16 Solutions to the Problems

16.1 Solutions to the Problems of Chapter 2

Problem 1

$$\begin{aligned}\text{dose} &= \frac{\text{absorbed energy}}{\text{unit mass}} \\ &= \frac{\text{activity} \times \text{energy per decay} \times \text{time}}{\text{mass}} \\ &= \frac{10^9\,\text{Bq} \times 1.5 \times 10^6\,\text{eV} \times 1.602 \times 10^{-19}\,\text{J/eV} \times 86\,400\,\text{s}}{10\,\text{kg}} \\ &= 2.08\,\text{J/kg} = 2.08\,\text{Gy}\ .\end{aligned}$$

Here, we have used the electron volt as unit for the energy, in addition to the standard unit of joule: $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$ (see also Footnote 1, page 21).

Problem 2

The decay in the body of the worker is composed of two components. The total effective decay constant λ_{eff} is

$$\lambda_{\text{eff}} = \lambda_{\text{phys}} + \lambda_{\text{bio}}\ .$$

Because of $\lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}}$ we get

$$T_{1/2}^{\text{eff}} = \frac{T_{\text{phys}}\,T_{\text{bio}}}{T_{\text{phys}} + T_{\text{bio}}} = 79.4\,\text{d}\ .$$

Using $\dot{D} = \dot{D}_0\,\text{e}^{-\lambda t}$ and $\dot{D}/\dot{D}_0 = 0.1$ we arrive at[1]

$$t = \frac{1}{\lambda}\ln\left(\frac{\dot{D}_0}{\dot{D}}\right) = \frac{T_{1/2}^{\text{eff}}}{\ln 2}\ \ln\left(\frac{\dot{D}_0}{\dot{D}}\right) = 263.8\,\text{d}\ .$$

A mathematically more demanding calculation allows to determine the dose the worker was exposed to over this period:

[1] The notation $\dot{D}_0$ describes the dose rate at $t = 0$. It does not represent the time derivative of the constant dose D_0 (which would be zero!).

C. Grupen, *Introduction to Radiation Protection*,
Graduate Texts in Physics, DOI 10.1007/978-3-642-02586-0_16,

$$
\begin{aligned}
D_{\text{total}} &= \int_0^{263.8\,\text{d}} \dot{D}_0\, \mathrm{e}^{-\lambda t}\, \mathrm{d}t \\
&= \dot{D}_0 \left(-\frac{1}{\lambda}\right) \mathrm{e}^{-\lambda t} \Bigg|_0^{263.8\,\text{d}} \\
&= \frac{\dot{D}_0}{\lambda} \left[1 - \mathrm{e}^{-\lambda \times 263.8\,\text{d}}\right] \ .
\end{aligned}
$$

Using

$$
\lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}^{\text{eff}}} = 8.7 \times 10^{-3}\,\text{d}^{-1}
$$

we get ($1\,\mu\text{Sv/h} = 24\,\mu\text{Sv/d}$)

$$
D_{\text{total}} = \frac{24\,\mu\text{Sv/d}}{\lambda}[1 - 0.1] = 2.47\,\text{mSv} \ .
$$

The 50-years committed dose equivalent $D_{50} = \int_0^{50\,\text{yrs}} \dot{D}(t)\,\mathrm{d}t$ is determined to be

$$
\begin{aligned}
D_{50} &= \int_0^{50\,\text{yrs}} \dot{D}_0\, \mathrm{e}^{-\lambda t}\, \mathrm{d}t = \frac{\dot{D}_0}{\lambda} \left[1 - \mathrm{e}^{-\lambda \times 50\,\text{yrs}}\right] \\
&\approx \frac{\dot{D}_0}{\lambda} = 2.75\,\text{mSv} \ .
\end{aligned}
$$

Problem 3

At a distance of 2 m the electrons from ^{60}Co decay are already absorbed (compare Figs. 3.4 and 4.4). Therefore only the γ dose is of interest. The specific γ dose constant for ^{60}Co is $\Gamma_\gamma = 3.41 \times 10^{-13}\,\frac{\text{Sv m}^2}{\text{Bq h}}$. Using the relation

$$
\dot{H} = \Gamma_\gamma \frac{A}{r^2}
$$

we get

$$
A = \frac{r^2}{\Gamma_\gamma}\,\dot{H} = \frac{(2\,\text{m})^2\,\text{Bq h} \times 10^{-4}\,\text{Sv}}{3.41 \times 10^{-13}\,\text{Sv m}^2\,\text{h}} = 1.17\,\text{GBq} \ .
$$

Problem 4

The solution of this problem can be obtained in two ways. After one half-life the activity has decayed by a factor of 2; that is, after two days corresponding to $48/6 = 8$ half-lives the activity is decreased by a factor of 2^8. Therefore the patient still 'radiates' after two days with an activity of

$$
A(48\,\text{h}) = A_0 \times 2^{-8} = 10^7\,\text{Bq}/2^8 = 39\,\text{kBq} \ .
$$

In an analogous way the time dependence of the activity can be represented by

$$A = A_0\,\mathrm{e}^{-t/\tau} \ .$$

Because of

$$\tau = \frac{T_{1/2}}{\ln 2} = 8.66\,\mathrm{h}$$

we arrive at the same result,

$$A(48\,\mathrm{h}) = A_0\,\mathrm{e}^{-\frac{48\,\mathrm{h}\times\ln 2}{T_{1/2}}} = 39\,\mathrm{kBq} \ .$$

16.2 Solutions to the Problems of Chapter 3

Problem 1

The electrostatic repulsion of protons reduces the binding energy of nuclei. The electrically neutral neutrons do not experience electrostatic forces and therefore have no electromagnetic effect on nuclear binding. A nucleus with relative proton excess or even equal number of protons and neutrons at high atomic masses has the tendency to transform some of the protons into neutrons, so that the nucleus gains higher stability.

Problem 2

Because of the non-linear relation between atomic number Z and mass number A nuclear fission nearly always leads to fission products with relatively large neutron excess. The shell model of the nucleus (magic numbers) explains why the fission is asymmetric in most cases (see Fig. 12.1). For example, fission of ^{235}U with slow neutrons may result in the production of highly excited ^{141}I and ^{95}Y nuclei. The reduction of the neutron excess can proceed by the emission of prompt or delayed neutrons or by transformation of some neutrons into protons by β^- decays, for example,

$$\begin{aligned}
^{141}\mathrm{I}^* &\longrightarrow {}^{140}\mathrm{I} + n \\
&\hookrightarrow {}^{140}\mathrm{Xe} + e^- + \bar{\nu}_e \\
&\quad\hookrightarrow {}^{139}\mathrm{Xe} + n \\
&\qquad\hookrightarrow {}^{139}\mathrm{Cs} + e^- + \bar{\nu}_e \\
&\qquad\quad\hookrightarrow {}^{139}\mathrm{Ba} + e^- + \bar{\nu}_e \\
&\qquad\qquad\hookrightarrow {}^{139}\mathrm{La} + e^- + \bar{\nu}_e \ .
\end{aligned}$$

^{139}La is the stable end product, where the proton and neutron numbers are balanced.

