Analysis of Four Wheeled Flexible Joint Robotic Arms with Application on Optimal Motion Design

M.H. Korayem, H.N. Rahimi, and A. Nikoobin

Abstract. Designing optimal motion is critical in several applications for mobile robot from payload transport between two given states in a prescribed time such that a cost functional is minimized. This paper deals with the problem of path design of wheeled non-holonomic robots with flexible joints, based on Pontryagin's minimum principle. The simplified case study of a Four Wheeled, two-link manipulator with joint elasticity is considered to study the method in generalized model. Nonlinear state and control constraints are treated without any simplifications or transforming them into sequences of systems with linear equations. By these means, the modeling of the complete optimal control problem and the accompanying boundary value problem is automated to a great extent. Performance of method is illustrated through the computer simulation.

Keywords: Flexible Joint Robotic Arm, Non-holonomic Constraints, Optimal Motion Design, Pontryagin's Minimum Principle.

1 Introduction

Mobile robotic arms consist of a mobile platform equipped with mechanical manipulators. If assumed that the mobile base does not slide then a non-holonomic constraint is imposed on the system hence the system is considered as a non-holonomic platform. Besides, in many industrial applications such as high-speed assembly and heavy load carrying, the joint flexibility exists in most manipulators in the drive transmission systems (transmission belts, gears, shafts, etc.), that usually neglected to analysis of such flexible joint systems [1].

Several research works have been carried out for mobile robot arms. A comprehensive literature survey on nonholonomic systems can be found in [2].

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However, little work has been reported on a comprehensive model that Encompass mobile manipulator with flexible joint. In [3] a computational technique for obtaining the maximum load-carrying capacity of robotic manipulators with joint elasticity is described while different base positions are considered, so finding the optimal path is not considered in it. Another algorithm to the maximum load determination via linearizing the dynamic equation and constraints in [4] is studied on the basis of Iterative Linear Programming (ILP) method for flexible mobile manipulators. But, because of difficulties of ILP method the flexibilities of joints are neglected either in the dynamic equation or simulation procedure.

Nowadays the advantages of optimal control theory are well established and a host of issues related to this technique have been studying specially in the field of optimal motion planning of robots [5]. Accordingly indirect solution of open-loop optimal control method was proposed to trajectory optimization of flexible link mobile manipulator in point-to-point motion [6]. In mentioned work, despite ILP based studies, boundary conditions are satisfied exactly, in addition the complete form of the obtained nonlinear equation is used. However, the joints are assumed rigid in addition to kinematics and dynamics of wheels are not considered in it.

In this paper with applying the optimal control approach path generating of four wheeled mobile manipulators is done by modeling the elasticity at each joint as a linear torsinal spring. The remaining of the paper is organized as follows. The development of kinematics and dynamics of wheeled mobile robotic arms for four wheels-two links robot is deal with, considering the joint flexibility in each joints in section 3. Subsequently, the optimal control problem that with implementing of Pontryagin's Minimum Principle supports the execution of the optimization solution of model is expressed as a brief review in section 4. Simulation is done and results are discussed in Section 5. Finally, the concluding remarks by highlighting the main advantages of the presented method are expressed in the last section.

2 Kinematic and Dynamic Analysis of System

The mobile manipulator consisting of differentially driven vehicle with flexible revolute joints robot arms is expressed in this section. With decide on choosing X_0Y_0 as the inertial co-ordinate frame (CF) and selecting proper subsequent frames then writing the transition equation between them, the kinematic equations are obtained.

The nonholonomic constraints states that the robot can only move in the direction normal to the axis of the driving wheels, in addition, the acceleration constraint should be considered in order to avoid slippage during the robot navigation. For a mobile robot with four differential wheels move on a planar surface these constraints can be derived as:

$$\begin{aligned} \dot{x}\cos(\alpha+\theta+\beta)+\dot{y}\sin(\alpha+\theta+\beta)+l\dot{\theta}\sin\beta+d(\dot{\theta}+\dot{\beta})=0 \\ -\dot{x}\sin(\alpha+\theta+\beta)+\dot{y}\cos(\alpha+\theta+\beta)+l\dot{\theta}\cos\beta+r\dot{\phi}=0 \end{aligned} \tag{1}$$

where θ is the robot's heading angle, l is the width of the platform, r is the radius of the wheels and ϕ is angular displacement of wheels.

