Amount of Information and Its Reliability in the Ranking of Atanassov's Intuitionistic Fuzzy Alternatives

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Abstract. In this paper we discuss the ranking of alternatives represented by elements of Atanassov's intuitionistic fuzzy sets, to be called A-IFSs, for short. That is, alternatives are elements of the universe of discourse with a degree of membership and a degree of non-membership assigned. First, we show disadvantages of some approaches known from the literature, including a straightforward method based on the calculation of distances from the ideal positive alternative which can be viewed as a counterpart of the approach in the traditional fuzzy setting. Instead, we propose an approach which takes into account not only the amount of information related to an alternative (expressed by a distance from an ideal positive alternative) but also the reliability of information represented by an alternative meant as how sure the information is.

1 Introduction

Atanassov's intuitionistic fuzzy sets (cf. Atanassov [2], [3]), to be called A-IFSs for brevity, which are a generalization of the fuzzy sets(Zadeh [35]) can be viewed as a tool that may help better model imperfect information, especially under imperfectly defined facts and imprecise knowledge. A-IFSs have found numerous applications in many areas, notably decision making. One of important, omnipresent problems in the context of decision making, and many other contexts, is the ranking of fuzzy (or intuitionistic fuzzy) alternatives (options), for instance obtained as a result of decision analysis, evaluation, aggregation, etc. The fuzzy and intuitionistic fuzzy alternatives may be understood in different ways, and in this paper we meant them, in the fuzzy case, as elements of the universe of discourse with their associated membership degrees, and, in the intuitionistic fuzzy case, as elements of a universe of discourse with their associated membership and non-membership degrees. We consider here the latter case, and then a natural interpretation in our context of decision making can be that each option fulfills a set of criteria to some extent $\mu(.)$ and, on the other hand, it does not fulfill this set of criteria to some extent $\nu(.)$. This clearly suggest that the alternatives can conveniently

E. Rakus-Andersson et al. (Eds.): Recent Advances in Decision Making, SCI 222, pp. 7–19.
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be expressed via Atanassov's intuitionistic fuzzy sets. For brevity, such alternatives will be called *intuitionistic fuzzy alternatives*.

The problem of ranking intuitionistic fuzzy alternatives may be solved under some additional assumptions only because there is no linear order among elements of the A-IFSs as opposed to that for fuzzy sets (Zadeh [35]) for which elements of the universe of discourse are naturally ordered because their membership degrees are real numbers from $[0, 1]$.

In the literature there are not many approaches for ranking the intuitionistic fuzzy alternatives. They were proposed by, for instance, Chen and Tan [5], Hong and Choi [7], Li et al. [8], [9], and Hua-Wen Liu and Guo-Jun Wang [10].

Here we propose another approach that is different in several respects.

First, we employ the representation of A-IFSs, which constitute the representation of intuitionistic fuzzy alternatives, taking into account all three functions: the membership function, non-membership function, and hesitation margin. Such a representation has proved to be effective and efficient in solving many problems giving intuitively appealing results (cf. e.g., Szmidt and Kacprzyk[28], [21], [30]), [31]) while constructing measures of a distance, similarity, entropy, etc. that play a crucial role in virtually all information processing tasks, notably those related to decision making.

Second, we propose an ordering function for ranking intuitionistic fuzzy alternatives which depends on two factors: the amount of information associated with an alternative (expressed by the distance from the ideal positive alternative), and the reliability of information (i.e. how sure an alternative is) – expressed by the hesitation margin.

As an example we present an application to a choice of a best course of action in the context of medical treatment.

2 A Brief Introduction to Intuitionistic Fuzzy Sets

One of the possible generalizations of a fuzzy set in X (Zadeh [35]), given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0, 1]$ is the membership function of the fuzzy set $A^{'}$, is Atanassov's intuitionistic fuzzy set (Atanassov [1], [2], [3]) A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2}$$

where: $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote the degree of membership and a degree of non-membership of $x \in A$, respectively.

Obviously, each fuzzy set may be represented by the following intuitionistic fuzzy set

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{4}$$

For each intuitionistic fuzzy set in X, we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

an *intuitionistic fuzzy index* (or a *hesitation margin*) of $x \in A$ and, it expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [15], [19], [28], entropy (Szmidt and Kacprzyk [21], [30]), similarity (Szmidt and Kacprzyk [31]) for the A-IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks. In this paper the hesitation margin is shown to be useful, if not indispensable, in ranking the intuitionistic fuzzy alternatives because it indicates how reliable (sure) the information represented by an alternative is.

