4 Interaction of Ionizing Radiation with Matter

"A careful analysis of the process of observation in atomic physics has shown that the subatomic particles have no meaning as isolated entities, but can only be understood as interconnections between the preparation of an experiment and the subsequent measurement."

Erwin Schrödinger

Charged particles (electrons, positrons, protons, helium nuclei, ...) will ionize matter in a direct way, in contrast to neutral particles (neutrons, neutrinos, ...) or short-wavelength electromagnetic radiation (X rays and γ rays), which will ionize only indirectly. Strictly speaking, radiation is never directly measured, rather it can only be detected via its interaction with matter. A large number of specific interaction processes exists. These interactions are characteristic for each of charged particles, neutrons, neutrinos, X rays, and γ radiation.

4.1 Detection of Charged Particles

In passing through matter, charged particles lose some kinetic energy by excitation of bound electrons and by ionization, e.g. for α particles:

$$\begin{aligned} \alpha + \text{atom} &\rightarrow \text{atom}^* + \alpha \\ &\quad \hookrightarrow \text{atom} + \text{photon} \quad \text{(excitation)} \ , \\ \alpha + \text{atom} &\rightarrow \text{atom}^+ + e^- + \alpha \qquad \text{(ionization)} \ . \end{aligned} \tag{4.1}$$

excitation and ionization: energy loss of charged particles

The energy loss of charged particles passing through matter depends on the particle velocity, its charge, and the properties of the traversed material. Slow particles spend a relatively large amount of time close to individual atoms; correspondingly, the interaction probability associated with a given energy transfer to the target atom is relatively large. If v is the particle velocity, one obtains for the energy transfer in collisions between charged particles

$$\Delta E \sim \frac{1}{v^2} \ . \tag{4.2}$$

Since the interaction is mediated by the electrical charges of the projectile (z) and the target (Z), the energy loss increases with larger

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charges of the involved particles:

$$\Delta E \sim z^2 \, Z \ . \tag{4.3}$$

Finally, the electric field of the charged particle is distorted at high velocities – corresponding to high energies – due to relativistic effects. This results in an increase of the energy loss at high energies.

Bethe–Bloch relation

These dependences are described in the framework of the Bethe–Bloch relation, which relates the energy loss ΔE to the traversed distance Δx in the following way:

$$\frac{\Delta E}{\Delta x} \sim \frac{1}{v^2} z^2 \frac{Z}{A} \ln(a \, E) \ , \tag{4.4}$$

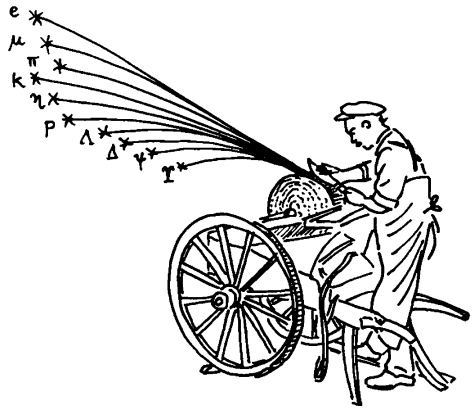
where

ΔE – energy deposited over the distance Δx,
v – velocity of the incident particle,
z – charge number of the projectile ($z_\alpha = 2$, $z_e = -1$),
Z, A – atomic number and mass number of the traversed material,
E – total energy of the projectile ($E = E_{\text{kin}} + m_0 c^2$) with E_{kin} – kinetic energy,
a – a material-dependent constant.

The average energy loss of relativistic electrons ($\approx$ MeV range) is approximately 2.5 keV/cm in air. That of α particles is much larger ($\approx$ 100 keV/mm in air) because of the higher charge ($\sim z^2$) and the lower velocities of α particles for energies typical for radioactive decays.

average energy loss

In the field of practical radiation protection it is sometimes desirable to consider only the local energy deposit, i.e. collisions with



"Creation of elementary particles in interactions."

relatively low energy transfer. The idea is that in collisions with large energy transfers long-range δ electrons are created which are weakly ionizing and therefore have only little biological effect, in contrast to the high ionization density generated by low energy transfers.[1] This led to the introduction of the concept of the linear energy transfer (LET). The linear energy transfer of charged particles is the ratio of the average energy loss ΔE, where only collisions with energy transfers smaller than a given cutoff parameter E_{cut} are considered, and the traversed distance Δx,

linear energy transfer (LET)

$$\mathrm{LET} = L_{E_{\mathrm{cut}}} = \left(\frac{\Delta E}{\Delta x}\right)_{E_{\mathrm{cut}}} . \tag{4.5}$$

The energy cut parameter is usually given in eV. A value of LET_{100} indicates that only collisions with energy transfers below 100 eV are considered. High linear-energy-transfer radiation corresponds to high biological effectiveness, and this type of radiation is very important for cancer treatment (see also Fig. 4.3).

Figure 4.1 shows the energy loss of electrons in air compared to the energy loss of protons and α particles. Since the energy loss varies considerably with energy, we have used the differential notation $\frac{\mathrm{d}E}{\mathrm{d}x}$ instead of $\frac{\Delta E}{\Delta x}$. Figures 4.2a and 4.2b show the average energy loss of electrons in copper (a) and tissue (b). The energy loss in thin absorbers is subject to large fluctuations which can be described by an asymmetric distribution ('Landau distribution').

proton therapy

specific energy loss

[1] The energy spectrum of ionization electrons falls off like $1/\epsilon^2$, where ϵ is the energy of the electrons knocked out from the atom. These electrons are also called 'knock-on electrons' or δ rays.

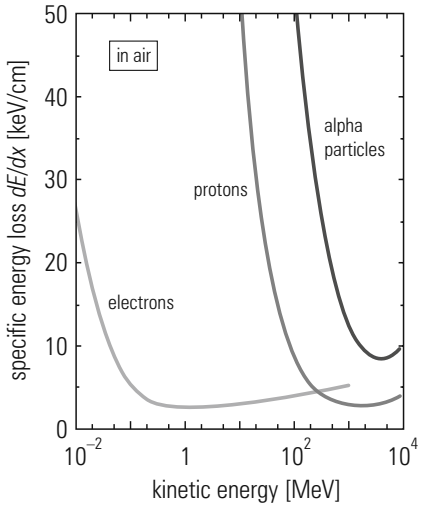


Figure 4.1
Energy loss of electrons, protons, and α particles in air as a function of energy

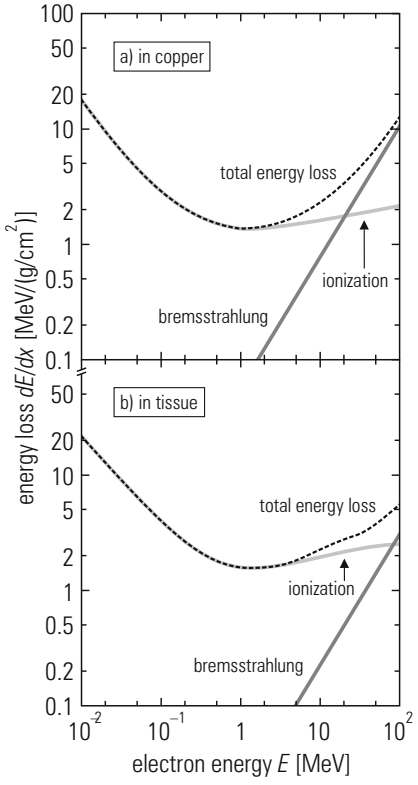


Figure 4.2
Specific energy loss of electrons in copper (a) and in tissue (b). The distance dx is not given in cm in this case, but in a nearly material-independent area density $\rho\,\mathrm{d}x$, where ρ is the density of the material. 1 g/cm^2 tissue corresponds almost exactly to 1 cm, while 1 g/cm^2 of copper is only 0.11 cm

range of charged particles

According to Eq. (4.4) the energy loss rises again at the end of the range of the particle (i.e. at low velocities). Figure 4.3 shows the relative energy loss (or dose, respectively) of negative pions, α particles, and nitrogen nuclei in comparison to ^{60}Co γ rays in tissue. As can be seen from this diagram, heavy ions are ideally suited for

tumor therapy

the irradiation of tumors. This is due to the fact that heavy ions exhibit, because of their large charge and their relatively low velocity,