Dynamic equation of rigid mobile manipulator is obtained in compact form as:

$$M(q)\ddot{q} + H(q,\dot{q}) + G(q) = U, \tag{2}$$

where M is the inertia matrix, H is the vector of Coriolis and centrifugal forces, G describes the gravity effects and U is the generalized force inserted into the actuator. Their details are omitted.

To model a flexible joint manipulator (FJM) the link positions are let to be in the state vector as is the case with rigid manipulators. Actuator positions must be also considered because in contradiction to rigid robots these are related to the link position through the dynamics of the flexible element. By defining the link number of a flexible joint manipulator is m, position of the i^{th} link is shown with $\theta_{2i-1} : i = 1,2,...,m$ and the position of the i^{th} actuator with $\theta_{2i} : i = 1,2,...,m$, it is usual in the FJM literature to arrange these angles in a vector as follows:

$$Q = [\theta_1, \theta_3, ...\theta_{2m-1} \mid \theta_2, \theta_4, ...\theta_{2m}]^T = [q_1^T, q_2^T]^T \tag{3}$$

So by adding the joint flexibility with considering the elastic mechanical coupling between the i^{th} joint and link is modeled as a linear torsional spring with constant stiffness coefficient k_i, the set of equation of motion comprising mobile base with both link and joint flexibility can be rearranged into the following form:

$$\begin{aligned} &M(q_1)\ddot{q}_1 + H(q_1,\dot{q}_1) + G(q_1) + K(q_1 - q_2) = 0 \\ &J\ddot{q}_2 + K(q_2 - q_1) = U \end{aligned} \tag{4}$$

where K=diag[$k_1, k_2, ..., k_m$] is a diagonal stiffness matrix which models the joint elasticity, J=diag[$J_1, J_2, ..., J_m$] is the diagonal matrix representing motor inertia

3 Defining of the Optimal Control Problem

This section identifies the basic content of the optimization problem in order to be deal with in path optimization procedure. In summary the optimal control problem which that uses for optimization solution has basic statements as:

- Finding a state function X and actuating inputs, i.e. the control variables U during the overall time t_f where the state equation describing the dynamic evolution of the multibody system over this time interval be specified as:

$$\dot{X}(t) = f(X(t), U(t)) \tag{5}$$

- Minimizing a performance criterion

$$J(u) = \int_{t_0}^{t_f} L(X(t), U(t))dt \tag{6}$$

This can combine, for instance, energy consumption, actuating torques, traveling time or bounding the velocity magnitude or maximum payload.

By defining $\overline{U}$ as a set of admissible control torque over the time interval the imposed bound of torque for each motor can be expressed as:

$$\overline{U} = \{U^- \le U \le U^+\} \tag{7}$$

The optimization problem is completed by the boundary conditions

$$X(t_0) = X_0 \quad , X(t_f) = X_f \tag{8}$$

which represent the characteristics of each joint at initial and final time.

By implementing Pontryagin's minimum principle for solving optimization problems the necessary conditions for optimality are obtained as stated on the basis of variational calculus. Defining the Hamiltonian function as:

$$H^*(X,U,Y,m_p,t) = Y^T f(X,U,t) + L(X,U,m_p,t) \tag{9}$$

in addition to costate time vector-function *Y(t)* that verifying the costate vector-equation (or adjoint system)

$$\dot{Y}^T = -\partial H^* / \partial X \tag{10}$$

and the minimality condition for the Hamiltonian as:

$$\begin{cases} \partial H^* / \partial U = 0 \ , \\ \dot{X} = \partial H^* / \partial Y \end{cases} \tag{11}$$

leads to transform the problem of optimal control into a non-linear multi-point boundary value problem, that it can be solved by numerical techniques.