The use of A-IFSs instead of fuzzy sets implies the introduction of another degree of freedom (non-memberships) into the set description. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge which leads to describing many real problems in a more adequate way. Applications of intuitionistic fuzzy sets to group decision making, negotiations, voting and other situations are presented in Szmidt and Kacprzyk [14], [16], [17], [20], [22], [24], [23], [25], [29], Szmidt and Kukier [32], [33]. (because of the different approaches presented in the works cited above, we are not able to discuss details here, and refer the interested reader directly to them).

2.1 Geometrical Representation

One of the possible geometrical representations of an intuitionistic fuzzy sets is given in Figure 1 (cf. Atanassov [3]). It is worth noticing that although we use a two-dimensional figure (which is more convenient to draw in our further considerations), we still adopt our approach (e.g., Szmidt and Kacprzyk [19], [28], [21], [30]), [31]) taking into account all three functions (membership, non-membership and hesitation margin values) describing an intuitionistic fuzzy set. Any element belonging to an intuitionistic fuzzy set may be represented inside an MNO triangle. In other words, the MNO triangle represents a surface where the coordinates of any element belonging to an A-IFS can be represented. Each point belonging to the MNO triangle is described by the three coordinates: (μ, ν, π). Points M and N represent crisp elements. Point $M(1, 0, 0)$ represents elements fully belonging to an A-IFS as $\mu = 1$, and may be seen as the representation of the ideal positive element. Point $N(0, 1, 0)$ represents elements fully not belonging to an A-IFS as $\nu = 1$, i.e. can be viewed as the ideal negative element. Point $O(0, 0, 1)$ represents elements about which we are not able to say if they belong or not belong to an A-IFS (intuitionistic fuzzy index $\pi = 1$). Such an interpretation is intuitively appealing and provides means for the representation of many aspects of imperfect information. Segment MN (where $\pi = 0$) represents elements belonging to the classic fuzzy sets ($\mu + \nu = 1$). For example, point $A(0.5, 0.5, 0)$ (Figure 1), like any element from segment MN represents an element of a fuzzy set. A line parallel to MN describes the elements with the same values of the hesitation margin. In Figure 1 we can see point $B(0.4, 0.4, 0.2)$ representing an element with the hesitation margin equal 0.2, like $D(0.1, 0.7, 0.2)$, $E(0.5, 0.3, 0.2)$ and all elements on the line pointed out by any two from B, E, D. The closer a parallel line to MN is to O, the higher the hesitation margin.

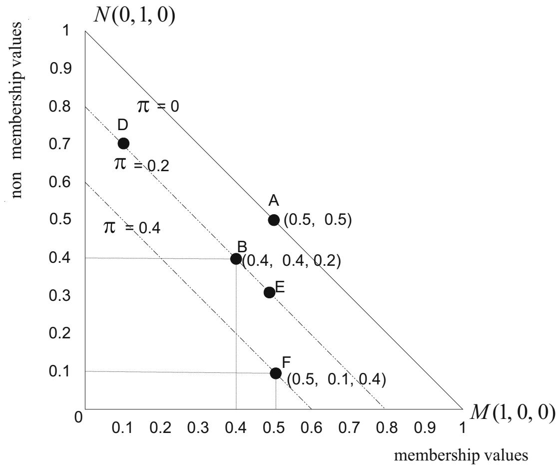

Fig. 1. Geometrical representation

Remark: We use the capital letters (e.g., A, B, C) for the geometrical representation of x_i's (Figure 1) on the plane. The same abbreviations (capital letters) mean in this paper the sets but we always explain the current meaning of a symbol used.

2.2 Distances between the A-IFSs

In Szmidt and Kacprzyk [19], Szmidt and Baldwin [12,13], and especially in Szmidt and Kacprzyk [28] it is shown why while calculating distances between the A-IFSs we should take into account all three functions describing the A-IFSs. In [28] not only the reasons why we should take into account all three functions are given but also some possible serious problems that can occur while taking into account two functions only and that can imply some serious conceptual and numerical difficulties.