Bragg peak

a very pronounced ionization peak ('Bragg peak') at the end of their range, which is a consequence of the z^2/v^2 dependence of the energy loss. Because of this physical phenomenon deep-seated tumors can be treated non-invasively with proton or nitrogen beams (or carbon beams) of variable energy. By vertical and horizontal beam-deflection techniques and by varying the energy of the heavy-ion or proton beam one can destroy very precisely regions of cancerous tissue while at the same time the healthy tissue is not strongly affected ('raster-scan technique').

microbeam radiation therapy

Another very promising technique for the treatment of tumors is microbeam radiation therapy (MRT). Here the tumor tissue is ir-

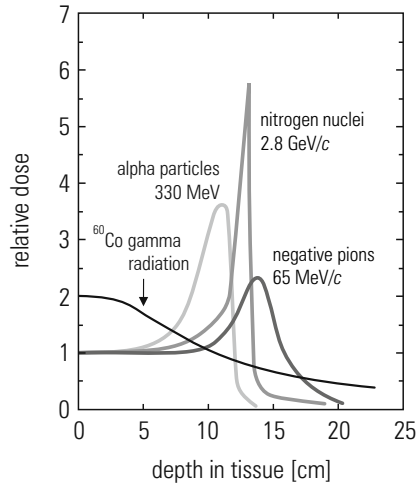


Figure 4.3
Relative dose as a function of depth in tissue for ^{60}Co γ rays, negative pions with momentum[1] 65 MeV/c, nitrogen nuclei (momentum 2.8 GeV/c), and α particles (kinetic energy 330 MeV)

radiated in microsteps along lines of typically 25 µm width and a mutual distance of 200 µm ('spatially fractionated irradiation'). The irradiation steps are implemented with the help of an intense synchrotron-radiation beam (see Sect. 9.1) operating at X-ray energies. An applied tissue dose per irradiated line element of about 600 Gy is lethal for the local tissue. The healthy tissue can withstand this way of irradiation by means of biological repair mechanisms much better than the cancerous tissue.

synchrotron radiation in the X-ray range

This technique is interesting because the treatment of metastatic tumors also appears possible, in contrast to the heavy-ion therapy which can only be applied for well-localized tumors. Therefore, heavy-ion therapy cannot be used for those metastatic tumors which are spread out.

The range of electrons in various materials and those of α particles in air is shown in Figs. 4.4 and 4.5. Electrons from radioactive sources have ranges of several meters in air, but they will already be stopped by a few mm of aluminum. The range of α particles with typical energies of around 5 MeV in air is approximately 4 cm. A sheet of paper is sufficient to absorb these α rays.

For higher energies additional energy losses come into play, like bremsstrahlung (see Fig. 4.2). Bremsstrahlung is produced if a charged particle is decelerated in the Coulomb field of an atomic nucleus. Because of the $1/m^2$ dependence of the bremsstrahlung

bremsstrahlung

[1] For massless or very energetic particles the energy is related to the momentum $p = m\,v$ via the relation $E = p\,c$. Therefore, the momentum is frequently measured in energy units per velocity of light, e.g. MeV/c or GeV/c.

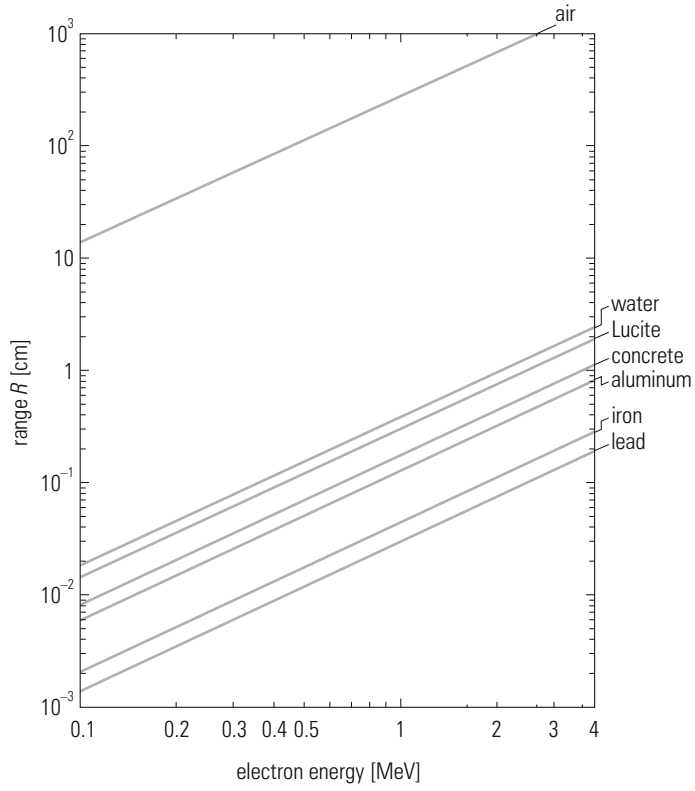


Figure 4.4
Range of electrons in various materials

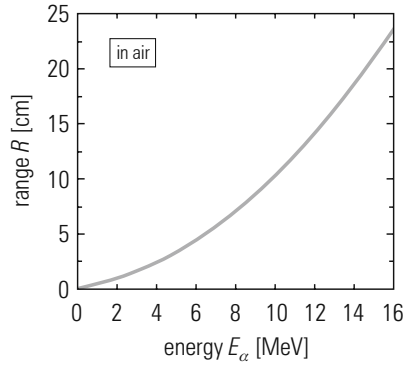


Figure 4.5
Range of α particles in air

energy loss this mechanism is practically only important in the field of radiation protection for the lightweight electrons ($m_e \approx \frac{1}{7300} m_\alpha$). For all other particles (protons, α particles, heavy nuclei) their high mass makes this type of energy-loss mechanism practically irrelevant in radiation physics. The energy loss of electrons in matter by bremsstrahlung can be parametrized by

Absorption of β rays

A Geiger–Müller counter allows the measurement of the absorption of β rays from a ^{90}Sr source. For this purpose an aluminum absorber of variable thickness is placed between source and detector. The count rate as a function of absorber thickness is shown in the figure. ^{90}Sr decays with a maximum β^- energy of 0.55 MeV into ^{90}Y, which itself transforms into ^{90}Zr with a transition energy of $E_{\beta_{\max}} = 2.28$ MeV (see Fig. 3.5). The electrons from the low-energy transition of the ^{90}Sr decay are absorbed relatively quickly (compare Fig. 4.4). The results of the absorption measurements for absorber thicknesses of more than 0.2 g/cm^2 Al (corresponding to 0.75 mm Al) are related to electrons from the ^{90}Y decay.
Even though the absorption of β rays cannot really be described by an exponential law from a theoretical viewpoint, an effective absorption coefficient for continuous β-ray emitters can still be given. A linear fit to the absorption data in the semilogarithmic plot allows to determine this effective absorption coefficient for β rays. Assuming that the experimental count rate follows an exponential,

$$N_i = N_0 \, \mathrm{e}^{-\kappa \, x_i} \qquad (i = 1, 2) \ ,$$

one can derive the absorption coefficient κ,

$$\kappa = \frac{\ln(N_1/N_2)}{x_2 - x_1} = 4.33 \, (\mathrm{g/cm^2})^{-1} \ .$$

The absorption coefficient for electrons from continuous β spectra can be approximated by the empirical relation

$$\kappa = 15/E_{\beta_{\max}}^{1.5}$$

($E_{\beta_{\max}}$ in MeV, κ in $(\mathrm{g/cm^2})^{-1}$) (see page 252). The κ value of 4.36 derived from this relation agrees well with the experimentally determined result.

