Guidelines for Low Mass and Low Inertia Dynamic Balancing of Mechanisms and Robotics

Volkert van der Wijk and Just L. Herder

Abstract. Dynamic balance of machines is important when, for example, high precision in combination with low cycle times is necessary. One of the major problems with dynamic balancing is that considerable mass and considerable inertia need to be added to the mechanism. So far, only a few studies have been carried out into the comparison of various dynamic balancing principles in order to reduce these additions. Based on the findings of these studies, this paper aims to formulate guidelines for the design of dynamically balanced mechanisms with low mass and low inertia additions. Furthermore, the influence of limited design space on the resulting mass and inertia is investigated.

1 Introduction

Whenever mechanisms and robots have to move at high speeds, dynamic motion of the machine elements induces vibrations (shaking forces and shaking moments) to their base and surroundings, causing noise, wear and fatigue [6], discomfort [3] and inaccuracy [12]. Dynamically balanced mechanisms (i.e. mechanisms of which both linear momentum and angular momentum are constant) however do not induce vibrations. Therefore, high accuracy in combination with low cycle times is possible. Further advantages include that in factories, machines do not influence one another and floors do not need a special construction to withstand machine's dynamic loads. For moving objects and vehicles (at the ground or in space), dynamic balance is important to maintain position, orientation and stability [5, 1].

One of the major problems with dynamic balancing is that a considerable amount of mass and a considerable amount of inertia need to be added to the mechanism [4, 12], while generally also the complexity of the mechanism is increased. As an

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example, Wu and Gosselin in 2005 [14] were the first to completely dynamically balance a 6-Degree-of-Freedom (DoF) spatial parallel mechanism. However to balance a payload of 50 grams, 15 counter-masses (CM) and 9 counter-inertias (CI, counter-rotating elements), totalling 4.5 kg, had to be added, yielding a ratio of balancer elements' total mass over payload of 90.

It is likely that this disadvantage is an important reason for the limited research and application interest in dynamic balancing. In (planar) machines, dynamic balancing is more common than in (spatial) robotics. To reduce the increase of mass and inertia, and hence complexity, to an acceptable level, dynamic balancing usually is done partly. The most common approaches include to only force balance some elements [6, 7], to dynamically balance some specific frequencies [2], and constrained optimization of design parameters [9]. However, to reach the accuracy in combination with the production speed that future mechanisms and robotics will need to have, complete dynamic balancing (i.e. eliminating all shaking forces and shaking moments) with a low addition of mass and inertia is necessary.

Only few studies and numerical experiments have been done concerned with the comparison of general balancing principles regarding their addition of mass and addition of inertia [4, 12, 13, 11]. These studies argue that, for the investigated balancing principles, dynamic balancing tends to increase the mass and inertia considerably. In the case that both low mass and low inertia are of concern, the balancing principle of 'duplicate mechanisms' (Fig. 1a), where three axial and mirror copies of the initial mechanism are added and altogether result in a dynamically balanced system, proved to be the most advantageous balancing principle. It also turned out that the use of 'Counter-Rotary Counter-Masses' (Fig. 1b), where the CM for force balancing also is used as CI for moment balancing, is more advantageous than the common practice of applying separate CI's for moment balancing (Fig. 1c).

The objective of this article is to formulate general guidelines for the development of dynamically balanced machinery that have a minimum of additional mass and a minimum of additional inertia, based on the literature cited above. In addition, the influence of the design space, i.e. the space that is available for the balancing elements, is taken into account, since in practise this often has a considerable influence on the performance.

There is a difference between the inertia of an element and the inertia of the mechanism. This paper deals with the minimization of the inertia of the mechanism, which is represented by the reduced inertia as defined in [8]. This is the inertia of the mechanism reduced to the input parameters, in fact the inertia that an actuator feels when driving the system.

The discussion of low mass and low inertia dynamic balancing is divided in three different sections. First the influence of the balancing principles and their application, i.e. the resulting balancing architecture, on the mass and inertia additions is discussed. The influence of the balancing parameters, i.e. the masses and dimensions of the elements, is treated in the second section. The third section investigates the influence of the design space on the mass and inertia additions in dynamically balanced mechanisms. After these three sections general guidelines for low mass and low inertia dynamic balancing are formulated and listed.

