Finite Element Method and Optimal Control Theory for Path Planning of Elastic Manipulators

M.H. Korayem, M. Haghpanahi, H.N. Rahimi, and A. Nikoobin

Abstract. Planning of robot trajectory is a very complex task that plays a crucial role in design and application of robots in task space. This paper is concerned with path planning of flexible robot arms for a given two-end-point task in point-to-point motion, based on indirect solution of optimal control problem. We employ the finite element method to modeling and deriving the dynamic equations of robot manipulator with flexible link, so in the presence of all nonlinear terms in dynamic equations open loop optimal control approach is a good candidate for generating the path that optimizes the end effector trajectory. Then the Hamiltonian function is formed and the necessary conditions for optimality are derived from the Pontryagin's minimum principle. The obtained equations establish a two point boundary value problem which is solved by numerical techniques. Finally, simulations for a two-link planar manipulator with flexible links are carried out to investigate the efficiency of the presented method. The results illustrate the power and efficiency of the method to overcome the high nonlinearity nature of the problem.

Keywords: Flexible Manipulator, Finite Element, Optimal Trajectory, Optimal Control.

1 Introduction

Flexible robot arms have some advantages over rigid ones, such as their capability to assure faster motions and a higher ratio of payload to arm weight. However, due to the flexible nature of the system, their dynamic equations are highly non-linear and complex.

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K. Nakamatsu et al. (Eds.): New Advan. in Intel. Decision Techno., SCI 199, pp. 117–126.
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Wang et al. have solved the optimal control problem with direct method using the B-Spline functions in order to determine the maximum payload of a rigid manipulator [1]. The assumed mode expansion method is used by Sasiadek and Green [2, 3] to derive the dynamic equation of fixed base flexible manipulator. In [4, 5] a formulation based on Iterative Linear Programming (ILP) is presented to determine the Maximum Allowable Dynamic Load (MADL) of flexible manipulators. Indeed, the linearizing procedure and its convergence to the proper answer is a challenging issue, especially when nonlinear terms are large and fluctuating, e.g. in problems with consideration of flexibility in links or having high speed motion. As a result, in none of the previous mentioned work which is based on the ILP method, the link flexibility has not been considered either in the dynamic equation or simulation procedure.

None of these published works have used Finite Element Method (FEM) to model and analysis for their systems. One of the main advantages of FEM over the most of other approximate solution methods to modeling the flexible links is the fact that in FEM the connection are supposed to be clamp-free with minimum two mode shape per link. Another significant advantage of FEM, especially over analytical solution techniques is the ease with which nonlinear conditions can be handled. The finite element method has been used to solve very complex structural engineering problems during the past years. The maximum payload of flexible mobile manipulator is determined along the given trajectory by using the finite element approach in [6], so finding the optimal path is not considered in it.

Optimal control can be used in both open loop and close-loop strategies. However, because of the off-line nature of the open loop optimal control in spite of the close-loop ones, many difficulties such as system nonlinearities and all types of constraints may be catered for and implemented easily, so it generally used in analyzing nonlinear systems such as trajectory optimization of different types of robots [7, 8]. It solved by direct and indirect approaches. But, since direct method leads to the approximate solution and this approach is time consuming and quite ineffective due to the large number of parameters involved [9], indirect methods is a good candidate for the cases where the system has a large number of degree of freedom or optimization of the various objectives is targeted [10].

Open-loop optimal control method is proposed as an approach for trajectory optimization of flexible link mobile manipulator for a given two-end-point task in point-to-point motion [11]. But in mentioned paper combined Euler–Lagrange formulation and assumed modes method is used for driving the equation of motions with considering the simply support mode shape and one mode per link. So beside the advantages of this paper over than ILP based ones it can not expressed realistically the behavior of links besides it connection to the motors.

