# Chapter 12
# Modelling the Economy as an Evolving Space of Flows

## Methodological Challenges

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## 12.1 Introduction

The spatial economy has increasingly come to be viewed, in the felicitous phrase of Manuel Castells (2000), as a *space of flows*. The mental picture we have of this economy is a motion picture, not a still shot. Moving along the links of various networks are ever greater quantities of people, goods, material, money, and information. Settlements, in turn, appear as increasingly interdependent nodes through which these vast quantities pass. The acceleration of flows through space can be accounted for largely by technological advances in communication and transportation and the emergence of far-flung value chains, which are driven by economizing behaviour, and abetted by increasingly liberal trade agreements and industrial deregulation (Wolf 2004).

Many authors have commented on how the spatial economy would seem to manifest characteristics of complex systems – and there are indeed similarities. Steven Durlauf, who has written extensively on economic complexity (both theoretical and empirical), defines complex systems as “those [systems composed] of a set of heterogeneous agents whose behaviour is interdependent and may be described as a stochastic process” (Durlauf 2005, p. 226). Durlauf sees the following four properties as distinguishing complex systems from other systems characterized by stochastic processes and interdependencies.

- *Nonergodicity* (also known as path dependence) or the property that conditional probability statements describing the system do not uniquely characterize the average or long-run behaviour of the system;
- *Phase transition*, or the property that small changes in parameters bring about qualitative changes in aggregate properties;[1]

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[1] See Anderson (1972).

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- *Emergent properties*, or properties that exist at a higher level of aggregation than the original description of the system; and
- *Universality*, or the property that the presence of a system characteristic is robust to alternative specifications of the system's microstructure (see Durlauf 2001, 2005).[2]

While these properties are potentially helpful in explaining and understanding spatial economic systems, their presence in such systems does not imply with necessity that these systems are complex.[3] In fact, it is an open question as to whether or not complexity in social systems has been established. Durlauf remarks that empirical evaluations of complexity are fraught with identification problems, although he sees the social interaction literature as containing the strongest evidence that forces giving rise to complexity are present. Ostrom's (2000) work on the resolution of collective action problems, Pettit's (1996) on the "Common Mind" and Schelling's (1960, 1971) on spatial segregation and the avoidance of calamitous international conflagrations all suggest compelling candidate examples. Still, the four properties Durlauf has identified do provide a useful benchmark for evaluating empirical work on complexity. And if the systems we are modelling are *potentially* complex, these properties ought to be realizable within our models.

Whether or not a "complexity sighting" has been positively confirmed, there are considerable methodological challenges to modelling an economy as a spatial system inclined toward complexity. In this chapter I will discuss a number of these and then illustrate how we might begin to take on some of these challenges in the case of modelling the evolution of commodity flows in the Midwest United States.

## 12.2 Methodological Challenges

If, for the sake of argument, we accept Durlauf's definition of a complex system and view the aggregate behaviours of (possibly, interdependent) networks involving many heterogeneous interdependent actors as our *explanandum*, or "that which is to be explained", we immediately encounter a number of challenges. Perhaps the first is to

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[2] There are, of course, other lists of properties characterizing complex systems. David K. Campbell's is as follows: nonlinearity, interaction, irreducibility (behaviour is lost if the system is broken up into parts), hierarchies (multiple scales in space–time), emergent /self-organizing behaviour (more is different), many nearly equivalent configurations, adaptation, life-like behaviour (learning), intelligent agents using if/then rules (Campbell 2000).

[3] Durlauf observes 'The disparate empirical strategies that have been employed to provide evidence on economic complexity have yet to integrate theoretical models of complexity with data analysis in such a way as to show how a given aggregate property is associated with interactions between agents in a way that allows for a plausible finding that a given environment is in fact complex' (2005, p. 240).

