Feng Bao Hui Li Guilin Wang (Eds.)

# Information Security Practice and Experience

5th International Conference, ISPEC 2009 Xi'an, China, April 2009 Proceedings



## Lecture Notes in Computer Science

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5th International Conference, ISPEC 2009 Xi'an, China, April 13-15, 2009 Proceedings



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## Preface

The 5th International Conference on Information Security Practice and Experience (ISPEC 2009) was held in Xi'an, China, April 13–15, 2009.

The ISPEC conference series is an established forum that brings together researchers and practitioners to provide a confluence of new information security technologies, including their applications and their integration with IT systems in various vertical sectors. In previous years, ISPEC has taken place in Singapore (2005), Hangzhou, China (2006), Hong Kong, China (2007), and Sydney, Australia (2008). For all sessions, as this one, the conference proceedings were published by Springer in the *Lecture Notes in Computer Science* series.

In total, 147 papers from 26 countries were submitted to ISPEC 2009, and 34 were finally selected for inclusion in the proceedings (acceptance rate 23%). The accepted papers cover multiple topics of information security and applied cryptography. Each submission was anonymously reviewed by at least three reviewers. We are grateful to the Program Committee, which was composed of more than 40 well-known security experts from 15 countries; we heartily thank them as well as all external reviewers for their time and valued contributions to the tough and time-consuming reviewing process.

In addition to the regular paper presentations, the program also featured four invited talks by Yupu Hu, from Xidian University, China; Youki Kadobayashi, from Nara Institute of Science and Technology, Japan; Mark Ryan, from the University of Birmingham, UK; and Gene Tsudik, from the University of California at Irvine, USA. We are grateful to them for accepting our invitation to speak at the conference.

The conference was organized and sponsored by Xidian University, China, co-organized by the School of Telecommunications Engineering, Xidian University, China; the Key Laboratory of Computer Networks and Information Security, Ministry of Education, China; and the National 111 Program of Introducing Talents of Discipline to Universities on Fundamental Theory and Technology of Modern Wireless Information Networks, China.

Special thanks are due to Ying Qiu for managing the review system, Libin Zhao for maintaining the conference website, and the Organizing Committee for dealing with local issues.

Last but not least, we would like to thank all the authors who submitted their papers to ISPEC 2009, and all the attendees from all over the world.

April 2009

Feng Bao Hui Li Guilin Wang

## **ISPEC 2009**

5th International Conference on Information Security Practice and Experience

> Xi'an, China April 13–15, 2009

Organized and Sponsored by Xidian University, China Co-organized by

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## Efficient and Provable Secure Ciphertext-Policy Attribute-Based Encryption Schemes

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Abstract. With a Ciphertext-Policy Attribute-Based Encryption (CP-ABE) scheme, a user's private key is associated with a set of attributes and the data is encrypted under an access policy defined by the message sender. A user can decrypt a ciphertext if and only if her attributes satisfy the access policy. In CP-ABE, since the message sender enforces the access policy during the encryption phase, the policy moves with the encrypted data. In this paper, we provide an efficient CP-ABE scheme which can express any access policy represented by a formula involving the and ( $\wedge$ ) and or ( $\vee$ ) operators. The scheme is secure under Decision Bilinear Diffie-Hellman (DBDH) assumption. Furthermore, we extend the expressiveness of the scheme by including the of operator in addition to  $\wedge$  and  $\vee$ . We provide a comparison with some existing CP-ABE schemes and show that our schemes are more efficient.

#### 1 Introduction

Public-key encryption is an asymmetric primitive which uses a pair of keys, where a private key which is kept secret and a public key which is widely distributed. If Alice wants to send a confidential message to Bob, she can encrypt the message with the public key of Bob and only Bob can decrypt the message using his private key. With a Public-Key Infrastructure (PKI), a public key must be obtained from, or at least be certified by the Trusted Third Party (TTP) of the PKI. With an Identity-Based Encryption (IBE) scheme [6][10][19], any string can be used to generate a public key without the involvement of the TTP. IBE thus creates a degree of flexibility that a PKI cannot offer. However, if Alice does not know the identity of her party, but instead she only knows certain attributes of the recipient, then the above solutions will not work.

The solution to this problem is provided by Attribute-Based Encryption (ABE) which identifies a user with a set of attributes **[17]**. In the seminal paper, Sahai and Waters use biometric measurements as attributes in the following way. A private key, associated with a set of attributes  $\omega$ , can decrypt a ciphertext encrypted with a public key, associated with a set of attributes  $\omega'$ , only if the sets  $\omega$  and  $\omega'$  overlap sufficiently as determined by a threshold value t. We refer to the scheme, proposed by Sahai and Waters, as the SW scheme. A more general policy to decide which attributes are required to decrypt a message is provided by an access tree. For example the access tree  $\tau = class1978 \wedge mycollege \lor myteacher$ 

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states that all students with the attribute *class*1978 who studied at *mycollege* as well as the teacher possessing the attribute *myteacher* would satisfy the policy.

There are two variants of ABE, namely Key-Policy based ABE (KP-ABE) [12] and Ciphertext-Policy based ABE (CP-ABE)[3][9]. In KP-ABE, the ciphertext is associated with a set of attributes and the private key is associated with the access tree. The message sender does not define the privacy policy and has no control over who has access to the data, except for defining the set of descriptive attributes necessary to decrypt the ciphertext. The trusted authority who generates user's private key defines the combination of attributes for which the private key can be used. In CP-ABE, the idea is reversed: the ciphertext is associated with the access tree and the message sender determines the policy under which the data can be decrypted, while the private key is associated with a set of attributes.

Pirreti et al. 16 give a construction and implementation of a modified SW scheme, which, compared to the original scheme, reduces computational overhead during the encryption and the key generation phases. Goyal et al. 12 introduce the idea of KP-ABE and propose a new scheme. In their scheme, when a user makes a secret key request, the trusted authority determines which combination of attributes must appear in the ciphertext for the user to decrypt. In essence, this scheme is an extension of the SW scheme, where, instead of using the Shamir **18** secret sharing technique in the private key, the trusted authority uses a more generalized form of secret sharing to enforce a monotonic access tree. Chase S constructs a multi-authority ABE scheme, which allows multiple independent authorities to monitor attributes and distribute secret keys. A related work to KP-ABE is a predicate encryption paradigm or searching on encrypted data 1157114. Predicate encryption has the advantages of providing ciphertext anonymity by hiding the access structures, however, the system is less expressive compared to schemes which leave the access structures in the clear. Smart 20 gives an access control data scheme which encrypts data to an arbitrary collection of identities using a variant of the Boneh-Franklin IBE scheme.

The first CP-ABE scheme proposed by Bethencourt *et al.* [3] uses threshold secret sharing to enforce the policy during the encryption phase. We refer to this scheme as the BSW scheme. The main drawback of the BSW scheme is that it requires polynomial interpolation to reconstruct the secret, thus many expensive pairing and exponentiation operations are required in the decryption phase. The scheme is secure in the generic group model, which provides evidence to the hardness of the problem, without giving security proof which reduces the problem of breaking the scheme to a well-studied complexity-theoretic problem. The CP-ABE scheme, proposed by Cheung and Newport [9], does not use threshold secret sharing but uses random elements to enforce the policy during the encryption phase. We refer to this scheme as the CN scheme. The CN scheme has two drawbacks. Firstly, it is not sufficiently expressive because it supports only policies with logical conjunction. Secondly, the size of the ciphertext and secret key increases linearly with the total number of attributes in the system. This makes the CN scheme inefficient. Goyal *et al.*  $\square$  give a "bounded" CP-ABE construction. The disadvantage of their scheme is that the depth of the access trees *d* under which messages can be encrypted is defined in the setup phase. Thus, the message sender is restricted to use only an access tree which has the depth  $d' \leq d$ .

*Contribution.* In this paper we aim at designing efficient CP-ABE schemes, which can be proven based on standard complexity-theoretic assumptions. More specifically, our contribution is twofold:

- We present a new technique for realizing CP-ABE without using Shamir's threshold secret sharing. We first show such a construction which is referred to as as the basic CP-ABE scheme. In this scheme, the message sender defines the privacy policy through an access tree which is n-ary tree represented by ∧ and ∨ nodes. Note that, realizing a scheme, which does not use threshold secret sharing, is important for resource constraint devices since calculating polynomial interpolations to construct the secret is computationally expensive. Compared to the CN scheme, our scheme requires fewer computations during the encryption, key generation and decryption phases.
- Next, we extend the basic CP-ABE scheme and provide a second CP-ABE scheme which uses Shamir's threshold secret sharing technique [18]. The access tree is an n-ary tree represented by  $\land$ ,  $\lor$  and *of* nodes. We compare the efficiency of our scheme with the BSW scheme and show that our scheme requires less computations in the key generation, encryption and decryption phases.

*Organization.* The rest of this paper is organized as follows. In Section 2 we review concepts of the access structure, secret sharing, CP-ABE, and bilinear pairing. In Section 3 we introduce the basic CP-ABE scheme which is secure under DBDH assumption in the selective-attribute model. In Section 4 we provide an extension to the basic CP-ABE scheme by including the *of* operator in addition to  $\wedge$  and  $\vee$ . In the last section we conclude the paper.

#### 2 Background Knowledge

In this section we introduce the notions related to access structure, secret sharing, the security definition of CP-ABE, and bilinear maps.

#### 2.1 Access Structure

We restate the definition of Access Structure in 2.

**Definition 1. (Access Structure).** Let  $\{P_1, P_2, \dots, P_n\}$  be a set of parties. A collection  $\mathbb{A} \subseteq 2^{\{P_1, P_2, \dots, P_n\}}$  is monotone if  $\forall B, C$ : if  $B \in \mathbb{A}$  and  $B \subseteq C$  then  $C \in \mathbb{A}$ . An access structure (respectively, monotone access structure) is a collection (respectively, monotone collection)  $\mathbb{A}$  of non-empty subsets of  $\{P_1, P_2, \dots, P_n\}$ , i.e.,  $A \subseteq 2^{\{P_1, P_2, \dots, P_n\}} \setminus \{\emptyset\}$ . The sets in  $\mathbb{A}$  are called the authorized sets, and the sets not in  $\mathbb{A}$  are called the unauthorized sets.

