Chapter 4
Dynamics of Innovation Fields with Endogenous Heterogeneity of People

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4.1 Introduction: Towards the New Economic Geography in the Brain Power Society

4.1.1 Welcome to the Brain Power Society

According to Lester Thurow at MIT, advanced countries are shifting from capitalism based on mass production of commodities to the *brain power society* in which creation of knowledge and information using brain power plays the central role (Thurow 1996). The concept of brain power society is essentially the same as that of the *C-society* advocated by Åke Andersson who maintains that advanced countries are leaving the industrial society (with its reliance on simplicity of production and products and the heavy use of natural resources and energy) and entering the C-society with and increasing reliance on creativity, communication capacity, and complexity of products (Andersson 1985). In this paper, the term "brain power society" is synonymous with the "C-society" of Åke Andersson.

The ultimate concern of this paper is the further development of the *New Economic Geography* (NEG) towards a more comprehensive theory of geographical economics in the age of brain power society, in which the dynamics of the spatial economy arise from the dual linkages in the economic and knowledge fields. Before elaborating this ultimate objective, let me explain briefly what is the so-called the New Economic Geography.

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4.1.2 The New Economic Geography and Its Future: Incorporating Dual Linkages in Economic and Knowledge Fields

Since about 1990 there has been a renaissance of theoretical and empirical work on economic geography. Among others, the pioneering work of Paul Krugman (1991) on the core–periphery model has triggered a new flow of interesting contributions to economic geography. The work represented by this new school of economics is called the New Economic Geography (NEG).[1] The hallmark of the NEG is the presentation of a unified approach to modeling a spatial economy characterized by a large variety of economic agglomeration – one that emphasizes the three-way interaction among increasing returns, transport costs (broadly defined), and the movement of productive factors – in which a general equilibrium model is combined with nonlinear dynamics and an evolutionary approach for equilibrium selection. Figure 4.1 represents the basic conceptual framework of the NEG.

The observed spatial configuration of economic activities is considered to be the outcome of a process involving two opposing types of forces, that is, *agglomeration* (or *centripetal*) forces and *dispersion* (or *centrifugal*) forces.[2] As a complicated balance of these two opposing forces, a variety of local agglomeration of economic

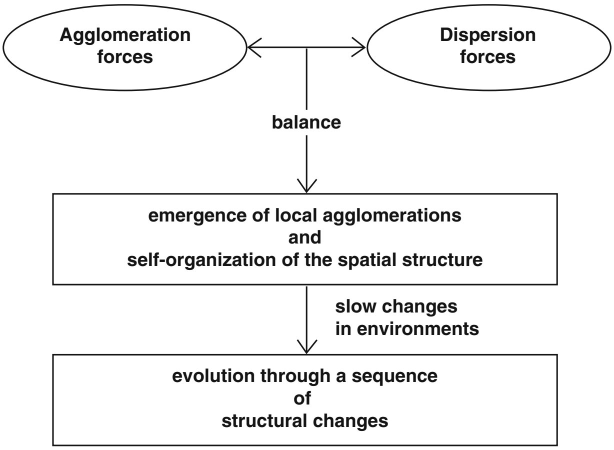

Fig. 4.1 The basic framework of the New Economic Geography

[1]See Fujita et al. (1999) for a comprehensive manifestation of this approach. See also Fujita and Thisse (2002) and Baldwin et al. (2003) for the recent development of the NEG. For an overview of the NEG, see Fujita and Krugman (2004), Fujita (2005), Fujita and Mori (2005).

[2]This hypothesis is not entirely new, of course. For, e.g., Zipf (1949) conjectured that the changing spatial configuration of economic activities was the outcome of the two sets of centripetal (unifying) and centrifugal (diversifying) forces.

activity emerges, and the spatial structure of the entire economy is self-organized. And, with the gradual changes in technological and socioeconomic environments, the spatial system of the economy experiences a sequence of structural changes, evolving towards an increasingly complex system.

In this framework, then, the two questions of obvious importance are:

Question 1: how to explain the agglomeration forces?
Question 2: how to explain the dispersion forces?

The answer to Question 2 is rather easy, for the concentration of economic activities at a location will naturally increase factor prices (such as land price and wage rate) and induce congestion effects (such as traffic congestion and air pollution as well as more severe competition among similar firms), which can be readily explained by the traditional economic theory. Thus, the principal concern of the NEG is Question 1, i.e., how to explain the agglomeration forces behind the formation of a large variety of spatial agglomeration such as cities and industrial districts.

In most models of the NEG so far, agglomeration forces arise solely from pecuniary externalities through linkage effects among consumers and industries, neglecting all other possible sources of agglomeration economies such as knowledge externalities and information spillovers. This has led to the opinion that the theories of the NEG have been too narrowly focused, ignoring as much of the reality as old trade theory.

I fully understand the concern. But, such a narrow focus of the NEG was designed in order to establish a firm micro-foundation of geographical economics based on modern tools of economic theory. It does not necessarily mean that the NEG is limited to such a narrow range of models and issues. On the contrary, its framework is widely open to further development. Indeed, recently many of such possibilities are being explored vigorously by many young scholars.[3]

That much said, however, I admit that there still remains a big room for further development of the NEG. In particular, there remains one type of agglomeration forces of which micro-foundations have seen little development so far, i.e., the linkages among people through the creation and transfer of knowledge, or in short, the *K-linkages*. (Hereafter, “knowledge” is defined broadly to include ideas and information.)

Traditionally, K-linkage effects have either been called “knowledge spillovers” or “knowledge externalities”. However, the term, “spillovers”, tends to have a connotation of passive effects. And, the term, “externalities”, tends to imply too many different things at once. So, in the remaining discussion, instead of knowledge spillovers or externalities, let me use the term, K-linkages, in order to emphasize that they represent the agglomeration forces resulting from the activities related to both the “creation of knowledge” and the “transfer of knowledge” or “learning” (either in an active way or a passive way). In contrast to the K-linkages, the traditional linkages through the production and transactions of (traditional) goods and services may be

[3]See those articles reviewed in Fujita and Mori (2005).

called the *E-linkages* (where "E" represents the economic activities in the traditional economics).