In our further considerations we will use the normalized Hamming distance between the A-IFSs A, B in $X = \{x_{1,}, \ldots, x_n\}$ (cf. Szmidt and Baldwin [12,13], Szmidt and Kacprzyk [19], [28]):

$$l_{IFS}(A, B) =$$
$$= \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \quad (6)$$

For (6) we have: $0 \leq l_{IFS}(A, B) \leq 1$. Clearly the normalized Hamming distance (6) satisfies the conditions of the metric.

3 Ranking the Alternatives

First, we will remind briefly some more relevant approaches known from the literature.

Chen and Tan [5] [1] proposed the concept of a score function for an intuitionistic fuzzy alternative $a = (\mu, \nu)$ meant as

$$S(a) = \mu - \nu, \tag{7}$$

and, clearly, $S(a) \in [-1, 1]$.

It follows immediately from (7) that the score function $S(a)$ alone is not enough for evaluating intuitionistic fuzzy alternatives as it produces the same result for such different intuitionistic fuzzy alternatives $a = (\mu, \nu)$ as, e.g.,: $(0.5, 0.4)$, $(0.4, 0.3)$, $(0.3, 0.2)$, $(0.1, 0)$ – for all of them $S(a) = 0.1$ which seems counterintuitive.

Then Hong and Choi [7] considered in addition to the score function as defined above, a so-called accuracy function H

$$H(a) = \mu + \nu, \tag{8}$$

where $H(a) \in [0, 1]$.

By making use of (7) and (8), Xu [34] proposed an algorithm ranking the intuitionistic fuzzy alternatives. We will present here its idea in the case of two alternatives a_i and a_j [34]:

- if $S(a_i) \leq S(a_j)$, then a_i is smaller than a_j;
- if $S(a_i) = S(a_j)$, then:
  - if $H(a_i) = H(a_j)$, then a_i and a_j represent the same information (are equal);
  - if $H(a_i) \leq H(a_j)$, then a_i is smaller than a_j.

However, the above ranking does not meet our expectation in many cases. Let us consider two intuitionistic fuzzy alternatives $a_1 = (0.5, 0.45)$ and $a_2 = (0.25, 0.05)$ for which we obtain $S(a_1) = 0.5 - 0.45 = 0.05$, $S(a_2) = 0.25 - 0.05 = 0.2$, suggesting that a_1 is smaller than a_2. But the information supplied by a_1 (i.e. $0.5 + 0.45 = 0.95$) is for sure greater than those supplied by a_2 (i.e. $0.25 + 0.05 = 0.3$). In other words, it is difficult to agree that a_1 is smaller than a_2. Later we will return to ranking the two intuitionistic fuzzy alternatives by the method we propose.

3.1 Ranking Alternatives via Distances from the Ideal Positive Alternative

In Section 2 we have mentioned some possible applications of the A-IFSs, among others, those related to voting. Now we will try to propose how to rank the voting alternatives expressed via intuitionistic fuzzy elements.

Let an element x belonging to an A-IFS characterized via (μ, ν, π) expresses a voting situation: μ means the proportion (from $[0, 1]$) of voters who vote for x, ν the proportion of those who vote against x, and π of those who abstain. The simplest idea to compare different voting situations (ranking the alternatives) seems to use a distance measure from the ideal voting situation $M = (x, 1, 0, 0)$ (100% voting for, 0% vote against and 0% abstain) to the alternatives considered. We will call M the ideal positive alternative.

[1] The score function in [5] is discussed for vague sets [6] but Bustince and Burillo [4] have proved that vague sets are equivalent to Atanassov's intuitionistic fuzzy sets.

Let
$A = (x, 0.5, 0.5, 0)$ – 50% vote for, 50% against, and 0% abstain,
$B = (x, 0.4, 0.4, 0.2)$ – 40% vote for, 40% vote against and 20% abstain,
$C = (x, 0.3, 0.3, 0.4)$ – 30% vote for, 30% vote against and 40% abstain.