1. *Agree on what stylized facts or empirical regularities are to be explained and what would count as an acceptable explanation*

If our investigations are motivated by policy concerns – say, for example, building and maintaining infrastructure to accommodate burgeoning flows of freight – we must concur on which of possibly many systems properties will be taken as indicators of system performance and identify which causal mechanisms need to be modelled. A second but related challenge is to

2. *Identify stable relationships or the "deep structure" of the system*

To be identifiable as a system, even a stochastic path-dependent evolutionary system must have some aspects which confer an enduring integrity to it through however many permutations it may pass. Without such a structure in mind, a modeller is just tracking passing phenomena. We may need to sharpen and reexamine the definitions of stability we apply to the study of dynamical systems (see Rotmans 2006). A third challenge facing modellers of an economy viewed as a space of flows involving interdependent heterogeneous actors is to

3. *Integrate within the same framework different conceptualizations of networks by the agents involved*

For example, firms involved in far-flung production networks and firms involved in freight logistics will view transportation infrastructure networks very differently – the former, focusing on nodes, will see potential sites for the disaggregated operations of sourcing, assembly, and distribution, whereas the latter, focusing on links, will see the means by which freight can be routed between distant nodes. Both perspectives are essential to a well formulated model of a dynamic game between shippers and carriers.

As Durlauf (2005) points out, any discussion of economic complexity entails identifying how system effects (possibly externalities of various sorts or emergent phenomena) result from the purpose-driven economizing behaviour of interacting agents.[4] Hence a fourth challenge our modellers face is to

4. *Relate micro-behavioural decision making and interaction by different types of agents to system effects*

Assuming we can overcome problems of identification and representation alluded to above, we know that determining whether or not a causal explanation is acceptable minimally entails making a fair comparison with other models based on competing explanations – that is, validating individual models of complex systems is important but does not constitute proving a theory, even provisionally (see Miller 1987). Hence we are challenged to

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[4] Some theorists of complexity would seem to argue against the possibility of doing just this. See Markose (2005) after Hayek (1945).

5. *Formulate models that enable fair comparisons of competing explanations*

Two related challenges follow logically. The first is to

6. *Determine which of the available competing explanations is better supported by the data*

While obvious, the sixth challenge is very demanding, since we often lack sufficient spatial time-series observations to estimate together the parameters of a complex economic systems model. At this point in time we cannot test empirically the logical implications of our most advanced theoretical formulations – spatial computable general equilibrium models; we can only view them as sources of interesting and suggestive information that is complementary to the outputs of other models. Work by Brock and Durlauf (2001) on discrete choice with social interactions between agents in well circumscribed neighbourhoods is an exception. This sixth challenge, then, calls for particularly creative responses. The second challenge that logically follows from the fifth is to

7. *Determine, when we encounter any of the benchmark indicators of system complexity, whether or not such markers of complexity as "lock-in", "path dependence", "power laws", or "red queen effects" are really occurring or if something else is going on*

There may be several observationally equivalent but logically incompatible explanations. As dynamic systems modellers who operate with a practical interest in understanding how systems operate – so that we can anticipate how policy interventions might steer system behaviour – our primary objective should probably not be to identify genuine manifestations of complexity, *unless* positively identifying these manifestations contributes to our ability to manage systems. With policy interventions in mind, an eighth challenge to modellers of the economy as a space of flows and path dependent development is to be able to

8. *Identify the extent to which there is local autonomy in interdependent networks*

What capacity remains for local or regional policy to make a difference – say, regarding networks for freight movement? Alternatively, if not at the local level, at what level of spatial resolution might policy interventions make a difference?

In the balance of this short chapter, I shall discuss an example of recent work, which illustrates an attempt to meet some of the challenges set out above.

## 12.3 Modelling the Evolution of Commodity-Flow Patterns in the Midwest United States

Developments in transportation and communications technologies have enabled firms to exploit *economies of scale* and *scope* by fragmenting production processes and dispersing activities to least-cost locations (Jones and Kierzkowski 2001).