2 Influence of Balancing Architecture

The architecture of the elements that are added to the mechanism for dynamic balancing, influences the addition of mass and the addition of inertia. The architecture of these balancing elements depends both on the choice of balancing principle with which the mechanism is balanced and how the chosen balancing principle is applied. In [12] and [13], from literature concerning planar and spatial, serial and parallel mechanisms, in total three fundamental and generally applicable balancing principles were found, which are shown in Fig. 1. Figure 1a shows the principle of duplicate mechanisms (DM). Although for moment balancing a single mirror copy would be sufficient, three horizontal and vertical mirror copies of the initial mechanism are necessary for the full elimination of the shaking forces in both horizontal and vertical direction. Figure 1c shows the principle of Separate Counter-Inertias (SCI), where a counter-mass (CM) (m^*) is used for force balancing only, while moment balancing is accomplished with a separate counter-inertia (CI) (I^*_{CI}). Figure 1b shows the principle of Counter-Rotary Counter-Masses (CRCM) where the moment is balanced by using the inertia of the CM (I^*) itself.

The influence of the architecture of these principles on the addition of mass and inertia depends on the contribution of the balancing elements to the dynamic balance [13]. For the SCI-principle, the CM does only contribute to the force-balance, however the inertia of the CM increases the inertia of the mechanism and since the inertia is balanced by the CI, the inertia of the CI increases also. For the CRCM principle, the CM contributes to both the force and moment balance. The inertia of the CM is directly used to balance the inertia of the mechanism.

The influence of the element contribution to the dynamic balance is also shown in [10], where a double pendulum is balanced with CRCMs as shown in Fig. 2. There are various ways to apply the CRCM principle to a double pendulum, for instance by simply stacking two CRCM-balanced links with the configuration of Fig. 1b.

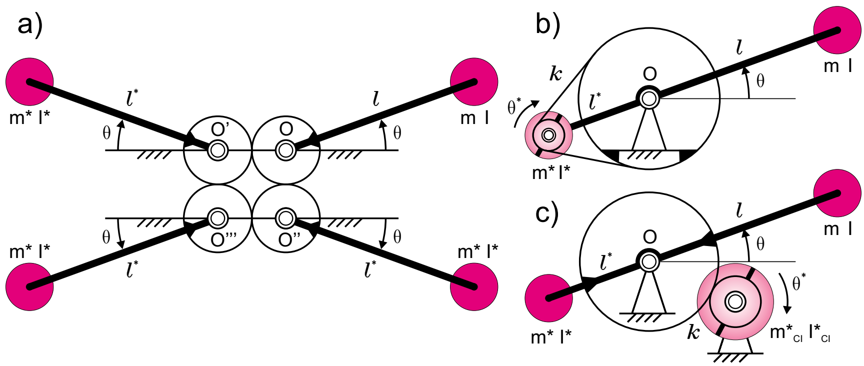

Fig. 1 General balancing principles, balancing by using (a) Duplicate Mechanisms (DM), (b) Counter-Rotary Counter-Masses (CRCM) and (c) Counter-Masses and Separate Counter-Inertias (SCI) (source [13]).

However the configuration of Fig. 2 showed to be most advantageous for low inertia addition, since the inertia I_2^* of the CRCM of link 2 is balancing the moment of link 2 for any motion of the linkage, while it does not influence the inertia I_1^* of the CRCM at link 1.

Regarding the addition of mass, both the CRCM and SCI principle have the same problem. Although the CM m_2^* contributes to the dynamic balance of link 2, it negatively influences CM m_1^*. In fact CM m_2^* itself must also be balanced, which is done by CM m_1^* and is an important source of mass addition. The reason for this is that CM m_2^* is balancing link 2 about a moving point, while if link 2 would be balanced about a fixed point directly, the problem would be eliminated.

An example of this is shown in [11], where a double pendulum is balanced with only one CRCM (as a balanced pantograph). The problem in this case is that, in order to balance both the force and moment of each link for any motion, the mass distribution is not constant, even though the CoM remains stationary at the fixed point O. This means that the inertia of the mechanism depends on the position and velocity of the mechanism. Hence it is not possible to balance the moment of this mechanism solely with mechanical elements (passive balancing). In [11] this problem is solved by actively counter-rotating the CRCM with an additional actuator (active balancing). The results indeed show that this approach is more advantageous than the passive approach with CRCMs.

The contribution of the balancing elements to the dynamic balance is not the reason why the DM principle turned out to be the most advantageous for low mass and low inertia dynamic balancing. Since one mirror copy is sufficient for balancing the moment of the mechanism, the two other copies only contribute to the force balance, however they increase the inertia of the mechanism. The advantage of the DM principle will become clear in the next section.