Problem 3

The range of a 100-keV electron in tissue is about 0.2 mm (see Fig. 4.4). It is still marginally acceptable to use the non-relativistic, classical relation between energy and velocity in this case. Its initial velocity can be worked out from

$$E_{\text{kin}} = \frac{1}{2} m_e \, v^2$$

to be

$$v = \sqrt{\frac{2\,E_{\text{kin}}}{m_e}} = \sqrt{\frac{2 \times 100\,\text{keV}}{511\,\text{keV}}}\, c \approx 1.9 \times 10^8 \,\frac{\text{m}}{\text{s}} \ .$$

c is the velocity of light in vacuum. This allows to estimate the traveling time t from

$$\begin{aligned} s &= \frac{1}{2}\, a\, t^2 \\ \text{and} \quad v &= a\, t \\ \text{to be} \quad t &= \frac{2\,s}{v} = 2.1 \times 10^{-12}\,\text{s} = 2.1\,\text{ps} \ . \end{aligned}$$

Problem 4

a) The initial kinetic energies are determined to be:

$$\begin{aligned} E_{\text{kin}}^{\text{tennis ball}} &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} \times 0.06\,\text{kg} \times \left(200\,\text{km/h} \times \frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)^2 \\ &= 92.6\,\frac{\text{kg}\,\text{m}^2}{\text{s}^2} = 92.6\,\text{joule} \ , \\ E_{\text{kin}}^{\alpha} &= 5 \times 10^6\,\text{eV} \times 1.602 \times 10^{-19}\,\text{J/eV} \\ &= 8 \times 10^{-13}\,\text{joule} \ , \\ E_{\text{kin}}^{\text{tennis ball}} / E_{\text{kin}}^{\alpha} &= 1.16 \times 10^{14} \ . \end{aligned}$$

b) The energy density ρ_E is the energy per unit volume:

$$\begin{aligned} \rho_E^{\text{tennis ball}} &= \frac{1}{2} m v^2 / \frac{4}{3} \pi r_{\text{T}}^3 = 6.4 \times 10^5\,\text{J/m}^3 \ , \\ \rho_E^{\alpha} &= 5\,\text{MeV} / \frac{4}{3} \pi r_{\alpha}^3 = 4.7 \times 10^{31}\,\text{J/m}^3 \ . \end{aligned}$$

The energy density in the α particle exceeds by far that of the tennis ball.

Problem 5

1 ton of ^{235}U contains

$$N_{\text{A}} \times \frac{10^3\,\text{kg}}{0.235\,\text{kg}} = 2.56 \times 10^{27}\,\text{nuclei}$$

(N_{A} – Avogadro number; 1 mol ^{235}U corresponds to 235 g).

Therefore we get for the mass loss Δm:

$$\Delta m = \frac{2.56 \times 10^{27}\,\text{nuclei} \times 235\,\frac{\text{MeV}}{\text{nucleus}} \times 1.6 \times 10^{-13}\,\frac{\text{J}}{\text{MeV}}}{c^2}$$
$$\approx 1\,\text{kg} \ .$$

The complete fission of one ton of ^{235}U results in a mass defect of about 1 kg. The efficiency of mass-to-energy conversion is therefore on the order of magnitude of 1‰.

16.3 Solutions to the Problems of Chapter 4

Problem 1

The half-value layer (tenth-value layer) is that thickness of an absorber which reduces the γ intensity by a factor of two (ten). Using

$$I = I_0\,\mathrm{e}^{-\mu x}$$

we get

$$x = \frac{1}{\mu}\,\ln\left(\frac{I_0}{I}\right) \ ,$$
$$x_{1/2} = \frac{1}{\mu}\,\ln\left(\frac{I_0}{I_0/2}\right) = 5.8\,\text{cm} \ ,$$
$$x_{1/10} = \frac{1}{\mu}\,\ln\left(\frac{I_0}{I_0/10}\right) = 19.2\,\text{cm} \ .$$

Problem 2

Photons from ^{60}Co γ decay are hardly absorbed in air. Therefore the inverse-square ($1/r^2$) law is a reasonable approximation for photons:

$$\dot{D}_\gamma(1\,\text{m}) = \dot{D}_\gamma(5\,\text{cm})\,\left(\frac{5}{100}\right)^2 = 12.5\,\frac{\mu\text{Sv}}{\text{h}} \ .$$

In contrast, electrons are strongly absorbed in air. Since the energy of electrons from ^{60}Co decay is rather low ($E_{\max} = 310\,\text{keV}$), almost no electrons will survive a distance of 1 m. Therefore we can safely assume

$$\dot{D}_\beta(1\,\text{m}) = 0 \ .$$

The estimate of the effect of electrons via their range is only an approximation. The energy spectrum of electrons in nuclear beta decay is continuous. A large number of low-energy electrons is already absorbed by very thin layers. Therefore the absorption of electrons from continuous spectra can also be approximated by an exponential attenuation law:

$$I_\beta(x) = I_\beta(0)\,\mathrm{e}^{-\kappa\,x}\ ;$$

κ is the linear absorption coefficient. For electron energies typical for the field of radiation protection ($0.1\,\mathrm{MeV} \le E_\beta \le 3.5\,\mathrm{MeV}$) κ can be approximated by the empirical relation

$$\kappa = \frac{15}{E_{\beta_{\max}}^{1.5}}$$

($E_{\beta_{\max}}$ in MeV and κ in $(\mathrm{g/cm^2})^{-1}$).

The exponential law for the absorption of electrons from continuous beta spectra can only be applied for thin absorbers. For absorber layers comparable to the range of electrons the exponential attenuation law fails. In these cases the beta intensities are significantly overestimated by the exponential law. In our example we get for the beta dose considering the electron absorption in 5 cm of air ($E_{\beta_{\max}} = 0.31\,\mathrm{MeV}$ for $^{60}\mathrm{Co}$):

$$\begin{aligned}\dot{D}_\beta(5\,\mathrm{cm}) &= \Gamma_\beta\,\frac{A}{r^2}\,\frac{I_\beta(5\,\mathrm{cm})}{I_\beta(0\,\mathrm{cm})}\\ &= 388\,\frac{\mathrm{mSv}}{\mathrm{h}}\,\mathrm{e}^{-\rho_{\mathrm{air}}\,\kappa\times 5\,\mathrm{cm}} = 222\,\frac{\mathrm{mSv}}{\mathrm{h}}\ .\end{aligned}$$

For a distance of 1 m this approximation gets questionable. The application of the absorption law would give in this case

$$\dot{D}_\beta(1\,\mathrm{m}) \le 388\,\frac{\mathrm{mSv}}{\mathrm{h}} \times \left(\frac{5\,\mathrm{cm}}{100\,\mathrm{cm}}\right)^2 \times \frac{I_\beta(1\,\mathrm{m})}{I_\beta(0\,\mathrm{cm})} = 0.013\,\frac{\mu\mathrm{Sv}}{\mathrm{h}}\ .$$

Here we used the $\le$ symbol, because the exponential overestimates the dose rate for large distances.

Problem 3

We have

$$I = I_0\,\mathrm{e}^{-\mu\,x\,\rho} \quad \text{with} \quad \mu = 0.07\,(\mathrm{g/cm^2})^{-1}$$

and

$$x = 1\,\mathrm{m} \mathrel{\widehat{=}} 100\,\mathrm{g/cm^2}\ .$$

From

$$\frac{I_0}{I} = \mathrm{e}^{\mu\,x\,\rho}$$

we get an absorption factor of 1097.