In this paper, for path planning of Elastic manipulators, an indirect solution of the optimal control problem is employed. Dynamic equations are derived using the FEM. Hamiltonian function is formed, and necessary conditions for optimality are obtained from the Pontryagin's minimum principle. These equations establish a Two Point Boundary Value Problem (TPBVP) solved by MATLAB. In comparison with other method the open-loop optimal control method does not require linearizing the equations, differentiating with respect to joint parameters and using of a fixed-order polynomial as the solution form. Finally, a two-link elastic manipulator is simulated to demonstrate the capability of the method.

2 Modeling of Robot Arms with Multiple Flexible Links

The finite element method is used to derive dynamic equations of flexible manipulators. The overall approach involves treating each link of the manipulator as an assemblage of *n* elements of length L_i . For each of these elements the kinetic energy T_{ij} and potential energy V_{ij} , (where *i* and *j* are referred to the number of links and the number of elements respectively) are computed in terms of a selected system of n generalized variables $q = (q_{1,} \quad q_{2,} \quad ..., \quad q_n)$ and their rate of change $\dot{q}$. These energies are then combined to obtain the total kinetic energy, *T*, and potential energy, *V*, for the entire system. Finally, using Lagrange equations the equations can be written in compact form as:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = U, \tag{1}$$

By defining the state vector as:

$$X = [X_1 \quad X_2]^T = [q \quad \dot{q}]^T \, , \tag{2}$$

Eq. (1) can be rewritten in state space form as:

$$\dot{X} = [\dot{X}_1 \quad \dot{X}_2]^T = [X_2 \quad N(X) + D(X)U] \, , \tag{3}$$

where $N = -M^{-1}(C(X_1, X_2) + G(X_1))$ and $D = M^{-1}$. Then optimal control problem is imposed to determine the position and velocity variable $X_1(t)$ and $X_2(t)$ and the joint torque *U(t)* which optimize a well-defined performance measure when the model is given in Eq. (3)

3 Formulation of the Optimal Control Problem

The basic idea to improve the formulation is to find the optimal path for a specified payload. For the sake of this, the following objective function is considered

$$\underset{U(t)}{Minimize}\, J_0 = \int_{t_0}^{t_f} L(X,U)dt \, , \tag{4}$$

where

$$L(X,U) = \frac{1}{2}\left\|X_2\right\|_W^2 + \frac{1}{2}\left\|U\right\|_R^2 . \tag{5}$$

Integrand *L(.)* is a smooth, differentiable function in the arguments, $\| X \|_K^2 = X^T KX$ is the generalized squared norm, *W* is symmetric, positive semi-definite ($k\times k$) weighting matrix and *R* is symmetric, positive definite ($k\times k$) matrix. The objective function specified by Eqs. (4) and (5) is minimized over the entire duration of the motion. The designer can decide on the relative importance among the angular position, angular velocity and control effort by the numerical choice of *W* and *R* which can also be used to convert the dimensions of the terms to consistent units. According to the Pontryagin's minimum principle, the following conditions must be satisfied,

$$\dot{X} = \partial H / \partial \psi \tag{6}$$

$$\dot{\psi} = -\partial H / \partial X \tag{7}$$

$$0 = \partial H / \partial U \tag{8}$$

where by defining the nonzero costate vector $\psi = \begin{bmatrix}\psi_1^T & \psi_2^T\end{bmatrix}^T$, the Hamiltonian function can be obtained as:

$$\mathrm{H(X,U,}\psi) = 0.5(\left\|X_2\right\|_W^2 + \left\|U\right\|_R^2) + \psi_1^T X_2 + \psi_2^T \left[N(X) + D(X)U\right]. \tag{9}$$

So, according to Eq. (7), the optimality conditions can be obtained by differentiating the Hamiltonian function with respect to states, costates and control as follows:

$$\begin{bmatrix}\dot{X}_1 & \dot{X}_2\end{bmatrix}^T = \begin{bmatrix}X_2 & N(X) + D(X)U\end{bmatrix}^T \tag{10}$$

$$\begin{bmatrix}\dot{\psi}_1 & \dot{\psi}_2\end{bmatrix}^T = -\begin{bmatrix}\partial H/\partial X_1 & \partial H/\partial X_2\end{bmatrix}^T \tag{11}$$

$$RU + D^T \psi_2 = 0 \tag{12}$$

The control values are limited with upper and lower bounds, so using Eq. (12) the optimal control are given by:

$$U = \begin{cases} U^+ & -R^{-1}D^T\psi_2 > U^+ \\ -R^{-1}D^T\psi_2 & U^- < -R^{-1}D^T\psi_2 < U^+ \\ U^- & -R^{-1}D^T\psi_2 < U^- \end{cases} . \tag{13}$$