Consequently, the production of most goods worldwide now takes place in a distributed pattern over many locations in which semi-finished goods are shipped from one *specialized* establishment to another. *What* activities are carried out and *where* they agglomerate appear to be path dependent – initial advantages are reinforced due to scale effects (Venables 2006). And with the increased use of just-in-time inventory management methods, all production is becoming more transport intensive. The obverse of this development is that most freight shipments are now between establishments of firms operating in the same industry. As a consequence, the industrial cores of many regional economies have become hollowed out and regional economies, both near-by and far-flung, have become increasingly interdependent through global supply chains (Munroe et al. 2007).

While the stylized facts of this story of the evolution of goods movement are generally acknowledged to be accurate, this story is a difficult one to model formally. Why should we be concerned to do so? Some familiar reasons are to

- Test theories (causal explanations)
- Forecast further evolution of goods movement for transportation infrastructure planning purposes and
- Conduct thought experiments of possible policy interventions

But also, a broad-based community of politicians, planners, municipal administrators, environmental groups, port facility managers, shippers, carriers, freight handlers, and labour unions is concerned about these developments, in large part because they lack a clear sense of how these interdependent developments are related and what they portend. Moreover, the design of effective policies to accommodate anticipated increases in freight movement and to promote public/private partnerships that can abate and mitigate deleterious externalities, requires a better understanding of how cost and incentive structures affect the form and functioning of supply chains. While empirically supported theoretical explanations of fragmentation at the firm and industry levels, public/private partnerships at urban and regional levels, and network externalities at the systems level are available, we still lack theories and models that explicitly link micro-behavioural decision making of producers (or shippers) and carriers with impacts on nodes as well as links in transportation networks (that is, with aggregate flows).

The principal objective of a recent research project, undertaken by the author jointly with Geoffrey Hewings, Gianfranco Piras, and Jürgen Scheffren at the University of Illinois – see Donaghy et al. (2006) – is to elaborate an empirically oriented framework that can characterize in large the evolution of goods movement, in which the current state of affairs, or a stylized version thereof, can arise. In so doing, we have drawn on contributions to the literatures on fragmentation (Jones and Kierzkowski 2001), the new economic geography (Krugman and Venables 1995), dynamic networks (Nagurney and Dong 2002), and commodity flow modelling (Wilson 1970; Batten and Boyce 1986; Friesz et al. 1998; Boyce 2002; Ham et al. 2005). A prototype model is sketched in the appendix to this chapter. In particular, we specify a non-cooperative dynamic game between shippers and carriers. The specification is such that both economies of scale and scope can be captured (if present) and competing explanations involving interdependent actors – for example, those of the new economic geographers (aggregationists), theorists of fragmentation (disaggregationists), and other theorists can be confronted with data.

Data availability remains the biggest concern checking modelling ambitions, especially with respect to calibration of parameters (but see Donaghy et al. 2006 for a discussion of possible solutions to the challenges encountered). Numerical solutions to the dynamic games framed by the model also will not be trivial but may be obtained in the case of large-scale models by employing a dynamic variational-inequality approach (see Nagurney and Dong 2002 for details). One may also be able to solve the explicit set of first-order necessary conditions for smaller-scale versions of the dynamic optimization problems as in Donaghy and Schintler (1998), using a custom package, such as Wymer's (2004) continuous-time systems modelling tools, WYSEA. Operationalization of the framework elaborated above is presently proceeding with a small proof-of-concept model and with a larger model of the Midwest United States.