16.4 Solutions to the Problems of Chapter 5

Problem 1

The measured charge ΔQ is related to the change of the voltage ΔU via the capacitor equation:

$$\begin{aligned}\Delta Q &= C\,\Delta U \\ &= 7 \times 10^{-12}\,\mathrm{F} \times 30\,\mathrm{V} = 210 \times 10^{-12}\,\text{coulomb}\ .\end{aligned}$$

The mass of the air is

$$m = \rho_\mathrm{L}\,V = 3.225 \times 10^{-3}\,\mathrm{g}\ .$$

This leads to an ion dose of

$$\begin{aligned}I = \frac{\Delta Q}{m} &= 6.5 \times 10^{-8}\,\frac{\mathrm{C}}{\mathrm{g}} \\ &= 6.5 \times 10^{-5}\,\frac{\mathrm{C}}{\mathrm{kg}}\ .\end{aligned}$$

Because of $1\,\mathrm{R} = 2.58 \times 10^{-4}\,\mathrm{C/kg}$ this corresponds to a dose of 0.25 roentgen. Since $1\,\mathrm{R} = 8.8\,\mathrm{mGy}$, we get

$$D = 2.2\,\mathrm{mGy}\ .$$

Problem 2

The true dead-time-corrected rate at a distance of $d_1 = 10\,\mathrm{cm}$ is

$$R_1^* = \frac{R_1}{1 - \tau\,R_1}\ .$$

Because of the inverse-square law ($\sim 1/r^2$) the true rate R_2^* at $d_2 = 30\,\mathrm{cm}$ is

$$R_2^* = \left(\frac{d_1}{d_2}\right)^2\,R_1^*\ ;$$

and because of $R_2^* = R_2/(1 - \tau\,R_2)$ one arrives at the equation

$$\left(\frac{d_1}{d_2}\right)^2\,\frac{R_1}{1 - \tau\,R_1} = \frac{R_2}{1 - \tau\,R_2}\ .$$

Solving for τ yields

$$\tau = \frac{\left(\frac{d_2}{d_1}\right)^2\,R_2 - R_1}{\left[\left(\frac{d_2}{d_1}\right)^2 - 1\right]\,R_1\,R_2} = 10\,\mu\mathrm{s}\ .$$

Problem 3

For the solid-angle factor we get

$$f_1 = \frac{\pi \left(\frac{d}{2}\right)^2}{4\pi r^2} = 5.625 \times 10^{-5}$$

and for the efficiency

$$f_2 = 0.08 \ ,$$

resulting in an activity of

$$A = \frac{(\text{count rate} - \text{background rate}) \times \text{min}/60\,\text{s}}{f_1\ f_2}$$
$$= 1.1 \times 10^7\,\text{Bq} = 300\,\mu\text{Ci} \ .$$

Problem 4

The initial contamination is N. After single wiping we have the residual contamination $N(1-\varepsilon)$; after the third decontamination step one has $N(1-\varepsilon)^3$. Therefore we get for N:

$$N = \frac{512\,\text{Bq/cm}^2}{(1-\varepsilon)^3} = 1000\,\text{Bq/cm}^2 \ .$$

The third decontamination step removes an area activity of

$$N(1-\varepsilon)^2\,\varepsilon = 128\,\text{Bq/cm}^2 \ .$$

16.5 Solutions to the Problems of Chapter 6

Problem 1

1 m^3 ground air contains a krypton activity of 1.1 Bq. However, 1 m^3 air contains only 1.1 ppm krypton, corresponding to a fraction of 1.1×10^{-6}. Therefore 1 m^3 of krypton has an activity of 10^6 Bq. This means that the exemption limit is already exhausted by an amount of 0.01 m^3 = 10 l.

A different, more formal approach to solve this problem, is presented by the following consideration: ^{85}Kr has a half-life of 10.4 yrs. Consequently, the decay constant is

$$\lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}} = 2.11 \times 10^{-9}\,\text{s}^{-1} \ .$$

Since the activity of ^{85}Kr in 1 m^3 of ground air is 1.1 Bq, the number of krypton-85 nuclei in 1 m^3 of air can be worked out from

$$A = \lambda\, N$$

to be

$$N = \frac{A}{\lambda} = 5.2 \times 10^8 \ .$$

1.1 ppm krypton, corresponding to 1.1 cm^3 in 1 m^3 air, leads to an amount of krypton of 4.125 mg (the density of krypton is $\rho = 3.75\,g/l$). This corresponds to $\frac{4.125\times10^{-3}}{83.8} = 4.92 \times 10^{-5}$ mol $\widehat{=}$ 2.96×10^{19} nuclei. (1 mol has 6.022×10^{23} atoms, Avogadro number.) This leads to an atomic abundance of ^{85}Kr of

$$\frac{5.2 \times 10^8}{2.9 \times 10^{19}} = 1.75 \times 10^{-11} \ .$$

The exemption limit is at 10^4 Bq ^{85}Kr, corresponding to $5.2 \times 10^8 \times \frac{10^4}{1.1} = 4.73 \times 10^{12}$ nuclei. Considering the isotopic abundance we get

$$\begin{aligned}\frac{4.73 \times 10^{12}}{1.78 \times 10^{-11}} &= 2.70 \times 10^{23} \text{ krypton atoms} \\ &\widehat{=} 0.448\,\text{mol} \widehat{=} 37.5\,\text{g} \widehat{=} 10 \text{ liters} \ .\end{aligned}$$

Problem 2

The activity of a substance is derived from

$$N = N_0\, e^{-\lambda t} = N_0\, e^{-t/\tau}$$

$$\text{to be} \quad A = -\frac{dN}{dt} = \lambda N = \frac{N}{\tau} \ .$$

For a mixture of isotopes the total activity is

$$\begin{aligned}A_{\text{total}} &= A_1(^{238}\text{U}) + A_2(^{235}\text{U}) + A_2(^{235}\text{U}) \\ &= \frac{N_1}{\tau_1} + \frac{N_2}{\tau_2} + \frac{N_3}{\tau_3} \\ &= N \left\{ \frac{0.992\,75 \ln 2}{T_{1/2}(^{238}\text{U})} + \frac{0.007\,195 \ln 2}{T_{1/2}(^{235}\text{U})} + \frac{0.000\,055 \ln 2}{T_{1/2}(^{234}\text{U})} \right\} \\ &= N \ \{1.53 + 0.0712 + 1.59\} \times 10^{-10}\,\text{yr}^{-1} \ .\end{aligned}$$

This leads to

$$\begin{aligned}N &= \frac{A_{\text{total}}}{3.19 \times 10^{-10}}\,\text{Bq yrs} = 3.135 \times 10^{13}\,\text{Bq yrs} \\ &= 9.886 \times 10^{20}\,\text{Bq s} = 9.886 \times 10^{20} \text{ nuclei} \ .\end{aligned}$$

Since the atomic mass is practically completely concentrated in the nucleus, and 1 mol uranium contains 6.022×10^{23} atoms, 9.886×10^{20} nuclei correspond to 1.64×10^{-3} mol.

1 mol uranium in its natural isotopic abundance has

$$(238\times0.992\,75+235\times0.007\,195+234\times5.5\times10^{-5})\,\text{g} = 237.98\,\text{g}.$$

Therefore the uranium fraction of the lump of ore is given by

$$M_{\text{chunk of uranium ore}} = 0.390\,\text{g} \ .$$

Since the density of uranium is rather large, $\rho_{\text{U}} = 18.95\,g/cm^3$, we are only dealing with a uranium volume of $0.0206\,cm^3 = 20.6\,mm^3$!

Problem 3

The maximum dose for workers of category A in controlled areas is 20 mSv/yr. A whole-body irradiation of 12 mSv and a liver exposure of 40 mSv leads to an effective whole-body dose of

$$H_{\text{eff}} = (12 + 0.05 \times 40)\,\text{mSv} = 14\,\text{mSv} \ ,$$

where 0.05 is the tissue weighting factor for the liver. To exhaust the maximum allowable whole-body dose an additional whole-body exposure of

$$H_{\text{max}} - 14\,\text{mSv} = 6\,\text{mSv}$$

would be acceptable.

If this dose were received exclusively on the lung, a maximum lung dose of

$$\frac{6\,\text{mSv}}{0.12} = 50\,\text{mSv}$$

would keep the total effective dose still within the limit of 20 mSv/yr.

16.6 Solutions to the Problems of Chapter 7

Problem 1

A work load of 40 hours a week and 52 weeks a year (no holiday assumed) leads to a total residence time of workers in a radiation area of 2080 hours per year. The legislator has fixed the annual residence time to 2000 hours per year. (Does this mean that the authorities allow only two weeks of holiday in a year?) This leads to an annual dose of 8 mSv for the given dose rate. Therefore this room must be considered as controlled area and the worker counts as belonging to the category-A group (dose range $6\,\text{mSv/yr} < \dot{D} \leq 20\,\text{mSv/yr}$).