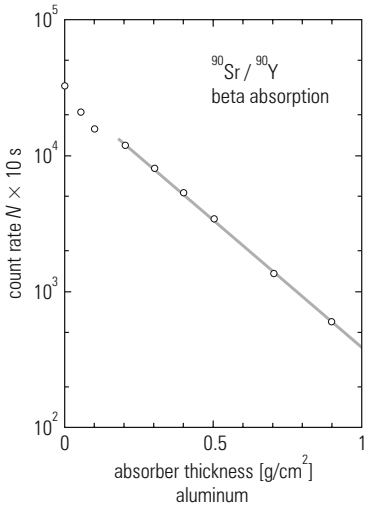


Figure 4.6
Absorption of electrons from a ^{90}Sr source in aluminum

Determination of the range of α rays

The range of α rays can be determined reasonably well with an expansion cloud chamber. α rays from an unsealed, collimated ^{226}Ra source are injected into a flat chamber, which is filled with a gas–vapor mixture (e.g. air–water vapor or argon–alcohol). By adiabatic expansion[3] of the chamber volume the temperature of the gas mixture is lowered, which brings the vapor into a supersaturated state. The vapor will condense as droplets on condensation nuclei, in this case on positive ions marking the ionization tracks of the α particles. The illuminated tracks of the α particles can easily be photographed and evaluated (see Fig. 4.7, left-hand part).

The determination of the range of α particles with electronic detectors is much more difficult. One has to take care that both the exit window of the source and the entrance window of the detector are extremely thin. A count-rate measurement of a ^{226}Ra source (α energy 4.8 MeV) with a Geiger–Müller counter leads to the results as shown in the right-hand part of Fig. 4.7.

Plotted is the count-rate dependence as a function of the absorber thickness between source and detector. The variable absorber thickness was realized by extremely thin polyethylene foils. If this count-rate dependence is differentiated, one can read the range of α particles from the position of the maximum of this differential curve (see lower insert in the figure). The result of 4.75 mg/cm^2, corresponding to 3.7 cm of air, agrees well with expectation.

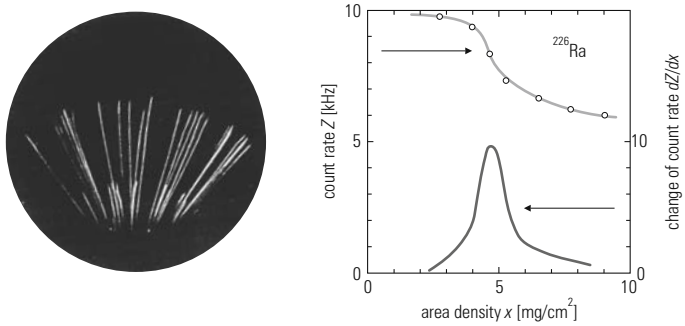


Figure 4.7
Tracks of 8.8 MeV and 6 MeV α particles in a cloud chamber (L. Meitner, K. Freitag; left); determination of the range of α rays by absorption measurements with thin polyethylene foils (right)

$$\frac{\Delta E}{\Delta x} \sim Z^2 E \ . \tag{4.6}$$

For high energies (for example, in lead, energies larger than 7.4 MeV, the 'critical energy'[2]) the radiation loss of electrons is larger than the energy loss by ionization and excitation (see Fig. 4.2).

shielding

prevention of bremsstrahlung

Electrons of high energy (several MeV, e.g. from an accelerator in the radiological department of a hospital) are best shielded with a sandwich consisting of a material of low atomic number followed by a layer of lead. In the absorber of low atomic number (e.g. plas-

[2] The critical energy is defined to be the energy, at which the energy loss by bremsstrahlung and the energy loss by ionization and excitation are the same.

[3] Any process that occurs without heat entering or leaving the system is called adiabatic.

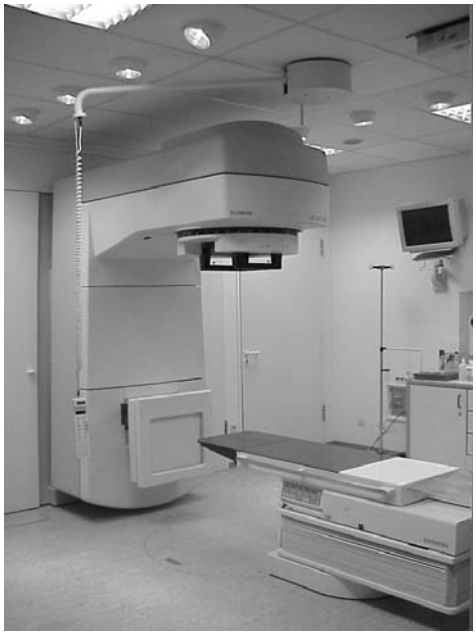


Figure 4.8
Linear accelerator, electron energy between 6 and 21 MeV with maximum photon energies of 6 to 21 MeV (type: Siemens KD-2, `www.ma.uni-heidelberg.de`)

tic) the electrons lose their energy by ionization and excitation almost without producing bremsstrahlung, and then the electrons are stopped in lead. In a pure lead absorber energetic electrons would produce bremsstrahlung against which it is very difficult to shield.

4.2 Detection of Neutrons

Neutrons, being neutral particles, cannot be measured directly. All detection techniques result in the production of charged particles in neutron interactions. These charged particles are then measured by the normal interaction processes like ionization, excitation, or scintillation.

neutron interactions

For neutrons with energies which are typical in the field of radiation protection ($E_{\mathrm{kin}} \leq 10\,\mathrm{MeV}$), the following detection reactions can be considered:

$$n + {}^{6}_{3}\mathrm{Li} \rightarrow \alpha + {}^{3}_{1}\mathrm{H} \ , \tag{4.7}$$

$$n + {}^{10}_{5}\mathrm{B} \rightarrow \alpha + {}^{7}_{3}\mathrm{Li} \ , \tag{4.8}$$

$$n + {}^{3}_{2}\mathrm{He} \rightarrow p + {}^{3}_{1}\mathrm{H} \ , \tag{4.9}$$

$$n + p \rightarrow n + p \ . \tag{4.10}$$

The cross sections[4] for these reactions depend strongly on the neutron energy (see Fig. 4.9).

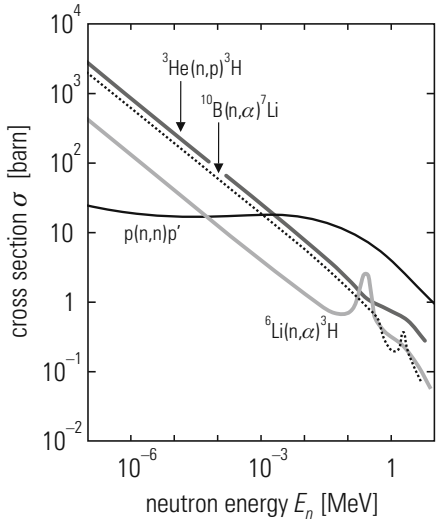


Figure 4.9
Interaction cross sections for neutron-induced reactions (1 barn = 10^{-24} cm^2)

boron trifluoride counter
^{3}He counter

Thermal neutrons (neutrons at room temperature, ≈ 0.025 eV) can easily be measured with ionization chambers or proportional counters, which are filled with gas mixtures containing boron trifluoride (BF_3). Because of the large cross section for the reaction (4.9) (see also Fig. 4.9) ^{3}He is also an attractive alternative for the detection of slow neutrons. Higher-energy neutrons must first be decelerated ('moderated') so that they can be recorded with reasonable detection efficiency.

moderation (deceleration)

The moderation of non-thermal neutrons is best done with materials which contain many 'free' protons, i.e. lots of hydrogen, because neutrons can transfer a large fraction of their energy to partners of the same mass while in collisions with heavy nuclei mostly only quasi-elastic scattering processes with low energy transfers occur. Paraffin and water are common choices as moderators.

Semiconductor counters or scintillators which contain lithium are equally well-suited for neutron detection; also proportional counters with gas fillings, which contain ^{3}He or hydrogen (say, in form of CH_4), can be used equally well for neutron detection.