4 Simulations

4.1 Background, Deriving the Equations

A mobile manipulator as depicted in Fig. 4.1 is considered to simulate the model. It consists of a symmetric wheeled mobile platform and a two-link manipulator mounted on top of the platform.

The platform moves by driving four independent wheels. The manipulator is constructed as a two-link planar arm with attached at the flexible joints. A concentrated payload of mass mp is connected to the second link.

This nonholonomic robot arm for motion in the plane has nine degrees of freedom that are arose from movement of platform- 3 DOFs-, with arms -2 DOFs- and wheels -4 DOFs-. Hence the question which naturally arises is: how to guarantee soft and well-organized movement encounter of seven redundant DOFs in system? A common answer is: by prescribing five kinematical constraints the redundancy resolution of system is ensured as well as previously specification of base trajectory during the motion. These constraints are consisted of four conditions

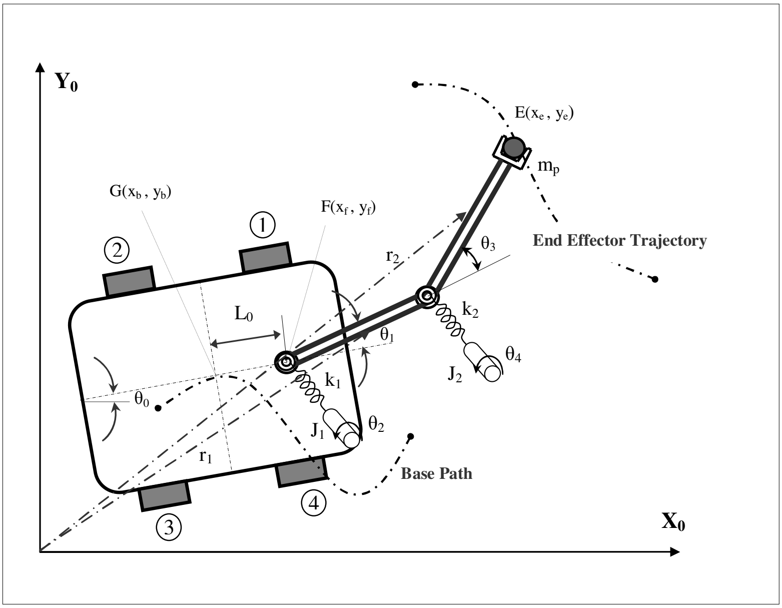

Fig. 4.1 Four-wheeled mobile manipulator with two flexible joint

associated to the avoidance of slippage of wheels during the robot navigation and last is related to the rolling without slipping condition Eq. (1).

The physical parameters of arms are summarized in the Table 4.1.

Table 4.1 Simulation parameters

Parameter	Value	Unit
Length of Links	$L_1 = L_2 = 0.7$	m
Mass of Links	$m_1 = m_2 = 6$	Kg
Spring constant	$k_1 = k_2 = 2000$	N/m
Moment of inertia	$J_1 = J_2 = 2$	Kg. m^2
Max. no Load Speed of Actuators	$w_{s1} = w_{s2} = 3$	rad/s
Actuator Stall Torque	$\tau_{s1} = \tau_{s2} = 70$	N.m

Also each wheel has 0.3 kg mass, 0.3 m length, 0.3 m radius and the mass of base is 2 kg.