Problem 2

The total activity is calculated to be

$$A_{\text{total}} = 100\,\text{Bq/m}^3 \times 4000\,\text{m}^3 = 4 \times 10^5\,\text{Bq} \ .$$

Consequently the original activity concentration in the reactor core was

$$A_0 = \frac{4 \times 10^5\,\text{Bq}}{500\,\text{m}^3} = 800\,\text{Bq/m}^3 \ .$$

Problem 3

The activity is given by

$$A = \lambda\, N = \frac{1}{\tau}\, N = \frac{\ln 2}{T_{1/2}}\, N \ ,$$

corresponding to

$$N = \frac{A\, T_{1/2}}{\ln 2} = 1.9 \times 10^{12}\ \text{cobalt nuclei}$$

and $m = N\, m_{\text{Co}} = 0.2\,\text{ng}$. Such a low amount of cobalt can hardly be detected by analytical chemistry.

16.7 Solutions to the Problems of Chapter 8

Problem 1

For a radioactive source to be considered leak-proof, the result of a wipe test of the source must not exceed an activity of 200 Bq. The measurement, however, yielded a larger value, namely,

$$\frac{20 \text{ counts per second}}{0.8 \times 0.1} = 250\,\text{Bq} \ .$$

Therefore the source cannot be considered leak-proof.

Problem 2

The activity per cm^2 is

$$\frac{\text{activity}}{\text{cm}^2} = \frac{R - R_0}{F\ \eta_1\ \eta_2} = 550\,\frac{\text{Bq}}{\text{cm}^2} \ .$$

Problem 3

The radiated power is

$$S = 10^{17}\,\text{Bq} \times 10\,\text{MeV} = 10^{24}\,\text{eV/s} = 160\,\text{kJ/s} \ ,$$

which leads to a temperature increase of

$$\begin{aligned} \Delta T &= \frac{\text{heat input}}{m\ c} \\ &= \frac{160\,\text{kJ/s} \times 86\,400\,\text{s/d} \times 1\,\text{d}}{120\,000\,\text{kg} \times 0.452\,\text{kJ/(kg K)}} \\ &= 255\,\text{K} \ . \end{aligned}$$

The temperature rise of 255 degrees leads to a final temperature of 275 °C.

Problem 4

The dose rate from a pointlike emitter of activity A at a distance r is given by

$$\dot{H} = \Gamma_\gamma\ \frac{A}{r^2} \ .$$

Using the specific γ dose constant of ^{60}Co (see Table 2.3),

$$\Gamma_\gamma(^{60}\text{Co}) = 3.41 \times 10^{-13}\,\frac{\text{Sv m}^2}{\text{Bq h}} \ ,$$

we can solve for the minimum distance r:

$$r \le \sqrt{\frac{\Gamma_\gamma\ A}{\dot{H}}} = 1.85\,\text{m} \ .$$

If we assume that the source has been dropped to the floor, and if we further assume that the dosimeter is carried at a height of 1 m, the search will cover a radial distance of $\sqrt{1.85^2 - 1}\,\text{m} = 1.55\,\text{m}$. Therefore it is sufficient to search for the radioactive source in places about 3 m apart.

Problem 5

We have

$$\dot{D} \sim \frac{1}{r^2} \ ;$$

with the result

$$\frac{\dot{D}(x)}{\dot{D}(100\,\mathrm{m})} = \frac{(100\,\mathrm{m})^2}{x^2} = \frac{25\,\mu\mathrm{Sv/h}}{1\,\mu\mathrm{Sv/h}} \ .$$

Solved for x we get

$$x = 20\,\mathrm{m} \ .$$

Problem 6

The decay of the radioactive uranium in the filter can be safely neglected. This leads to the activity

$$A = m\,\kappa\,\eta\,t = 40.32\,\mathrm{MBq} \ .$$

If the decay of the long-lived uranium isotope is taken into account ($T_{1/2} = 4.5 \times 10^9$ yrs, decay constant $\lambda = \frac{\ln 2}{T_{1/2}} = 1.76 \times 10^{-14}\,\mathrm{h}^{-1}$), the solution of the problem is more demanding. In this case we have to solve the following differential equation for the time-dependent activity:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = m\,\kappa\,\eta - \lambda\,A$$

with the result[2]

$$A = \frac{m\,\kappa\,\eta}{\lambda}\left(1 - \mathrm{e}^{-\lambda\,t}\right) \ .$$

Because of $\lambda\,t \ll 1$ and $A = \frac{m\,\kappa\,\eta}{\lambda}\,(1 - (1 - \lambda\,t + \cdots)) \approx m\,\kappa\,\eta\,t$ the above numerical result is practically unchanged.

Problem 7

According to the rules laid down in Chap. 8 (radiation safety) the dose rate at the outside surface of goods for II–yellow cargo-container types is in the range

$$5\,\mu\mathrm{Sv/h} \leq \dot{D} \leq 0.5\,\mathrm{mSv/h} \ .$$

The transport index $t = 0.3$ means that $\dot{D}(1\,\mathrm{m}) = 0.3\,\mathrm{mrem/h}$ corresponding to $\dot{D}(1\,\mathrm{m}) = 3\,\mu\mathrm{Sv/h}$.

16.8 Solutions to the Problems of Chapter 9

Problem 1

The average energy loss of α particles of $1.37\,\mathrm{MeV/(mg/cm^2)}$ can be converted into a range of

[2] compare supplementary information No. 1, Chap. 8

$$\lambda = \frac{5\,\mathrm{MeV}}{1.37\,\mathrm{MeV/(mg/cm^2)}} = 3.65\,\mathrm{mg/cm^2} \ .$$

Only this distance is relevant for neutron production. The neutron production probability per incident α particle is then

$$\begin{aligned}\phi &= \frac{N_\mathrm{A}[\mathrm{mol}^{-1}] \times \mathrm{g}^{-1}}{A}\,\sigma\,\lambda \\ &= \frac{6.022 \times 10^{23}}{9} \times 3 \times 10^{-25} \times 3.65 \times 10^{-3} = 7.3 \times 10^{-5} \ .\end{aligned}$$

According to this calculation 73 neutrons are generated for one million of incident α particles.

Problem 2

The solid-angle–area product of the coincidence arrangement can be approximated by

$$\Omega A = \frac{F_1\ F_2}{d^2} = \frac{100 \times 100\,\mathrm{cm}^4}{(50\,\mathrm{cm})^2} = 4\,\mathrm{cm}^2\,\mathrm{sr} \ .$$

The flux of cosmic-ray muons is (see Eq. (9.12))

$$\phi = 8 \times 10^{-3}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1} \ ,$$

giving $\phi\ \Omega A = 0.032\,\mathrm{s}^{-1}$, which corresponds to 2765/d. For a signal width of 100 µs the total dead time is

$$\tau = 2765 \times 10^{-4}\,\mathrm{s} \approx 0.28\,\mathrm{s}$$

corresponding to a negligibly small fraction of 3.2×10^{-6} of the total measurement time.