#### 2.2 Secret Sharing Schemes

In designing our CP-ABE schemes we will make use of two different secretsharing schemes: unanimous consent control by modular addition scheme and the Shamir secret sharing scheme.

Unanimous Consent Control by Modular Addition Scheme. In a unanimous consent control by modular addition scheme 15, there is a dealer who splits a secret s into t shares in a such way that all shares are required to reconstruct the secret s. To share the secret s,  $0 \le s \le p-1$  for some integer p, the dealer generates a t-1 random numbers  $s_i$  such that  $1 \le s_i \le p-1$ ,  $1 \le i \le t-1$  and  $s_t = s - \sum_{i=1}^{t-1} s_i \mod p$ . The secret s is recovered by:  $s = \sum_{i=1}^{t} s_i$ . Shares  $s_i$ ,  $1 \le i \le t$  are distributed to parties  $P_i$ ,  $1 \le i \le t$ . For each party  $P_i$ ,  $1 \le i \le t$ , the shares are random numbers between 0 and p-1, thus no party has any information about s except the dealer.

Shamir's Secret Sharing Scheme. In Shamir's secret sharing technique [18] a secret s is divided into n shares in a such way that any subset of t shares, where  $t \leq n$ , can together reconstruct the secret; no subset smaller than t can reconstruct the secret. The technique is based on polynomial interpolation where a polynomial y = f(x) of degree t - 1 is uniquely defined by t points  $(x_i, y_i)$ . The details of the scheme are as follows:

- 1. Setup. The dealer D wants to distribute the secret s > 0 among t users.
  - 1) *D* chooses a prime p > max(s, n), and defines  $a_0 = s$ .
  - 2) *D* selects t-1 random coefficients  $a_1, \ldots, a_{t-1}, 0 \le a_j \le p-1$ , and defines the random polynomial over  $\mathbb{Z}_p$ ,  $f(x) = \sum_{j=0}^{t-1} a_j x^j$ .

3) D computes  $s_i = f(i) \mod p$ , and sends securely the share  $s_i$  to user  $p_i$  together with the public index i.

2. Pooling of shares. Any group of t or more users pool their distinct shares  $(x, y) = (i, s_i)$  allowing computation of the coefficients  $a_j$  of f(x) by Lagrange interpolation,  $f(x) = \sum_{i=0}^{t-1} l_j(x)$  where  $l_j(x) = \prod_{1 \le j \le t, j \ne i} \frac{x - x_j}{x_i - x_j}$ . The secret is  $f(0) = a_0 = s$ .

#### 2.3 Ciphertext-Policy ABE

According to the definition given in [3], a CP-ABE schemes consist of four polynomial time algorithms:

- $\mathsf{Setup}(k)$ . The setup algorithm takes as input a security parameter k and outputs the public parameters pk and a master key mk.
- Keygen $(\omega, mk)$ . The algorithm takes as input the master key mk and a set of attributes  $\omega$ . The algorithm outputs a secret key  $sk_{\omega}$  associated with  $\omega$ .
- Encrypt $(m, \tau, pk)$ . The encryption algorithm takes as input a message m, an access tree  $\tau$  representing an access structure, and the public key pk. The algorithm will return the ciphertext  $c_{\tau}$  such that only users who have the secret key generated from the attributes that satisfy the access tree will be able to decrypt the message.

-  $\text{Decrypt}(c_{\tau}, sk_{\omega})$ . The decryption algorithm takes as input a ciphertext  $c_{\tau}$ , a secret key  $sk_{\omega}$  associated with  $\omega$ , and it outputs a message m or an error symbol  $\perp$ .

The semantic security against chosen-plaintext attack (CPA) is modeled in the selective-attribute (sAtt) model, where the adversary must provide the challenge access tree he wishes to attack before he receives the public parameters from the challenger. Suppose, that the adversary in the lnit phase chooses the challenge access tree  $\tau^* = (A \land B) \lor C$ . In Phase1, the adversary can make secret key requests to Keygen oracle for any attribute set  $\omega$  with the restriction that attributes  $A, B, C \notin \omega$ . The selective-attribute (sAtt) model can be considered to be analogous to the selective-ID model [4] used in identity-based encryption schemes, in which the adversary commits ahead of time the  $ID^*$  it will attack, and where the adversary can make secret key requests to Keygen oracle for any ID such that  $ID \neq ID^*$ .

The game is carried out between a challenger and an adversary  $\mathcal{A}$ , where the challenger simulates the protocol execution and answers queries from  $\mathcal{A}$ . Specifically, the game is as follows:

- 1. Init. The adversary chooses the challenge access tree  $\tau^*$  and gives it to the challenger.
- 2. Setup. The challenger runs Setup to generate (pk, mk) and gives the public key pk to the adversary  $\mathcal{A}$ .
- 3. Phase1.  $\mathcal{A}$  makes a secret key request to the Keygen oracle for any attribute set  $\omega = \{a_j | a_j \in \Omega\}$ , with the restriction that  $a_j \notin \tau^*$ . The challenger returns Keygen $(\omega, mk)$ .
- 4. Challenge.  $\mathcal{A}$  sends to the challenger two messages  $m_0, m_1$ . The challenger picks a random bit  $b \in \{0, 1\}$  and returns  $c_b = \mathsf{Encrypt}(m_b, \tau^*, pk)$ .
- 5. Phase2. A can continue querying Keygen with the same restriction as during Phase1.
- 6. Guess.  $\mathcal{A}$  outputs a guess  $b' \in \{0, 1\}$ .

**Definition 2.** A CP-ABE scheme is said to be secure against an adaptive chosen-plaintext attack (CPA) if any polynomial-time adversary has only a negligible advantage in the IND-sAtt-CPA game, where the advantage is defined to be  $\epsilon = |\Pr[b' = b] - \frac{1}{2}|$ .

#### 2.4 Review of Pairing

We briefly review the basis of bilinear pairing. A pairing (or, bilinear map) satisfies the following properties:

- 1.  $\mathbb{G}_0$  and  $\mathbb{G}_1$  are two multiplicative groups of prime order p.
- 2. g is a generator of  $\mathbb{G}_0$ .
- 3.  $\hat{e} : \mathbb{G}_0 \times \mathbb{G}_0 \to \mathbb{G}_1$  is an efficiently-computable bilinear map with the following properties:

- Bilinear: for all  $u, v \in \mathbb{G}_0$  and  $a, b \in \mathbb{Z}_p$ , we have  $\hat{e}(u^a, v^b) = \hat{e}(u, v)^{ab}$ . - Non-degenerate:  $\hat{e}(q, q) \neq 1$ .

 $\mathbb{G}_0$  is said to be a bilinear group if the group operation in  $\mathbb{G}_0$  can be computed efficiently and if there exists a group  $\mathbb{G}_1$  and an efficiently-computable bilinear map  $\hat{e}$  as defined above.

The Decision Bilinear Diffie-Hellman (DBDH) problem is defined as follows. Given  $g, g^a, g^b, g^c \in \mathbb{G}_0$  as input, the adversary must distinguish a valid tuple  $\hat{a}(g,g)^{abc} \in \mathbb{G}_1$  from the random element  $Z \in \mathbb{G}_1$ . An algorithm  $\mathcal{A}$  has advantage  $\epsilon$  in solving the Decision Bilinear Diffie-Hellman (DBDH) problem in  $\mathbb{G}_0$  if:

$$|\Pr[\mathcal{A}(g, g^a, g^b, g^c, \hat{e}(g, g)^{abc}) = 0] - \Pr[\mathcal{A}(g, g^a, g^b, g^c, Z) = 0]| \ge \epsilon$$

Here the probability is over the random choice of  $a, b, c \in \mathbb{Z}_p$ , the random choice of  $Z \in \mathbb{G}_1$ , and the random bits of  $\mathcal{A}$  (the adversary is a nondeterministic algorithm).

**Definition 3.** We say that the  $(t, \epsilon)$ -DBDH assumption holds if no t-time algorithm has advantage at least  $\epsilon$  in solving the DBDH problem in  $\mathbb{G}_0$ .

#### 3 The Basic CP-ABE Construction

In this paper, the access tree is a n-ary tree, in which leaves are attributes and inner nodes are  $\wedge$  and  $\vee$  boolean operators. Intuitively, the access tree is a policy which specifies which combination of attributes can decrypt the ciphertext. Consider the following example where a patient wants to specify access restrictions on his medical data. The patient can enforce the access policy in the encryption phase. Each member from the medical staff who has enough attributes should be able to decrypt the encrypted message. For instance, a patient wants to allow his data to be seen by Doctor A who works at Department A or by Doctor B who works at Department B. Using boolean operators the patient defines the following access policy:  $\tau_{Data} = (Doc.A \wedge Dep.A) \vee (Doc.B \wedge Dep.B)$ .

To decrypt the ciphertext which is encrypted according to the  $\tau_{Data}$  access tree, the decryptor must possess a private key, which is associated with the attribute set which satisfies  $\tau_{Data}$ . To determine whether or not an access tree is satisfied, we interpret each attribute as a logical variable. Possession of the secret key for the corresponding attribute makes the logical variable true. If the decryptor does not possess the attribute, the variable is false. For the policy above there are several different sets of attributes that can satisfy the access tree, such as the secret key associating with the attribute set {Doc.A, Dep.A}, the secret key associating with the attribute set {Doc.B, Dep.B}, or the secret key associating with all attributes defined in the access tree.

#### 3.1 Description of the Basic Scheme

The polynomial time algorithms of the basic CP-ABE scheme are defined as follows.