Certainly, the method of calculating distances between two A-IFSs A and B using the membership and non-membership values only (9) does not work properly (cf. Szmidt and Kacprzyk [19], [28], Szmidt and Baldwin [12], [13]) in this case, too:

$$l_2(A, B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|) \tag{9}$$

The results from (9), i.e., the distances for the above voting alternatives represented by points A, B, C (cf. Figure 2) from the ideal positive alternative represented by $M(1, 0, 0)$ are, respectively:

$$l_2(M, A) = 0.5(|1 - 0.5| + |0 - 0.5|) = 0.5 \tag{10}$$
$$l_2(M, B) = 0.5(|1 - 0.4| + |0 - 0.4|) = 0.5 \tag{11}$$
$$l_2(M, C) = 0.5(|1 - 0.3| + |0 - 0.3|) = 0.5 \tag{12}$$

The results seems to be counterintuitive as (9) suggests that all the alternatives (represented by) A, B, C seem to be “the same”. On the other hand, the normalized Hamming distance (6) taking into account besides the membership and non-membership the hesitation margin too, gives:

$$l_{IFS}(M, A) = 0.5(|1 - 0.5| + |0 - 0.5| + |0 - 0|) = 0.5 \tag{13}$$
$$l_{IFS}(M, B) = 0.5(|1 - 0.4| + |0 - 0.4| + |0 - 0.2|) = 0.6 \tag{14}$$
$$l_{IFS}(M, C) = 0.5(|1 - 0.3| + |0 - 0.3| + |0 - 0.4|) = 0.7 \tag{15}$$

The results (13)–(15) seem to reflect our intuition: alternative A seems to be the best in the sense that the distance $l_{IFS}(M, A)$ is the smallest (we know for sure that 50% vote for, 50% vote against). The situation is given in Figure 2. The alternative represented A is just a fuzzy alternative (A lies on MN where the values of the hesitation margin are equal 0). On the other hand, alternatives B and C are “less sure” (with the hesitation margin equal 0.2, and 0.4, respectively).

However, a weak point in the ranking of alternatives by calculating the distances from the ideal positive alternative represented by M is that for a given value of the membership function, (6) gives just the same value (for example, if the membership value μ is equal 0.8, for any intuitionistic fuzzy element, i.e. such that its non-membership degree ν and hesitation margin π fulfill $\nu + \pi = 0.2$, is equal 0.2). It is shown in Figure 3, a and b. To better see this, the distances (6) for any alternative from M (Figure 3a) are presented for μ and ν for the whole range $[0, 1]$ (instead for $\mu + \nu \leq 1$ only). For the same reason (to better see the effect), in Figure 3b) the contour plot of the distances (6) is given only for the range of μ and ν for which $\mu + \nu \leq 1$).

The conclusion is that the distances from the ideal positive alternative alone do not make it possible to rank the alternatives in the intended way.

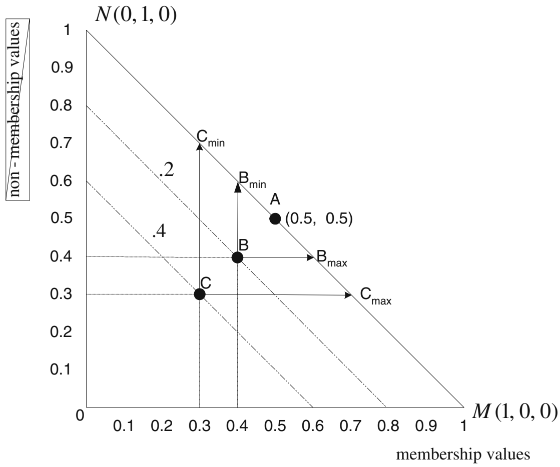

Fig. 2. Geometrical representation of IFSs

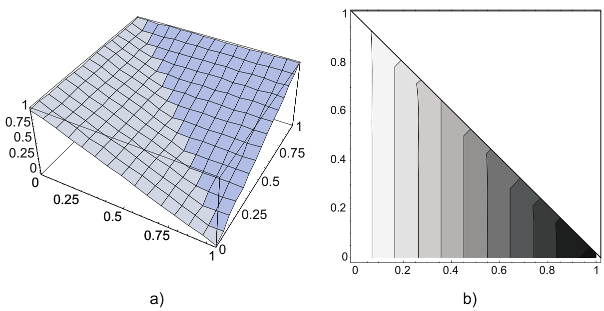

Fig. 3. a) Distances (6) of any IFS element from ideal alternative M; b) contour plot

3.2 A New Method for Ranking Alternatives

Let us analyze the sense of a voting alternative (expressed via an intuitionistic fuzzy element) using the operators of (cf. Atanassov [3]): *necessity* ($\Box$), *possibility* ($\Diamond$), $D_\alpha(A)$ and $F_{\alpha,\beta}(A)$ given as:

- The *necessity* operator ($\Box$)

$$\Box A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle | x \in X\} \tag{16}$$

- The *possibility* operator ($\Diamond$)

$$\Diamond A = \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle | x \in X\} \tag{17}$$