Using such a terminology, we may imagine that *the agglomeration forces in the real world arise from the dual effects of E-linkages and K-linkages*. In this context, we conjecture that the role of K-linkages has been becoming increasingly more dominant recently. Yet, developing the micro-foundations of K-linkages seems to be the most challenging task, largely left for young scholars in the future. This paper represents my modest efforts with my colleagues towards this objective.

Needless to say, there has been a great amount of conceptual studies on knowledge externalities/spillovers in a spatial context, starting with Marshall (1890), and including more recent pioneering work such as Jacobs (1969), Andersson (1985) and Lucas (1988) in an urban context, and Porter (1998) in the context of industrial clusters. Yet, it would be fair to say that there is a lot of room left for advancing the micro-foundations of K-linkages in space. Particularly, in developing the micro-foundations of K-linkages, "creation of knowledge" must be clearly distinguished from "transfer of knowledge" or "learning". Furthermore, for the creation of new ideas, cooperation among heterogeneous people is essentially important. Yet, through communication and migration, the degree of heterogeneity of people in a region changes over time. Thus, the nature of K-linkages is essentially dynamic, and hence their full-fledged treatment requires a dynamic framework as elaborated in the next section.

4.1.3 Dynamics of Innovation Fields Through the Endogenous Heterogeneity of Brains

Figure 4.2 represents abstractly the cooperative process of knowledge creation by two persons, i and j, when they meet and collaborate to create new ideas (or new knowledge) together.

The left circle, K_i, represents *the state of knowledge*, or just *knowledge*, of person i (at the time of meeting), whereas the right circle, K_j represents the knowledge of person j. The overlapping area, C_{ij}, represents their knowledge in common, or just common knowledge,[4] whereas the left area, $D_{ij} = K_i - C_{ij}$, shows the *differential knowledge of person i* from j, the right area $D_{ji} = K_j - C_{ij}$ the *differential knowledge of person j* from i. Through mutual communication and discussion based on the common knowledge C_{ij}, the two persons endeavor to develop new ideas by combining their differential knowledge D_{ij} and D_{ji}. This joint process of knowledge creation can

[4]Here, "common knowledge" represents simply the short expression of "the knowledge in common" or "mutual knowledge". It is not the term used in game theory.

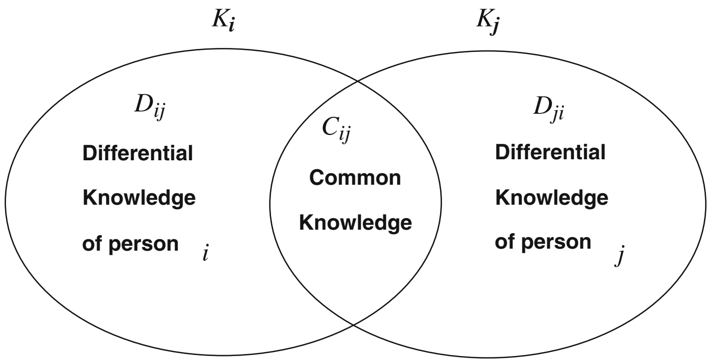

Fig. 4.2 Cooperative process of knowledge creation

be expected to be most productive when the proportions of the three components, i.e., the common knowledge (C_{ij}), the differential knowledge of person i (D_{ij}), and the differential knowledge of person j (D_{ji}), are well balanced. A sufficient amount of common knowledge is necessary for effective communication between two persons. Furthermore, if one person does not have a sufficient amount of differential knowledge, there is little motivation for the other person to meet and collaborate. In other words, too much common knowledge means little heterogeneity or originality in the collaboration, unable to yield enough synergy.

Therefore, in general, *for a cooperative process of knowledge creation by a group of people to be productive, both a sufficient heterogeneity and a sufficient common base in their states of knowledge are essential*. When such a delicate balance in their states of knowledge holds, an unexpected synergy may be created from their close collaboration.

Actually, this observation is not entirely new. We have, e.g., an old Chinese saying, “San ge chou pi jiang, Di ge Zhuge Liang” which roughly means “With three ordinary persons getting together, splendid ideas will come out”.

However, any nice saying must be taken with caution, for it may imply an antinomy. Concerning the previous Chinese saying, we may continue: “But, after three ordinary persons meeting for three months, no more splendid idea will come out”.

Likewise, returning to Fig. 4.2 even when the two persons have initially a sufficient heterogeneity in their states of knowledge, if they continue a close cooperation in knowledge creation, their heterogeneity may keep shrinking. This is because the very cooperative process of knowledge creation results in *the expansion of their common knowledge* through both the sharing of newly created ideas and the transfer of differential knowledge to each other. Thus, unless some additional complementary mechanisms are not working, the cooperative process of knowledge creation among the same group of people tends to become less productive eventually.

4.2 The Model

Building upon what has been discussed above, in this section, I present a micro-model of knowledge creation through the interaction of a group of people, which has been developed by Berliant and Fujita (2007).[5] In describing the model, the analogy between partner dancing and working jointly to create and exchange knowledge is useful, so we will use terms from these activities interchangeably. We assume that it is not possible for more than two persons to meet or dance at one time, though more than one couple can dance simultaneously. When agents meet, they create new, shared knowledge, thus building up knowledge in common. When agents are not meeting with each other, their knowledge base grows more different. The fastest rate of knowledge creation occurs when common and differential knowledge is in balance.[6]

Specifically, suppose that there exist N persons in the economy. Consider a given time t, and focus on two persons i and j. And, let in terms of Fig. 4.1, $n_{ij}^d(t)$ be the size of D_{ij}, the differential knowledge of person i from j; $n_{ij}^c(t)$ be the size of C_{ij}, the common knowledge for person i and j; $n_{ji}^d(t)$ be the size of D_{ji}, the differential knowledge of person j from i. And let

$$n_i(t) = n_{ij}^c(t) + n_{ij}^d(t), \tag{4.1}$$

$$n_j(t) = n_{ij}^c(t) + n_{ji}^d(t), \tag{4.2}$$

so that $n_i(t)$ represents the size of K_i, the knowledge of person i at time t; $n_j(t)$ the size of K_j, the knowledge of person j at time t.