- 1.  $\mathsf{Setup}(k)$ : On input of the security parameter k, this algorithm generates the following.
  - (a) Generate a bilinear group  $\mathbb{G}_0$  of prime order p with a generator g and bilinear map  $\hat{e} : \mathbb{G}_0 \times \mathbb{G}_0 \to \mathbb{G}_1$
  - (b) Generate the attribute set  $\Omega = \{a_1, a_2, \dots a_n\}$ , for some integer n, and random elements  $\alpha, t_1, t_2 \dots t_n \in \mathbb{Z}_p$ .

Let  $y = \hat{e}(g,g)^{\alpha}$  and  $T_j = g^{t_j}$   $(1 \leq j \leq n)$ . The public key is  $pk = (\hat{e}, g, y, T_j (1 \leq j \leq n))$  and the master secret key is  $mk = (\alpha, t_j (1 \leq j \leq n))$ . 2. Keygen $(\omega, mk)$ : The algorithm performs as follows.

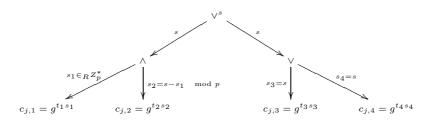
- 2. Reygen( $\omega$ , *nin*). The algorithm performs as follows.
  - (a) Select a random value  $r \in \mathbb{Z}_p$  and compute  $d_0 = g^{\alpha r}$ .
  - (b) For each attribute  $a_j$  in  $\omega$ , compute  $d_j = g^{rt_j^{-1}}$ .
  - (c) Return the secret key  $sk_{\omega} = (d_0, \forall a_j \in \omega : d_j)$
- 3. Encrypt $(m, \tau, pk)$ : To encrypt a message  $m \in \mathbb{G}_1$  the algorithm proceeds as follows:
  - (a) First level encryption: Select a random element  $s \in \mathbb{Z}_p$  and compute  $c_0 = g^s$  and

$$c_1 = m \cdot y^s = m \cdot \hat{e}(g,g)^{\alpha s}$$

- (b) Second level encryption: Set the value of the root node of τ to be s, mark all child nodes as un-assigned, and mark the root node assigned. Recursively, for each un-assigned non-leaf node, do the following:
  - If the symbol is  $\wedge$  and its child nodes are marked un-assigned, we use a unanimous consent control by modular addition scheme to assign a value to each child node. To do that, for each child node except the last one, assign a random value  $s_i$  where  $1 \leq s_i \leq p-1$ , and to the last child node assign the value  $s_t = s - \sum_{i=1}^{t-1} s_i \mod p$ . Mark this node assigned.
  - If the symbol is  $\lor$ , set the values of each child node to be s. Mark this node assigned.

Values of the leaves of  $\tau$  are used to produce ciphertext components.

- (c) For each leaf attribute  $a_{j,i} \in \tau$ , compute  $c_{j,i} = T_j^{s_i}$  where *i* denotes the index of the attribute in the access tree. The index values are uniquely assigned to leave nodes in an ordering manner for a given access structure.
- (d) Return the ciphertext  $c_{\tau} = (\tau, c_0, c_1, \forall a_{j,i} \in \tau : c_{j,i}).$
- In the following figure, we show an example of assigning secret shares  $s_i$  to the access tree  $\tau = (T_1 \wedge T_2) \vee (T_3 \vee T_4)$ .



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- 4.  $\mathsf{Decrypt}(c_{\tau}, sk_{\omega})$ : If  $\omega$  does not satisfy  $\tau$ , return  $\bot$ , otherwise the algorithm chooses the smallest set  $\omega' \subseteq \omega$  (we assume that this can be computed efficiently by the decryptor) that satisfies  $\tau$  and performs as follows:
  - (a) For every attribute  $a_i \in \omega'$ , compute

$$\begin{split} \prod_{a_j \in \omega'} \hat{e}(c_{j,i}, d_j) &= \prod_{a_j \in \omega'} \hat{e}(T_j^{s_i}, g^{rt_j^{-1}}) \\ &= \prod_{a_j \in \omega'} \hat{e}(g^{t_j s_i}, g^{rt_j^{-1}}) \\ &= \hat{e}(g, g)^{rs} \end{split}$$

(b) Compute

$$\hat{e}(c_0, d_0) \cdot \hat{e}(g, g)^{rs} = \hat{e}(g^s, g^{\alpha - r}) \cdot \hat{e}(g, g)^{rs}$$
$$= \hat{e}(g^s, g^{\alpha})$$

(c) Return m', where

$$m' = \frac{c_1}{\hat{e}(g^s, g^\alpha)}$$
$$= \frac{m \cdot \hat{e}(g, g)^{\alpha s}}{\hat{e}(g^s, g^\alpha)}$$
$$= m$$

#### 3.2 Security and Efficiency Analysis

The proposed scheme is proven IND-sAtt-CPA secure under the DBDH assumption. For more details of the proof, the reader can refer to the full version of this paper **13**.

The number of calculations in the Encryption algorithm depends on the number of attributes in the access tree  $\tau$ . The encryption requires  $|\tau| + 1$  exponentiations in  $\mathbb{G}_0$  and one exponentiation in  $\mathbb{G}_1$ . The number of calculations in the KeyGen algorithm depends on the number of attributes in the set  $\omega$  that the user has, namely  $|\omega| + 1$  exponentiations in  $\mathbb{G}_0$ . The number of calculations in the decryption algorithm depends on the number of attributes in the attribute set  $\omega'$ . The decryption requires  $|\omega'| + 1$  pairing operations,  $|\omega'|$  multiplications, but no exponentiations in  $\mathbb{G}_1$ .

In Table **1**, we compare our CP-ABE scheme with the CN scheme. We count the number of calculations in the encryption, key generation, and decryption phases. Compared to the CN scheme, our scheme requires fewer computations in the encryption, key generation and decryption phase.

	Our Scheme			The CN Scheme		
	$\operatorname{Exp.}(\mathbb{G}_0)$	$\operatorname{Exp.}(\mathbb{G}_1)$	Pairing	$\operatorname{Exp.}(\mathbb{G}_0)$	$\operatorname{Exp.}(\mathbb{G}_1)$	Pairing
Encrypt	$ \tau +1$	1	/	$ \Omega +1$	1	/
Keygen	$ \omega +1$	/	/	$ \Omega +1$	/	/
Decrypt		/	$ \omega' +1$	/	/	$ \Omega +1$
	$\Omega$ is the set of all attributes defined in the Setup phase					
	$\begin{array}{l} \tau \text{ is the access tree} \\ \omega \text{ is the set of attributes the user has, } \omega \subseteq \varOmega \\ \omega' \text{ the set of attributes satisfying the access tree, } \omega' \subseteq \omega \end{array}$					

Table 1. The Comparison with the CN Scheme

#### 4 Extension of the Expressiveness

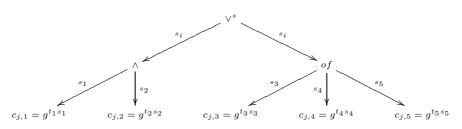
In the basic scheme, the access tree is a n-ary tree represented by  $\wedge$  and  $\vee$  nodes, which allows the user who performs encryption to express any privacy policy using boolean formulas. Similar to the BSW scheme, we would like to have an n-ary access tree which supports the *of* operator. The essential idea is to allow the encryptor to define the minimum number of attributes from a given list of attributes that the decryptor has to posses in order to decrypt the message. For instance, to decrypt the ciphertext encrypted under the policy  $\tau = 2$  of (class1978, mycollege, myteacher), the decryptor must have at least two out of three attributes. We extend the basic CP-ABE scheme to support the of operator as follows:

- 1. Setup and KeyGen are the same as in basic CP-ABE scheme.
- 2.  $\mathsf{Encrypt}(m,\tau,pk):$  To encrypt a message  $m\in\mathbb{G}_1$  the algorithm proceeds as follows:
  - (a) First level encryption: Select a random element  $s \in \mathbb{Z}_p$  and compute  $c_0 = g^s$  and

$$c_1 = m \cdot y^s$$
$$= m \cdot \hat{e}(g, g)^{\alpha s}$$

- (b) Second level encryption: Set the value of the root node to be *s*, mark all child nodes as un-assigned, and mark the root node assigned. Recursively, for each un-assigned non-leaf node, do the following:
  - If the symbol is of (threshold operator), and its child nodes are marked un-assigned, the secret s is divided using (t, n) Shamir's secret sharing technique where  $t \neq n$ , and n is the total number of child nodes and t is the number of child nodes necessary to reconstruct the secret. To each child node a share secret  $s_i = f(i)$  is assigned. Mark this node assigned.
  - If the symbol is  $\wedge$ , and its child nodes are marked un-assigned, the secret s is divided using (t, n) Shamir's secret sharing technique where t = n, and n is the number of the child nodes. To each child node a share secret  $s_i = f(i)$  is assigned. Mark this node assigned.

- If the symbol is  $\lor$ , and its child nodes are marked un-assigned, the secret s is divided using (t, n) Shamir's secret sharing technique where t = 1 and n is the number of the child nodes. To each child node a share secret  $s_i = f(i)$  is assigned. Mark this node assigned.
- Values of the leaves of  $\tau$  are used to produce ciphertext components.
- (c) For each leaf attribute  $a_{j,i} \in \tau$ , compute  $c_{j,i} = T_j^{s_i}$ , where *i* denotes the index of the attribute in the access tree.
- (d) Return the ciphertext:  $c_{\tau} = (\tau, c_0, c_1, \forall a_{j,i} \in \tau : c_{j,i}).$
- In the following figure, we show an example of assigning secret shares  $s_i$  to the access tree  $\tau = (T_1 \wedge T_2) \vee 2$  of  $(T_3, T_4, T_5)$ .