Problem 3

Tissue consists essentially of water. From 1 mol of water (18 g H_2O) one obtains $N = N_\mathrm{A} \times 2\,\mathrm{g}/(18\,\mathrm{g/mol}) = 6.69 \times 10^{22}$ 'quasi-free' protons per gram (N_A – Avogadro number). The average mean free path of 5-MeV neutrons in tissue is $\lambda = \frac{1}{N\sigma\rho} \approx 15\,\mathrm{cm}$. This means that such a neutron interacts on average only once in the human body. Depending on the scattering angle in the (n, p) reaction an energy between 0 and 5 MeV is transferred, which corresponds on average to half the neutron energy. This energy will be deposited in a volume of $15\,\mathrm{cm} \times 1\,\mathrm{cm}^2$. This leads to an energy dose of

$$D(\mathrm{J/kg}) = \frac{N\ \sigma\ \Delta E\ d\ \rho}{m} \ .$$

The 'target thickness' is $d = 15\,\mathrm{cm}$ and its density is $\rho = 1\,\mathrm{g/cm}^3$. The irradiated tissue per cm^2 has a mass of 15 g. Therefore the dose per incident neutron is given by

$$D(\mathrm{Gy}) = \frac{1}{15 \times 10^{-3}\,\mathrm{kg}} \times 6.69 \times 10^{22}\,\frac{1}{\mathrm{g}} \times 10^{-24}\,\mathrm{cm}^2 \times 2.5\,\mathrm{MeV} \times 1.602 \times 10^{-13}\,\frac{\mathrm{J}}{\mathrm{MeV}} \times 15\,\frac{\mathrm{g}}{\mathrm{cm}^2} = 2.68 \times 10^{-11}\,\mathrm{Gy}\ .$$

For a neutron flux of $10^6/\mathrm{cm}^2$ and a radiation weighting factor of 10 for 5-MeV neutrons one obtains

$$H(\mathrm{Sv}) = D(\mathrm{Gy}) \times 10^6 \times 10\,\frac{\mathrm{Sv}}{\mathrm{Gy}} = 268\,\mu\mathrm{Sv}\ .$$

16.9 Solutions to the Problems of Chapter 10

Problem 1

The accelerating voltage has to be chosen in such a way that the shortest wavelength is $\lambda_{\min} \leq 0.5\,\text{Å}$. Because of

$$e\,U = h\nu = \frac{h\,c}{\lambda_{\min}}$$

(ν – frequency, h – Planck constant) we get

$$U \geq \frac{h\,c}{\lambda_{\min}\,e} = 24\,816\,\mathrm{V} \approx 25\,\mathrm{kV}\ .$$

Problem 2

The absorption of X rays is described by

$$I = I_0\,\mathrm{e}^{-\mu\,x} \Rightarrow \mathrm{e}^{\mu\,x} = \frac{I_0}{I}\ .$$

Solved for x we get

$$x = \frac{1}{\mu}\,\ln\left(\frac{I_0}{I}\right) = 30.7\,\frac{\mathrm{g}}{\mathrm{cm}^2}$$

or, equivalently,

$$x^* = \frac{x}{\rho_{\mathrm{Al}}} = 11.4\,\mathrm{cm}\ .$$

Problem 3

1.) $\dot{D}(\mathrm{concrete} + \mathrm{lead}) = \dot{D}(\mathrm{concrete})\,\mathrm{e}^{-\mu_{\mathrm{Pb}}\,x}$.

This leads to

$$\mathrm{e}^{\mu_{\mathrm{Pb}}\,x} = \frac{\dot{D}(\mathrm{concrete})}{\dot{D}(\mathrm{concrete} + \mathrm{lead})}\ .$$

This can be solved for x to give

$$x = 0.59\,\mathrm{cm}\ .$$

2.) First of all we have to work out the dose rate without any shielding:

$$\dot{D}(\text{without shielding}) = \dot{D}_0 = \dot{D}(20\,\text{cm concrete}) \times \mathrm{e}^{\mu_{\text{concrete}}\, y} \ ;$$

if $y = 20\,\text{cm}$ is the thickness of the concrete shielding, we get

$$\dot{D}_0 = 282\,\text{mSv/h} \ .$$

For a shielding exclusively of lead one would obtain:

$$\dot{D}(x\,\text{cm Pb}) = \dot{D}_0\, \mathrm{e}^{-\mu_{\text{Pb}}\, x} = 1\,\mu\text{Sv/h} \ ,$$

$$\mathrm{e}^{\mu_{\text{Pb}}\, x} = \frac{282}{10^{-3}} = 2.82 \times 10^5$$

with the result (see above)

$$x = 1.14\,\text{cm} \ .$$

16.10 Solutions to the Problems of Chapter 11

Problem 1

At a distance of one meter practically only the γ activity counts, because both the α particles and the β particles have been almost completely absorbed. Most of the α particles will even not have been able to escape from the ground. Based on a γ activity of 500 Bq/kg and an assumed average specific γ dose constant for the mixed radiation field[3] of $10^{-13}\ \frac{\text{Sv m}^2}{\text{Bq h}}$, we arrive at a dose rate of

$$\begin{aligned}\dot{H} &= 10^{-13}\,\frac{\text{Sv m}^2}{\text{Bq h}} \times 500\,\text{Bq/kg} \times 1\,\text{m}^{-2} \\ &= 5 \times 10^{-11}\,\frac{\text{Sv}}{\text{h kg}} \ .\end{aligned}$$

Under the assumption that persons are irradiated from one ton of the soil, this leads to a dose rate of 5×10^{-8} Sv/h and an annual dose of about 0.5 mSv.

Problem 2

With modern medical X-ray machines patients are exposed to a whole-body dose of 0.1 mSv or even less. A stay at an altitude of 3000 m at average geographical latitudes leads to a dose rate due to cosmic rays of about 0.1 µSv/h corresponding to 67 µSv in four weeks. If also the effect of terrestrial radiation is considered (about

[3] Standard soil usually contains the radioisotopes ^{40}K, ^{226}Ra, and ^{232}Th, dominated mostly by ^{40}K. Averaged over these three isotopes the given specific γ dose constant is a reasonable approximation.

40 µSv in four weeks), one arrives at a total dose which is comparable to the exposure from an X-ray of the chest. However, it has to be mentioned that older X-ray machines may lead to higher exposures. Also, it has to be considered that the dose from an X-ray is received in a very short time interval (ms) with the consequence of a much higher dose rate compared to the effect of natural radiation.[4]

Problem 3

$$D(\text{CASTOR}) = 30\,\mu\text{Sv/h} \times 10\,\text{h} = 0.3\,\text{mSv} \ ,$$
$$\overline{D}(\text{Black Forest}) \approx 5\,\text{mSv/yr} \ .$$

This means that people accompanying a single CASTOR transport will get a dose that corresponds roughly to 6% of the annual dose one gets when living in the Black Forest and only 1.5% of the permitted maximum annual dose of workers of category A in controlled areas.

16.11 Solutions to the Problems of Chapter 12

Problem 1

$$\Delta T = \frac{Q}{c \cdot m} = \frac{0.1 \times 10^9\,\text{W} \times 0.7 \times 200\,\text{s}}{0.116 \times 10^3\,\text{J/(kg K)} \times 50 \times 10^3\,\text{kg}} = 2414\,\text{K} \ .$$

This means that the water evaporates completely and also the reactor core will melt (melting point of uranium 1132.2 °C).

Problem 2

Energy gain $\Delta E = 50\,\text{kW} \times 5 \times 10^5\,\text{yrs} = 7.884 \times 10^{17}\,\text{J} = 4.92 \times 10^{36}\,\text{eV}$. Per fission process about 200 MeV are liberated. Thus the number of fission processes is

$$N = 2.46 \times 10^{28} \ .$$

Given the mass of an ^{235}U nucleus of $m = 235 \times 1.66 \times 10^{-27}\,\text{kg} = 3.90 \times 10^{-25}\,\text{kg}$, the total amount of uranium ^{235}U processed is

$$M = m\,N = 9597\,\text{kg} \approx 9.6\,\text{tons} \ .$$

Problem 3

$$N(^{238}\text{U}) = N_0(^{238}\text{U})\ \exp(-\lambda_1\,t) \ ,$$
$$N(^{235}\text{U}) = N_0(^{235}\text{U})\ \exp(-\lambda_2\,t) \ ;$$
$$N_0(^{235}\text{U}) = N_0(^{238}\text{U}) \, , \ N(^{238}\text{U}) = 0.9965 \, , \ N(^{235}\text{U}) = 0.0035 \, ;$$

[4] For more information about exposures from X-ray examinations see `www.radiologyinfo.org/en/info.cfm?pg=chestrad`. Detailed comparisons with radiation exposures from natural sources are given in the 'safety page', which is linked to the given web page.

$$r = \frac{N(^{238}\mathrm{U})}{N(^{235}\mathrm{U})} = \exp((\lambda_2 - \lambda_1)\, t) \ ;$$

$$t = \frac{1}{\lambda_2 - \lambda_1} \ln r \ ;$$

$$\lambda_2(^{235}\mathrm{U}) = 3.08 \times 10^{-17}\,\mathrm{s}^{-1} \ , \quad \lambda_1(^{238}\mathrm{U}) = 4.87 \times 10^{-18}\,\mathrm{s}^{-1} \ ;$$

$$t = 2.18 \times 10^{17}\,\mathrm{s} \approx 6.9 \times 10^{9}\,\mathrm{yrs} \ .$$

This means that this deposit would be older than the solar system. In other words, a fraction of the observed ^{235}U must have been transformed by fission processes.