Knowledge is a set of ideas that are possessed by a person at a particular time. However, knowledge is not a static concept. New knowledge can be produced either individually or jointly, and ideas can be shared with others. But all of this activity takes time.

Now we describe the components of the rest of the model. To keep the description as simple as possible, we focus on just two agents, i and j. At each time, each faces a decision about whether or not to meet with others. If two agents want to meet at a particular time, a meeting will occur. If an agent decides not to meet with anyone at a given time, then the agent produces separately and also creates new knowledge separately, away from everyone else. If two persons do decide to meet at a given time, then they collaborate to create new knowledge

[5] See Berliant and Fujita (2007) for the further elaboration of the following model.

[6] For simplicity, we employ a deterministic framework. It seems possible to add stochastic elements to the model, but at the cost of complexity. It should also be possible to employ the law of large numbers to a more basic stochastic framework to obtain equivalent results.

together. Here we limit the scope of our analysis to knowledge creation as opposed to knowledge transfer.[7]

What do the agents know when they face the decision about whether or not to meet a potential partner j at time t? Each person knows both $K_i(t)$ and $K_j(t)$. In other words, each person is aware of his own knowledge and is also aware of others' knowledge. Thus, they also know $n_i(t)$, $n_j(t)$, $n^c_{ij}(t) = n^c_{ji}(t)$, $n^d_{ij}(t)$, and $n^d_{ji}(t)$ (for all $j \neq i$) when they decide whether or not to meet at time t. The notation for whether or not a meeting of persons i and j actually occurs at time t is: $\delta_{ij}(t) = \delta_{ji}(t) = 1$ if a meeting occurs and $\delta_{ij}(t) = \delta_{ji}(t) = 0$ if no meeting occurs at time t. For convenience, we define $\delta_{ii}(t) = 1$ when person i works in isolation at time t, and $\delta_{ii}(t) = 0$ when person i meets with another person at time t.

Next, we must specify the dynamics of the knowledge system and the objectives of the people in the model in order to determine whether or not two persons decide to meet at a particular time. The simplest piece of the model to specify is what happens if there is no meeting between person i and anyone else, so i works in isolation. Let $a_{ii}(t)$ be the rate of creation of new ideas created by person i in isolation at time t (this means that i meets with itself). Then we assume that

$$a_{ii}(t) = \alpha n_i(t) \quad \text{when} \quad \delta_{ii}(t) = 1, \tag{4.3}$$

where α is a positive constant. So we assume that if there is no meeting at time t, individual knowledge grows at a rate proportional to the knowledge already acquired by an individual.

If a meeting occurs between i and j at time t ($\delta_{ij}(t) = 1$), then joint knowledge creation occurs, and it is governed by the following dynamics:[8]

$$a_{ij}(t) = \beta \left[n^c_{ij}(t) n^d_{ij}(t) n^d_{ji}(t) \right]^{\frac{1}{3}} \quad \text{when} \quad \delta_{ij}(t) = 1 \ \text{ for } j \neq i, \tag{4.4}$$

where β is a positive constant. So when two people meet, joint knowledge creation occurs at a rate proportional to the normalized product of their knowledge in common, the differential knowledge of i from j, and the differential knowledge of j from i. The rate of creation of new knowledge is highest when the proportion of ideas in common, ideas exclusive to person i, and ideas exclusive to person j are split evenly. Ideas in common are necessary for communication, while ideas exclusive to one person or the other imply more heterogeneity or originality in

[7]In an earlier version of this paper, Berliant and Fujita (2004, available at http://econpapers.hhs.se/paper/wpawuwpga/0401004.htm), we have worked out the details of the model with both knowledge creation and transfer when there are only two persons, and found no essential difference in the results. However, in the N person case, it is necessary to keep track of more details of who knows which ideas, and thus the model becomes very complex. This extension is left to future work.

[8]See Berliant and Fujita (2007, Sect. 4.6) for a more general form of joint knowledge creation.

the collaboration. If one person in the collaboration does not have exclusive ideas, there is no reason for the other person to meet and collaborate.

Whether a meeting occurs or not, there is production in each period for both persons. *Felicity in that time period is defined to be the quantity of output.*[9] Define $y_i(t)$ to be production output (or felicity) for person i at time t. Normalizing the coefficient of production to be 1, we take

$$y_i(t) = n_i(t). \tag{4.5}$$

So,

$$\dot{y}_i(t) = \dot{n}_i(t).$$

By definition,

$$\frac{\dot{y}_i(t)}{y_i(t)} = \frac{\dot{n}_i(t)}{n_i(t)} \tag{4.6}$$

which represents the rate of growth of income.

We now describe the dynamics of the system, dropping the time argument. Let us focus on agent i, as the expressions for the other agents are analogous.

$$\dot{y}_i = \dot{n}_i = \sum_{j=1}^{N} \delta_{ij} a_{ij}, \tag{4.7}$$

$$\dot{n}_{ij}^c = \delta_{ij} a_{ij} \quad \text{for} \quad \text{all } j \neq i, \tag{4.8}$$

$$\dot{n}_{ij}^d = \sum_{k \neq j} \delta_{ik} a_{ik} \quad \text{for} \quad \text{all } j \neq i. \tag{4.9}$$

Equation (4.7) means that the increase in the knowledge of person i is the sum of the knowledge created in isolation and the knowledge created jointly with someone else. Equation (4.8) means that the increase in the knowledge in common for persons i and j equals the new knowledge created jointly by them. This is based on our previous assumption that there is no transfer of existing knowledge between agents even when they are meeting together. Finally, (4.9) means that all the knowledge created by person i either in isolation or jointly with persons other than person j becomes a part of the differential knowledge of person i from person j.