- 3.  $\mathsf{Decrypt}(c_{\tau}, sk_{\omega})$ : If  $\omega$  does not satisfy  $\tau$ , return  $\bot$ , otherwise the algorithm chooses the smallest set  $\omega' \subseteq \omega$  that satisfies  $\tau$  and performs as follows:
  - (a) For every attribute  $a_j \in \omega'$ , compute

$$\begin{split} \prod_{a_j \in \omega'} \hat{e}(c_{j,i}, d_j)^{l_i(0)} &= \hat{e}(T_j^{s_i}, g^{rt_j^{-1}})^{l_i(0)} \\ &= \prod_{a_j \in \omega'} \hat{e}(g^{t_j s_i}, g^{rt_j^{-1}})^{l_i(0)} \\ &= \prod_{a_j \in \omega'} \hat{e}(g, g)^{rs_i l_i(0)} \\ &= \hat{e}(g, g)^{rs} \end{split}$$

 $l_i(0)$  is a Lagrange coefficient and can be computed by everyone who knows the index of the attribute in the access tree.

(b) Compute

$$\hat{e}(c_0, d_0) \cdot \hat{e}(g, g)^{rs} = \hat{e}(g^s, g^{\alpha - r}) \cdot \hat{e}(g, g)^{rs}$$
$$= \hat{e}(g^s, g^{\alpha})$$

(c) Return m', where

$$m' = \frac{c_1}{\hat{e}(g^s, g^\alpha)} = \frac{m \cdot \hat{e}(g, g)^{\alpha s}}{\hat{e}(g^s, g^\alpha)} = m$$

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	Our Scheme		The BSW Scheme			
	$\operatorname{Exp.}(\mathbb{G})$	$Exp.(\mathbb{G}_1)$	Pairing	$\operatorname{Exp.}(\mathbb{G})$	$\operatorname{Exp.}(\mathbb{G}_1)$	Pairing
Encrypt	$ \tau  + 1$	1	/	$2 \tau +1$	1	/
KeyGen	$ \omega +1$	/	/	$2 \omega +1$	/	/
Decrypt	/	$ \omega' $	$ \omega' +1$	/	$ \omega' $	$2 \omega' $
	(Note:) $\Omega$ is the set of all attributes defined in the Setup phase			etup phase		
	$\tau$ is the access tree					
	$\omega$ is the set of attributes the user has, $\omega \subseteq \Omega$					
	$\omega'$ the set of attributes satisfying the access tree, $\omega' \subseteq \omega$			tree, $\omega' \subseteq \omega$		

 Table 2. The Comparison with the BSW scheme

The proposed scheme is proven IND-sAtt-CPA secure under the DBDH assumption as shown in the full version of this paper **[13]**. In Table **[2]**, we give a comparison of the efficiency of our extended CP-ABE scheme with the BSW scheme. Compared to the BSW scheme, our scheme requires fewer computations in the encryption, key generation and decryption phases.

#### 5 Conclusion and Future Work

We have shown how to improve the efficiency of a CP-ABE scheme. Firstly, we presented a new technique to construct a CP-ABE scheme which does not use threshold secret sharing. The encryptor specifies the policy in the encryption phase using an n-ary tree which consists from  $\lor$  and  $\land$  nodes. Secondly, we presented a modified scheme which is more expressive compared to the basic scheme. In the modified scheme, the policy can be expressed as an n-ary access tree which consists of  $\lor$ ,  $\land$  and of nodes. We have shown that these schemes require less computations than some other similar schemes.

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## A Ciphertext-Policy Attribute-Based Encryption Scheme with Constant Ciphertext Length

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**Abstract.** An Attribute-Based Encryption (ABE) is an encryption scheme, where users with some attributes can decrypt ciphertexts associated with these attributes. However, the length of the ciphertext depends on the number of attributes in previous ABE schemes. In this paper, we propose a new Ciphertext-Policy Attribute-Based Encryption (CP-ABE) with constant ciphertext length. Moreover, the number of pairing computations is also constant.

**Keywords:** Attribute-based encryption, Ciphertext-Policy, Constant Ciphertext Length.

#### 1 Introduction

A user identity (such as the name, e-mail address and so on) can be used for accessing control of some resources. For example, in Identity-Based Encryption (IBE) schemes such as [4],6], an encryptor can restrict a decryptor to indicate the identity of the decryptor. An Attribute-Based Encryption (ABE) is an encryption scheme, where users with some attributes can decrypt the ciphertext associated with these attributes. The first ABE scheme has been proposed in [13], which is inspired by IBE. Although IBE schemes have a restriction such that an encryptor only indicates a single decryptor, in ABE schemes, an encryptor can indicate many decryptors by assigning common attributes of these decryptors such as gender, age, affiliation and so on. There are two kinds of ABE, Key-Policy ABE (KP-ABE) and Ciphertext-Policy ABE (CP-ABE). KP-ABE [9][13] are schemes such that each private key is associated with an access structure. CP-ABE [2][7][8][12][14] are schemes such that each ciphertext is associated with an access structure. This means that an encryptor can decide who should or should not be allowed to decrypt. However, in all previous ABE schemes [2][7][8][12][13][14].

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the length of the ciphertext depends on the number of attributes. Also, the number of pairing computations depends on the number of attributes. A Predicate Encryption Scheme (PES), where secret keys correspond to predicates, and where ciphertexts are associated with attributes, has been proposed in [5,10]. It is shown that PES can be regarded as a kind of CP-ABE (see Appendix A and B in [12] for details). However, both the [5] and [10] schemes also have the same problems, in that the length of the ciphertext and the number of pairing computations are not constant.

**Contribution.** In this paper, for the first time we propose a CP-ABE with constant length of ciphertext and constant length of the number of pairing computations. The access structure used in our CP-ABE is constructed by AND-gates on multi-valued attributes. This is a subset of the access structures used in [7]12]. Although previous CP-ABE schemes [2]7[8]12[14] can complement our access structures, the length of the ciphertext depends on the number of attributes. This means that, until our work, to the best of our knowledge, there has been no scheme that enables a constant ciphertext length with AND-gates on multi-valued attributes.

**Organization:** The paper is organized as follows: Some definitions are presented in Section 2. The previous scheme is introduced in Section 3. Our scheme is described in Section 4. The security proof is presented in Section 5. Efficiency comparisons are made in Section 6.

### 2 Preliminary

In this section, some definitions are presented. Note that  $x \in_R S$  means x is randomly chosen for a set S.

#### 2.1 Bilinear Groups and Complexity Assumption

**Definition 1** (Bilinear Groups). *Bilinear groups and a bilinear map are defined as follows:* 

- 1.  $\mathbb{G}_1$  and  $\mathbb{G}_T$  are cyclic groups of prime order p.
- 2.  $g_1$  is a generator of  $\mathbb{G}_1$ .
- 3. *e* is an efficiently computable bilinear map  $e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_T$  with the following properties.
  - Bilinearity : for all  $u, u', v, v' \in \mathbb{G}_1$ , e(uu', v) = e(u, v)e(u', v) and e(u, vv') = e(u, v)e(u, v').
  - Non-degeneracy :  $e(g_1, g_1) \neq 1_{\mathbb{G}_T}$  ( $1_{\mathbb{G}_T}$  is the  $\mathbb{G}_T$ 's unit).

**Definition 2** (**DBDH assumption**). The Decision Bilinear Diffie-Hellman (DBDH) problem in  $\mathbb{G}_1$  is a problem, for input of a tuple  $(g_1, g_1^a, g_1^b, g_1^c, Z) \in \mathbb{G}_1^4 \times \mathbb{G}_T$  to decide  $Z = e(g_1, g_1)^{abc}$  or not. An algorithm  $\mathcal{A}$  has advantage  $\epsilon$  in solving DBDH problem in  $\mathbb{G}_1$  if  $Adv_{DBDH}(\mathcal{A}) := |\Pr[\mathcal{A}(g_1, g_1^a, g_1^b, g_1^c, e(g_1, g_1)^{abc}) = 0] - \Pr[\mathcal{A}(g_1, g_1^a, g_1^b, g_1^c, e(g_1, g_1)^z) = 0]| \geq \epsilon(\kappa)$ , where  $e(g_1, g_1)^z \in \mathbb{G}_T \setminus \{e(g_1, g_1)^{abc}\}$ . We say that the DBDH assumption holds in  $\mathbb{G}_1$  if no PPT algorithm has an advantage of at least  $\epsilon$  in solving the DBDH problem in  $\mathbb{G}_1$ .

#### 2.2 Definition of Access Structures

Several access structures such as the threshold structure **[13]**, the tree-based access structure **[28]**, AND-gates on positive and negative attributes with wildcards **[7]**, AND-gates on multi-valued attributes with wildcards **[12]**, and the linear access structure **[14]** are used in previous ABE schemes. In our scheme, the sum of master keys are used to achieve the constant ciphertext length. Therefore, we use AND-gates on multi-valued attributes (which can be represented by using the sum of master keys) as follows:

**Definition 3.** Let  $\mathcal{U} = \{att_1, \ldots, att_n\}$  be a set of attributes. For  $att_i \in \mathcal{U}$ ,  $S_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,n_i}\}$  is a set of possible values, where  $n_i$  is the number of possible values for  $att_i$ . Let  $L = [L_1, L_2, \ldots, L_n]$ ,  $L_i \in S_i$  be an attribute list for a user, and  $W = [W_1, W_2, \ldots, W_n]$ ,  $W_i \in S_i$  be an access structure. The notation  $L \models W$  expresses that an attribute list L satisfies an access structure W, namely,  $L_i = W_i$   $(i = 1, 2, \ldots, n)$ .

The number of access structures is  $\prod_{i=1}^{n} n_i$ . For each  $att_i$ , an encryptor has to explicitly indicate a status  $v_{i,*}$  from  $S_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,n_i}\}$ .