## 12.4 Conclusions

In the foregoing we have identified a number of challenges that face modellers attempting to characterize an economy in terms of spatial-dynamic networks. We have also discussed a model that would enable us to meet some of these challenges – including:

- Integrating different conceptualizations of networks by the agents involved (in a dynamic game)
- Relating micro-behavioural decision making and interaction by different types of agents to system effects
- Formulating models that enable fair comparisons of competing explanations, hence
- Determining which of the available competing explanations is better supported by available data

The other four challenges identified

- Agreeing on what empirical regularities are to be explained and what would count as an acceptable explanation
- Identifying stable relationships or the "deep structure" of the system
- Determining if benchmark indicators of complexity – when encountered – indicate real complexity and
- Identifying the extent to which there is local autonomy in interdependent networks

are less easy to meet and will require further discussion within the scientific community, such as is promoted in this volume.[5]

# Appendix. A
# Dynamic Commodity Flow Model of Donaghy et al. (2006)

We adopt the following notation to characterize network flows. Nodes of the network through which goods are shipped are indexed by $l$ and $m$. Links joining such nodes are indexed by $a$ and routes comprising contiguous links are indexed by $r$. The length of some link $a$ connecting two nodes is denoted by $d_{\mathrm{a}}$. If link $a$ is part of route $r$ connecting nodes $l$ and $m$, an indicator variable $\delta^{a}_{lmr}$ assumes the value 1.0. It is 0 otherwise. The length of a given route from some node $l$ to another node $m$, $D_{\mathrm{tmr}}$, is given by the sum of link distances along the route:

$$D_{lmr} \equiv \sum_{a} d_a \delta^{a}_{lmr}. \tag{12.1}$$

Turning to quantities shipped through the network, we index sectors engaged in production in the spatial economy by $i$ and $j$. Types of final demand will be indexed by $k$. Let $X^{i}_{l}$ denote the total output (in dollars) of sector $i$ produced at node $l$, $x^{ij}_{lm}$ denote interindustry sales from sector $i$ at location $l$ to sector $j$ at location $m$, and $\mathrm{FD}^{ik}_{lm}$ denote final demand of type $k$ at location $m$ for sector $i$'$s$ product at location $l$. The physical flow of sector $i$'$s$ product from $l$ to $m$ along route $r$ is $h^{i}_{lmr}$. This quantity is obtained by converting the value flow along route $r$ from dollars to tons by means of the ratio of total annual interregional economic flow to total annual physical flow, $q^{i}_{x}$. The total physical flow of all commodities shipped on a link $a$ via all routes using the link is given by

$$f_a \equiv \sum_{i} \sum_{lmr} h^{i}_{lmr} \delta^{x}_{lmr}, \tag{12.2}$$

and the periodic flow capacity of link $a$ is denoted by $k_a$. Conditions that the network must satisfy at any point in time are as follows.

*Material balance constraint*

$$X^{i}_{l} = \sum_{m} \sum_{j} x^{ij}_{lm} + \sum_{m} \sum_{k} FD^{ik}_{lm}, \forall i, \forall l. \tag{12.3}$$

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[5] But see Donaghy and Richard (2006) on identifying the deep structure of an evolving system of demand for international currencies, and Piras et al. (2007) on explicitly testing for types of evolutionary dynamics.

*Conservation of flows constraint*

$$\sum_r h^i_{lmr} = \sum_j x^{ij}_{lm}/q^i_x + \sum_k \mathrm{FD}^{ik}_{lm}/q^i_x, \forall i, \forall l, \forall m.. \tag{12.4}$$

*Link capacity constraint*

$$\sum_i \sum_{lmr} h^i_{lmr}\delta^a_{lmr} = f_a \le k_a, \forall a. \tag{12.5}$$

*Non-negativity and feasibility conditions*

$$fa \ge 0, \forall a; h^i_{lmr} \ge 0, \forall i, \forall l, \forall m, \forall r; x^{ij}_{lm} > 0, \forall i, \forall j, \forall l, \forall m. \tag{12.6}$$

Equation (12.3) ensures that shipments from industry $i$ in location $l$ do not exceed production by the industry in that location, while (12.4) reconciles physical and value flows. Inequality (12.5) ensures that flows along links do not exceed capacities and the conditions given in (12.6) ensure that the distribution of goods throughout the network is feasible.[6]