By definition, it is also the case that

$$\sum_{j=1}^{N} \delta_{ij} = 1.$$

[9]Given that the focus of this paper is on *knowledge creation* rather than production, we use the simplest possible form for the production function.

Furthermore, on the equilibrium path it is necessary that

$$\delta_{ij} = \delta_{ji} \quad \text{for all} \quad i \text{ and } j.$$

Concerning the rule used by an agent to choose their best partner, to keep the model tractable in this first analysis, we assume a myopic rule. At each moment of time t, person i would like a meeting with person j when the increase in their rate of output while meeting with j is highest among all potential partners, including himself. Note that we use the *increase in the rate of output* $\dot{y}_i(t)$ rather than the rate of output $y_i(t)$ since in a continuous time model, the rate of output at time t is unaffected by the decision made at time t about whether to meet. As we are attempting to model close interactions within groups, we assume that at each time, the myopic persons interacting choose a core configuration.

In order to analyze our dynamic system, we first divide all of our equations by the total number of ideas possessed by i and j:

$$n^{ij} = n_{ij}^d + n_{ji}^d + n_{ij}^c \tag{4.10}$$

and define new variables

$$m_{ij}^c \equiv m_{ji}^c = \frac{n_{ij}^c}{n^{ij}} = \frac{n_{ji}^c}{n^{ij}},$$

$$m_{ij}^d = \frac{n_{ij}^d}{n^{ij}}, \quad m_{ji}^d = \frac{n_{ji}^d}{n^{ij}}.$$

By definition, m_{ij}^d represents the percentage of ideas exclusive to person i among all the ideas known by person i or person j. Similarly, m_{ij}^c represents the ideas known in common by persons i and j among all the ideas known by the pair. From (4.10), we obtain

$$1 = m_{ij}^d + m_{ji}^d + m_{ij}^c. \tag{4.11}$$

Then, using (4.7)–(4.9) and (4.11), we can rewrite the income growth rate, (4.6), as follows:[10]

$$\frac{\dot{y}_i}{y_i} = \frac{\dot{n}_i}{n_i} = \delta_{ii}\alpha + \sum_{j \neq i} \delta_{ij} \frac{\beta\left[(1 - m_{ij}^d - m_{ji}^d) m_{ij}^d m_{ji}^d\right]^{\frac{1}{3}}}{1 - m_{ji}^d}, \tag{4.12}$$

[10]For details of the analyses in the rest of this paper, see Berliant and Fujita (2007).

where

$$\dot{m}_{ij}^{d} = \alpha(1 - m_{ij}^{d})\left[\delta_{ii}\left(1 - m_{ji}^{d}\right) - \delta_{jj}m_{ij}^{d}\right] - \delta_{ij}m_{ij}^{d}\beta\left[\left(1 - m_{ij}^{d} - m_{ji}^{d}\right)m_{ij}^{d}m_{ji}^{d}\right]^{\frac{1}{3}}$$
$$+ \left(1 - m_{ij}^{d}\right)\left(1 - m_{ji}^{d}\right)\sum_{k\neq i,j}\delta_{ik}\frac{\beta\left[\left(1 - m_{ik}^{d} - m_{ki}^{d}\right)m_{ik}^{d}m_{ki}^{d}\right]^{\frac{1}{3}}}{1 - m_{ki}^{d}}$$
$$- \left(1 - m_{ij}^{d}\right)m_{ij}^{d}\sum_{k\neq i,j}\delta_{jk}\frac{\beta\left[\left(1 - m_{jk}^{d} - m_{kj}^{d}\right)m_{jk}^{d}m_{kj}^{d}\right]^{\frac{1}{3}}}{1 - m_{kj}^{d}} \qquad \text{for } i,j = 1,2,\ldots,N\,. \tag{4.13}$$

At time t, since $y_i(t)$ is a state-variable, maximizing $\dot{y}_i(t)$ is equivalent to maximizing the growth rate, $\dot{y}_i(t)/y_i(t)$. Hence, at each moment of time, the equilibrium values of δ_{ij} $(i,j = 1,2,\ldots,N)$ are to be determined as the core of the game in which each agent wishes to maximize the growth rate of income given by (4.12). Thus, the dynamics of the system are described in terms of m_{ij}^{d} $(i,j = 1,2,\ldots,N)$ only.

4.3 Equilibrium Dynamics

4.3.1 The General Framework

The model with only two people is very limited. Either two people are meeting or they are each working in isolation. With more people, the dancers can be partitioned into many pairs of dance partners. Within each pair, the two dancers are working together, but pairs of partners are working simultaneously. This creates more possibilities in our model, as the knowledge created within a dance pair is not known to other pairs. Thus, knowledge differentiation can evolve between different pairs of dance partners. Furthermore, the option of switching partners is now available.

We limit ourselves to the case where N is divisible by 4. This is a square dance on the vertices of the Hilbert cube. When the population is not divisible by 4, our most useful tool, symmetry, cannot be used to examine dynamics. Although this may seem restrictive, when N is large, asymmetries apply only to a small fraction of the population, and thus become negligible. In the general case, we impose the assumption of pairwise symmetric initial heterogeneity conditions for all agents.

The initial state of knowledge is symmetric among the dancers, and given by

$$n_{ij}^{c}(0) = n^{c}(0) \quad \text{for all } i \neq j, \tag{4.14}$$

$$n_{ij}^{d}(0) = n^{d}(0) \quad \text{for all } i \neq j. \tag{4.15}$$

At the initial state, each pair of dancers has the same number of ideas, $n^c(0)$, in common. Moreover, for any pair of dancers, the number of ideas that one dancer knows but the other does not know is the same and equal to $n^d(0)$. Given that the initial state of knowledge is symmetric among the four dancers, it turns out that the equilibrium configuration at any time also maintains the basic symmetry among the dancers.