By decomposing the system dynamic Eq. (4) into redundant and non-redundant parts and considering components associated with non-redundant ones the resultant equations in state space formed as:

$$\dot{x}_{2i-1} = x_{2i}\ ,\ \dot{x}_{2i} = f_2(i)\ \ ;\ i = 1, \cdots, 4 \tag{12}$$

where $X = [X_1 \quad X_2 \quad X_3 \quad X_4]^T$ is a state vector as:

$$\begin{aligned} X_1 &= Q^T = [x_1 \quad x_3 \quad x_5 \quad x_7]^T, \\ X_2 &= \dot{Q}^T = [x_2 \quad x_4 \quad x_6 \quad x_8]^T. \end{aligned} \tag{13}$$

The boundary condition can be expressed as follows:

$$\begin{aligned} &x_1(0) = x_5(0) = 1.5\, rad\,,\ x_3(0) = x_7(0) = 2\, rad\,; \\ &x_1(f) = x_5(f) = -1\, rad\,, x_3(0) = x_7(f) = 1\, rad\,; \\ &x_{2i}(0) = x_{2i}(f) = 0, \quad i = 1...4 \end{aligned} \tag{14}$$

By controlling all active joints so as to achieve the best dynamic coordination of joint motions, while minimizing the actuating inputs together bounding the velocities can ensure soft and efficient functioning while improving the manipulator working performances. For this reason the objective function is formed as:

$$L = \frac{1}{2}\left(r_1 u_1^2 + r_2 u_2^2 + \sum_{i=1}^{8} w_i x_i^2 \right) \tag{15}$$

Then, by considering the costate vector as $Y = [y_1 \quad y_2 \quad ... \quad y_8]$, the Hamiltonian function can be expressed from as:

$$H = \frac{1}{2}\left(r_1 u_1^2 + r_2 u_2^2 + \sum_{i=1}^{8} w_i x_i^2 \right) + \sum_{i=1}^{8} y_i \dot{x}_i \ , \tag{16}$$

where $\dot{x}_i\,, i = 1,\ldots,8$ can be substituted from Eq. (12).

In order to derive the equations associated with optimality conditions, penalty matrices can be selected as follows:

$$\begin{aligned} W_1 &= diag(w_1, w_3, w_5, w_7) \\ W_2 &= diag(w_2, w_4, w_6, w_8) \\ R &= diag(r_1, r_2) \end{aligned} \tag{17}$$

Using Eq. (10), differentiating the Hamiltonian function with respect to the states, result in costate equations as follows:

$$\dot{y}_i = -\frac{\partial H}{\partial x_i}\ , \ i = 1,\cdots,8 \tag{18}$$

Control functions are computing by differentiating the Hamiltonian function with respect to control and setting the derivative equal to zero. After using the extreme bound of control for each motor, by substituting the obtained control equations into (12) and (18), obtained 16 nonlinear ordinary differential equations that with

16 boundary conditions given in (14), constructs a two point boundary value problem that can be solved using the BVP4C command in MATLAB®.

4.2 Path Optimization

These simulations are carried out at three cases. In the first case, path planning is performed for generating the minimum vibration trajectory; second case is deal with the designing of minimum effort path, the effect of joint stiffness in performance characteristics of robot is investigated in last simulation.

In this simulations the payload is considered to be 3 kg and it must be carried from an initial to final point during the overall time t_f = 1.5s, while the mobile base is initially at point (x_0 = 0.6m, y_0 = 0.8m, θ_0 = 0) and moves to final position (x_f = 1.4m, y_f = 1.2m, θ_0 (end) = 0.44 rad).

4.3 Minimum Effort Trajectory

In this case finding the optimal path with minimum effort is considered, therefore, because of the matter that increasing W_2 decreases the proportion of weighting matrix R and the result of this is increasing the control values, penalty matrices can be considered to be R=diag (1), W_1=[1] and W_2=[0]. End effector trajectories in the XY plane are shown in Fig. 4.2. Figs. 4.3 and 4.4 show the angular position and velocity of joints with respect to time respectively. The computed torque is plotted in Fig. 4.5. Angular position and velocity of wheels are shown in figure 4.6. Wheels 1 with 2 also 3 with 4 have the same figures because of symmetry in system.