**Differences between the previous AND-gate structures [7]12] and ours** If  $n_i = 2$  (i = 1, 2, ..., n), then our structure is the same as the access structures **[7]** excluding wildcards. In **[12]**, an access structure W is defined as  $W = [W_1, W_2, ..., W_n]$  for  $W_i \subseteq S_i$ , and  $L \models W$  is defined as  $L_i \in W_i$  (i = 1, 2, ..., n). This means that our access structure is a subset of these in **[7]12]**. However, even if previous CP-ABE schemes **[7]12]** use our access structure, then the length of the ciphertext depends on the number of attributes.

#### 2.3 Ciphertext-Policy Attribute-Based Encryption Scheme (CP-ABE)

CP-ABE is described using four algorithms, Setup, KeyGen, Encrypt and Decrypt **[7]**.

#### **Definition 4.** *CP-ABE*

- Setup: This algorithm takes as input the security parameter  $\kappa$ , and returns a public key PK and a master secret key MK.
- KeyGen: This algorithm takes as input PK, MK and a set of attributes L, and returns a secret key  $SK_L$  associated with L.
- Encrypt: This algorithm takes as input PK, a message M and an access structure W. It returns a ciphertext C with the property that a user with  $SK_L$ can decrypt C if and only if  $L \models W$ .
- **Decrypt:** This algorithm takes as input PK, C which was encrypted by W, and  $SK_L$ . It returns M if  $SK_L$  is associated with  $L \models W$ .

#### 2.4 Selective Game for CP-ABE

We use the definition of "Selective Game" for CP-ABE [7]. This CP-ABE game captures the indistinguishability of messages and the collusion-resistance of secret keys, namely, attackers cannot generate a new secret key by combining their secret keys. To capture the collusion-resistance, multiple secret key queries can be issued by the adversary  $\mathcal{A}$  after the challenge phase. This means that  $\mathcal{A}$  can issue the KeyGen queries  $L_1$  and  $L_2$  such as  $(L_1 \not\models W^*) \land (L_2 \not\models W^*)$  and  $(L_1 \cup L_2) \models W^*$ . This collusion-resistance is an important property of CP-ABE scheme, which has not been considered in the Hierarchical IBE (HIBE) scheme such as [3].

#### **Definition 5.** Selective Game for CP-ABE

Init: The adversary A sends the challenge access structure  $W^*$  to the challenger. Setup: The challenger runs Setup and KeyGen, and gives PK to A.

- **Phase 1:** A sends an attribute list L to the challenger for a KeyGen query, where  $L \not\models W^*$ . The challenger answers with a secret key for these attributes. Note that these queries can be repeated adaptively.
- Challenge:  $\mathcal{A}$  sends two equal-length messages  $M_0$  and  $M_1$  to the challenger. The challenger chooses  $\mu \in_R \{0, 1\}$ , and runs  $C^* = \mathsf{Encrypt}(PK, M_\mu, W^*)$ . The challenger gives the challenge ciphertext  $C^*$  to  $\mathcal{A}$ .
- **Phase 2:** Same as Phase 1. A sends L to the challenger for a KeyGen query. The challenger answers with a secret key for these attributes. Note that  $L \not\models W^*$ , and these queries can be repeated adaptively.
- Guess:  $\mathcal{A}$  outputs a guess  $\mu' \in \{0, 1\}$ .

The advantage of  $\mathcal{A}$  is defined as  $Adv(\mathcal{A}) := |\Pr(\mu' = \mu) - \frac{1}{2}|$ .

#### 3 The Previous CP-ABE

In this section, we summarize the previous CP-ABE  $[\mathcal{I}]$ . Let  $\mathcal{U} = \{\neg att_1, \ldots, \neg att_n\}$ a set of negative attributes for a set of attributes  $\mathcal{U}$ . We refer to attributes  $att_i \in \mathcal{U}$ and their negations  $\neg att_i$  as literals. Let  $W = \bigwedge_{att_i \in I} a\bar{t}t_i$  be an access structure, where  $I \subseteq \mathcal{U}$  and  $a\bar{t}t_i$  is either  $att_i$  or  $\neg att_i$ . The public key elements  $T_i, T_{n+i}, T_{2n+i}$ correspond to the three properties of  $att_i$ , namely, positive, negative and don't care.

Protocol 1. CP-ABE [CN07] [7]

Setup(1<sup> $\kappa$ </sup>): A trusted authority TA chooses a prime number p, a bilinear group  $\mathbb{G}_1$ with order p, a generator  $g_1 \in \mathbb{G}_1$ ,  $y \in_R \mathbb{Z}_p$  and  $t_i \in_R \mathbb{Z}_p$  (i = 1, 2, ..., 3n), and computes  $Y = e(g_1, g_1)^y$  and  $T_i = g_1^{t_i}$  (i = 1, 2, ..., 3n). TA outputs  $PK = (e, g_1, Y, T_1, ..., T_{3n})$  and  $MK = (y, t_1, ..., t_{3n})$ .

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KeyGen(PK, MK, S): Every att<sub>i</sub>  $\notin S$  is implicitly considered to be a negative attribute. TA chooses  $r_i \in_R \mathbb{Z}_p$  (i = 1, 2, ..., n), sets  $r = \sum_{i=1}^n r_i$ , and computes  $\hat{D} = g_1^{y-r}$ . TA computes  $D_i$  and  $F_i$  as follows:

$$D_i = \begin{cases} g_1^{\frac{r_i}{t_i}} & (att_i \in S) \\ g_1^{\frac{r_i}{t_{n+i}}} & (att_i \notin S) \end{cases}, \ F_i = g_1^{\frac{r_i}{t_{2n+i}}} (att_i \in \mathcal{U}) \end{cases}$$

TA outputs  $SK = (\hat{D}, \{D_i, F_i\}_{i \in [1,n]})$ . Encrypt(PK, M, W): Let  $W = \bigwedge_{att_i \in I} a\bar{t}t_i$ . An encryptor chooses  $s \in_R \mathbb{Z}_p$ , and computes  $\tilde{C} = M \cdot Y^s$  and  $\hat{C} = g_1^s$ . The encryptor computes  $C_i$  as follows:

$$C_{i} = \begin{cases} T_{i}^{s} & (a\bar{t}t_{i} = att_{i}) \\ T_{n+i}^{s} & (a\bar{t}t_{i} = \neg att_{i}) \\ T_{2n+i}^{s} & (att_{i} \in \mathcal{U} \setminus I) \end{cases}$$

The encryptor outputs  $C = (W, \tilde{C}, \hat{C}, \{C_i\}_{i \in [1,n]}).$ 

**Decrypt**(PK, C, SK): A decryptor computes the pairing  $e(C_i, D_i)$  (att<sub>i</sub>  $\in I$ ) and  $e(C_i, F_i)$  (att<sub>i</sub>  $\notin I$ ) as follows:

$$e(C_i, D_i) = \begin{cases} e(g_1^{t_i \cdot s}, g_1^{\frac{r_i}{t_i}}) & (a\bar{t}t_i = att_i) \\ e(g^{t_{n+i} \cdot s}, g_1^{\frac{r_i}{t_{n+i}}}) & (a\bar{t}t_i = \neg att_i) \end{cases} = e(g_1, g_1)^{r_i \cdot s} & (att_i \in I) \\ e(C_i, F_i) = e(g_1^{t_{2n+i} \cdot s}, g_1^{\frac{r_i}{t_{2n+i}}}) = e(g, g)^{r_i \cdot s} & (att_i \notin I) \end{cases}$$

Then  $\tilde{C}/(e(\hat{C},\hat{D})\prod_{i=1}^{n}e(g_1,g_1)^{r_i\cdot s}) = M \cdot e(g_1,g_1)^{sy}/e(g_1,g_1)^{s(y-r)}e(g_1,g_1)^{sr} = M$  holds.

To compute  $e(g_1, g_1)^{sr}$ , the decryptor has to compute either  $e(C_i, D_i)$  or  $e(C_i, F_i)$ for each *i*. This means that all  $C_i$  are included in a ciphertext, and thus the length of a ciphertext depends on the number of attributes. This scheme does not provide for adding new attributes after **Setup**. If some attributes are added after **Setup**, then some users who have already obtained the secret key can decrypt a ciphertext which one must not be able to decrypt. For example, let  $\mathcal{U} =$  $\{att_1, att_2\}$ , and assume that a user U has secret keys of  $att_1$  and  $att_2$ , and that a ciphertext C is associated with  $W = att_1 \land att_2$ . Then, U can decrypt a ciphertext associated with  $att_1 \land att_2 \land att_3$  without a secret key of  $att_3$ . Concretely, U ignores a part of the ciphertext for  $att_3$ . CP-ABE schemes which enable the addition of new attributes after **Setup** have been proposed in BSW07 [2] and NYO08 [12] (which is the second construction of the NYO08 paper). If a user wants to decrypt a ciphertext with an access structure including newly added attributes, then the user must obtain a new secret key (including newly added attributes) from the trusted authority again. However, the security proof of both schemes contains no reduction, namely, it is proven under the generic group heuristic.

#### 4 Our Construction

In this section, we propose a constant ciphertext length CP-ABE with a function of adding new attributes after **Setup**. Let  $\mathbb{G}_1$  and  $\mathbb{G}_T$  be cyclic groups of prime order p and  $e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_T$  be a bilinear map. Let  $\mathcal{U} = \{att_1, \ldots, att_n\}$ be a set of attributes;  $S_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,n_i}\}$  be a set of possible values with  $n_i = |S_i|; L = [L_1, L_2, \ldots, L_n]$  ( $L_i \in S_i$ ) be an attribute list for a user; and  $W = [W_1, W_2, \ldots, W_n]$  ( $W_i \in S_i$ ) be an access structure.