[4] A measure for the probability that a collision will occur between the incoming particle and the target, expressed as the effective area presented by the particle and the target in this particular interaction. If r_P is the radius of the projectile and r_T that of the target, the geometrical cross section is $\sigma = \pi(r_P + r_T)^2$.

reaction	threshold energy (MeV)
fission of ^{234}U	0.3
fission of ^{236}U	0.7
$n + {}^{31}\mathrm{P} \rightarrow p + {}^{31}\mathrm{Si}$	0.72
$n + {}^{32}\mathrm{S} \rightarrow p + {}^{32}\mathrm{P}$	0.95
fission of ^{238}U	1.3
$n + {}^{27}\mathrm{Al} \rightarrow p + {}^{27}\mathrm{Mg}$	1.9
$n + {}^{56}\mathrm{Fe} \rightarrow p + {}^{56}\mathrm{Mn}$	3.0
$n + {}^{27}\mathrm{Al} \rightarrow \alpha + {}^{24}\mathrm{Na}$	3.3
$n + {}^{24}\mathrm{Mg} \rightarrow p + {}^{24}\mathrm{Na}$	4.9
$n + {}^{65}\mathrm{Cu} \rightarrow 2n + {}^{64}\mathrm{Cu}$	10.1
$n + {}^{58}\mathrm{Ni} \rightarrow 2n + {}^{57}\mathrm{Ni}$	12.0

Table 4.1
Threshold reactions for determination of neutron energies

measurement of neutron energy

threshold counter

For applications in the field of radiation protection, it is also important to know the neutrons' energy, because the radiation weighting factors, and thereby the relative biological effectiveness, depend on it. For this purpose one uses neutron threshold detectors. Such a detector consists of a carrier foil that is coated with an isotope which only reacts with neutrons above a certain energy. The charged particles created in these reactions can be measured with plastic detectors via an appropriate etching technique. As plastic material usually polycarbonate foils are used. The local radiation damage is made visible by etching the plastic foil with sodium hydroxide (NaOH).

Table 4.1 contains suitable threshold reactions for the determination of the neutron energy.

4.3 Detection of Photons

photoelectric effect
Compton effect
pair production

Just as with neutrons, photons must first produce charged particles in an interaction process, which are then normally detected via processes of ionization, excitation, and scintillation. The interactions of photons are fundamentally different from those of charged particles since in a photon interaction process the photon is either completely absorbed (photoelectric effect, pair production) or scattered through relatively large angles (Compton effect).

It is possible to consider for charged particles a certain range – depending on their energy. In contrast, this is not possible for photons; only statistical probabilities can be given for the intensity attenuation when photons penetrate layers of material.

attenuation of photons

The intensity I_0 of a monoenergetic photon beam is attenuated in a certain layer of material of thickness x according to

Absorption of γ rays

The absorption of γ rays from a ^{60}Co source was investigated with a scintillation counter. For this purpose the electrons, with a maximum energy of 0.31 MeV were absorbed by a thin aluminum absorber. The remaining γ count rate is plotted in Fig. 4.10 as a function of the lead-absorber thickness. It is obvious that the energetic γ rays (1.17 MeV and 1.33 MeV, see Fig. 3.4) are only slowly absorbed in lead. The cross section for photon interactions has a minimum around these energies (see Fig. 4.12). This minimum is determined by the relatively low probability for the Compton effect which dominates in this energy range.
A fit to the experimental data in form of a straight line in the semilogarithmic plot leads to a value for the mass absorption coefficient of γ rays of average energy of 1.25 MeV in lead of

$$\mu_\mathrm{a} = \frac{\ln(I_1/I_2)}{x_2 - x_1} = 0.37\,\mathrm{cm}^{-1}$$

corresponding to $\mu = 0.033\,(\mathrm{g/cm^2})^{-1}$, in excellent agreement with the expectation.

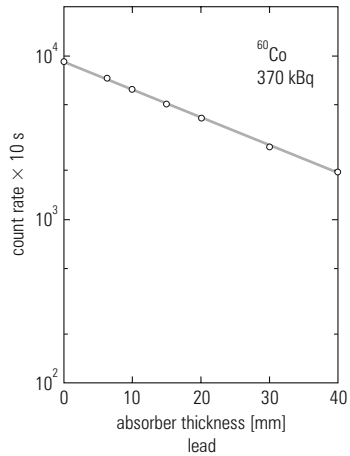


Figure 4.10
Absorption of γ rays from a ^{60}Co source in lead

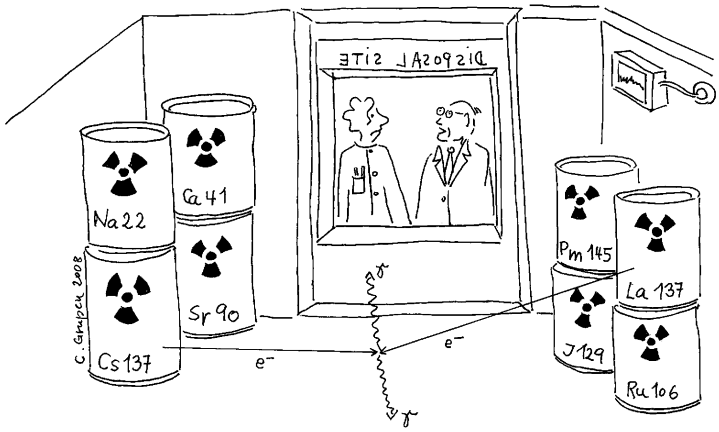
(see Appendix Q)

$$I = I_0\,\mathrm{e}^{-\mu x}\ , \tag{4.11}$$

where μ is the mass attenuation coefficient. This coefficient involves the probability of the different photon interaction processes. In the photoelectric effect and in pair production the photon is completely absorbed, while in the Compton process the photon 'survives' – although with reduced energy. Therefore the effect of photon absorption has to be distinguished from the photon intensity attenuation. For this purpose a Compton scattering coefficient is defined,

Compton scattering

$$\mu_\mathrm{cs} = \frac{E'_\gamma}{E_\gamma}\,\mu_{\text{Compton effect}}\ . \tag{4.12}$$



"We store β^+ emitters together with β^- emitters so that they annihilate one another."

In this relation E_γ and E'_γ are the energies of the photons before and after the scattering in the Compton process. The Compton absorption coefficient is then the difference between the total probability for the Compton effect and the Compton scattering coefficient:

absorption of photons

$$\mu_{\text{ca}} = \mu_{\text{Compton effect}} - \mu_{\text{cs}} \ . \tag{4.13}$$

Using the abbreviations $\mu_{\text{photoelectric effect}} = \mu_{\text{pe}}$, $\mu_{\text{pair production}} = \mu_{\text{pp}}$, and $\mu_{\text{Compton effect}} = \mu_{\text{c}}$ the mass attenuation coefficient μ for photons is

Compton absorption

$$\mu = \mu_{\text{pe}} + \mu_{\text{pp}} + \mu_{\text{c}} \ , \tag{4.14}$$

where $\mu_{\text{c}} = \mu_{\text{cs}} + \mu_{\text{ca}}$, and the mass absorption coefficient μ_{a} is given by

$$\mu_{\text{a}} = \mu_{\text{pe}} + \mu_{\text{pp}} + \mu_{\text{ca}} \ . \tag{4.15}$$

The mass attenuation coefficient μ is related to the cross sections for the various interaction processes of photons according to

$$\mu = \frac{N_{\text{A}}}{A} \sum_i \sigma_i \ , \tag{4.16}$$

where σ_i is the atomic cross section for the process i, A the atomic weight, and N_{A} the Avogadro constant. An equivalent relation holds for the mass absorption coefficient μ_a.

The cross section for the photoelectric effect, which is the liberation of an electron from the atomic shell,

photoelectric effect

$$\gamma + \text{Atom} \rightarrowtail \text{Atom}^+ + e^- \ , \tag{4.17}$$

depends very strongly on the atomic number of the absorber and the energy of the photon. Away from the absorption edges[5] the cross

absorption edge

section for the photoelectric effect is given by

$$\sigma_{\text{photo}} \sim \frac{Z^5}{E_\gamma^{3.5}} \ . \tag{4.18}$$

The photoelectric effect dominates at low energies (e.g. in the X-ray regime) and for heavy absorbers (lead, tungsten). In most cases the photoionization proceeds in the K shell (corresponding to 80% of the total photoelectric cross section).

The cross section in the vicinity of the absorption edges[5], where the photon energy coincides with the element's specific binding energy in the respective electronic shell, deviates quite strongly from the cross section as given in Eq. (4.18). Consequently the energy dependence of this cross section in the vicinity of the absorption edges must be considerably modified.