- Operator $D_\alpha(A)$ (where $\alpha \in [0,1]$)

$$D_\alpha(A) = \{\langle x,\ \mu_A(x) + \alpha\pi_A(x),\ \nu_A(x)(1-\alpha)\pi_A(x)\rangle\, | x \in X\} \tag{18}$$

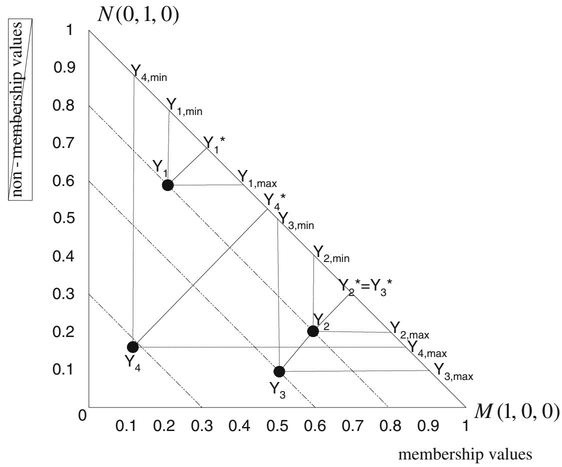

Fig. 4. Ranking alternatives Y_i

- Operator $F_{\alpha,\beta}(A)$ (where $\alpha, \beta \in [0, 1]; \alpha + \beta \leq 1$)

$$F_{\alpha,\beta}(A) = \{\langle x,\ \mu_A(x) + \alpha\pi_A(x), \nu_A(x)\beta\pi_A(x)\rangle \, | x \in X\} \tag{19}$$

For example, for alternative $B(0.4, 0.4, 0.2)$ we obtain $\Box B = B_{min}$, where $B_{min} = (0.4, 0.6)$, and $\Diamond B = B_{max}$, where $B_{max} = (0.6, 0.4)$ (Figure 2). Operator $F_{\alpha,\beta}(A)$ makes it possible for alternative B to become any alternative represented in triangle $BB_{max}B_{min}$. A similar reasoning leads to the conclusion that alternative C (Figure 2) might become any alternative represented in triangle $CC_{max}C_{min}$, and alternative $O(0, 0, 1)$ (because of the hesitation margin equal 1) may become any alternative (the whole area of the triangle MNO).

Having the above considerations in mind we could say that the smaller the area of the triangle $Y_iY_{i,min}Y_{i,max}$ (Figure 4) the better alternative Y_i from a set Y of the alternatives considered. Alternatives having their representations on segment MN are the best in the sense that:

- the hesitation margin is equal 0 here, which means that the alternatives are fully reliable in the sense of the information represented, and
- the alternatives are ordered – the closer an alternative to ideal positive alternative $M(1, 0, 0)$, the better it is (it is an obvious fact as fuzzy alternatives are univocally ordered).

The above reasoning suggests that a promising way of ranking the intuitionistic fuzzy alternatives Y_i with the same values of π_i is converting them into the fuzzy alternatives (which may be easily ranked). For alternatives Y_i with different values of π_i the simplest way to rank the alternatives seems to be to use information carried by triangles $Y_iY_{i,min}Y_{i,max}$.

Y_i^* indicates the amount of information connected with Y_i (the amount of information is indicated by "the position" of triangle $Y_iY_{i,min}Y_{i,max}$ inside triangle

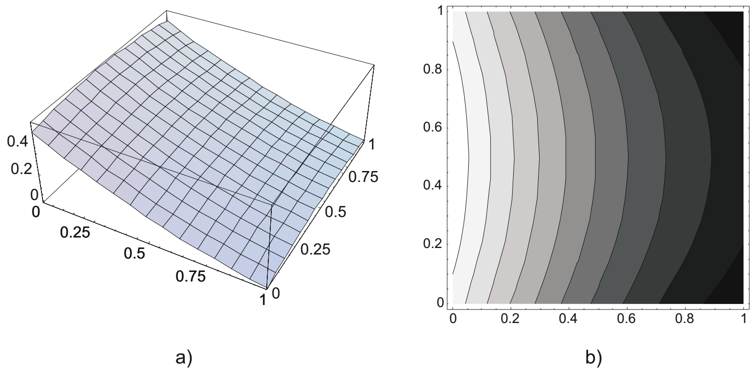

Fig. 5. a) $R(Y_i)$ as a function of a distance Y_i^* from M and a hesitation margin; b) contour plot

MNO – expressed by the projection on segment MN). The value of the hesitation margin π_{Y_i} indicates how sure (reliable) is the information represented by Y_i^*.