The actuators which are used for medium and small size manipulators are the permanent magnet D.C. motor. The torque speed characteristic of such D.C. motors may be represented by the following linear equation:

$$U^{+} = K_1 - K_2 X_2 \ , U^{-} = -K_1 - K_2 X_2 \,. \tag{14}$$

Where $K_1 = [\tau_{s1} \ \tau_{s2} \cdots \tau_{sn}]^T$, $K_2 = dig[\tau_{s1}/\omega_{m1} \cdots \tau_{sn}/\omega_{mn}]$, $\dot{\theta} = [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \ldots \quad \dot{\theta}_n]^T$, τ_s is the stall torque and ω_m is the maximum no-load speed of the motor. The boundary values will be expressed as below:

$$\begin{aligned} X_1(0) = X_{10},\ X_2(0) = X_{20} \quad ; \\ X_1(t_f) = X_{1f},\ X_2(t_f) = X_{2f} \end{aligned} \tag{15}$$

In this formulation, for a specified payload value, 4*m* differential equations given in Eq. (11) are used to determine the 4*m* state and costate variables. The set of differential Eq.(11), the control law Eq. (13), and the boundary conditions construct a standard form of TPBVP, which is solvable with available commands in different software such as MATLAB or MATEMATHICA.

4 Simulation for a Flexible Planar Manipulator

In this Section, simulations are carried out for a two-link planar flexible manipulator as shows in figure 4.1. This manipulator must carry a concentrated payload with mass of 1 kg during the overall time $t_f = 1$ second.

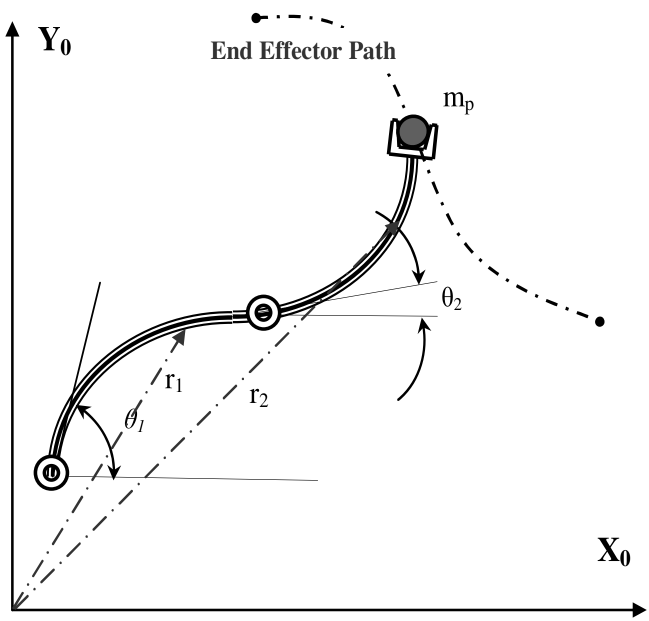

Fig. 4.1 Two-link manipulator with flexible links

The necessary parameters of the flexible manipulator are summarized in the Table 4.1.