#### 4.1 Proposed Scheme

#### Protocol 2. Our CP-ABE Scheme with Constant Ciphertext Length

- Setup(1<sup> $\kappa$ </sup>): A trusted authority TA chooses a prime number p, a bilinear group ( $\mathbb{G}_1, \mathbb{G}_T$ ) with order p, a generator  $g_1 \in \mathbb{G}_1$ ,  $h \in \mathbb{G}_1$ ,  $y \in_R \mathbb{Z}_p$  and  $t_{i,j} \in_R \mathbb{Z}_p$  ( $i \in [1, n], j \in [1, n_i]$ ). TA computes  $Y = e(g_1, h)^y$  and  $T_{i,j} = g_1^{t_{i,j}}$ ( $i \in [1, n], j \in [1, n_i]$ ). TA outputs  $PK = (e, g_1, h, Y, \{T_{i,j}\}_{i \in [1, n], j \in [1, n_i]})$  and  $MK = (y, \{t_{i,j}\}_{i \in [1, n], j \in [1, n_i]})$ . Note that  $\forall L, L'$  ( $L \neq L'$ ),  $\sum_{v_{i,j} \in L} t_{i,j} \neq \sum_{v_{i,j} \in L'} t_{i,j}$  is assumed
- KeyGen(PK, MK, L): TA chooses  $r \in_R \mathbb{Z}_p$ , outputs  $SK_L = (h^y(g_1^{\sum_{v_{i,j} \in L} t_{i,j}})^r, g_1^r)$ , and gives  $SK_L$  to a user with L.
- Encrypt(PK, M, W): An encryptor chooses  $s \in_R \mathbb{Z}_p$ , and computes  $C_1 = M \cdot Y^s$ ,  $C_2 = g_1^s$  and  $C_3 = (\prod_{v_{i,j} \in W} T_{i,j})^s$ . The encryptor outputs  $C = (W, C_1, C_2, C_3)$ .

 $\mathsf{Decrypt}(PK, C, SK_L)$ : A decryptor computes what follows:

$$\frac{C_1 \cdot e(C_3, g_1^r)}{e(C_2, h^y(g_1^{\sum_{v_{i,j} \in L} t_{i,j}})^r)} = \frac{M \cdot e(g, h)^{sy} e(g_1, g_1)^{sr \sum_{v_{i,j} \in W} t_{i,j}}}{e(g_1, h)^{sy} e(g_1, g_1)^{sr \sum_{v_{i,j} \in L} t_{i,j}}} = M$$

#### 4.2 Construction of Secret Keys $t_{i,j}$

In our scheme,  $\sum_{v_{i,j} \in L} t_{i,j} \neq \sum_{v_{i,j} \in L'} t_{i,j}$  is assumed. If there exist L and L' $(L \neq L')$  such that  $\sum_{v_{i,j} \in L} t_{i,j} = \sum_{v_{i,j} \in L'} t_{i,j}$ , a user with the attribute list L' can decrypt a ciphertext associated with W, where  $L' \not\models W$  and  $L \models W$ . Remark that the assumption holds with overwhelming probability  $\frac{p(p-1)\cdots(p-(N-1))}{p^N} > \frac{(p-(N-1))^N}{p^N} = (1 - \frac{N-1}{p})^N > 1 - \frac{N(N-1)}{p} > 1 - \frac{N^2}{p}$ , where  $N := \prod_{i=1}^n n_i$ . Therefore, if each secret key  $t_{i,j}$  is chosen at random from  $\mathbb{Z}_p$ , then our assumption is natural.

#### 5 Security Analysis

## **Theorem 1.** Our scheme satisfies the indistinguishability of messages under the DBDH assumption.

Proof. We suppose that the adversary  $\mathcal{A}$  wins the selective game for CP-ABE with the advantage  $\epsilon$ . Then we can construct an algorithm  $\mathcal{B}$  that breaks the DBDH assumption with the advantage  $\frac{\epsilon}{2}(1-\frac{N^2}{p})$ , where  $N := \prod_{i=1}^n n_i$  is the number of expressed access structures. The DBDH challenger selects  $a, b, c, z \in_R$  $\mathbb{Z}_p, \nu \in_R \{0, 1\}$ , and  $g_1$ , where  $\langle g_1 \rangle = \mathbb{G}_1$ . If  $\nu = 0$ , then  $Z = e(g_1, g_1)^{abc}$ . Otherwise, if  $\nu = 1$ , then  $Z = e(g_1, g_1)^z$ . The DBDH challenger gives the DBDH instance  $(g_1, g_1^a, g_1^b, g_1^c, Z) \in \mathbb{G}_1^4 \times \mathbb{G}_T$  to  $\mathcal{B}$ . First,  $\mathcal{B}$  is given the challenge access structure  $W^*$  from  $\mathcal{A}$ . Let  $W^* = [W_1^*, \ldots, W_n^*]$ .  $\mathcal{B}$  selects  $u \in_R \mathbb{Z}_p^*$ , and sets  $h = g_1^u$  and  $Y = e(g_1^a, (g_1^b)^u) = e(g_1, h)^{ab}$ . Moreover,  $\mathcal{B}$  selects  $t'_{i,j} \in_R \mathbb{Z}_p$   $(i \in [1, n], j \in [1, n_i])$ , and sets  $t_{i,j} = t'_{i,j}$  (in the case where  $v_{i,j} = W_i^*$ ) and  $t_{i,j} = bt'_{i,j}$ (in the case where  $v_{i,j} \neq W_i^*$ ), and computes public keys  $T_{i,j}$   $(i \in [1, n], j \in [1, n_i])$  as follows:

$$T_{i,j} = g_1^{t_{i,j}} = \begin{cases} g_1^{t'_{i,j}} & (v_{i,j} = W_i^*) \\ (g_1^b)^{t'_{i,j}} & (v_{i,j} \neq W_i^*) \end{cases}$$

 $\mathcal{B}$  gives  $PK = (e, g_1, h, Y, \{T_{i,j}\}_{i \in [1,n], j \in [1,n_i]})$  to  $\mathcal{A}$ . For KeyGen query L, there exists  $v_{i,\ell}$  such that  $v_{i,\ell} = L_i \wedge v_{i,\ell} \neq W_i^*$ , since  $L \not\models W^*$ . Therefore,  $\sum_{v_{i,j} \in L} t_{i,j}$  can be represented as  $\sum_{v_{i,j} \in L} t_{i,j} = T_1 + bT_2$ , where  $T_1, T_2 \in \mathbb{Z}_p$ . Note that both  $T_1$  and  $T_2$  are represented by the sum of  $t'_{i,j}$ . Therefore,  $\mathcal{B}$  can compute  $T_1$  and  $T_2$ .  $\mathcal{B}$  chooses  $\beta \in_R \mathbb{Z}_p$ , sets  $r := \frac{\beta - ua}{T_2}$ , and computes  $SK_L = ((g_1^b)^{\beta}g_1^{\frac{T_1}{T_2}\beta}(g_1^a)^{-\frac{T_1u}{T_2}}, g_1^{\frac{\beta}{T_2}}(g_1^a)^{-\frac{u}{T_2}})$ . We show that  $SK_L$  is a valid secret key as follows:

$$\begin{split} (g_1^b)^\beta g_1^{\frac{T_1}{T_2}\beta}(g_1^a)^{-\frac{T_1u}{T_2}} &= g_1^{uab} \cdot g_1^{-uab}(g_1^b)^\beta g_1^{\frac{T_1}{T_2}\beta}(g_1^a)^{-\frac{T_1u}{T_2}} \\ &= g_1^{uab} \cdot g_1^{\frac{T_1}{T_2}(\beta-ua)} \cdot g_1^{b(\beta-ua)} \\ &= g_1^{uab}(g_1^{T_1} \cdot g_1^{bT_2})^{\frac{\beta-ua}{T_2}} \\ &= g_1^{uab}(g_1^{T_1+bT_2})^{\frac{\beta-ua}{T_2}} \\ &= h^y(g_1^{\sum_{v_{i,j} \in L} t_{i,j}})^r, \end{split}$$

and

$$g_1^{\frac{\beta}{T_2}}(g_1^a)^{-\frac{u}{T_2}} = g_1^{\frac{\beta-ua}{T_2}} = g_1^r$$

If  $T_2 = 0 \mod p$ , then  $\mathcal{B}$  aborts. If  $T_2 = 0 \mod p$  holds, then there exists L such that  $\sum_{v_{i,j} \in L} t_{i,j} = \sum_{v_{i,j} \in W^*} t_{i,j}$  holds. Therefore, this probability is at most  $\frac{N^2}{p}$ . See Section 4.2 for details. For the challenge ciphertext,  $\mathcal{B}$  chooses

	PK	MK	SK	Ciphertext
SW05 13	$n \mathbb{G}_1  +  \mathbb{G}_T $	$(n+1) \mathbb{Z}_p $	$r_2 \mathbb{G}_1 $	$r_1 \mathbb{G}_1  +  \mathbb{G}_T $
GPSW06 9	$n \mathbb{G}_1 + \mathbb{G}_T $	$(n+1) \mathbb{Z}_p $	$r_2 \mathbb{G}_1 $	$r_1 \mathbb{G}_1 + \mathbb{G}_T $
CN07 7	$(3n+1) \mathbb{G}_1  +  \mathbb{G}_T $	$(3n+1) \mathbb{Z}_p $	$(2n+1) \mathbb{G}_1 $	$(n+1) \mathbb{G}_1  +  \mathbb{G}_T $
BSW07 2	$3 \mathbb{G}_1  +  \mathbb{G}_T $	$ \mathbb{Z}_p  +  \mathbb{G} $	$(2n+1) \mathbb{G}_1 $	$(2r_2+1) \mathbb{G}_1 + \mathbb{G}_T $
NYO08 12	$(2N'+1) \mathbb{G}_1 + \mathbb{G}_T $	$(2N'+1) \mathbb{Z}_p $	$(3n+1) \mathbb{G}_1 $	$(2N'+1) \mathbb{G}_1 + \mathbb{G}_T $
W08 14	$2 \mathbb{G}_1  +  \mathbb{G}_T $	$ \mathbb{G}_1 $	$(1+n+r_2) \mathbb{G}_1 $	$(1+r_1n) \mathbb{G}_1 + \mathbb{G}_T $
Our scheme	$(2N'+3) \mathbb{G}_1 + \mathbb{G}_T $	$(N'+1) \mathbb{Z}_p $	$2 \mathbb{G}_1 $	$2 \mathbb{G}_1  +  \mathbb{G}_T $