When all dancers are pairwise symmetric to each other, that is, when

$$m_{ij}^d = m_{ji}^d \quad \text{for all } i \neq j \tag{4.16}$$

the income growth rate (4.12) is simplified as

$$\frac{\dot{y}_i}{y_i} = \frac{\dot{n}_i}{n_i} = \delta_{ii}\alpha + \sum_{j \neq i} \delta_{ij} g(m_{ij}^d) \tag{4.17}$$

and the dynamics (4.13) can be rewritten as

$$\begin{aligned} \frac{\dot{m}_{ij}^d}{1 - m_{ij}^d} &= \alpha\left[\delta_{ii}\left(1 - m_{ij}^d\right) - \delta_{jj} m_{ij}^d\right] - \delta_{ij} m_{ij}^d g(m_{ij}^d) \\ &\quad + \left(1 - m_{ij}^d\right) \sum_{k \neq i,j} \delta_{ik} g(m_{ik}^d) - m_{ij}^d \sum_{k \neq i,j} \delta_{jk} g(m_{jk}^d), \end{aligned} \tag{4.18}$$

where the function $g(m)$ is defined as

$$g(m) = \beta\left[\left(1 - \frac{m}{1-m}\right)\left(\frac{m}{1-m}\right)^2\right]^{\frac{1}{3}} \tag{4.19}$$

which represents the growth rate when the two persons meet. Figure 4.3 illustrates the graph of the function $g(m)$ as a bold line for $\beta = 1$.

Differentiating $g(m)$ yields, we can readily see that

$$g'(m) \gtrless 0 \quad \text{as } m \lessgtr \frac{2}{5} \text{ for } m \in \left(0, \frac{1}{2}\right). \tag{4.20}$$

Thus, $g(m)$ is strictly quasi-concave on $\left[0, \frac{1}{2}\right]$, achieving its maximal value at $m^B = \frac{2}{5}$; we call the latter the "Bliss Point". It is the point where the rate of increase in income or utility is maximized for each person.

Next, taking the case of $N = 4$, we illustrate the possible equilibrium configurations, noting that the equilibrium configuration can vary with time. Figure 4.4 gives the possibilities at any fixed time for $N = 4$. Given that the initial state of knowledge is symmetric among the four dancers, as noted above, the equilibrium configuration at any time also maintains the basic symmetry among dancers.

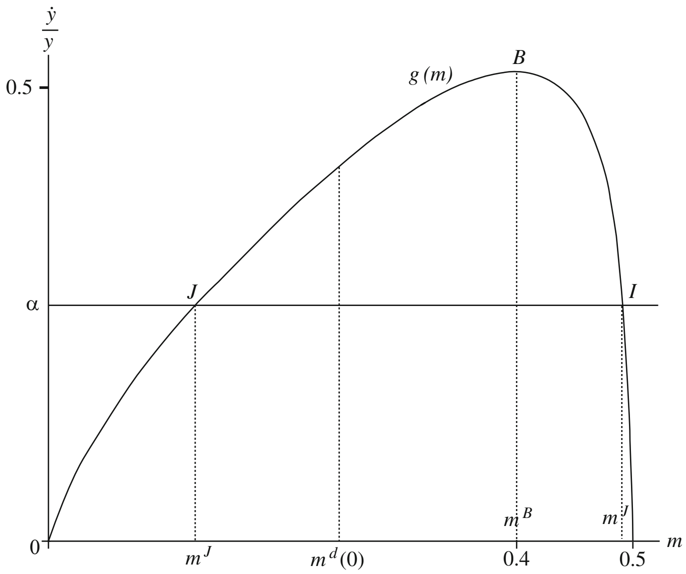

Fig. 4.3 The $g(m)$ curve and the bliss point when $\beta = 1$

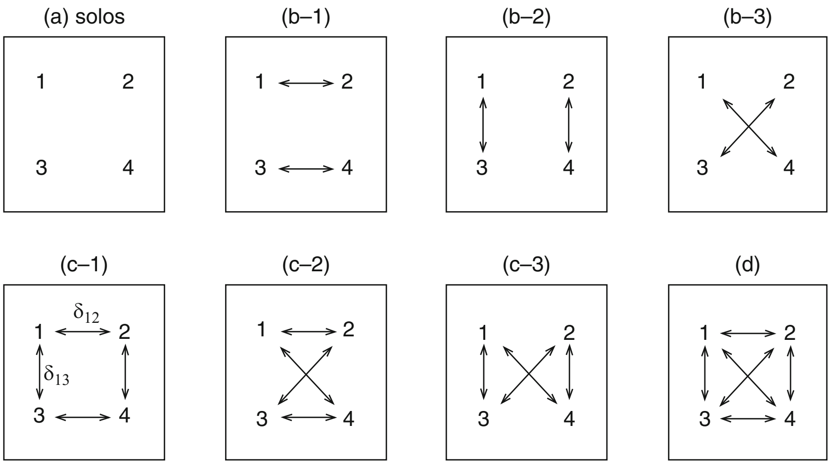

Fig. 4.4 Possible equilibrium configurations when N=4

Panel (a) in Fig.4.4 represents the case in which each of the four dancers is working alone, creating new ideas in isolation. Panels (b-1)–(b-3) represent the three possible configurations of partner dancing, in which each of the two couples dance separately but simultaneously. In panel (b-1), for example, 1 and 2 dance together. At the same time, 3 and 4 dance together.

Although panels (a)–(b-3) represent the basic forms of dance with four persons, it turns out that the equilibrium path often requires a mixture of these basic forms. That is, on the equilibrium path, people wish to change partners as frequently as possible. The purpose is to balance the number of different and common ideas with partners as best as can be achieved. This suggests a square dance with rapidly changing partners on the equilibrium path.