Compton effect

The Compton effect describes the photon scattering off quasi-free electrons of an atom,

$$\gamma + e^-_{\text{at rest}} \rightarrow \gamma' + e^-_{\text{fast}} \ . \tag{4.19}$$

The scattering probability is proportional to the number of potential scattering partners in the atom ($\sim Z$). The energy dependence of the Compton scattering cross section can be parametrized by

$$\sigma_{\text{Compton}} \sim Z \, \frac{\ln E_\gamma}{E_\gamma} \ . \tag{4.20}$$

Compton edge

For reasons of energy and momentum conservation the photon can only transfer energies up to a maximum value onto the electron ('Compton edge').

Pair production is the conversion of a photon into an electron–positron pair in the Coulomb field of an atomic nucleus,

$$\gamma + \text{nucleus} \rightarrow \text{nucleus}' + e^+ + e^- \ . \tag{4.21}$$

pair production

There is a threshold energy for this process because the photon has to create the rest masses m_e of the electron and the positron. One might think that the threshold energy would be just $E_\gamma > 2m_e c^2$; actually the threshold energy must be somewhat larger than twice the electron mass because also the nucleus, which acts as a scattering partner, will suffer a small recoil and thereby takes away a certain

[5] The cross section experiences a rapid increase at the absorption edges if the photon energy coincides with the excitation or ionization energy of a shell electron. The excitation energies are given by Moseley's law (see Sect. 9.2 'Photon Sources').

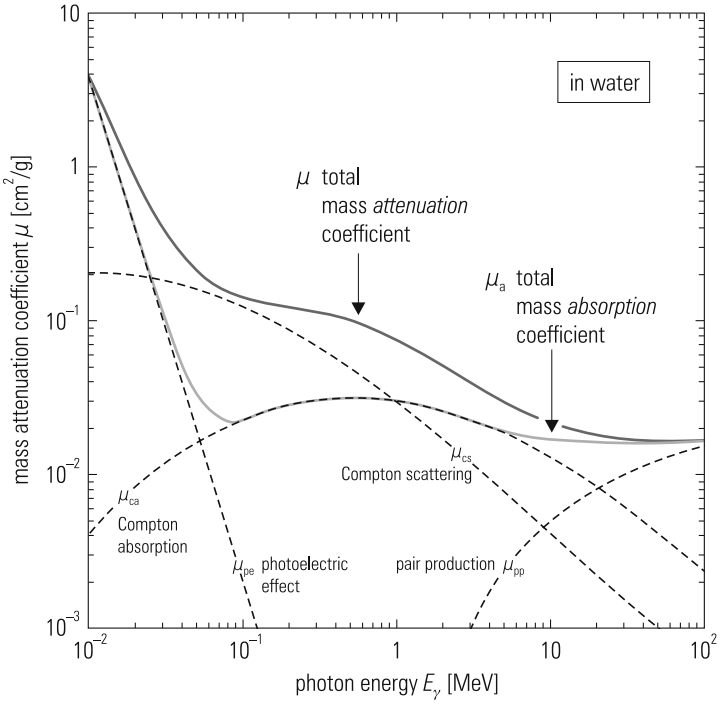


Figure 4.11
Energy dependence of the mass attenuation coefficient μ and mass absorption coefficient μ_a for photons in water. μ_{pe} describes the photoelectric effect, μ_{pp} pair production, μ_{cs} Compton scattering, and μ_{ca} Compton absorption. μ_a is the total mass absorption coefficient ($\mu_a = \mu_{pe} + \mu_{pp} + \mu_{ca}$) and μ the total mass attenuation coefficient ($\mu = \mu_{pe} + \mu_{pp} + \mu_c$ with $\mu_c = \mu_{cs} + \mu_{ca}$)

energy and momentum.[6] The cross section for pair production can be parametrized by

$$\sigma_{\text{pair}} \sim Z^2 \ln E_\gamma \quad . \tag{4.22}$$

For high energies (in the ultra-relativistic limit, $E_\gamma \gg 100\,\text{MeV}$) the cross section approaches an energy-independent constant value.

Figure 4.11 shows the mass attenuation and mass absorption coefficients for water as absorber and Fig. 4.12 those for lead. The mass absorption coefficient can be measured equally well in the unit cm^{-1} or in $(\text{g/cm}^2)^{-1}$, where

attenuation coefficient
absorption coefficient

$$\mu(\text{cm}^{-1}) = \mu\big((\text{g/cm}^2)^{-1}\big)\,\rho$$

(ρ – density of the absorber in g/cm^3).

As a consequence of the photoelectric and Compton effect an electron is missing in the atomic shell. If this vacancy is filled up by electrons from higher shells, the excitation energy of the shell can be emitted in form of characteristic X rays or by Auger electrons (see Chap. 3, page 27).

characteristic X rays
Auger electrons

[6] The exact value for the threshold energy is $E_\gamma = 2m_e c^2 + 2m_e^2 c^2 / m_{\text{nucleus}}$. For practically all nuclei the second term can be neglected. If, however, the pair production happens in the Coulomb field of an electron, the threshold energy is $E_\gamma = 4m_e c^2$.

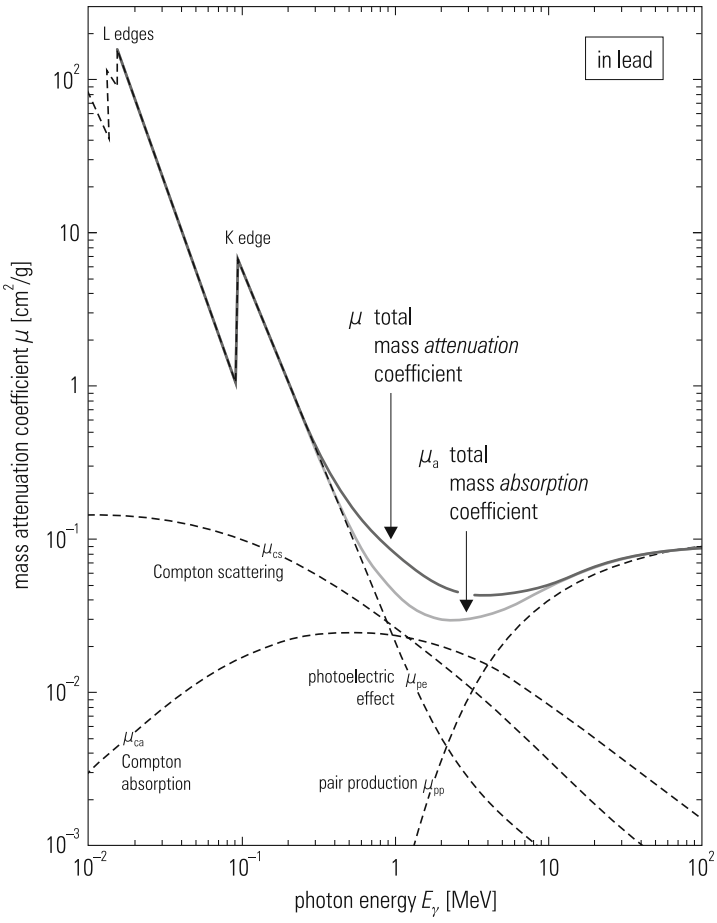


Figure 4.12
Energy dependence of the mass attenuation coefficient μ and mass absorption coefficient μ_a for photons in lead. μ_{pe} describes the photoelectric effect, μ_{pp} pair production, μ_{cs} Compton scattering, and μ_{ca} Compton absorption. μ_a is the total mass absorption coefficient ($\mu_a = \mu_{pe} + \mu_{pp} + \mu_{ca}$) and μ the total mass attenuation coefficient ($\mu = \mu_{pe} + \mu_{pp} + \mu_c$ with $\mu_c = \mu_{cs} + \mu_{ca}$)

4.4 Supplementary Information

Example 1

cosmic-ray muons

Energetic cosmic-ray muons will deposit about $2\,\mathrm{MeV/(g/cm^2)}$ in tissue. How many charge-carrier pairs will be created in a human germ cell (diameter 0.05 mm) if it is hit by a cosmic-ray muon and if the energy for the production of a charge-carrier pair is 30 eV?