In summary, an architecture in which all balancing elements contribute to both the force and moment balance and where the mechanism is force balanced about a

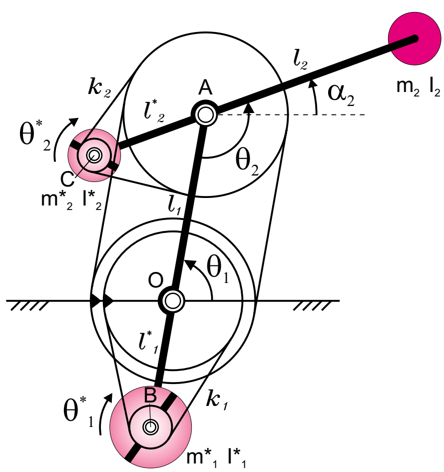

Fig. 2 CRCM principle applied to a double pendulum in a low inertia configuration. Because the CRCM of link 2 at point C is driven by a chain connection with a gear that is mounted on the base at O, the inertia I_2^* of this CRCM does not influence the inertia I_1^* of the CRCM at link 1 (source [10]).

stationary point directly is most advantageous for dynamic balancing with low mass and low inertia addition. This applies to any mechanism, from single-DoF planar to multi-DoF spatial. Expanding the idea of combining the function of force balancing and the function of moment balancing into the same element to the architecture of the mechanism itself, it could be stated that for low mass and a low inertia dynamically balanced mechanisms all the mechanism links should be, besides kinematically relevant, also dynamically relevant by contributing to the dynamic balance.

3 Influence of Balancing Parameters

From the studies in [11], [12] and [13] it can be concluded that the main parameters of the balancing elements of concern for the addition of mass and addition of inertia are the position of the CM with respect to its center-of-rotation (CoR) and the transmission ratio of the counter-rotation with the rotation of the mechanism. The former parameter determines the mass of the CM and the mechanism inertia to be balanced while the latter determines the inertia of the CI.

Two other parameters that influence the mass and inertia addition are the mass distribution (inertia) of the CM that is not used for moment balancing and the mass of the CI that is not used for force balancing. While the former influences the mechanism inertia to be balanced, the latter influences the total mass of the system. Of a CM that is also used as CI, the mass distribution is determined solely by the main parameters.

Table 1 Equations for force balance, moment balance, mechanism inertia and total mass for the CRCM and the SCI principle of the mechanisms in Fig. 1 (source [13]).

	CRCM principle	SCI principle
Force Balance:	$ml = m^*l^*$	$ml = m^*l^*$
Dynamic Balance:	$I+ml^2+m^*l^{*2}+kI^* = 0$	$I+ml^2+I^*+m^*l^{*2}+kI^*_{cr} = 0$
Mechanism Inertia:	$I_\theta^{red} = I+ml^2+m^*l^{*2}+k^2I^*$	$I_\theta^{red} = I+ml^2+I^*+m^*l^{*2}+k^2I^*_{cr}$
Total Mass:	$m_{tot} = m+m^*$	$m_{tot} = m+m^*+m^*_{cr}$

In [13] the influence of these parameters becomes most clear by using the mechanisms of Fig. 1 as an example. The equations for the force balance, the moment balance, the mechanism inertia and the total mass of the mechanisms are given in Table 1. Substituting the force balance and moment balance equations in the equations of the total mass and mechanism inertia results for the CRCM principle in

$$m_{tot} = m\left(1+\frac{l}{l^*}\right) \tag{1}$$

$$I_\theta^{red} = I+ml^2+mll^*+k\left(I+ml^2+mll^*\right) \tag{2}$$

and for the SCI principle in

$$m_{tot} = m\left(1+\frac{l}{l^*}\right)+m^*_{CI} \tag{3}$$

$$I^{red}_\theta = I+ml^2+mll^*+I^*+k\left(I+ml^2+mll^*+I^*\right) \tag{4}$$

For the DM principle the equations for the mechanism inertia and total mass are

$$m_{tot} = 4m \tag{5}$$

$$I^{red}_\theta = 4(I+ml^2) \tag{6}$$

In this example the position of the CM is represented by l^* and the transmission ratio by k. For the SCR principle it is visible that besides the influence of l^* and k, the mass of the CI m^*_{CI} influences the total mass and the mass distribution of the CM I^* influences the inertia of the mechanism, while these influences do not exist for the CRCM principle since the inertia of the CM is used as CI. The equations for the DM principle do not have any balancing parameters. The only possibility is to copy the initial mechanism three times.

For both the CRCM as the SCI principle holds that a large transmission ratio results into a large mechanism inertia. According to the equation of the moment balance in Table 1, a large transmission ratio results into a small inertia of the CI. This implies that for a low addition of mechanism inertia, the inertia of the CI should be large. The explanation for this is the quadratic appearance of the transmission ratio in the equations of the mechanism inertia.