4.4 Minimum Speed Trajectory

In this case study, the problem of bounding the velocity of joints in minimum magnitude is considered and the obtained results are compared with minimum effort ones. Simulations are performed for selecting weighting factors as: R=diag (1), W_1 = [1] and W_2 = [1000]. The results of these simulations are illustrated in same figures of case 1. as shown in figures: a) by increasing the W_2 the angular position change to approach approximately to a straight line, b) as it can be seen, for the minimum speed trajectory, the oscillation amplitudes in velocity curves has been reduced considerably, but the magnitude of motor torques has been increased. It means that for achieving a smoother path, more effort must be applied, c) it is clearly observed that the minimum speed path is smoother than the minimum effort path, d) As shown in figures, in this method, via changing the penalty matrices values, various optimal trajectories with different specifications can be obtained, Consequently, this method enables the designer to compromise between different objectives by considering the proper penalty matrices.

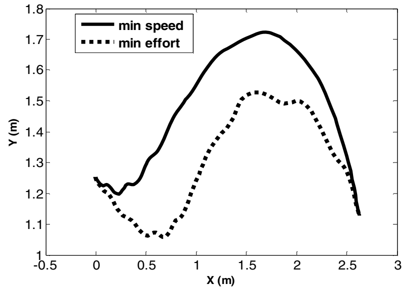

Fig. 4.2 End effector trajectory in XY plane

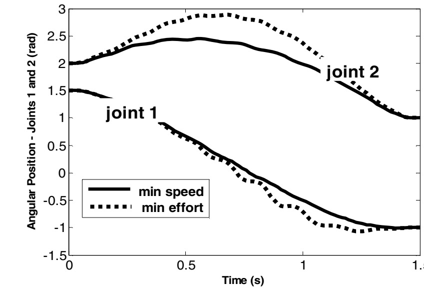

Fig. 4.3 Angular positions of joints

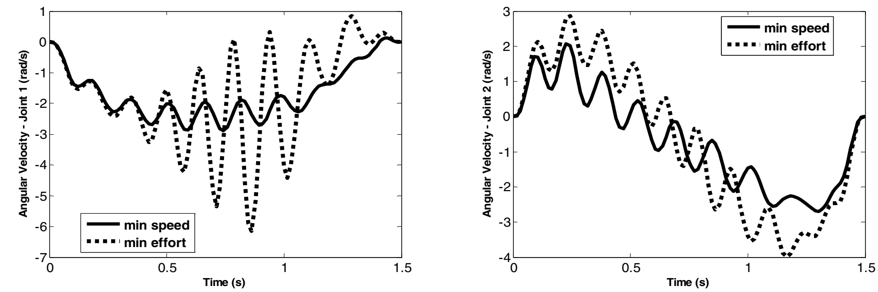

Fig. 4.4 Angular velocities of joints

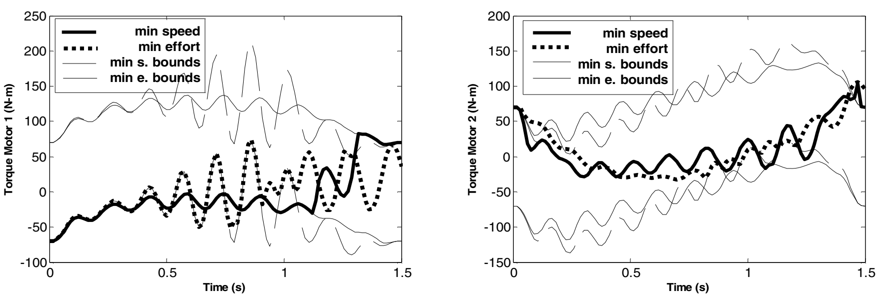

Fig. 4.5 Torque of motor 1

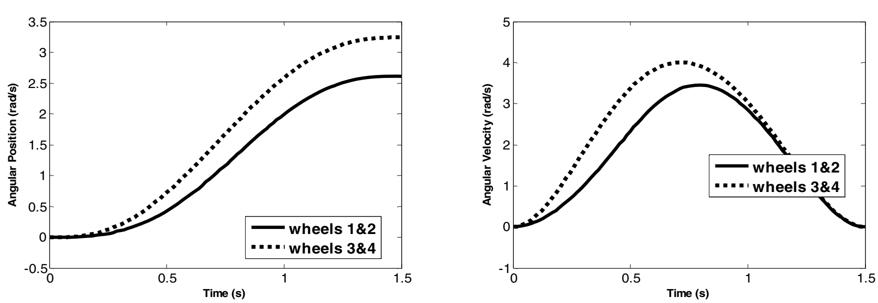

Fig. 4.6 Angular position and velocity of wheels

4.5 Different Joint Stiffness Trajectory

In this section, the effect of joint stiffness in performance characteristics of robot is investigated. Penalty matrices are considered to be :W_1=W_2=[1] and R=diag(1). The values of K using in simulation are given in Table 4.2.

Table 4.2 The values of *K* used in simulation