The density of human tissue is about $1\,\mathrm{g/cm^3}$; therefore, a specific energy loss of $2\,\mathrm{MeV/(g/cm^2)}$ corresponds to a deposited energy of $2\,\mathrm{MeV/cm} = 0.2\,\mathrm{keV/\mu m}$. Let us approximate the cell by a cylindrical volume the height of which is assumed to correspond to the cell diameter. This leads to a deposited energy of 10 keV for the given cell diameter of $0.05\,\mathrm{mm} = 50\,\mu\mathrm{m}$, which in turn leads to a number of $10\,000\,\mathrm{eV}/30\,\mathrm{eV} = 333$ charge-carrier pairs.

mutations

Mutations can only be caused if a chromosome within a germ-cell nucleus will be hit. For an effective diameter for ionization in the chromosomes of about 0.5 µm the hit probability for a certain chromosome, if the cell has been hit, is

chromosome aberrations

$$\frac{(0.5\,\mu\text{m})^2}{(50\,\mu\text{m})^2} = 10^{-4} \ .$$

cosmic rays (muons)

Since the cosmic-ray-muon rate at sea level corresponds to about 1 muon/(cm^2 min), the hit probability for a particular chromosome is

$$\begin{aligned}\frac{(0.5\,\mu\text{m})^2}{10^8\,\mu\text{m}^2\ \text{min}} &= 2.5 \times 10^{-9}\ \text{ per minute} \\ &= 0.0013\ \text{ per year}\end{aligned}$$

corresponding to 1 hit every 761 years.

chromosome hits

A hit chromosome is the condition for the creation of a mutation, however, not every hit leads to a mutation. Very few 'successful' hits on chromosomes lead to a mutation which the cell can survive, i.e., most of the hits on chromosomes create mutations which cause the cell to die.

Example 2

As well as the activity A, the term specific activity A^* is used, particularly when dealing with the classification of nuclear waste. The specific activity is the activity per unit mass. For example, the specific activity of ^{54}Mn is worked out in the following way: the half-life of ^{54}Mn is 312 days;

$$A^* = -\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda\, N = \frac{1}{\tau}\, N \ .$$

In this relation N is the number of atoms per gram,

$$\begin{aligned}A^* &= \frac{\ln 2}{T_{1/2}}\, N = \frac{\ln 2}{T_{1/2}}\, \frac{\text{Avogadro constant}}{\text{atomic weight in grams}} \\ &= \frac{0.6931}{312 \times \underbrace{24 \times 3600}_{\text{seconds per day}}} \times \frac{6.022 \times 10^{23}}{54\ \text{gram}} = 2.87 \times 10^{14}\, \frac{\text{Bq}}{\text{g}} \ .\end{aligned}$$

Since low-level detectors can easily measure count rates of 1 Bq, one obtains a sensitivity for ^{54}Mn of

$$m_{\min} = \frac{1\ \text{Bq}}{2.87 \times 10^{14}\ \text{Bq/g}} = 3.49 \times 10^{-15}\ \text{g} \ .$$

detection sensitivity

This shows that very small quantities of a radioisotope are detectable.

specific activity

In a normal nuclear physics laboratory radioactive sources with activities on the order of 10^6 Bq are handled. This would correspond to a mass of 3.5 nanograms for ^{54}Mn. Since masses of such small amount cannot easily be handled technically, the radioisotope is frequently mixed with a chemical equivalent but inactive isotope, e.g. ^{55}Mn. If the radioisotope ^{54}Mn with an activity of 10^6 Bq (corresponding to 3.5 ng) were mixed with 10 mg ^{55}Mn, the specific activity for this sample would be

$$A^* = \frac{10^6\,\mathrm{Bq}}{10\,\mathrm{mg} + 3.5\,\mathrm{ng}} = 10^8\,\mathrm{Bq/g} \ .$$

The low ^{54}Mn fraction in such a sample would be very difficult to detect with techniques of analytical chemistry.

Let us now assume that 1 mg ^{54}Mn is used for a radioactive source (i.e. a very strong source). The activity of this sample then is

$$A = 2.87 \times 10^{11}\,\mathrm{Bq} = 7.7\,\mathrm{Ci} \ .$$

What kind of energy dose would be measured under these circumstances at a distance of 30 cm in tissue of 1 cm thickness?

The energy flux per cm^2 at a distance of $r = 30$ cm is

$$W = A\,\epsilon\,\Omega \ ,$$

where

solid angle

ϵ – energy per decay (842-keV photon) and

Ω – solid-angle fraction $= \frac{1}{4\,\pi\,r^2}$.

$1/r^2$ dependence

Therefore, one gets

$$W = 2.87 \times 10^{11}\,\mathrm{Bq} \times 0.842\,\mathrm{MeV} \times \frac{1}{4\,\pi\,30^2\,\mathrm{cm}^2}$$
$$= 2.13 \times 10^7\,\frac{\mathrm{MeV}}{\mathrm{s\,cm}^2} \ .$$

The mass absorption coefficient for 842 keV photons in tissue is about $\mu = 0.035\,(\mathrm{g/cm}^2)^{-1}$ (see Fig. 4.11; water and tissue are comparable in this case); i.e., of the original photon intensity only the fraction

$$\frac{N}{N_0} = 1 - \mathrm{e}^{-\mu\,x\,\rho} = 1 - \mathrm{e}^{-0.035\,(\mathrm{g/cm}^2)^{-1} \times 1\,\mathrm{cm} \times 1\,\mathrm{g/cm}^3} = 0.0344$$

per g/cm^2 is absorbed in tissue of 1 cm thickness. This corresponds to an absorbed energy of

$$W^* = W\,\frac{N}{N_0} = 0.735 \times 10^6\,\frac{\mathrm{MeV}}{\mathrm{s\,g}}$$

Experimental determination of the distance law

A 370 kBq ^{60}Co source is mounted at variable distances from a scintillation counter. The attenuation of γ rays of energy 1.17 MeV and 1.33 MeV (see Fig. 3.4) from the ^{60}Co isotope in air can be neglected. The count rate as a function of the distance between source and scintillator is plotted in double logarithmic scale in Fig. 4.13. A line fit to the data leads to a slope of -2, which means that the count rate N varies like

$$N \sim r^{-2} \ ,$$

since such a dependence will provide the measured slope in the log–log scale:

$$\ln N \sim -2 \ \ln r.$$

The fact that the count rate decreases inversely proportional to the square of the distance follows easily from a consideration of the solid angle: the total solid angle (surface of a sphere) corresponds to $4\pi r^2$. A detector of area A at a distance r records only the solid-angle fraction $A/4\pi r^2$. Therefore, the count rate varies like $N \sim 1/r^2$. For radiation protection one learns from this behavior: keeping one's distance from a radioactive source is a very effective protection against radiation.

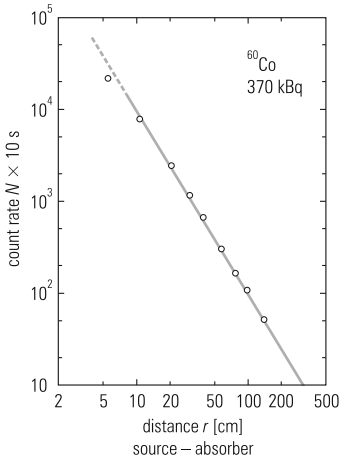


Figure 4.13
Distance law ($1/r^2$ law) for γ radiation from a ^{60}Co source

(1 cm^3 tissue $\widehat{=}$ 1 gram; 1 cm tissue $\widehat{=}$ 1 g/cm^2),

$$\begin{aligned} W^* &= 0.735 \times 10^6 \times 1.6 \times 10^{-13} \times 10^3 \,\mathrm{J/(s\,kg)} \\ &= 1.18 \times 10^{-4} \,\mathrm{Gy/s} \qquad (1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}) \ , \end{aligned}$$

which leads to a dose rate of

$$\dot{D} = 0.42\,\mathrm{Gy/h} \ .$$

Because the dose rate varies with the inverse square of the distance, for a distance of 1 m one has

$$\dot{D}_{1\,\mathrm{m}} = \dot{D}_{0.3\,\mathrm{m}} \times \frac{(0.3\,\mathrm{m})^2}{(1\,\mathrm{m})^2} = 38\,\mathrm{mGy/h} \ .$$

Eventually, the deposited energy will be transformed into heat. What kind of heat power $\dot{Q}$ corresponds to a dose rate of 38 mGy/h?