Table 1. Size of each value

Table 2. Computational time of each algorithm

	Enc.	Dec.
SW05 13	$r_1 \mathbb{G}_1 + 2\mathbb{G}_T$	$r_1 C_e + (r_1 + 1) \mathbb{G}_T$
GPSW06 9	$r_1 \mathbb{G}_1 + 2\mathbb{G}_T$	$r_1 C_e + (r_1 + 1) \mathbb{G}_T$
CN07 7	$(n+1)\mathbb{G}_1 + 2\mathbb{G}_T$	$(n+1)C_e + (n+1)\mathbb{G}_T$
BSW07 2	$(2r_1+1)\mathbb{G}_1+2\mathbb{G}_T$	$2r_1C_e + (2r_1+2)\mathbb{G}_T$
NYO08 12	$(2N'+1)\mathbb{G}_1+2\mathbb{G}_T$	$(3n+1)C_e + (3n+1)\mathbb{G}_T$
W08 14	$(1+3r_1n)\mathbb{G}_1+2\mathbb{G}_T$	$(1+n+r_1)C_e + (3r_1-1)\mathbb{G}_1 + 3\mathbb{G}_T$
Our scheme	$(n+1)\mathbb{G}_1 + 2\mathbb{G}_T$	$2C_e + 2\mathbb{G}_T$

 $\mu \in_{R} \{0,1\}, \text{ computes } C_{1}^{*} = M_{\mu} \cdot Z^{u}, C_{2}^{*} = g_{1}^{c} \text{ and } C_{3}^{*} = (g_{1}^{c})^{\sum_{v_{i,j} \in W^{*}} t_{i,j}^{'}},$ and sends  $(C_{1}^{*}, C_{2}^{*}, C_{3}^{*})$  to  $\mathcal{A}$ . Finally,  $\mathcal{A}$  outputs  $\mu' \in \{0,1\}$ .  $\mathcal{B}$  outputs 1 if  $\mu' = \mu$ , or outputs 0 if  $\mu' \neq \mu$ . If  $Z = e(g_{1}, g_{1})^{abc}$ , then  $(C_{1}^{*}, C_{2}^{*}, C_{3}^{*})$  is a valid ciphertext associated with  $W^{*}$ . Therefore,  $\mathcal{A}$  has the advantage  $\epsilon$ . Hence,  $\Pr[\mathcal{B} \to 1|Z = e(g_{1}, g_{1})^{abc}] = \Pr[\mu' = \mu|Z = e(g_{1}, g_{1})^{abc}] = \frac{1}{2} + \epsilon$ . Otherwise, if  $Z = e(g_{1}, g_{1})^{z}$ ,  $\mathcal{A}$  has no advantage to distinguish a bit  $\mu$ , since all parts of the challenge ciphertext when  $\mu = 0$  and when  $\mu = 1$  have the same distributions. Hence,  $\Pr[\mathcal{B} \to 0|Z = e(g_{1}, g_{1})^{z}] = \Pr[\mu' \neq \mu|Z = e(g_{1}, g_{1})^{z}] = \frac{1}{2}$ . It follows that  $\mathcal{B}$ 's advantage in the DBDH game is  $\frac{\epsilon}{2}(1 - \frac{N^{2}}{p})$ .

Although a symmetric bilinear map is required in this proof, our scheme can be proven with an asymmetric bilinear map such as the Weil or Tate pairing  $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$  over MNT curves  $\square$ , where  $\mathbb{G}_1$  and  $\mathbb{G}_2$  are distinct groups. Then the indistinguishability of messages can be proven under the DBDH assumption over  $\mathbb{G}_2$   $\square$ .

#### 6 Comparison

Let PK, MK, SK and Ciphertext be the size of the public key, of the master key, of the secret key, and the ciphertext length excluding the access structure, respectively. Moreover, Enc. and Dec. are the computational times of encryption and decryption, respectively. We use the terms DBDH, DMBDH [13] and D-Linear [12] to refer to the Decision Bilinear Diffie-Hellman assumption, the

	Policy	Recipient Anonymity	Assumption
SW05 13	Key	No	DMBDH
GPSW06 9	Key	No	DBDH
CN07 7	Ciphertext	No	DBDH
BSW07 2	Ciphertext	No	Generic Group Model
NYO08 12	Ciphertext	Yes	DBDH, D-Linear
W08 14	Ciphertext	No	DBDH
Our scheme	Ciphertext	No	DBDH

Table 3. Some properties of ABE schemes

 Table 4. Expressiveness of policy

SW05 13	Threshold Structure
GPSW06 9	Tree-based Structure
CN07 7	AND-gates on positive and negative attributes with wildcards
BSW07 2	Tree-Based Structure
W08 14	Linear Structure
NYO08 12	AND-gates on multi-valued attributes with wildcards
Our scheme	AND-gates on multi-valued attributes

**Table 5.** Performance Results for n = 3

	Enc. Time	Dec. Time
CN07 7	$0.028 \mathrm{sec}$	0.031sec
NYO08 12	0.032 sec	$0.078 \mathrm{sec}$
Our scheme	0.015 sec	0.015 sec

Decision Modified Bilinear Diffie-Hellman assumption and the Decision Linear assumption, respectively. The notation  $|\mathbb{G}|$  is the bit-length of the element which belongs to  $\mathbb{G}$ . Let the notations  $k\mathbb{G}$  and  $kC_e$  (where  $k \in \mathbb{Z}_{>0}$ ) be the k-times calculation over the group  $\mathbb{G}$  and pairing, respectively. Let  $\mathcal{U} = \{att_1, att_2, \ldots, att_n\}$ be the set of attributes. Let  $\gamma_1$  ( $|\gamma_1| = r_1$ ) be a set of attributes associated with the ciphertext, and  $\gamma_2$  ( $|\gamma_2| = r_2$ ) a set of attributes associated with the secret key. Actually,  $\gamma_2$  is different for each user. Let  $N' := \sum_{i=1}^n n_i$  be the total number of possible statements of attributes. The computational time over  $\mathbb{Z}_p$  is ignored as usual.

Our scheme is efficient in that the ciphertext length and the costs of decryption do not depend on the number of attributes. Especially, the number of pairing computations is constant. No previous schemes provide these properties. An access structure is constructed by AND-gates on multi-valued attributes defined in section 2.2, which is a subset of the access structures in [12]. Although previous CP-ABE schemes [2]7]8]12]14] can complement our access structures, the length of the ciphertext depends on the number of attributes. To the best of our knowledge, our scheme is the first constant ciphertext length CP-ABE with AND-gates on multi-valued attributes. In future work, we plan to construct a CP-ABE with both a constant ciphertext length and more flexible structures, such as linear structures.

Our scheme does not provide recipient anonymity. Some parts of a ciphertext for attributes  $(C_2, C_3) = (g_1^s, g_1^{s \sum_{v_{i,j} \in W} t_{i,j}})$  is a DDH (Decision Diffie-Hellman)-tuple. Therefore, some information about attributes is exposed. Concretely, for an access structure W', an attacker can run the DDH test  $e(C_2, \prod_{v_{i,j} \in W'} T_{i,j}) \stackrel{?}{=} e(C_3, g_1)$ . Then, the attacker can determine whether an encryptor used the policy W' or not. We expect that our scheme will enable the property of the hidden encryptor-specified policies when a DDH-hard bilinear group is applied. However, we could not give the proof of security. Added to this, our scheme is inefficient in that the size of public key grows linearly with the number of attributes. There are rooms for argument on these points.

The CN07 scheme [7], the NYO08 scheme [12] and ours are implemented with the same access structure  $\{v_{1,1}, v_{2,1}, v_{3,1}\}$ , by using the Pairing-Based Cryptography (PBC) Library ver. 0.4.18 [15]. The performance results are shown in Table 5. Our experiment was performed by using a PC with an Intel(R) Core(TM)2 Duo CPU P8400 2.26GHz Windows Vista Home Premium Edition Service Pack 1. The execution of our scheme takes a very small amount of time, which is quite feasible for practical implementation. When n = 3, our decryption algorithm is approximately twice as fast as that of the CN07 scheme, and approximately five times faster than that of the NYO08 scheme.

#### 7 Conclusion

In this paper, we propose a constant ciphertext length CP-ABE with AND-gates on multi-valued attributes. Moreover, the number of pairing computations is also constant. To the best of our knowledge, this is the first such construction.

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## RSA-Based Certificateless Public Key Encryption

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**Abstract.** Certificateless Public Key Cryptography was first introduced by Al-Riyami and Paterson in order to eliminate the inherent key-escrow problem of Identity-Based Cryptography. In this paper, we present a new practical construction of certificateless public key encryption scheme without paring. Our scheme is, in the random oracle model, provably secure under the assumption that the RSA problem is intractable.

Keywords: Certificateless public key encryption, RSA.

#### 1 Introduction

In order to solve the key escrow problem that is inherent in identity-based cryptography (IBC) [20], while at the same time, eliminate the use of certificates in the traditional public key cryptography (PKC), Al-Riyami and Paterson [1] introduced the concept of certificateless public key cryptography (CL-PKC). Different from IBC, a user's public key in CL-PKC is no longer an arbitrary string; instead, the public key is generated by the user based the user's secret information as well as a partial private key obtained from a trusted authority called Key Generation Center (KGC). As such, public keys in CL-PKC do not need to be explicitly certified. Note here that the KGC does not know the user's private keys since they contain secret information generated by the users themselves, thereby removing the escrow problem in IBC.