This last observation is the reason why the DM principle proved to be the most advantageous for the combination of low mass and low inertia dynamic balancing. The transmission ratio of the DM principle is relatively low (-1) and the inertia of the counter rotating elements is large, since their inertias depend on the length of the links which have a relatively large size. Generally, links can be characterized by having a relatively large inertia with respect to their mass.

A large l^* results for both the CRCM as the SCI principle into a low total mass but also into a large inertia of the mechanism. This means that there is a trade off between the addition of mass and the addition of inertia.

The comparing studies of [11], [12] and [13] were limited to 2-dof mechanisms. With multi-DoF planar or spatial mechanism, where DoFs influence one another, optimal values for the mechanism inertia and the total mass are likely to exist. Whenever the parameter values for the minimum inertia are different than for the minimum mass, a trade off remains. This still needs further investigation.

4 Influence of Design Space

The previous section showed that for low mass addition, CMs should be placed far away from their CoRs and for a low inertia addition the inertia of the CIs should be large, which by keeping the mass low means a large size. However in practise the dimensions of the balancing elements are limited by the design space. The comparative studies did not take this into consideration yet. Two interesting questions however arise. What is the optimal design of the mechanism within a specific design space and in relation with the other machine components, what is the optimal design space?

Figure 3a shows the CRCM principle of Fig. 1b, however here the CRCM is designed as a ring to obtain a large inertia. Also the transmission ratio is kept small. If the workspace of this manipulator by full rotation is the circumscribed circle about the mechanism, it is not possible to place other machine components within this space since they would interfere with the manipulator. However, it is possible to place the balancing elements in this area. By choosing the design space to be equal to the workspace, no concessions need to be done for the design of other machine parts.

If the target of the configuration of Fig. 3a is to have a low inertia, l^* and I^* have to be large. However by increasing the link length l^* the size of the CRCM decreases and hence the inertia of the CRCM decreases. Increasing the size of the CRCM results into a decreasing l^*. This means that by taking the design space into account, an optimum for the addition of inertia to the mechanism will exist.

Figure 3b shows a situation in which the design space is outside the workspace and might depend on, for instance, the location of the other machine components or the available space behind the machine in the factory. The target is to design the most advantageous mechanism for low mass and low inertia dynamic balancing. In Fig. 3b the configuration is shown that has the largest CRCM for a large inertia. Although both increasing and decreasing the length l^* makes the CRCM become smaller, it does not mean that the lowest mechanism inertia is found in this position.

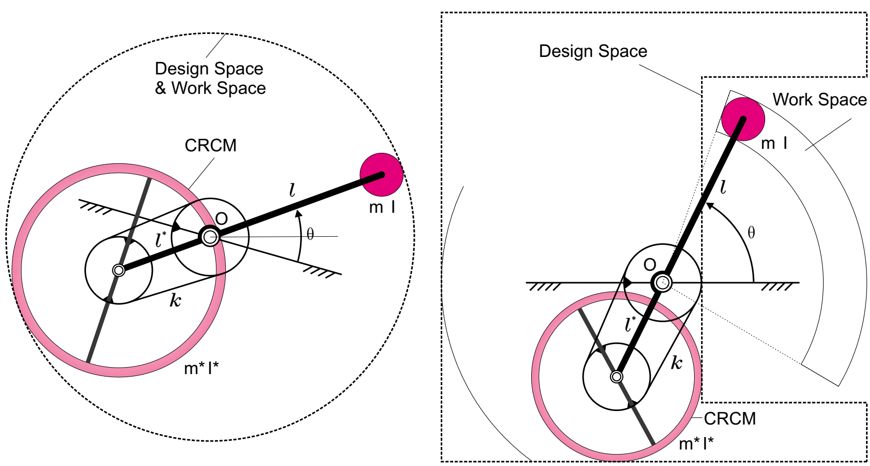

Fig. 3 Design limitations due to the available space for the dynamic balancing elements. (a) Design space equal to workspace; (b) Design space outside the workspace.

The ring-shaped CRCM of Fig. 3 requires a large empty space. In fact the CRCM can attain any shape as long as its CoM is at the CoR of the CRCM. The CRCM can for in instance be a link as shown in Fig. 4b with transmission ratio $k=-1$. The DM principle already showed that counter-rotating links (CRL) are advantageous. Although the length of this CRL is larger than the diameter of the ring shaped CRCM, the design space that is necessary for the balancing elements is much smaller. With this solution it is possible to place other machine components nearby.