Problem 4

The dose rate after one day can be scaled using the given power law: a) $\dot{D}(t = 30\,\mathrm{h}) = \dot{D}(t_0) \cdot (\frac{30}{6})^{-1.2} = 1.45\,\mathrm{mSv/h}$.
The dose obtained in a 10-hour period 30 hours after the accident is obtained by integration: b) $D(t = 30\,\mathrm{h}\text{–}40\,\mathrm{h}) = \int_{t_1}^{t_2} \dot{D}(t)\,\mathrm{d}t$
$= \dot{D}(t_0)\, \frac{1}{t_0^{-\alpha}}\, (t_1^{-0.2} - t_2^{-0.2}) \times 5 = 12.16\,\mathrm{mSv}$.

16.12 Solutions to the Problems of Chapter 13

Problem 1

The effective half-life for ^{137}Cs in humans is

$$T_{1/2}^{\mathrm{eff}} = \frac{T_{1/2}^{\mathrm{phys}}\, T_{1/2}^{\mathrm{bio}}}{T_{1/2}^{\mathrm{phys}} + T_{1/2}^{\mathrm{bio}}} = 109.9\,\mathrm{days} \ .$$

The residual amount of ^{137}Cs after three years can be determined in two different ways:

a) the time interval of three years corresponds to $\frac{3\times365}{109.9} = 9.9636$ half-lives:

$$\mathrm{activity}(3\,\mathrm{yrs}) = 4 \times 10^6 \times 2^{-9.9636} = 4006\,\mathrm{Bq} \ ;$$

b) on the other hand, the time dependence of the activity yields the same result:

$$\mathrm{activity}(3\,\mathrm{yrs}) = 4 \times 10^6 \times \mathrm{e}^{-3\,\mathrm{yrs}\times\ln 2/T_{1/2}^{\mathrm{eff}}} = 4006\,\mathrm{Bq} \ .$$

Problem 2*

Let us assume the pancreas to be spherical. For a density of $\rho = 1\,\mathrm{g/cm^3}$ the radius of the pancreas is worked out from

$$50\,\mathrm{cm}^3 = \frac{4}{3}\,\pi\, r^3 = V_r$$

* The solution of this problem is mathematically demanding.

to be $r = 2.29\,\text{cm}$. The range of 45-keV electrons can be read from Fig. 4.4 by extrapolation to be $R = 5 \times 10^{-3}\,\text{cm}$. Only those electrons in the spherical shell between r and $r - R$ can escape, where $R \ll r$.

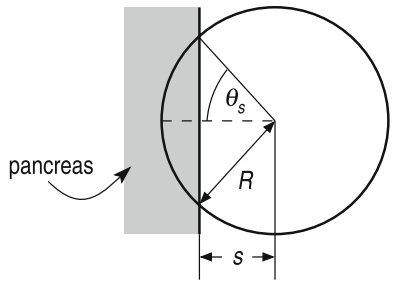
The activity $3.25\,\mu\text{Ci}$ can be converted to $120\,\text{kBq}$. For an average beta energy of $45\,\text{keV}$ an amount of $\langle E \rangle = 5.4 \times 10^9\,\text{eV}$ is deposited per second in the human body. This corresponds to

$$\langle \dot{E} \rangle = 8.7 \times 10^{-10}\,\text{J/s} \ .$$

For the inner part of the pancreas (positions with radii smaller than $r - R$) this leads to a dose rate of

$$\dot{D}_0 = \frac{8.7 \times 10^{-10}\,\text{J/s}}{50 \times 10^{-3}\,\text{kg}} = 1.74 \times 10^{-8}\,\frac{\text{Sv}}{\text{s}} = 62.64\,\mu\text{Sv/h} \ .$$

We now assume that all electrons have the fixed range R. Let s be the distance of a point outside the pancreas from its surface. Then only those rays from a spherical surface with radius R lying inside the pancreas contribute to the dose rate at this point. For $R \ll r$ one can replace the spherical surface of the pancreas by a plane.



The dose rate scales with the ratio of the solid angle Ω_s to the full one (4π); it is integrated in spherical coordinates, where the polar angle varies from 0 to θ_s, which is identified through $\cos\theta_s = s/R$:

$$\begin{aligned}\Omega_s &= 2\pi \int_0^{\theta_s} \sin\theta \, \mathrm{d}\theta = 2\pi \, [-\cos\theta]_0^{\theta_s} = 2\pi(1 - \cos\theta_s) \\ &= 2\pi \left(1 - \frac{s}{R}\right) \ .\end{aligned}$$

Therefore, the dose-rate fraction $f_s = \Omega_s/4\pi$ varies linearly with the distance to the surface, $f_s = (1 - s/R)/2$. This result is also valid for the inner part, $-R \le s < 0$. On average, for the outside the dose rate is scaled by $f_{\text{out}} = (f_0 + f_R)/2 = 1/4$ to be $\dot{D}_2 = \dot{D}_0/4$. On the inside the averaged factor f_{in} is obtained by using the volumes in question, $f_{\text{in}} = (V_r - 4\pi r^2 R f_{\text{out}})/V_r = 1 - 3 f_{\text{out}} R/r \approx 0.9984$, leading to the dose rate $\dot{D}_1 = f_{\text{in}} \dot{D}_0 = 0.9984\ \dot{D}_0$.

Problem 3*

^{226}Ra has a physical half-life of 1600 years. Therefore the effective half-life is

$$T_{1/2}^{\text{eff}} = \frac{T_{1/2}^{\text{phys}}\, T_{1/2}^{\text{bio}}}{T_{1/2}^{\text{phys}} + T_{1/2}^{\text{bio}}} = 299.8\,\text{days}\ ,$$

i.e., it is dominated by the biological half-life.

The old activity unit 1 curie was defined as activity of 1 g ^{226}Ra. The incorporated amount of 1 µg ^{226}Ra corresponds to an activity of

$$A_0 = 3.7 \times 10^{10}\,\frac{\text{Bq}}{\text{g}} \times 1 \times 10^{-6}\,\text{g} = 3.7 \times 10^{4}\,\text{Bq}\ .$$

In the radium decay chain to the stable lead isotope ^{208}Pb a total amount of 33 MeV of α rays, 3 MeV of β rays, and 1.5 MeV of γ rays is liberated. Considering the high biological effectiveness of α rays (radiation weighting factor w_R = 20) the weighted equivalent energy deposition is worked out to be about 665 MeV per 1 Bq ^{226}Ra.