In the sequel we shall assume that at each location $l$ the behavior of all establishments engaged in production in a given industrial sector can be characterized by a *representative establishment*.[7] Following Dixit and Stiglitz (1977), we further assume that firms operating the establishments act as monopolistic competitors of the Chamberlinian sort: they are output-level and input-price takers and they set output prices by a mark-up over marginal cost (which equals average cost in equilibrium). For a firm with an establishment producing in sector $i$ at location $l$, the mark-up, $\pi^i_l$, is given in terms of the price-elasticity of demand for $X^i_l$, $\sigma^i_l$, as

$$\pi^i_l = [\sigma^i_l/(\sigma^i_l - 1)].$$

Under the assumption of Chamberlinian monopolistic competition, the spatial markets in which firms compete are sufficiently competitive – barriers to entry are sufficiently low – so as to drive to a very low margin, if not zero, profits earned by firms from production of commodities at all locations.

Each local representative establishment is assumed to produce its output according to a two-level C.E.S. – constant elasticity of substitution – technology (Sato 1967). This fungible output can be used in production of other commodities or absorbed in final demand (in the forms of household and government consumption, investment,

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[6] The assumption that all $x^{ij}_{lm}$ are positive is an assumption of convenience to ensure that marginal products, specified in (12.9) below, are defined. But given the level of sectoral aggregation of available commodity-flow data, this should be of no consequence.

[7] Hence we are allowing for the possibility that firms may have multiple establishments located in different areas.

and export). At the first level, inputs of each industrial type procured locally and non-locally are aggregated into input bundles:

$$c_m^{ij} = \gamma_m^{ij} \left[ \sum_l \theta_{lm}^{ij} (x_{lm}^{ij})^{-\varepsilon_m^{ij}} \right]^{-1/\varepsilon_m^{ij}}, \forall i, \forall j, \forall m. \tag{12.7}$$

In (12.7), $c_m^{ij}$ is a bundle of inputs produced by representative establishments operating in industry $i$ at various locations $l$ used by the representative establishment in industry $j$ in its production activities at location $m$. The parameters $\gamma_m^{ij}, \theta_{lm}^{ij}$, and $\varepsilon_m^{ij}$ have standard interpretations as *scale*, *factor-intensity* and *substitution* parameters (see Ferguson 1969).

At the second level of the production function, total output by a representative establishment in a given industry in a given location is produced from the commodity bundle aggregates at the first level and labor and capital services, $L_m^j$ and $K_m^j$. At the second level, we allow explicitly for the possibility of increasing returns to scale in production at the establishment, regardless of the number of varieties aggregated in the commodity bundles, by employing a generalized C.E.S. function in which $\kappa_m^j \geq 1.0$ is the scale parameter (see Henderson and Quandt 1980).

$$X_m^j = \beta_m^j \left[ \sum_i \alpha_m^{ij} (c_m^{ij})^{-\rho_m^j} + \alpha_m^{Lj} (L_m^j)^{-\rho_m^j} + \alpha_m^{Kj} (K_m^j)^{-\rho_m^j} \right]^{-\kappa_m^j/\rho_m^j}. \tag{12.8}$$

Again, the parameters of this function have their standard interpretations. The marginal product (in terms of good $j$) at location $m$ of a unit of good $i$ produced at and shipped from location $l$ is

$$\frac{\partial X_m^j}{\partial x_{lm}^{ij}} = \frac{\partial X_m^j}{\partial c_m^{ij}} \frac{\partial c_m^{ij}}{\partial x_{lm}^{ij}} = \frac{\kappa_m^j \alpha_m^i}{(\beta_m^j)^{\rho_m^j/\kappa_m^j}} \frac{(X_m^j)^{(\kappa_m^j+\rho_m^j)/\kappa_m^j}}{(c_m^{ij})^{(\rho_m^j+1)}} \frac{\theta_{lm}^{ij}}{(\gamma_m^{ij})^{\varepsilon_m^{ij}}} \left( \frac{c_m^{ij}}{x_{lm}^{ij}} \right)^{\varepsilon_m^{ij}+1}. \tag{12.9}$$