Table 4.1 Simulation parameters

Parameter	Value	Unit
Length of Links	$L_1 = L_2 = 1$	m
Mass of Links	$m_1 = m_2 = 5$	Kg
Moment of Area of Links	$I_1 = I_2 = 5e\text{-}9$	m^4
Module of Elasticity of Links	$E_1 = E_2 = 2e10$	$Kg.m^2$
Max. no Load Speed of Actuators	$w_{s1} = w_{s2} = 3.5$	rad/s
Actuator Stall Torque	$\tau_{s1} = \tau_{s2} = 30$	N.m

By defining the state vectors as follows:

$$X_1 = Q^T = \begin{bmatrix} x_1 & x_3 & x_5 & x_7 & x_9 & x_{11} \end{bmatrix}^T, \quad X_2 = \dot{Q}^T = \begin{bmatrix} x_2 & x_4 & x_6 & x_8 & x_{10} & x_{12} \end{bmatrix}^T. \tag{16}$$

The sate space form of Eq. (16) can be written as:

$$\dot{x}_{2i-1} = x_{2i} \ , \ \dot{x}_{2i} = F_2(i) \quad ; i = 1\ldots 6 , \tag{17}$$

where F_2(i) can be obtained from Eq. (3). And the boundary condition can be expressed as:

$$\begin{aligned} & x_1(0) = \pi/2 \ \text{rad} , \ x_3(0) = 2\times\pi/3 \ \text{rad} \\ & x_1(t_f) = \pi/6 \ \text{rad}, \quad x_3(t_f) = \pi/3 \ \text{rad} \\ & x_{2i}(0) = x_{2i}(t_f) = 0, \quad i = 1\ldots 6 \\ & x_5(0) = x_5(t_f) = x_7(0) = x_7(t_f) = 0 \end{aligned} \tag{18}$$

In order to derive the equations associated with optimality conditions, penalty matrices can be selected as:

$$W = diag(w_1, w_2, w_3, w_4, w_5, w_6); \quad R = diag(r_1, r_2). \tag{19}$$

So the objective function is obtained by substituting Eq. (19) Into Eq. (5) as below

$$\mathrm{L} = \frac{1}{2}\left(\mathrm{r}_1 \mathrm{u}_1^2 + \mathrm{r}_2 \mathrm{u}_2^2 + \sum_{\mathrm{i}=1}^{6} \mathrm{w}_\mathrm{i} \mathrm{x}_{2\mathrm{i}}^2 \right). \tag{20}$$

Then, by considering the costate vector as $\varphi = [\varphi_1 \quad \varphi_2 \quad \ldots \quad \varphi_{12}]$, the Hamiltonian function can be expressed from as:

$$H = \frac{1}{2}\left(\mathrm{r}_1 \tau_1^2 + \mathrm{r}_2 \tau_2^2 + \sum_{\mathrm{i}=1}^{6} \mathrm{w}_\mathrm{i} \mathrm{x}_{2\mathrm{i}}^2 \right) + \sum_{i=1}^{12} \varphi_i \dot{x}_i \ , \tag{21}$$

where $\dot{x}_i, i = 1,\ldots,12$ can be substituted from Eq. (17). Using Eq. (11) differentiating the Hamiltonian function with respect to the states, result in costate equations as follows:

$$\dot{\varphi}_i = -\frac{\partial H}{\partial x_i}\ , \ i = 1,\cdots,12 \tag{22}$$

The control function in the admissible interval can be computed using Eq. (11), by differentiating the Hamiltonian function with respect to the torques (τ_1, τ_2) and setting the derivative equal to zero.

Then, by applying motors torque limitation, the optimal control becomes:

$$PU_i = \begin{cases} U_i^{+} & U_i > U^{+} \\ U_i & otherwise \\ U_i^{-} & U_i < U^{-} \end{cases} ; i = 1,2\,. \tag{23}$$

After that, from Eq. (14) the extrimal bound of control for each motor becomes:

$$\begin{aligned} U_1^{+} &= k_{11} - k_{12} x_2 \quad ; \quad U_1^{-} = -k_{11} - k_{12} x_2 \\ U_2^{+} &= k_{21} - k_{22} x_4 \quad ; \quad U_2^{-} = -k_{21} - k_{22} x_4 \end{aligned} \tag{24}$$

Consequently, substituting computed control equations (23) and (24) into Eqs. (17) and (22), obtain 16 nonlinear ordinary differential equations that with 16 boundary conditions given in Eq. (18), constructs a two point boundary value problem. A number of methods exist for solving these problems including shooting, collocation, and finite difference methods. In this study, BVP4C command in MATLAB® which is based on the collocation method is used to solve the obtained problem. The details of the numerical technique used in MATLAB to solve the TPBVP are given in [12]. This problem can be solved using the BVP4C command in MATLAB®.