$$\dot{Q} = 38 \times 10^{-3} \frac{\mathrm{W\,s}}{\mathrm{kg\,h}} = \frac{38 \times 10^{-3}}{3600} \frac{\mathrm{W}}{\mathrm{kg}}$$
$$= 1.06 \times 10^{-5} \frac{\mathrm{W}}{\mathrm{kg}} \mathrel{\widehat{=}} 1.06 \times 10^{-8}\,\mathrm{W/cm^3} \ ,$$

Heating by ionizing radiation?

since the density of tissue was assumed to be $\rho = 1\,\mathrm{g/cm^3}$. Expressed in units of calories (1 cal is the quantity of heat, which is required to warm up 1 g of water by 1° from 14.5°C to 15.5°C; 1 cal = 4.186 J), this corresponds to a heat flow of

$$\dot{Q} = 9 \times 10^{-3} \frac{\mathrm{cal}}{\mathrm{kg\,h}} \ ,$$

which results in a remarkably low increase of the temperature. Assuming an exposure time of one hour the deposited energy would have been

$$Q = 9 \times 10^{-3} \frac{\mathrm{cal}}{\mathrm{kg}} \ ,$$

which would lead, according to

$$\Delta T = \frac{Q}{c} \ , \quad c - \text{specific heat of tissue} \left(= 1 \frac{\mathrm{cal}}{\mathrm{g\,K}} \right) \ ,$$

to an increase in the body temperature of the exposed person by

$$\Delta T = 9 \times 10^{-6}\,\mathrm{K} = 9\,\mu\mathrm{K} \ .$$

These numbers clearly demonstrate that the danger of nuclear radiation cannot be traced back to an increase in the body temperature.

Example 3

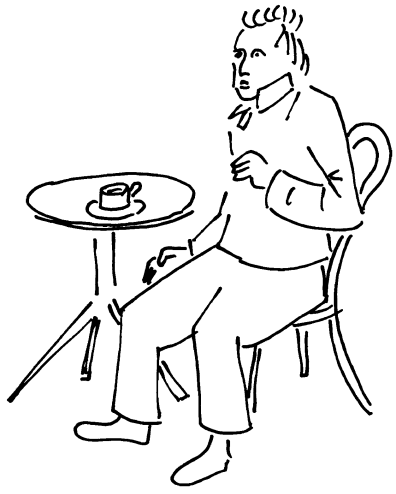
range of electrons

The range of electrons in water is about 1 cm. How is it possible to estimate the range of electrons in air given this information?

The range depends on the absorber material as Z/A. Since for both water and air this ratio is very similar[7], the range in air can be estimated from the density ratio (see also Fig. 4.4):

$$R_{\mathrm{air}} \approx R_{\mathrm{water}} \frac{\rho_{\mathrm{water}}}{\rho_{\mathrm{air}}} = 775\,\mathrm{cm} \ .$$

[7] The effective nuclear charge Z_{eff} and the effective atomic weight A_{eff} can be calculated from $Z_{\mathrm{eff}} = \Sigma f_i Z_i$, resp. $A_{\mathrm{eff}} = \Sigma f_i A_i$, where f_i are the mass fractions. For water one gets $Z_{\mathrm{eff}} = 6.8$ and $A_{\mathrm{eff}} = 14.3$ with $Z_{\mathrm{eff}}/A_{\mathrm{eff}} = 0.47$; for air one has 0.5.



"Oh my God, a cup of coffee amounts to 200 J/kg!"

Example 4

A researcher in a nuclear physics laboratory has forgotten to store a ^{60}Co source in a lead safe. As a consequence the source is lying around unshielded. The activity of the source is assumed to be 370 kBq (10 µCi). ^{60}Co emits per decay one electron with a maximum energy of 310 keV and two γ rays with energies 1.17 and 1.33 MeV (energy sum 2.5 MeV).

Work out the dose rate which students in the laboratory would receive, if their average distance from the source were 3 m. What kind of dose would they get per day in the nuclear physics lab assuming 8 hours exposure? Estimate the energy dose they would have received in the lab in comparison to the radioactivity from the natural environment.

To do this calculation (which is more an estimation) one should proceed in the following way:

1. First one should work out the effect of the electrons. Their range is approximated by **electron range**

$$R = 0.526\, E_{\mathrm{kin}}/\mathrm{MeV} - 0.095 \left(\mathrm{g/cm^2} \right) \ .$$

2. For the absorption probability of γ rays in 3 m of air one assumes a mass absorption coefficient in air of $\mu = 0.03\,\mathrm{cm^2/g}$ for an average energy $\langle E_\gamma \rangle = 1.25\,\mathrm{MeV}$. **$\gamma$ absorption**
3. What kind of fraction of γ rays will interact in the student? One should assume that the student consists essentially of water which is assumed to be tissue equivalent. The corresponding

Absorption of β and γ rays of isotopes from the ^{226}Ra decay chain

In the ^{226}Ra decay chain γ rays up to 2 MeV and β rays up to 3 MeV occur. The experimental results of absorption measurements (see Fig. 4.14) show that the β component is already completely absorbed after 3 mm of aluminum while photons have only been attenuated by about a factor of 2 even after 5 cm of aluminum. The attenuation coefficients can be described quite reasonably by exponential functions as can be seen from the semi-logarithmic plot of the data.

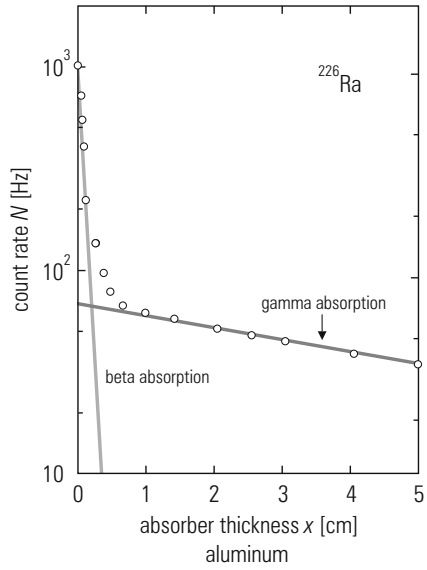


Figure 4.14
Absorption of β and γ rays from a ^{226}Ra source in aluminum

mass attenuation coefficient is $\mu_{\text{water}}(\langle E_\gamma \rangle = 1.25\,\text{MeV}) = 0.06\,\text{cm}^2/\text{g}$.

4. Then, one should estimate which fraction of the γ energy will be transferred to the student in this type of interaction, on average. Additionally, one should assume that for the photoelectric effect and pair production 100% of the energy is transferred in contrast to the Compton effect where only 50% is transferred.

solid-angle effect

5. One also must consider the solid angle (from the source) which the student occupies.
6. Then the relative relative biological effectiveness of β and γ rays should be considered.

relative dose

7. After the above calculation it will be interesting to know the dose rate, the total dose, and the dose ratio compared to the annual dose due to natural radioactivity (assumed to be about 2.3 mSv/yr).

The empirical formula for the range of electrons requires that the kinetic energy E is inserted in MeV; in this case one obtains the range R in $\mathrm{g/cm^2}$:

$$R = 0.526\, E_{\mathrm{kin}} - 0.095\, \left(\mathrm{g/cm^2}\right)$$
$$= (0.526 \times 0.31 - 0.095)\ \mathrm{g/cm^2} = 0.068\,\mathrm{g/cm^2}\ .$$

To work out the range in air the density[8] of air is required, $\rho_{\mathrm{air}} = 1.29 \times 10^{-3}\,\mathrm{g/cm^3}$; this leads to

$$r = \frac{R}{\rho_{\mathrm{L}}} = \frac{0.068}{1.29} \times 10^3\,\mathrm{cm} = 52.8\,\mathrm{cm}\ .$$

effect of electrons

Since the maximum range of electrons in air is only about 50 cm, students which have an average distance of 3 m will not be exposed to electron radiation. For the following, therefore, we can neglect the effect of electrons.