To make further progress with an explanation of economic behavior, we need to introduce prices as well as technology. Let $p_m^j$ denote the f.o.b. (or mill) price of a unit of industry $j$s output at location $m$ and $p_{lm}^i$ the delivered price of a unit of intermediate good $i$ at $m$. Then, defining $w_m^j$ and ucc$_m^j$ as the wage rate and user cost of capital in industry $j$ at location $m$, the mill price of this good under Chamberlinian monopolistic competition is given by

$$p_m^j = \pi_m^j \left[ \sum_i \sum_l p_{lm}^i \cdot x_{lm}^{ij} + w_m^j \cdot L_m^j + ucc_m^j \cdot K_m^j \right] / X_m^j, \forall j, \forall m. \tag{12.10}$$

The *delivered price* at location $m$ of a good $i$ produced at location $l$, $p_{lm}^i$, includes the *unit cost of transport* by a carrier from location $l$ to location $m$, $\vartheta_{lm}^{ti}$, which is set by the carrier. Collecting these various price components, the delivered price of a unit of good $i$ at location $m$ will be

$$p_{lm}^{i} = p_{l}^{i} + \vartheta_{lm}^{ti}, \forall l, \forall m, \forall i. \tag{12.11}$$

Defining several new variables for the time rates of change in installed capacity (net of depreciation), in interindustry and interregional commodity flows, in employment, and the f.o.b. goods price that is,

$$I_{m}^{j} = \dot{K}_{m}^{j},\ a_{lm}^{xij} = \dot{x}_{lm}^{ij},\ a_{m}^{Lj} = \dot{L}_{m}^{j},\ \text{and}\ a_{m}^{pj} = \dot{p}_{m}^{j},$$

the intertemporal optimization decision of a representative establishment in sector $j$ at location $m$ is to choose $I_{m}^{j}, a_{lm}^{xij}, a_{m}^{Lj}$ and $a_{m}^{pj}$ so as to minimize the present value of costs of operation at *and adjustment to* equilibrium levels of capital, intermediate goods, and labor:[8]

$$\begin{aligned}
&\int_{t_0}^{t_1} \mathrm{e}^{-\lambda_{\mathrm{m}}^{\mathrm{sj}}\mathrm{t}} \Bigg\{ \sum_{\mathrm{i}} \sum_{\mathrm{l}} p_{lm}^{i} \cdot x_{lm}^{ij} + w_{m}^{j} L_{m}^{j} + \mathrm{ucc}_{m}^{j} K_{m}^{j} + q_{m}^{j} I_{m}^{j} + \frac{\omega_{m}^{Kj}}{2} (I - \upsilon_{m}^{Kj} (K_{m}^{j} * - K_{m}^{j}))^2 \\
&+ \sum_{i} \sum_{l} \frac{\omega_{lm}^{xij}}{2} (a_{lm}^{xij} - \upsilon_{lm}^{xij} (x_{lm}^{ij} * - x_{lm}^{ij}))^2 + \frac{\omega_{m}^{Lj}}{2} (a_{m}^{Lj} - \upsilon_{m}^{Lj} (L_{m}^{j} * - L_{m}^{j}))^2 \\
&+ \frac{\omega_{m}^{pj}}{2} (a_{m}^{pj} - \upsilon_{m}^{pj} (p_{m}^{j} * - p_{m}^{j}))^2 \, \mathrm{d}t,
\end{aligned} \tag{12.12}$$

subject to the following identities

$$\dot{K}_{m}^{j} = I_{m}^{j}, \tag{12.13}$$

$$\dot{x}_{lm}^{ij} = a_{lm}^{xij}, \forall i, \forall l, \tag{12.14}$$

$$\dot{L}_{m}^{j} = a_{m}^{Lj}, \tag{12.15}$$

$$\dot{p}_{m}^{j} = a_{m}^{pj},^{9} \tag{12.16}$$

and (12.3) and the non-negativity condition on $x_{lm}^{ij}$ in (12.6). In objective functional (12.12), $\lambda_{m}^{sj}$ denotes the temporal discount rate of representative establishment $j$ in location $m$, the equilibrium price level is given by (12.10), and the (atemporal)