In this simulation, the payload is considered to be 1 kg and the purpose is to find the optimal path between initial and final point of payload in such a way that the minimum amount of control value can be applied and the angular velocity values of motors be bounded in 2.

By considering the penalty matrices as *W*= (2, 2, 0, 0, 0, 0) and *R*=diag (0.1), the first path with appropriate amount of control value is determined, but the angular velocities are greater than 2 rad/s. Therefore for decreasing the velocities, *W* must be increased. A range of values of *W*= (*w, w*, 0, 0, 0, 0) used in simulation are given in Table 4.2.

Table 4.2 The values of w used in simulation

Case	1	2	3
w	2	50	500

Figs. 4.2 and 4.3 show the angular velocities of the first and second joints. It can be found that by increasing the *w*, maximum values of angular velocity reduce from -8.6 rad/s to -1.6 rad/s.

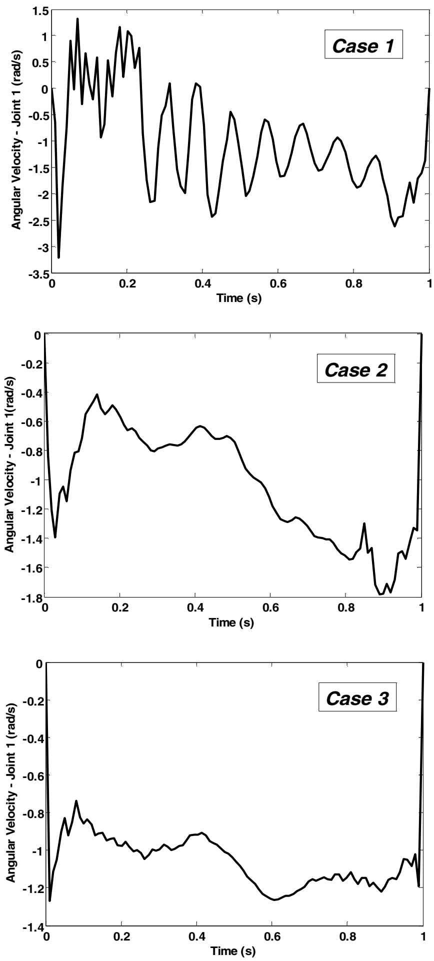

Fig. 4.2 Angular velocity-joint 1

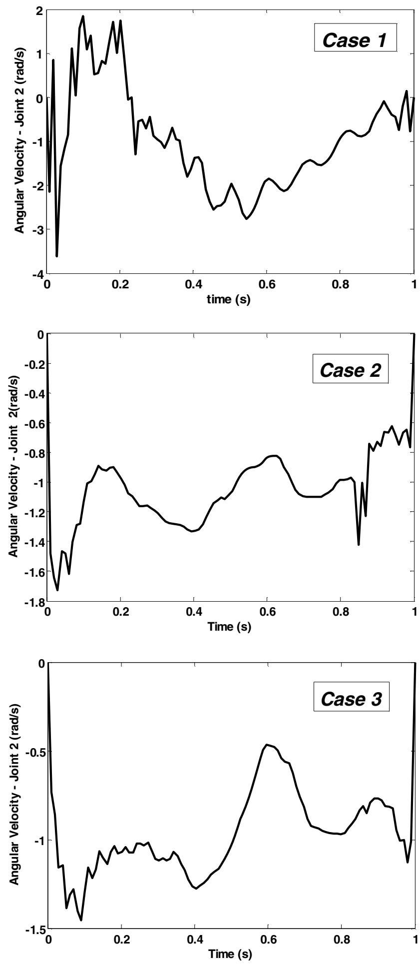

Fig. 4.3 Angular velocity-joint 2

Computed torqueses of motors are plotted in Figs. 4.4 and 4.5. As it can be seen, increasing the *w* causes to raise the torques. This result is predictable, because increasing the *w*, decreases the proportion of R and the result of this is increasing the control values.