case	1	2	3
K	Diag(500)	Diag(1000)	Diag(5000)

Fig. 4.7 presents angular velocities of the results. It is observed from figure that increasing the joint stiffness caused the reducing oscillatory behavior of system. In addition, it can be seen growing the elasticity in joints enlarges bounds of velocity. The angular velocities of links and motors in case 2 are given in Fig. 4.8. It shows that both the link angular velocities have deviations from their respective motor angular velocities. Thus, it is clear that joint flexibility significantly affects the link vibrations.

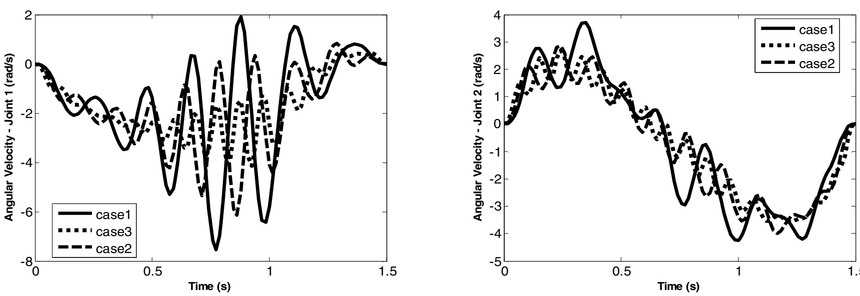

Fig. 4.7 Angular velocities of joints

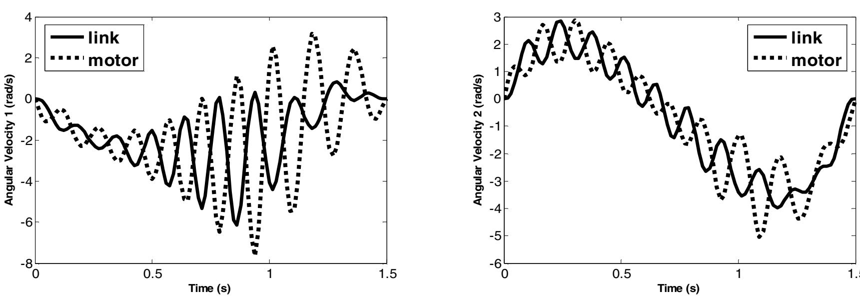

Fig. 4.8 Angular velocities of links and motors

5 Conclusions

In this paper, formulation of designing the path for wheeled mobile flexible joints manipulator in point-to-point motion is presented based on Pontryagin's minimum principle. Therefore, an efficient solution on the basis of TPBVP solution is proposed to optimize the path in order to achieve the predefined objective. The simulations on path planning of the minimum vibration trajectory, designing of minimum effort path and investigate on effect of joint stiffness in performance characteristics of robot are performed. Results show by defining the proper objective function and changing the penalty matrices can be achieve the desired requirements. The obtained results illustrate the power and efficiency of the method to overcome the high nonlinearity nature of the optimization problem which with other methods, it may be very difficult or impossible. The optimal trajectory and corresponding input control obtained using this method can be used as a reference signal and feed forward command in control structure of such manipulators.

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