Please refer to panels (c-1)–(c-3) in Fig. 4.4. Each of these panels represents square dancing where a dancer rotates through two fixed partners as fast as possible in order to maximize the instantaneous increase in their income. In panel (c-1), for example, dancer 1 chooses dancers 2 and 3 as partners, and rotates between the two partners under equilibrium values of δ_{12} and δ_{13} such that $\delta_{12} + \delta_{13} = 1$. Dancers 2, 3 and 4 behave analogously. In order for this type of square dance to take place, of course, all four persons must agree to follow this pattern. Finally, panel (d) depicts square dancing in which each dancer rotates though all three possible partners as fast as possible. That is, for all $i \neq j$, $\delta_{ij} \in (0, 1)$, and for all i, $\delta_{ii} = 0$ and $\sum_{j \neq i} \delta_{ij} = 1$.

At this point, it is useful to remind the reader that we are using a myopic core concept to determine equilibrium at each point in time. In fact, it is necessary to sharpen that concept in the model with N persons. When there is more than one vector of strategies that is in the myopic core at a particular time, namely more than one vector of joint strategies implies the same, highest first derivative of income for all persons, the one with the highest second derivative of income is selected. The justification for this assumption is that at each point in time, people are attempting to maximize the flow of income.

Now we are ready to investigate the actual equilibrium path, depending on the given initial composition of knowledge, which is common for all pairs i and j ($i \neq j$). In Fig. 4.3, let m^J and m^I be defined on the horizontal axis at the left intersection and the right intersection between the $g(m)$ curve and the horizontal line at height α, respectively.

$$m^d_{ij}(0) = m^d(0) = \frac{n^d(0)}{n^c(0) + 2n^d(0)}.$$

In the remainder of this paper, we assume that

$$\alpha < g(m^B) \tag{4.21}$$

so as to avoid the trivial case of all agents always working in isolation.

Figure 4.5 provides a diagram explaining our main result.

The top horizontal line represents the initial common state $m^d(0)$, while the bottom horizontal line represents the final common state or sink point, $m^d(\infty)$. There are four regions of the initial state that result in four different sink points, which are explained in turn below.

Case 1:

$$0 < m^d(0) \leq 2/5 = m^B$$

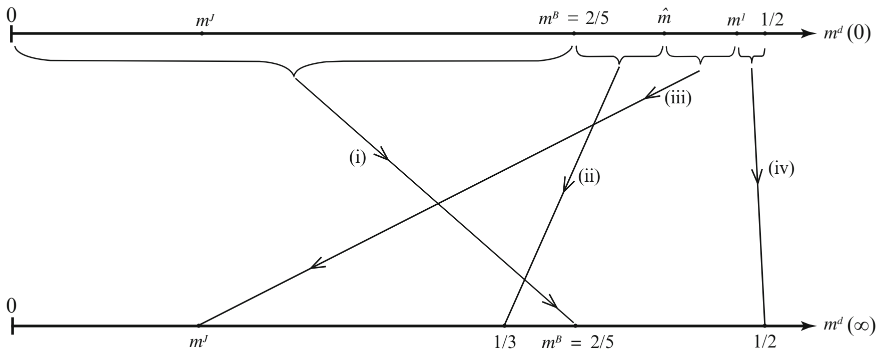

Fig. 4.5 Correspondence between the initial point $m^d(0)$ and the long-run equilibrium point $m^d(\infty)$

First suppose that the initial state is such that

$$m^J < m^d(0) \leq m^B.$$

Then, since $g(m_{ij}^d(0)) = g(m^d(0)) > \alpha$ for any possible dance pairs consisting of i and j, no person wishes to dance alone at the start. However, since the value of $g(m_{ij}^d(0))$ is the same for all possible pairs, all forms of (b-1) to (d) in Fig. 4.4 are possible equilibrium dance configurations at the start. To determine which one of them will actually take place on the equilibrium path, we must consider the second derivative of income for all persons.

In general, consider any time at which all persons have the same composition of knowledge:

$$m_{ij}^d = m^d \quad \text{for all } i \neq j, \tag{4.22}$$

where

$$g(m^d) > \alpha.$$

Focus on person i; the equations for other persons are analogous. Since person i does not wish to dance alone, it follows that

$$\delta_{ii} = 0 \quad \text{and} \quad \sum_{j \neq i} \delta_{ij} = 1. \tag{4.23}$$

Substituting (4.22) and (4.23) into (4.17) yields

$$\frac{\dot{y}_i}{y_i} = g(m^d).$$

Likewise, substituting (4.22) and (4.23) into (4.18) and arranging terms gives

$$\dot{m}_{ij}^d = (1 - m^d) g(m^d) \left[1 - 2m^d - \left(1 - m^d\right)\delta_{ij}\right]. \tag{4.24}$$

Since the income growth rate $\dot{y}/y$ above is independent of the values of δ_{ij} $(j \neq i)$, in order to examine what values of δ_{ij} $(j \neq i)$ person i wishes to choose, we must consider the time derivative of $\dot{y}_i/y_i$. From this second-order condition, we can show that each person, say i, chooses the optimal strategy such that

$$\delta_{ij} = \frac{1}{N-1} \quad \text{for all } j \neq i. \tag{4.25}$$

The vector of optimal strategies is the same for all persons. Thus, all persons agree to a square dance in which each person rotates through all $N - 1$ possible partners while sharing the time equally.