Figures 3 and 4 showed some approaches, that are also suitable for multi-DoF planar and spatial mechanisms, of integrating dynamic balancing into machines. The best solution however, still depends on the wishes of the designer since a trade off between mass addition and inertia addition must be made. To assist the designer with this choice, [12] introduces the Mass-Inertia factor μ which is defined as

$$\mu = w_M \cdot \hat{m} + \sum_j w_j \cdot \hat{I}_j \tag{7}$$

and weights the relative addition of mass and the relative addition of inertia with to be chosen weight factors w_M and w_j respectively. $\hat{m}$ and $\hat{I}_j$ are the mass ratio and the inertia ratio of input parameter j which are calculated with $\hat{m} = \frac{m_{tot}}{m^o_{tot}}$ and $\hat{I}_j = \frac{I_j^{red}}{I_j^{red,o}}$. m^o_{tot} and $I_j^{red,o}$ are the total mass and the mechanism inertia per input parameter before balancing, respectively, and m_{tot} and I_j^{red} are the total mass and mechanism inertia after balancing, respectively. For the lowest Mass-Inertia factor, the balanced mechanism is optimal for low mass and low inertia. An optimization of the balanced mechanism for low mass and low inertia including also the design space may be most useful when the complete mechanism is taken into account, including all other machine parts.

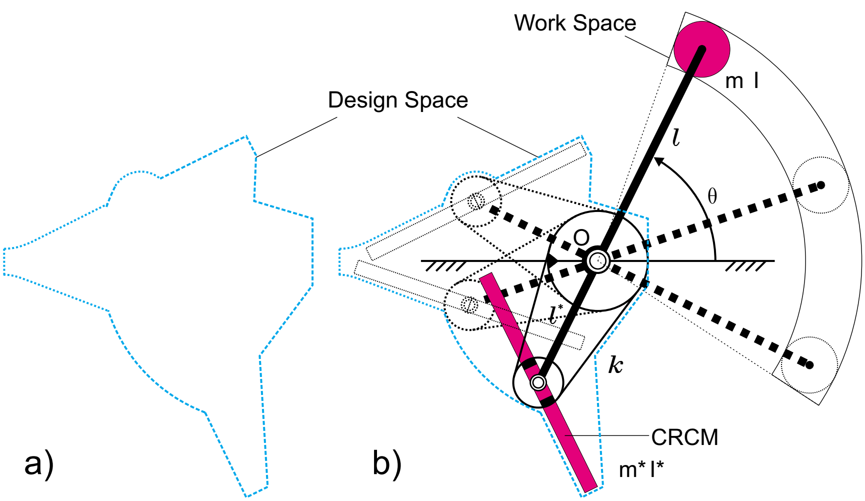

Fig. 4 Optimizing the mass and inertia addition together with the design space.

5 Guidelines for Low-Mass and Low-Inertia Dynamic Balancing

Minimal addition of mass and minimal addition of inertia

- For a minimal addition of mass and inertia, all elements of the mechanism including links, counter-masses, and counter-rotary counter-masses have to contribute to both the force balance and the moment balance of the mechanism for any motion of the mechanism. For existing mechanisms this applies to the additional elements while for the design of new mechanisms it applies to all elements. Mechanism links have to be positioned such that they counter-rotate with other links which means that dynamic balancing already starts within the kinematics of the unbalanced mechanism.
- For minimal addition of mass and inertia, the design space of the mechanism that is available for the position and the motion of balancing elements has to be maximal.
- For minimal addition of mass and inertia, the use of separate counter-rotations must be omitted.

Minimal addition of mass

- For minimal addition of mass, counter-masses and counter-rotary counter-masses have to be placed at maximum distance from their center of rotation.
- For minimal addition of mass, the balancing elements (counter-masses, counter-rotary counter-masses, counter-rotating links) have to be positioned such that they do not need to be balanced by other counter-masses.

Minimal addition of inertia

- For minimal addition of inertia, counter-masses and counter-rotary counter-masses have to be placed at minimum distance from their center of rotation.
- For minimal addition of inertia, counter-masses that are fixed to their link and elements that do not contribute to the moment balance must have minimal inertia.
- For minimal addition of inertia, geared counter-rotating elements themselves must have maximal inertia.
- For minimal addition of inertia, the gear- and transmission ratios of the counter-rotating elements have to be minimal.

Minimal addition of mass or inertia

- A trade off between the addition of mass and the addition of inertia must be made. The designer has to decide the relative importance of each for the intended purpose.

6 Conclusion

Based on the results of some comparative studies into several balancing principles and by taking into account the limitations of the design space, general guidelines for designing dynamically balanced mechanisms that have a low mass and a low inertia were formulated. Although a trade off between the addition of mass and the addition of inertia seems unavoidable, by optimization of the balancing parameters and the design space choices can be made judiciously.

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