Y_i^* are the orthogonal projections of Y_i on MN. Szmidt and Kacprzyk [18] considered such an orthogonal projection of the intuitionistic fuzzy elements belonging to an intuitionistic fuzzy set A. This orthogonal projection may be obtained via operator $D_\alpha(A)$ (18) with parameter α equal 0.5.

It is worth noticing that all the elements from segment OA (Figure 2) are transformed by $D_{0.5}(A)$ (18) into $A(0.5, 0.5)$ which reflects a lack of differences between the membership and non-membership, no matter which the value of the hesitation margin is.

In this context, a reasonable measure R that can be used for ranking the alternatives (represented by) Y_i seems to be

$$R(Y_i) = 0.5(1 + \pi_{Y_i}) l_{IFS}(M, Y_i^*) \tag{20}$$

where $l_{IFS}(M, Y_i^*)$ is the distance (6) from the ideal positive alternative $M(1, 0, 0)$, Y_i^* is the orthogonal projection of Y_i on MN. Constant 0.5 was introduced in (20) to ensure that $0 < R(Y_i) \leq 1$. The values of function R for any intuitionistic fuzzy element are presented in Figure 5a, and the counterpart contour plot – in Figure 5b. Unfortunately, the results obtained (20) do not rank the alternatives in the intended way. (The maximum value of (20) is not obtained for the alternative $(0, 0, 1)$ but for $(0, 1/2, 1/2)$.)

A better measure R that can be used for ranking the alternatives (represented by) Y_i seems to be

$$R(Y_i) = 0.5(1 + \pi_{Y_i}) l_{IFS}(M, Y_i) \tag{21}$$

where $l_{IFS}(M, Y_i)$ is the distance (6) Y_i from ideal positive alternative $M(1, 0, 0)$.

Equation (21) tells us about the "quality" of an alternative – the lower the value of $R(Y_i)$, (21), the better the alternative in the sense of the amount of positive information included, and reliability of information.

The best is alternative $M(1, 0, 0)$ for which $R(M) = 0$. For alternative $N(0, 1, 0)$ we obtain $R(N) = 0.5$ (alternative N is fully reliable as the hesitation margin is equal 0, but the distance $l_{IFS}(M, N) = 1$). Alternative A (Figures 1, 2) gives $R(A) = 0.25$.

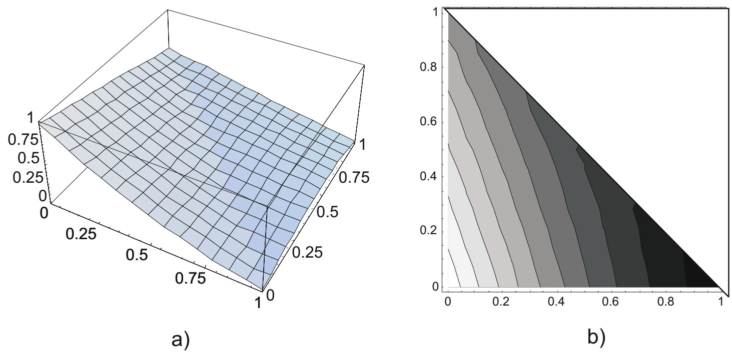

Fig. 6. a) $R(Y_i)$ as a function of a distance Y_i from M and a hesitation margin; b) contour plot

In general, on MN, the values of R decrease from 0.5 (for alternative N) to 0 (for the best alternative M). The maximal value of R, i.e. 1, we obtain for $O(0, 0, 1)$ for which both the distance from M and hesitation margin are equal 1 (alternative O "indicates" the whole triangle MNO). All other alternatives Y_i "indicate" smaller triangles $Y_iY_{i,min}Y_{i,max}$ (Figure 4), so their counterpart values of R are smaller (better in the sense of the amount of the reliable information).

The values of function R (21) for any intuitionistic fuzzy element are presented in Figure 6a, and the counterpart contour plot – in Figure 6b. Considering the numbers obtained via R (21), we may notice that the value 0.25 obtained for the alternative (0.5, 0.5, 0) constitutes the "border" of the "interesting" alternatives – in the sense of the amount of the positive knowledge.