The intuition behind this result is as follows. The condition $m^d < 2/5 \equiv m^B$ means that the dancers have relatively too many ideas in common, and thus they wish to acquire ideas that are different from those of each possible partner as fast as possible. That is, when $m^J < m_{ij}^d = m^d < m^B$ in Fig. 4.3, each dancer wishes to move the knowledge composition m_{ij}^d to the right as quickly as possible, thus increasing the growth rate $g(m_{ij}^d)$ as fast as possible. This means that when $m^J < m^d(0) = m_{ji}^d(0) < 2/5\,(= m^B)$ for all $i \neq j$, on the equilibrium path, the square dance with $\delta_{ij} = 1/(N-1)$ for all $i \neq j$ takes place at the start. Then, since the symmetric condition (4.22) holds thenceforth, the same square dance will continues as long as $m^J < m^d < 2/5\,(= m^B)$. The dynamics of this square dance are as follows. Setting $m_{ij}^d = m^d$ and $\delta_{ij} = 1/(N-1)$ in (4.24), we obtain

$$\dot{m}^d = (1 - m^d) g(m^d) \frac{(N-2) - (2N-3)m^d}{N-1}. \tag{4.26}$$

Setting $\dot{m}^d = 0$ and considering that $m^d < 1$, we obtain the sink point

$$m^{d*} = \frac{N-2}{2N-3}. \tag{4.27}$$

Surprisingly, when $N = 4$, $m^{d*} = 2/5 = m^B$. The value of $\dot{m}^d$ is positive when $m^d < m^B = 2/5$, and zero if $m^d = 2/5$. Hence, beginning at any point $m^d(0) = 2/5$, the system moves to the right, eventually settling at the bliss point m^B.

Since the right hand side of (4.27) is increasing in N,

$$m^{d*} = \frac{N-2}{2N-3} > 2/5 \equiv m^B \quad \text{when } N > 4. \tag{4.28}$$

Hence, when $N > 4$ and N is divisible by 4, beginning at any point $m^J < m^d(0) < 2/5$, the system moves to the right and reaches $m^B = 2/5$ in finite time. *When N agents reach the bliss point m^B, they break into groups of* 4 *to maintain heterogeneity at the bliss point.*

Next, when $0 \leq m^d(0) < m^J$, it is obvious that the four persons work alone until they reach m^J. Then they follow the path explained above, eventually reaching m^B.

Case 2:

$$m^B < m^d(0) \leq \hat{m}$$

Next, let us consider the dynamics of the system when it begins to the right of $m^B = 2/5$ but to the left of $\hat{m} < m^I$.[11] The equilibrium process takes the following three phases.

Phase 1: Since the initial state reflects a higher degree of heterogeneity than the bliss point, the dancers want to increase the knowledge they have in common as fast as possible, which leads to *couple dances*. Thus, person i wishes to choose any partner, say k, and set $\delta_{ik} = 1$, whereas $\delta_{ij} = 0$ for all $j \neq k$. The situation is the same for all dancers. Hence, without loss of generality, we can assume that N persons agree at time 0 to form the following combination of partnerships:

$$P_1 \equiv \{\{1,2\}, \{3,4\}, \{5,6\}, \ldots, \{N-1, N\}\} \tag{4.29}$$

and initiate pairwise dancing such that

$$\delta_{ij} = \delta_{ji} = 1 \quad \text{for } \{i,j\} \in P_1, \qquad \delta_{ij} = \delta_{ji} = 0 \quad \text{for } \{i,j\} \notin P_1. \tag{4.30}$$

The same pairwise dance, however, cannot continue too long by the following reason. On one hand, the proportion of differential knowledge for each couple, say $\{1,2\}$, decreases with time, making the partnership less productive eventually. On the other hand, the proportion of the differential knowledge increases for any pair of persons, say $\{1,3\}$, who are not dancing together. Thus, eventually, the *shadow partnership* $\{1,3\} \notin P_1$ becomes more productive than the *actual partnership* $\{1,2\} \in P$. Thus, there exists a *switching time* t' at which each dancer switches to a new partner.

Phase 2: One example of new equilibrium partnerships at the switching time t' is given by

$$P_2 \equiv \{\{1,3\}, \{2,4\}, \{5,7\}, \{6,8\}, \ldots, \{N-3, N-1\}, \{N-2, N\}\} \tag{4.31}$$

meaning that the first four persons form a group and exchange partners, the next four persons form another group and switch partners, and so on. (There exist many other possibilities for equilibrium partnerships to be chosen by N dancers at time t'. It turns out, however, that the essential characteristics of equilibrium dynamics are

[11] For the determination $\hat{m}$, see Berliant and Fujita (2007).

not affected by the choice at time t'. Hence, let us assume that N persons agree to choose the new partnerships P_2 at time t'.)

It turns out, however, that these new partnerships last only for a limited time. To examine this point, let us notice that in the dance form P_2, each group of four persons is isolated from everyone else. Thus, in the sequel, we focus on the dynamics of a four-person group, 1, 2, 3 and 4. Under the partnership P_2, since $m^d_{12}(t)$ is increasing with time while $m^d_{13}(t)$ is decreasing, there exists a time t'' at which $m^d_{12}(t)$ and $m^d_{13}(t)$ become the same,

$$m^d_{12}(t'') = m^d_{13}(t'')m^B \tag{4.32}$$

which can be shown to occur in the left of the bliss point m^B. Thus, if partnerships $\{1,3\}$ and $\{2,4\}$ were maintained beyond time t'', then it would follow from (4.20) that

$$g(m^d_{12}(t)) > g(m^d_{13}(t)) \quad \text{for } t > t''.$$

This implies that the same partnerships cannot be continued beyond t''. To see what form of dance will take place after t'', first note that dancers cannot go back to the previous form of partnerships $\{1,2\}$ and $\{3,4\}$. If they did so, then the proportion of the knowledge in common for the actual partners $\{1,2\}$ would increase, while the proportion of the differential knowledge for the shadow partnership $\{3,4\}$ would increase. This means that the following relationship,

$$m^d_{12}(t) < m^d(t'') < m^d_{13}(t) < m^B$$

holds immediately after t'', and thus

$$g(m^d_{12}(t)) < g(m^d_{13}(t))$$

which contradicts the assumption that $\{1,2\}$ is the actual partnership. Furthermore, relation (4.32) implies that under any possible partnership, the following inequality

$$g(m^d_{13}(t)) > g(m^d_{14}(t))$$

holds immediately after t''. Thus, immediately after time t'', the equilibrium dance cannot include partnerships $\{1,4\}$ and $\{2,3\}$. Hence, provided that $g(1/3) > \alpha$, we can see from Fig. 4.4 that the only possible equilibrium configuration immediately after t'' is a square dance in the form (c-1) involving a rapid rotation of non-diagonal partnerships, $\{1,2\}$, $\{1,3\}$, $\{2,4\}$ and $\{3,4\}$. That is, for dancer 1, $\delta_{11} = 0$ and $\delta_{1j} = \frac{1}{2}$ if $j = 2$ or 3, $\delta_{14} = 0$. Analogous expressions hold for the other dancers.