The instantaneous dose rate is

$$\dot{H} = \frac{A\ W}{m}\ ,$$

where A – activity, W – liberated energy per Bq, and m – body mass,

$$\dot{H} = A_0\,\frac{W}{m}\,\exp\left\{-\frac{\ln 2}{T_{1/2}^{\text{eff}}}\,t\right\}\ .$$

The 50-years commitment dose equivalent is obtained by integration to be

$$\begin{aligned} H_{50} &= \int_0^{50\,\text{yrs}} \dot{H}(t)\,\text{d}t \\ &= -\frac{A_0\ W}{m}\,\frac{T_{1/2}^{\text{eff}}}{\ln 2}\,\exp\left\{-\left((\ln 2)\,/T_{1/2}^{\text{eff}}\right)\,t\right\}\Big|_0^{50\,\text{yrs}} \\ &= \frac{A_0\ W\ T_{1/2}^{\text{eff}}}{m\ \ln 2}\left(1 - \exp\left\{-\frac{\ln 2}{T_{1/2}^{\text{eff}}} \times 50\,\text{yrs}\right\}\right)\ . \end{aligned}$$

Using $A_0 = 3.7 \times 10^4\,\text{Bq}$ and $W = 665\,\text{MeV} = 665 \times 1.6 \times 10^{-13}\,\text{J} = 1.064 \times 10^{-10}\,\text{J}$ as well as $T_{1/2}^{\text{eff}} = 299.8\,\text{d} \times 86\,400\,\text{s/d} = 2.59 \times 10^7\,\text{s}$ and $m = 50\,\text{kg}$ we get

$$H_{50} = 2.94 \times \left(1 - 5 \times 10^{-19}\right)\,\text{Sv} = 2.94\,\text{Sv}\ .$$

* The solution of this problem is mathematically demanding.

It is obvious that predominantly lips and tongues were affected by this activity with the consequence that the female dial painters contracted cancer of the lips and tongue and frequently died of heavy radiation damage. Some of the women even intentionally painted their lips and teeth with the radium solution when going to a rendezvous, because the radium caused such a romantic glow in the dark. It is reported that the bodies of the deceased women shone in the dark of the morgue, which is probably an exaggeration.

16.13 Solutions to the Problems of Chapter 14

Problem 1

Such an estimate is subject to substantial uncertainties.[5] This is also related to the fact that individual exposures have not been recorded, even not for the personnel involved in the cleanup ('liquidators'). It is almost unbelievable that suitable radiation detectors were not available. The received radiation doses were only estimated after the fact based on symptoms of radiation sickness and chromosome aberrations in blood samples of the exposed workers. It has also to be considered that the radiation sensitivity of children is significantly higher than that of adults. This comes about because of the increased mitosis in the period of growth, which tends to multiply radiation damage in the body of the children.

Further it has to be considered that the Chernobyl cloud spread over the whole northern hemisphere. For the following estimate we break down the northern hemisphere into five regions. For the increased radiation sensitivity of children we assume a factor of five (which is probably a little pessimistic).

Based on this rather crude estimate we would expect about 10 000 cases of leukemia in children and about the same number of incidents in adults worldwide. In contrast, at the present time, 23 years after the catastrophe in Chernobyl, no significant increase in leukemia rates is observed. However, the thyroid-gland cancer rate in children in the area around Chernobyl has clearly increased.

In view of the non-observation of an increase of leukemia rates (at the time of 2009) the relatively high 'predicted' cancer rates shown in the table could indicate that either the assumed doses or the risk factor were too high. According to the literature the radiation risk factors have changed over the years from previously lower values (ICRP-26 (1977): risk factor for leukemia 0.20% per sievert) to the presently assumed ones (ICRP-60 (1990): risk factor for

[5] I appreciate the information given by Dipl. Phys. Helmut Kowalewsky, Berlin, about the actual number of cases of leukemia and cancer of the thyroid gland in Russia and the Ukraine.

region	number of inhabitants	average estimated dose	expected incidents of leukemia
Chernobyl; 'liquidators'	700 000	0.1 Sv	350
Belorussia and Ukraine;			
children	600 000	20 mSv	300
adults	2.4 Mio.	20 mSv	240
remaining Russia;			
children	60 Mio.	5 mSv	7500
adults	240 Mio.	5 mSv	6000
Western Europe;			
children	100 Mio.	0.5 mSv	1250
adults	400 Mio.	0.5 mSv	1000
USA, Japan, China, . . .;			
children	680 Mio.	0.1 mSv	1700
adults	2 700 Mio.	0.1 mSv	1350

Table 16.1
Estimate of possible leukemia rates based on the given doses and the new risk factors (ICRP-60 (1990)) as shown in Table 13.1[6]

leukemia 0.50% per sievert). In the light of the observed current leukemia rates the new radiation risk factors as presented in Table 13.1 appear to be too pessimistic.

Problem 2

The specific dose constants for β and γ rays of ^{60}Co are

$$\Gamma_\beta = 2.62 \times 10^{-11} \times \frac{\mathrm{Sv\,m^2}}{\mathrm{Bq\,h}} \ ,$$

$$\Gamma_\gamma = 3.41 \times 10^{-13} \times \frac{\mathrm{Sv\,m^2}}{\mathrm{Bq\,h}} \ .$$

For the irradiation of the hands the β dose dominates. Let us assume an average distance of 10 cm and a handling time of the source of 60 seconds. This would lead to a partial-body dose of

$$H_\beta = \Gamma_\beta \frac{A}{r^2} \Delta t = 2.62 \times 10^{-11} \times \frac{3.7 \times 10^{11}}{0.1^2} \times \frac{1}{60} \,\mathrm{Sv} = 16.1 \,\mathrm{Sv} \ .$$

On the other hand, the whole-body dose essentially depends on γ rays from the ^{60}Co source. For an average distance of 0.5 m and an exposure time of 5 minutes the whole-body dose is estimated to be

$$H_\gamma = \Gamma_\gamma \frac{A}{r^2} \Delta t = 42 \,\mathrm{mSv} \ .$$

[6] Based on information from the report of L. A. Iljin: Atomnaja Energija, Volume 92, Issue 2, 2002.

Actually, such an accident happened to technicians in Saintes, France, in 1981. Under no circumstances they should have handled the strong source with their bare hands. Due to the severe radiation damage the hands of two technicians had to be amputated. For a third technician the amputation of three fingers was necessary.

Problem 3

360 kg uranium in 30 m^3 of water corresponds to a concentration of 12 g/l. This is a factor of $\frac{12\,\mathrm{g}}{15\,\mu\mathrm{g}} = 800\,000$ larger than the WHO-recommended limit for drinking water.

Natural uranium mainly consists of the isotopes ^{234}U (0.0055%, $T_{1/2} = 2.455 \times 10^5$ yrs), ^{235}U (0.7204%, $T_{1/2} = 7.038 \times 10^8$ yrs), and ^{238}U (99.2742%, $T_{1/2} = 4.468 \times 10^9$ yrs). Given the isotopic abundances and the half-lives the total activity of 12 g natural uranium contained in one liter of water is 305 kBq. In depleted uranium the amount of ^{234}U and ^{235}U is reduced resulting in an estimated activity of $\approx$ 200 kBq. This corresponds to 200 MBq/m^3. Compared to the ICRP limit for discharges from nuclear power plants this is higher by a factor of $\frac{2\times10^8}{3000} \approx 67\,000$. Naturally, the uranium concentration was rapidly decreased when it was discharged into the rivers.

Typical values for the activity of uranium isotopes in tap water in Europe are around 10 to 20 Bq/m^3. Apart from the radioactivity, uranium solutions are also highly toxic from the chemical point of view.

16.14 Solutions to the Problems of Chapter 15

Problem 1

In Maxwell's theory the current density j, the magnetic field H, and the electrical field in a poorly conducting medium are related by

$$\mathrm{rot}\,\boldsymbol{H} = \boldsymbol{j} + \boldsymbol{j}_\mathrm{V} \ , \quad \boldsymbol{j}_\mathrm{V} = \varepsilon\,\varepsilon_0\,\frac{\partial \boldsymbol{E}}{\partial t}$$

(ε, ε_0 – relative and absolute dielectric constant). The currents in a poorly conducting medium can be neglected. Only the displacement current density $\boldsymbol{j}_\mathrm{V}$ contributes. In one period (50 Hz, corresponding to 20 ms) the electric field varies according to a sine function ($\boldsymbol{E} = \boldsymbol{E}_0 \sin\omega t$). From $E = 0$ up to the first maximum it takes 5 ms.