The photons will also be partially absorbed in the three meter distance, but by no means completely. The initial intensity I_0 will be reduced to the intensity $I\,(3\,\mathrm{m})$ according to

$$I\,(3\,\mathrm{m}) = I_0\, \mathrm{e}^{-\mu\, \rho_{\mathrm{L}}\, x}\ .$$

Consequently, the absorption probability in air is given by

$$\alpha = 1 - \frac{I}{I_0} = 1.1\%\ .$$

We clearly see that γ rays are only very weakly absorbed in air.

effect of photons

Let us assume that the standard student is $l = 1.80\,\mathrm{m}$ large, has a weight of $m = 75\,\mathrm{kg}$, and an effective 'absorber thickness' of $d = 15\,\mathrm{cm}$. This leads to a width of the standard student of $b = 27.7\,\mathrm{cm}$ assuming the known density of tissue and water and a mass of $m = 75\,\mathrm{kg}$. The fraction of energy absorbed by the student is then worked out as

$$\beta = 1 - \mathrm{e}^{-0.06\times 15\times 1} = 59.3\%\ .$$

That is to say about 40.7% of photons will pass through the student without interaction.

For the estimation of the energy absorption it is sufficient to consider only the Compton process since in this energy range this is the dominant one. For the energies in question the cross section for pair production can be completely neglected anyway. In Compton scattering only 50% of the energy is transferred to the target.

[8] The density depends on pressure and temperature, for this example standard temperature and pressure conditions are assumed: $p = 1013\,\mathrm{hPa}$ and $T = 293\,\mathrm{K}$ (hPa = hectopascal, $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$.)

solid-angle argument

The solid angle (from the source) which the student occupies can be estimated from the illustration in Fig. 4.15.

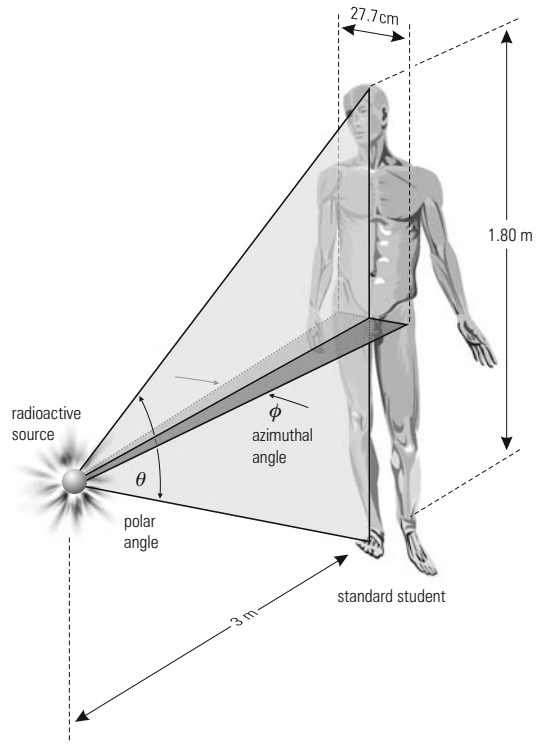


Figure 4.15
Illustration of the solid angle (from the source) which the student occupies

The angles θ and ϕ are calculated using this diagram to be[9]

$$\theta = 2 \times \arctan\left(\frac{0.9}{3}\right) = 33.4^\circ = 0.583\,\text{rad}\ ,$$

$$\phi = 2 \times \arctan\left(\frac{0.1385}{3}\right) = 5.30^\circ = 0.093\,\text{rad}\ .$$

This leads to a solid angle Ω' of

$$\Omega' = \theta\,\phi = 0.054\,\text{sterad}\ .$$

Since the student is not irradiated by electrons from the β decay, it is not necessary to consider a possible different biological effectiveness of β and γ rays. Anyway, it is the same for β and γ rays and it is equal to unity (see Chap. 2, Table 2.1).

[9] The trigonometric function 'tangent' describes the ratio of the opposite side to the adjacent side of a right-handed triangle. The arctan (on pocket calculators mostly given as $\tan^{-1}$) is the inverse function of tangent.

What we want to know is the energy-dose rate, the total dose, and the dose ratio to the annual dose due to natural radiation,

$$\text{dose rate}: \quad \dot{D} = \frac{W\,P}{m} \ ,$$

where

W – the absorbed energy per decay by the body of the student,
m – mass,
P – activity, including solid-angle and absorption factors.

$$\begin{aligned} P\,W &= \underbrace{\frac{1}{2}}_{\text{Compton}} \times \underbrace{\frac{\Omega'}{\Omega}}_{\text{solid angle}} \times \underbrace{\frac{59.3}{100}}_{\text{student}} \times \underbrace{\frac{98.9}{100}}_{\text{air}} \times 2.5\,\text{MeV} \times 10\,\mu\text{Ci} \\ &= \frac{1}{2} \times \frac{0.054}{4\,\pi} \times 0.593 \times 0.989 \times 2.5 \times 10^{6} \\ &\quad \times 1.602 \times 10^{-19} \times 10^{-5} \times 3.7 \times 10^{10}\,\frac{\text{J}}{\text{s}} \\ &= 1.87 \times 10^{-10}\,\frac{\text{J}}{\text{s}} \ . \end{aligned}$$

The dose rate, therefore, is calculated as

$$\begin{aligned} \dot{D} &= \frac{1.87 \times 10^{-10}}{75}\,\frac{\text{J}}{\text{s\,kg}} = 2.49 \times 10^{-12}\,\frac{\text{Gy}}{\text{s}} \\ &= 2.49 \times 10^{-12}\,\frac{\text{Sv}}{\text{s}} \ , \quad \text{because RBE} = 1 \ . \end{aligned}$$

The total dose on a work day of 8 h ≡ 28 800 s is given by

$$G = \dot{D}\,t = 2.49 \times 10^{-12} \times 28\,800\,\text{Sv} = 0.072\,\mu\text{Sv} \ .$$

The radiation level due to natural radioactivity is about 2.3 mSv per year, which leads to an exposure rate of 7.3×10^{-11} Sv per second, or 2.1 µSv in 8 h. This demonstrates that natural radiation leads to an exposure which is a factor of 30 larger than due to the unshielded source in the laboratory. This additional exposition can be almost neglected. The unshielded source would be only effective at very short distances, since in that case the solid angle will be much larger, and the effect of the electrons might also have to be considered.

Figure 4.16
Telescope probe for activity and contamination measurements at large distances (Graetz X50 DE detector with Geiger–Müller probe DE, GRAETZ radiation measurement technique GmbH)

determination of dose rates

dose comparison

Summary

Charged particles lose their energy in matter mainly by ionization and excitation. For electrons one has to consider the additional energy loss by bremsstrahlung. Because of the relatively large energy loss of charged particles, their range in matter is relatively short in most cases. In contrast to this, photons are attenuated only weakly when passing through matter, especially for the energies in the MeV range which are typical for the field of radiation protection. External radiation, therefore, usually consists of γ rays. Special care must be taken with neutrons, which have a relatively large range because they are electrically neutral. From the point of view of radiation protection neutrons are very unpleasant because by hitting the nuclei of cells they can create a substantial radiation damage.

4.5 Problems

Problem 1

The attenuation coefficient for 1-MeV gamma rays is assumed to be $0.12\,\mathrm{cm}^{-1}$ for concrete. What is the half-value and the tenth-value thickness?

Problem 2

The total beta-dose rate $\dot{D}_\beta$ of a pointlike 1-mCi ^{60}Co source at a distance of 5 cm is estimated by

$$\dot{D}_\beta = \Gamma_\beta \frac{A}{r^2} = 2.62 \times 10^{-11} \frac{\mathrm{Sv\,m^2}}{\mathrm{Bq\,h}} \times \frac{10^{-3} \times 3.7 \times 10^{10}\,\mathrm{Bq}}{0.05^2} = 388\,\mathrm{mSv/h}\ .$$

The total gamma-dose rate at the same position is

$$\dot{D}_\gamma = \Gamma_\gamma \frac{A}{r^2} = 3.41 \times 10^{-13} \frac{\mathrm{Sv\,m^2}}{\mathrm{Bq\,h}} \times \frac{10^{-3} \times 3.7 \times 10^{10}\,\mathrm{Bq}}{0.05^2} = 5\,\mathrm{mSv/h}\ .$$

Work out $\dot{D}_\beta$ and $\dot{D}_\gamma$ at a distance of 1 m.

Problem 3

The total attenuation coefficient (mass attenuation coefficient) for photons of 1 MeV in water is

$$\mu = 0.07\left(\mathrm{g/cm^2}\right)^{-1}\ .$$

By what factor will monoenergetic gamma radiation of 1 MeV be attenuated behind 1 m of water?