Phase 3: The dynamics for this square dance under the form (c-1) are as follows. We set

$$m^d_{ij} \equiv m^d \quad \text{for } \{i,j\} \in P_2.$$

Then, since conditions (4.22) and (4.23) hold also in the present context, setting $\delta_{ij} = 1/2$ in (4.24), we get

$$\dot{m}^d = (1 - m^d) g(m^d) \frac{1 - 3m^d}{2},$$

which is negative when $m^d > \frac{1}{3}$, and zero if $m^d = \frac{1}{3}$. Thus, beginning at any point $m^d(t'') > \frac{1}{3}$, the system moves to the left, eventually settling at $m^d = \frac{1}{3}$.
Case 3:

$$\hat{m} < m^d(0) \leq m^J$$

Next suppose $m^d(0)$ is such that $\hat{m} < m^d(0) \leq m^J$. As in Case 2, dancers are more heterogeneous than at the bliss point, so they would like to increase the knowledge they hold in common through couple dancing, for example using configuration (b-1) in Fig. 4.4. The initial phase of Case 3 is the same as the initial phase of Case 2. However, since $g(m^d_{12}(t)) > g(m^d_{13}(t))$ for all t before $m^d_{12}(t)$ reaches m^J, whereas $g(m^d_{12}(t)) > \alpha > g(m^d_{13}(t))$ when $m^d_{12}(t)$ reaches m^J. So each dancer keeps their original partner as the system climbs up to B and on to J. When the system reaches $m^d(t) = m^J$, each dancer uses fractional δ_{ij} to attain m^J by switching between working in isolation and dancing with their original partner.
Case 4:

$$m^J < m^d(0) \leq 1/2$$

Finally, suppose $m^d(0) > m^J$. Then, $g(m^d(0))\alpha$, and hence there is no reason for anyone to form a partnership. Thus, each person dances alone forever, and eventually reaches $m^d = 1/2$.

Compiling all four cases, we obtain the result summarized in Fig. 4.5. There are important remarks to be made about the result. First, the sink point changes discontinuously with changes in the initial conditions. Second, from each set of initial conditions, the N persons eventually divide into many separate groups between which no interaction occurs. Thus, from an initial state that is symmetric, we obtain an equilibrium path featuring asymmetry. Third, concerning the welfare properties of the equilibrium path, the most surprising result is with Case 1. That is, *whenever $m^d(0) < m^B$, the equilibrium path either approaches (when $N = 4$) or reaches in finite time (when $N > 4$) the most productive state, m^B*. Clearly, initial heterogeneity plays an important role in the efficiency properties of the equilibrium path. What distinguishes Case 1, aside from a relatively homogeneous beginning, is that the dancers can switch partners rapidly enough to increase heterogeneity while at the same time maximizing the increase in output. That is because each agent spends $1/(N-1)$ of the time dancing with any particular agent, and

$(N-2)/(N-1)$ of the time dancing with others. This is what leads to the most productive state.[12]

Bearing in mind the limitations of the model, it may have empirical relevance. The main result may explain the agglomeration of a large number of small firms in Higashi Osaka or in Ota ward in Tokyo, each specializing in different but related manufacturing services. Another example is the third Italy, which produces a large variety of differentiated products. Yet another example is the restaurant industry in Berkeley, California. In each case, tacit knowledge accumulated within firms plays a central role in operation of the firms.

4.4 Conclusion

We have presented a micro-model of knowledge creation through the interaction of a group of people. Our model incorporates two key aspects of the cooperative process of knowledge creation: (1) heterogeneity of people in their state of knowledge is essential for successful cooperation in the joint creation of new ideas, while (2) the very process of cooperative knowledge creation affects the heterogeneity of people through the accumulation of knowledge in common. The model features myopic agents in a pure externality model of interaction. Surprisingly, in the general case for a large set of initial conditions we find that the equilibrium process of knowledge creation may converge to the most productive state, where the population splits into smaller groups of optimal size; close interaction takes place within each group only. This optimal size is larger as the heterogeneity of knowledge is more important in the knowledge production process. Equilibrium paths are found analytically, and they are a discontinuous function of initial heterogeneity.

However, what we have done so far is, in effect, to open Pandora's box, scattering around a great number of new problems to be investigated further. Indeed, to take our model more realistic and interesting, we must extend it by considering/introducing various new elements such as knowledge transfer, knowledge structures and hierarchies, side payments and the markets for ideas, foresights and strategic behavior, and uncertainty and stochastic elements. In particular, we must return to our original motivation for this model, as stated in the introduction. That is, location seems to be an essential feature of knowledge creation and transfer, so regions and migration are important, along with urban economic concepts more generally. Thus, incorporating locations/regions in our model, we may be able to move one step closer to our ultimate objective of developing a comprehensive

[12]Here, it is natural ask why the optimal group size in knowledge production is four. Actually, using a more general functional form of joint knowledge production, Berliant and Fujita (2007) shown that when differential knowledge is relatively more important than common knowledge in knowledge production, the optimal group size is larger.

theory of geographical economics in the brain power society, in which the dual linkages in the economic and knowledge fields work in unison.

As the model becomes more realistic and hence more complex, however, its analytical tractability reaches the limit soon. Eventually, thus, we must appeal to computer simulations. In particular, the evolutionary process of knowledge creation and transfer may be simulated with the help of multi-agent-based simulation.

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