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[8] We assume adjustment for a variable $y(t)$ towards a target value $y{*}(t)$ according to $\mathrm{d}y(t)/\mathrm{d}t = a\,(y{*}-y)$. For description of such an adaptive approach based on decision rules see Marcellino and Salmon (2002) and Scheffran (2001).

[9] Note that there are now four state equations (12.13)–(12.16). Note also that the objective functional, which involves derivatives of what would be logical control variables for the shippers, introduces *integral action*.

equilibrium (cost-minimizing) levels of capital, intermediate goods, and labor are given by

$$x_{lm}^{ij}* = \left[ \frac{\theta_m^{ij}}{(\gamma_m^{ij})^{\varepsilon_m^{ij}}} \frac{\kappa_m^j}{\pi_m^j} \frac{\alpha_m^{ij}}{(\beta_m^j)^{\rho_m^j/\kappa_m^j}} \frac{p_m^j}{p_{lm}^i} \frac{(X_m^j)^{(\kappa_m^j+\rho_m^j)/\kappa_m^j}}{(c_m^{ij})^{(\rho_m^j+1)}} \right]^{1/(1+\varepsilon_m^{ij})} c_m^{ij}, \tag{12.17}$$

$$L_m^j* = \left[ \frac{\kappa_m^j}{\pi_m^j} \frac{\alpha_m^{Lj}}{(\beta_m^j)^{\rho_m^j/\kappa_m^j}} \frac{p_m^j}{w_m^i} \right]^{1/(1+\rho_m^j)} (X_m^j)^{(\kappa_m^j+\rho_m^j)/(\kappa_m^j+\kappa_m^j\rho_m^j)}, \tag{12.18}$$

$$K_m^j* = \left[ \frac{\kappa_m^j}{\pi_m^j} \frac{\alpha_m^{Kj}}{(\beta_m^j)^{\rho_m^j/\kappa_m^j}} \frac{p_m^j}{\mathrm{ucc}_m^i} \right]^{1/(1+\rho_m^j)} (X_m^j)^{(\kappa_m^j+\rho_m^j)/(\kappa_m^j+\kappa_m^j\rho_m^j)}.^{10} \tag{12.19}$$

We now make an assumption analogous to that made above concerning representative establishments: we assume that at each location $l$ there is a *representative carrier* which (1) takes as given quantities of goods to be transported from $l$ to other locations $m$ and the prevailing cost structure of goods movement, and (2) sets prices of carriage by commodity, origin, and destination and determines the routing pattern. The intertemporal optimization decision of a representative carrier at location $l$ is, then, to determine a time-varying schedule of prices, $\vartheta_{lm}^{ti}$, for shipping commodities from its respective location $l$ to establishments and sources of final demand (households, government agencies, etc.) at all other locations $m$, $x_{lm}^{ij}$ and $\mathrm{FD}_{lm}^{ik}$, and time-varying flows of commodities along available routes $r$, $h_{lmr}^i$, so as to maximize the present value of its anticipated stream of net revenues over the time horizon $t_0$ to $t_1$,

$$\int_{t_0}^{t_1} \mathrm{e}^{-\lambda_l^c t} \left\{ \sum_i \sum_m \vartheta_{lm}^{ti} \left(\sum_j x_{lm}^{ij} + \sum_k \mathrm{FD}_{lm}^{ik}\right) - \sum_i \sum_m \sum_r h_{lmr}^i D_{lmr} p_{lmr}^{ti} \right\} \mathrm{d}t, \tag{12.20}$$

subject to (12.4) and inequalities (12.5) and (12.6). In (12.20), $\lambda_l^c$ is the temporal discount rate of the representative carrier at location $l$. Also in (12.20), $p_{lmr}^{ti}$ denotes the cost to the carrier of delivering a ton of commodity $i$ from location $l$ to location $m$ via route $r$, and is assumed to be determined by the following cost relationship,