Let us return to the ranking of two alternatives (which were ranked counter-intuitively by the algorithm presented in [34] as shown in the beginning of Section 3), i.e., $Y_1 = (0.5, 0.45, 0.05)$ and $Y_2 = (0.25, 0.05, 0.7)$ (we stress here that we take into account all three values: the degrees of membership, non-membership and hesitation margin). From (21) we obtain: $R(Y_1) = 0.26$, $R(Y_2) = 0.64$ which means that Y_1 is better than Y_2 (previously, from the algorithm [34] Y_2 was better/bigger than Y_1). Obviously, Y_1 is not a "good" option as $R(Y_1)$ is bigger than 0.25 which follows from the fact that the non-membership value is quite big (equal 0.45). It might mean that we would not accept option Y_1. But option Y_2 seems even less interesting – with the smaller membership value (equal 0.25 instead of 0.5 for Y_1), and with the bigger hesitation margin (equal 0.7 instead of 0.05 for Y_1).

Example 1. Let us consider the ranking of six medical treatments, $C1$ – $C6$, affecting a patient in the following way:

- $C1 : (0.6, 0.2, 0.2)$ – influences in a positive way 60% of symptoms, in a negative way – 20% of symptoms, and its impact is unknown (was not confirmed) in a case of 20% of symptoms;
- $C2 : (0.7, 0.3, 0)$ – influences in a positive way 70% symptoms, in a negative way – 30% of symptoms, and its impact is unknown (was not confirmed) in case of 0% of symptoms;

Table 1. Ranking alternatives by R (21) – results for the data from Example1

No.	$C_i : (\mu_i, \nu_i, \pi_i)$	$R_E(C_i)$
1	$C1 : (0.6, 0.2, 0.2)$	0.240
2	$C2 : (0.7, 0.3, 0)$	0.150
3	$C3 : (0.7, 0.15, 0.15)$	0.173
4	$C4 : (0.775, 0.225, 0)$	0.113
5	$C5 : (0.8, 0.1, 0.1)$	0.110
6	$C6 : (0.8, 0.2, 0)$	0.100

- $C3 : (0.7, 0.15, 0.15)$ – influences in a positive way 70% of symptoms, in a negative way – 15% of symptoms, and its impact is unknown (was not confirmed) in case of 15% of symptoms;
- $C4 : (0.775, 0.225, 0)$ – influences in a positive way 77.5% of symptoms, in a negative way – 22.5% of symptoms, and its impact is unknown (was not confirmed) in case of 0% of symptoms;
- $C5 : (0.8, 0.1, 0.1)$ – influences in a positive way 80% of symptoms, in a negative way – 10% of symptoms, and its impact is unknown (was not confirmed) in case of 10% of symptoms;
- $C6 : (0.8, 0.2, 0)$ – influences in a positive way 80% of symptoms, in a negative way – 20% of symptoms, and its impact is unknown (was not confirmed) in case of 0% of symptoms;

The ranking of $C1, \ldots, C6$ from (21) is given in Table 1 – from the worst one, $C1$ to the best one, $C6$.

In general, the ranking function R (21) is constructed by strongly taking into account the lack of knowledge. Let us consider the pair: $C1$ and $C2$. In the case of $C1$ the lack of knowledge is 0.2, so that theoretically, we might expect "on the average" that the hesitation margin representing the lack of knowledge will be divided equally between the membership function and non-membership function giving as a result the case $C2$. But if we wish to avoid the most disadvantageous cases, we will rank $C2$ higher so as to avoid the possibility which might by implied by $C1$, namely: $(0.6, 0.4, 0)$ (while all the hesitation margin is added to the non-membership function). The best result which could happen (if the hesitation margin is added to the membership function of $C1$), namely $(0.8, 02, 0)$, (i.e. case $C6$ ranked as the best one – $R(C6) = 0.1$) does not influence the ranking of $C1(21)$.

Just the same situation can be observed for the pairs: $C3$ and $C4$, and next for $C5$ and $C6$. The existence of the non-zero hesitation margin influences negatively the ranking.

The obtained results seem to meet our expectations pretty well.

4 Conclusions

We have proposed a new method of ranking intuitionistic fuzzy alternatives. The method takes into account the amount of the information (both positive and negative) associated with an alternative (measured by a distance to the positive ideal alternative), and how reliable the information is (which is measured by the alternative's hesitation margin).

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