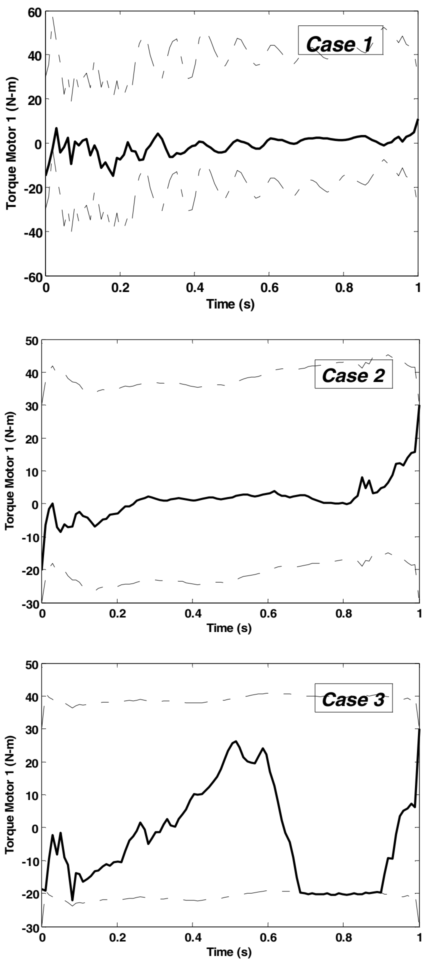

Fig. 4.4 Torque of motor 1

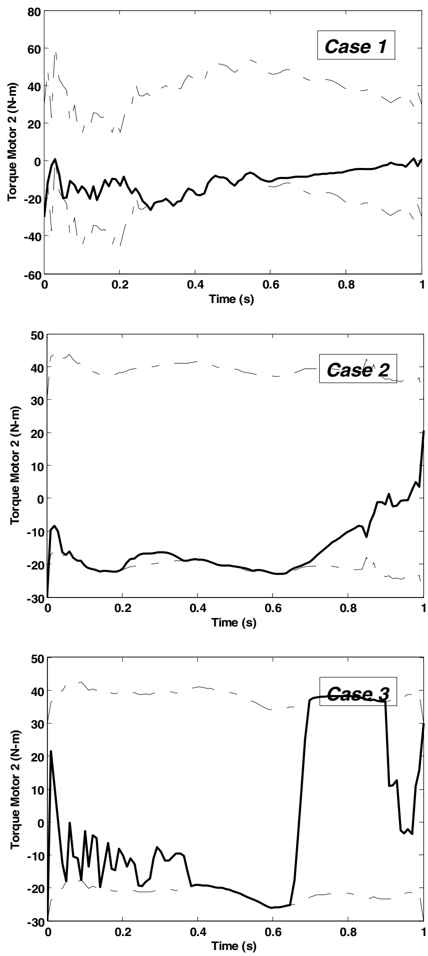

Fig. 4.5 Torque of motor 2

As show in figures (4.2- 4.5), there is not the solution that satisfies all the desired objectives simultaneously, e.g. the optimal path with minimum effort has maximum velocity and the optimal path with minimum velocity has maximum effort. Consequently, in this method, designer compromises between different objectives by considering the proper penalty matrices.

5 Conclusions

In this paper, formulation for the path planning of flexible robot arms in point-to-point motion, based on the open-loop optimal control approach is presented. Dynamic equations are derived using finite element method and an efficient solution on the basis of TPBVP is proposed. In comparison with other method the open-loop optimal control method does not require linearizing the equations, differentiating with respect to joint parameters and using of a fixed-order polynomial as the solution form. Moreover via changing the penalty matrices values, various optimal trajectories with different specifications can be obtained which able the designer to select a suitable path through a set of obtained paths. By a simulation study the application and validity of the algorithm is investigated. Results illustrate the power and capability of the method to overcome the high nonlinearity nature of the optimization problem in spite of using complete form of obtained nonlinear equations.

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