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[10] Note that, with the generalized C.E.S. technology with increasing returns to scale at the second level, the rate of technological substitution between input bundles remains the same as in the case of constant returns to scale, as does the expansion path. Consequently, the cost function dual to the technology manifests all the usual regularity properties of a well-behaved cost function. These properties include the cost function being *non-negative* in input prices and output, *non-decreasing* in input prices and output, *concave* and *continuous* in input prices, *positively linear homogeneous* in input prices (so only relative prices matter), and supportive of *Shephard's lemma* (see Chambers 1988).

$$p^{ti}_{lmr} = p^t \cdot D^{\xi_{1i}-1}_{lmr} h^{i^{\xi_{2i}}-1}_{lmr}, \quad \text{where} \quad \xi_{1i}, \xi_{2i} < 1.0, \forall i, \forall l, \forall m, \forall r, \tag{12.21}$$

where $p^t$ denotes the industry average ton-mile price of shipping a commodity. Cost relationship (12.21) implies that unit transport costs decline with distance and with total weight of shipment.

We shall further assume that volumes of final demand for goods at various locations are affected by the prices carriers set (through the delivered price) and that carriers are aware of this dynamic. The implied feedback relationship can be captured by defining final demand of type $k$ at location $m$ for good $i$ produced at location $l$ as

$$\mathrm{FD}^{ik}_{lm} = \widetilde{\mathrm{FD}}^{ik}_{lm}(p^i_{lm}/\bar{p}^i_{lm})^{-b^{ik}_{lm}}, \tag{12.22}$$

in which $\widetilde{\mathrm{FD}}^{ik}_{lm}$ is the volume of exogenously given final demand of type $k$ at location $m$ for good $i$ produced at location $l$ when the (normalized) delivered price is constant and $\bar{p}^i_{lm}$ is a period-average or reference delivered price.[11]

When taken over all producers and carriers, the first-order necessary conditions for the solution to the above joint intertemporal optimization problem – including the network constraints (12.3)–(12.6) – correspond to a non-cooperative (Nash) game in which each player takes all others' strategic behaviors as given (the first-order conditions are provided in the appendix to Donaghy et al. 2006). Given the curvature properties of the functional forms employed, a solution to the non-cooperative game should exist and should be unique (questions about the stability of the solution remain). Variations on the game set out above can also, and will be, investigated.

Note that the present set-up differs from the usual commodity-flow model formulation in that producers are minimizing transportation costs of inputs used in production along with other input costs, instead of minimizing shipping costs of supplying the market (cf. Boyce 2002). Carriers seek maximal profits through optimal route selection. The present set-up also differs from other formulations of dynamic games of shippers and carriers in that considerations of transportation costs influence production decisions (cf. Friesz and Holguin-Veras 2005).

Realism would dictate that in applied research on the evolution of goods movement and associated systems effects, transportation modes such as rail, air, or water should also be explicitly introduced, as Ham et al. (2005) have done for a static model of interregional commodity shipments and transportation network flows. This should not present great difficulties and would enable the basic model to support simulation and dynamic gaming exercises whose intent is to examine infrastructure policies.

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[11] The definition of $\mathrm{FD}^{ik}_{lm}$ given in (12.22) should be substituted for all occurrences of the variable in other relationships of the model.

A more satisfying and more complete modeling framework would account for the evolution of final demand components (including exports) and the evolution of labor markets. An expenditure system for a representative household could be introduced along the lines of a modified almost ideal demand system (MAIDS) (see Cooper and McLaren 1992). Capacity expansion of establishments should also be related to purchases of capital goods from other producers. Changes along these lines would bring the model within the ambit of spatial computable general equilibrium frameworks.

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