|g_{nr}\|^2 - \|g_{ni}\|^2 = \exp\left(-2\pi\left(\left(\frac{u_0}{a}\right)^2 + \left(\frac{v_0}{b}\right)^2\right)\right) \tag{25}$$

is obtained. Eq. 25 pinpoints that $\|g_{nr}\|^2 - \|g_{ni}\|^2 > 0$ for any $a > 0$, $b > 0$, $u_0 \in \Re$ and $v_0 \in \Re$ but $\lim_{u_0/a\to\infty} \|g_{nr}\|^2 - \|g_{ni}\|^2 = 0$ and $\lim_{v_0/b\to\infty} \|g_{nr}\|^2 - \|g_{ni}\|^2 = 0$. For any fixed u_0 and v_0, $\lim_{a\to 0, b\to 0} \|g_{nr}\|^2 - \|g_{ni}\|^2 = 0$. In this case, Gabor filter becomes a pair of Fourier functions.

4 Error Bounds for Gabor Phase and Magnitude

In addition to the sufficient condition for errorless detection, Eq. 12 also implies that if *k*-value defined as $k = \frac{\|g_{ni}\|^2}{\|g_{nr}\|^2}$ is close to 1, P_1 and M_1 can be considered as approximations of ϕ and $A_Z/2$, respectively. To estimate the quality of these approximations, their error bounds are needed.

Let us consider the error bound for phase first. Given a Gabor filter, *k* is fixed. P_1 depends on both ϕ and *k*. If ϕ is regarded as an independent variable, $|P_1 - \phi|$ is bounded by $\max_{\phi} |P_1 - \phi|$. Since $|P_1 - \phi|$ is non-differentiable at the point, $P_1 = \phi$, two cases, $P_1 > \phi$ and $P_1 < \phi$ are considered separately. For $P_1 > \phi$,

$$\frac{d}{d\phi}\left|P_1-\phi\right| = \frac{d}{d\phi}\left(\tan^{-1}(k\tan(\phi))-\phi\right) \tag{26}$$

$$= \frac{1}{1+k^2\tan^2(\phi)}k\sec^2(\phi)-1 \tag{27}$$

$$= \frac{k-(\cos^2(\phi)+k^2(1-\cos^2(\phi)))}{\cos^2(\phi)+k^2\sin^2(\phi)}\ . \tag{28}$$

Eq. 26 uses $P_1=\tan^{-1}(k\tan(\phi))$ from Eq. 12. Simplifying $k-(\cos^2(\phi)+k^2(1-\cos^2(\phi)))=0$, $\cos(\phi)=\pm\sqrt{\frac{k}{1+k}}$ is obtained. In this simplification, k is assumed not equal to 1. Using the equality $\cos^2(\phi)+\sin^2(\phi)=1$, we obtain $\sin(\phi)=\pm\sqrt{\frac{1}{1+k}}$. To identify the maximum, the second order derivative is computed.

$$\frac{d^2}{d^2\phi}\left|P_1-\phi\right| = \frac{d}{d\phi}\left(\frac{k}{\cos^2(\phi)+k^2\sin^2(\phi)}-1\right) \tag{29}$$

$$= \frac{d}{d\phi}\left(\frac{k}{k^2+(1-k^2)\cos^2(\phi)}-1\right) \tag{30}$$

$$= \frac{k(1-k^2)2\cos(\phi)\sin(\phi)}{(k^2+(1-k^2)\cos^2(\phi))^2} \tag{31}$$

Since $(k^2+(1-k^2)\cos^2(\phi))^2$ and $k(1-k^2)$ are always positive, the sign of $\frac{d^2}{d^2\phi}\left|P_1-\phi\right|$ depends on $\cos(\phi)\sin(\phi)$. If $\phi_0=\arg\max_{\phi}\left|P_1-\phi\right|$, $\cos(\phi_0)\sin(\phi_0)<0$. Therefore, when $\cos(\phi_0)$ is equal to $\sqrt{\frac{k}{1+k}}$, $\left(-\sqrt{\frac{k}{1+k}}\right)$, $\sin(\phi_0)$ is equal to $-\sqrt{\frac{1}{1+k}}$, $\left(\sqrt{\frac{1}{1+k}}\right)$. In either case, $\tan(\phi_0)=-\frac{1}{\sqrt{k}}$. Substituting it into $P_1-\phi$, the bound

$$P_1-\phi \le \tan^{-1}\left(\frac{1}{\sqrt{k}}\right)-\tan^{-1}(\sqrt{k})\ , \tag{32}$$

is obtained.

For the case $P_1<\phi$, repeating the previous derivation, $\tan\phi=\frac{1}{\sqrt{k}}$ is obtained. Therefore,

$$\phi - P_1 \le \tan^{-1}(\frac{1}{\sqrt{k}}) - \tan^{-1}(\sqrt{k}) \ . \tag{33}$$

Combining Eqs. 32 and 33, the error bound,

$$|P_1 - \phi| \le \tan^{-1}(\frac{1}{\sqrt{k}}) - \tan^{-1}(\sqrt{k}) \ , \tag{34}$$

is finally derived. Although in the derivation, k is assumed not equal to 1, this bound is also true for k=1. When k=1, $P_1 = \phi$ according to Section 2. Thus, $|P_1 - \phi| = 0$ and $\tan^{-1}(\frac{1}{\sqrt{k}}) - \tan^{-1}(\sqrt{k}) = 0$.

For magnitude, ratio bound, $\frac{\|M_1(Z, g_n)\|}{A_Z}$ is more useful because it is independent of the signal magnitude, A_z. Using $\|g_{nr}\|^2 \ge \|g_{ni}\|^2$ from Eq. 25 and Eq. 13, the bound,

$$\|g_{ni}\|^2 \le \frac{\|M_1(Z, g_n)\|}{A_Z} \le \|g_{nr}\|^2 \ , \tag{35}$$

is obtained. The error bounds given in this section show that when k is close to 1, the current Gabor phase and magnitude can be considered as approximations of the target information. In other words, these bounds uncover the meanings of Gabor features, phase and magnitude. They are approximated phase and magnitude of the corresponding Gabor atom in signals.

5 Conclusion

Using Gabor atom as a target signal, this paper uncovers the problem of information mix-up in the current Gabor phase and magnitude that causes detection errors. If a sufficient condition that k-value is equal to 1 is fulfilled, the corresponding Gabor filter can achieve errorless detection. When it is less than 1, the current Gabor phase and magnitude can be regarded as approximations of the target information and the quality of these approximations is controlled by k-value. This paper also points out that the Gabor features commonly employed in pattern recognition systems are approximated phase and magnitude of the corresponding Gabor atom in signals.

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