Using $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m and $\varepsilon(\mathrm{water}) = 81$ as tissue-equivalent medium, one finds for the current density

$$\begin{aligned} j_\mathrm{V} &= 81 \times 8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}} \times 5 \times 10^3\,\frac{\mathrm{V}}{\mathrm{m}} \times \frac{1}{5 \times 10^{-3}\,\mathrm{s}} \\ &= 7.2 \times 10^{-4}\,\frac{\mathrm{A\,s}}{\mathrm{V\,m}} \times \frac{\mathrm{V}}{\mathrm{m\,s}} = 7.2 \times 10^{-4}\,\frac{\mathrm{A}}{\mathrm{m}^2} \\ &= 7.2 \times 10^{-8}\,\frac{\mathrm{A}}{\mathrm{cm}^2} \approx 0.1\,\mu\mathrm{A/cm}^2 \ . \end{aligned}$$

Problem 2

The electrical field of a conductor with respect to the Earth's surface (assumed to be flat) can be reasonably well approximated by:

$$E = \frac{2}{\ln(2h/r_\mathrm{D})}\frac{U}{h} \ .$$

The electrical field of a free wire varies with distance from the wire like $1/r$. The related potential can be worked out by integration from the surface of the wire (Radius $r_\mathrm{D} \ll h$, height h of the wire above ground) effectively up to twice the height h (because of the mirror charge, one has voltage zero on the ground; the mirror charge also causes the factor of 2 in the numerator). For an assumed wire radius of $r_\mathrm{D} = 1\,\mathrm{cm}$ we get

$$\begin{aligned} E &= \frac{2}{\ln(2 \times 30\,\mathrm{m}/1\,\mathrm{cm})} \times \frac{220 \times 10^3\,\mathrm{V}}{30\,\mathrm{m}} = \frac{2}{8.7} \times 7.3\,\mathrm{kV/m} \\ &\approx 1.7\,\mathrm{kV/m} \ . \end{aligned}$$

In many countries the allowed limit is at $20\,\mathrm{kV/m}$, well above the obtained result.

The magnetic field strength is worked out from

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = I$$

to be ($r = h$)

$$H = \frac{I}{2\pi r} = \frac{10^3\,\mathrm{A}}{2\pi \times 30\,\mathrm{m}} = 5.3\,\mathrm{A/m} \ ,$$

corresponding to

$$B = \mu\mu_0 H = 4\pi \times 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2} \times 5.3\,\frac{\mathrm{A}}{\mathrm{m}} = 6.7 \times 10^{-6}\,\frac{\mathrm{V\,s}}{\mathrm{m}^2} = 6.7\,\mu\mathrm{T} \ ,$$

well below the natural magnetic field of the Earth.

Problem 3

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = \int \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{A} \ \Rightarrow \ H = \frac{I}{2\pi r} \ ,$$

$$P = 100\,\mathrm{W} = U\ I \ ,$$

$$I = \frac{P}{U} = \begin{cases} 8.33\,\mathrm{A} & \text{, halogen lamp} \\ 0.455\,\mathrm{A} & \text{, 220 V bulb} \end{cases} \ ,$$

$$\Rightarrow \quad H = \begin{cases} 1.33\,\mathrm{A/m} \ \widehat{=}\ 1.67\,\mu\mathrm{T} & \text{, halogen lamp} \\ 0.072\,\mathrm{A/m} \ \widehat{=}\ 90.5\,\mathrm{nT} & \text{, 220 V bulb} \end{cases} \ .$$

In both cases the magnetic field strength is acceptable.

Formulary

activity

1 becquerel (Bq) = 1 decay per second
1 curie (Ci) = 3.7×10^{10} Bq
1 Bq = 27×10^{-12} Ci = 27 pCi

energy dose

(or simply **dose**) D (Gy), $\quad 1\,\mathrm{Gy} = 1\,\mathrm{J/kg}$

dose equivalent

H (Sv), $\quad H = w_R\, D$

w_R – radiation weighting factor (see Table 2.1)

ion dose

$I = 1\,\mathrm{C/kg}$

1 roentgen (R) = 2.58×10^{-4} C/kg $\widehat{=}$ 8.8 mGy

dose

$$D = \Gamma\, \frac{A}{r^2}\, t \quad \text{(for a pointlike emitter)}$$

Γ – dose constant in units of Sv m^2/(Bq h)
A – activity in Bq; t – time in hours (h);
r – distance in meters (m)

dose rate

$$\dot{D} = \Gamma\, \frac{A}{r^2} \quad \text{(for a pointlike emitter)}$$

decay law

$$N = N_0\, \mathrm{e}^{-\lambda t}$$

λ – decay constant in s^{-1}; $\lambda = \ln 2 / T_{1/2} = 1/\tau$
$T_{1/2}$ – half-life
τ – lifetime

effective half-life

$$T_{1/2}^{\mathrm{eff}} = \frac{T_{1/2}^{\mathrm{phys}}\, T_{1/2}^{\mathrm{bio}}}{T_{1/2}^{\mathrm{phys}} + T_{1/2}^{\mathrm{bio}}}$$

$T_{1/2}^{\mathrm{phys}}$ – physical half-life
$T_{1/2}^{\mathrm{bio}}$ – biological half-life

effective whole-body dose equivalent

$$E = H_{\mathrm{eff}} = \sum_T w_T \sum_R w_R D_{T,R}$$

w_T – tissue weighting factor (see Table 2.2)
w_R – radiation weighting factor (see Table 2.1)
One has to sum over the relevant radiation fields R and tissues T.

$k = \frac{\Delta E}{\Delta m}$ **kerma**

ΔE – sum over the initial kinetic energies of all charged particles liberated in a volume element by indirectly ionizing radiation
$\Delta m = \rho \, \Delta V$ – mass element (ρ – density)

$I = I_0 \, \mathrm{e}^{-\mu x}$ **attenuation law for γ rays**

μ – mass attenuation coefficient

$I = I_0 \, \mathrm{e}^{-\mu_\mathrm{a} x}$ **absorption law for γ rays**

μ_a – mass absorption coefficient

$N = N_0 \, \mathrm{e}^{-\kappa x}$
$\kappa = 15/E_{\beta_\mathrm{max}}^{1.5}$ $\quad (E_{\beta_\mathrm{max}}$ in MeV, κ in $(\mathrm{g/cm^2})^{-1})$ **empirical absorption law for β rays**

$R[\mathrm{g/cm^2}] = 0.526 \times E_\mathrm{kin}[\mathrm{MeV}] - 0.095$ **empirical range for electrons**

$$A = \frac{\text{measured activity} - \text{background rate}}{\text{detection efficiency}}$$ **activity determination**

$$\sum_i \frac{A_i}{A_i^\mathrm{max}} \leq 1$$ **exemption limit for several radioactive sources**

A_i – activity of material i
A_i^max – exemption limit of material i

$$x_{1/2} = \frac{\ln 2}{\mu}$$ **half-value thickness**

$$x_{1/10} = \frac{\ln 10}{\mu}$$ **tenth-value thickness**

$N_\mathrm{true} = \frac{N}{1 - N\,\tau}$ **dead-time correction**

N – measured count rate
τ – dead time

$f(N, \mu) = \frac{\mu^N \mathrm{e}^{-\mu}}{N!}$ **Poisson distribution**

μ – average value, $\mu = \frac{1}{k} \sum_{i=1}^{k} N_i$

$f(N, \mu) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(N-\mu)^2}{2\sigma^2}\right)$ **Gaussian distribution**

σ – root-mean-square deviation