Since the introduction of CL-PKC [1], many concrete constructions of certificateless public key encryption (CL-PKE) schemes have been proposed. The schemes in [3]6[21]22 were proven secure in the random oracle model [4] while the schemes in [14] and [18] are secure without the random oracles.

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There were also efforts to construct generic CL-PKC schemes. The first generic CL-PKE scheme was proposed in [23] and was later shown in [16] to be insecure under the model of [1]. In [5], the authors extended the concept of key encapsulation mechanism to IBE and CL-PKE, and built generic constructions of identity-based key encapsulation mechanism and certificateless public key encapsulation mechanism.

One notable feature in the research of CL-PKE has been the development of a number of alternative security models that are substantially weaker than the original model of  $\blacksquare$ . These different models are summarized by Dent  $\blacksquare$ . Moreover, Dent et al.  $\blacksquare$  presents a generic construction as well as a concrete construction for certificateless encryption schemes that are provably secure against strong adversaries in the standard model.

Au et al. [2] pointed out the weakness of the previous security models and analyzed some previous schemes under an enhanced malicious KGC model. They showed that the CL-PKE scheme in [16] is secure against malicious KGC attacks under random oracle assumption. Hwang and Liu [13] proposed a new CL-PKE scheme which is secure against malicious KGC attacks. Its security is proven in the standard model. In addition, Huang and Wong [12] proposed a generic construction of certificateless encryption which is proven secure against malicious KGC attacks in the standard model.

**Other Related Work.** Gentry **10** introduced a different but related concept named certificate based encryption (CBE). This approach is closer to the context of a traditional PKI model as it involves a certification authority (CA) providing an implicit certification service for clients' public keys. Liu et al. proposed the first self-generated-certificate public key encryption (SGC-PKE) scheme in **14**, which defends the DoD attack that exists in CL-PKE. Lai and Kou **15** proposed a SGC-PKE scheme without using pairing.

**Contribution.** In spite of the recent advances in implementation technique, the paring computation is still considered as expensive compared with the "standard" operations such as modular exponentiations in finite fields. Baek et al. proposed the first CL-PKE scheme without pairing, which was related to the early works on the self-certified keys **1119**.

In this paper, inspired by the identity-based key agreement protocol proposed by Okamoto and Tanaka [17] and whose security relies on the RSA problem, we present a new CL-PKE scheme without paring. Due to the extensive deployment of RSA, our scheme is better off in compatibility with the existing cryptosystems. In addition, in [3], the Type I adversary is not allowed to replace the challenge identity's public key, which is the main attacking means of Type I adversary. Compared with the scheme in [3], our scheme does not have this limitation.

**Organization.** The rest of the paper is organized as follow. We give some related definitions in Section 2. The model of CL-PKE is also reviewed in this section. The proposed CL-PKE scheme and its security analysis is presented in Section 3. Finally concluding remarks are given in Section 4.

# 2 Preliminaries

### 2.1 Computational Problems

**Definition 1.** The RSA problem is, given a randomly generated RSA modulus n, an exponent e and a random z, to find  $y \in \mathbb{Z}_n^*$  such that  $y^e = z$ .

**Definition 2.** The Computational Diffie-Hellman (CDH) problem in  $\mathbb{Z}_n^*$  is, given p,q,n (where p = 2p'+1, q = 2q'+1, n = pq with p',q' being two equal-length large primes),  $g \in \mathbb{Z}_n^*$  of order  $p'q', g^a$  and  $g^b$  for uniformly chosen  $a, b \in \mathbb{Z}_n^*$ , to compute  $g^{ab}$ .

## 2.2 Certificateless Public Key Encryption

A generic CL-PKE is a tuple of algorithm described as follows 3:

- Setup: Takes as input a security parameter  $\kappa$  and outputs a common parameter *params* and a master secret *msk*.
- PartialKeyExtract: Takes as input *params*, *msk* and an identity ID. It outputs a partial private key  $d_{ID}$ .
- SetSecretValue: Takes as input *params* and an identity ID. It outputs a secret value  $s_{\text{ID}}$ .
- SetPrivateKey: Takes as input *params*,  $d_{ID}$  and  $s_{ID}$ . It outputs a private key  $SK_{ID}$ .
- SetPublicKey: Takes as input *params*,  $d_{ID}$  and  $s_{ID}$ . It outputs a public key PK<sub>ID</sub>.
- Enc: Takes as input *params*, a message m, a receiver's identity ID and  $\mathsf{PK}_{\mathsf{ID}}$ . It outputs a ciphertext c.
- Dec: Takes as input *params*,  $SK_{ID}$  and a ciphertext *c*. It outputs a message *m* or the failure symbol  $\perp$ .

We insist that CL-PKE satisfies the obvious correctness requirement that decapsulation "undoes" encapsulation.

Note that, the above model of CL-PKE is slightly weaker than the original one given in [I] as a user must authenticate herself to the KGC in order to obtain a partial private key to create a public key, while the original CL-PKE model does not require a user to contact the KGC to setup her public keys. However, as argued in [3], this modified model preserves the unique property of CL-PKE that no certificates are required in order to guarantee the authenticity of public keys, which is the main motivation of CL-PKE.

Security Model. There are two types of adversaries  $\blacksquare$ . Type I adversary models an "outsider" adversary, who does not have the KGC's master secret key but it can replace public keys of arbitrary identities with other public keys of its own choices. It can also obtain partial and full secret keys of arbitrary identities. Type II adversary models an "honest-but-curious" KGC, who knows the master secret key (hence it can compute partial secret key by itself). It is still allowed to obtain full secret key for arbitrary identities but is not allowed to replace public keys at any time. Security in CL-PKE is defined using the following game between an attack algorithm  $\mathcal{A}$  and a challenger.

- **Setup.** The challenger runs the **Setup** algorithm and gives  $\mathcal{A}$  the resulting system parameter *params*. If  $\mathcal{A}$  is of Type I, the challenger keeps the master secret key *msk* to itself; otherwise, it gives *msk* to  $\mathcal{A}$ .
- Query phase 1 The adversary  $\mathcal{A}$  adaptively issues the following queries:
  - **Public-Key-Request** query: On input an identity ID, the challenger runs SetPublicKey(*params*,  $d_{ID}$ ,  $s_{ID}$ ), where the partial private key  $d_{ID}$  and the secret value  $s_{ID}$  of the identity ID are obtained from PartialKeyExtract and SetSecretValue, respectively, and forwards the result to the adversary.
  - **Partial-Key-Extract** query: On input an identity ID, the challenger runs PartialKeyExtract(*params*, *msk*, ID) and returns the result to  $\mathcal{A}$ . Note that it is only useful to Type I adversary.
  - **Private-Key-Request** query: On input an identity ID, the challenger runs SetPrivateKey(*params*,  $d_{\text{ID}}$ ,  $s_{\text{ID}}$ ), where the partial private key  $d_{\text{ID}}$  and the secret value  $s_{\text{ID}}$  of the identity ID are obtained from PartialKeyExtract and SetSecretValue, respectively, and forwards the result to the adversary. It outputs  $\perp$  if the uesr's public key has been replaced in the case of Type I adversary.
  - Public-Key-Replace query (for Type I adversary only): On input an identity and a valid public key, it replaces the associated user's public key with the new one.
  - Dec query: On input a ciphertext and an identity, returns the decrypted message using the private key corresponding to the current value of the public key associated with the identity of the user.
- **Challenge** query: After making a polynomial number of queries,  $\mathcal{A}$  outputs two messages  $m_0, m_1$  and an identity  $\mathsf{ID}^*$ . The challenger picks a random bit  $\beta \in \{0, 1\}$ , sets  $c^* = \mathsf{Enc}(parmas, m_\beta, \mathsf{ID}^*, \mathsf{PK}_{\mathsf{ID}^*})$  and sends  $c^*$  to  $\mathcal{A}$ .

Query phase 2  $\mathcal{A}$  makes a new sequence of queries.

- **Guess**  $\mathcal{A}$  outputs a bit  $\beta'$ . It wins the game if  $\beta' = \beta$  under the following conditions:
  - At any time, ID\* has not been submitted to the Private-Key-Request query.
  - $-(c^*, \mathsf{ID}^*, \mathsf{PK}_{\mathsf{ID}^*})$  have not been submitted to the **Dec** query.
  - If it is Type I adversary,  $\mathsf{ID}^*$  cannot be equal to an identity for which both the public key has been replaced and the partial private key has been extracted.

We define  $\mathcal{A}$ 's advantage in attacking the certificateless public key encryption CL-PKE as

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{CL}\text{-}\mathsf{PKE}} = |\mathsf{Pr}[\beta = \beta^{'}] - \frac{1}{2}|.$$

**Definition 3.** We say that a certificateless public key encryption CL-PKE is  $(t, q_{pub}, q_{par}, q_{prv}, q_d, \epsilon)$ -IND-CCA secure against Type I (resp. Type II) adversary  $A_I$  (resp.  $A_{II}$ ), if for all t-time algorithms  $A_I$  (resp.  $A_{II}$ ) making at most  $q_{pub}$  Public-Key-Request queries,  $q_{par}$  Partial-Key-Extract queries,  $q_{prv}$  Private-Key-Request queries and  $q_d$  Dec queries, have advantage at most  $\epsilon$  in winning the above game.

IND-CPA security is defined similarly, but with the restriction that the adversary cannot make **Dec** queries.

**Definition 4.** We say that a certificateless public key encryption CL-PKE is  $(t, q_{pub}, q_{par}, q_{prv}, \epsilon)$ -IND-CPA secure, if it is  $(t, q_{pub}, q_{par}, q_{prv}, 0, \epsilon)$ -IND-CCA secure.

# 3 Our Scheme

Our CL-PKE scheme is inspired by the RSA-based key agreement protocol **17** introduced by Okamoto and Tanaka. We first present our scheme and then show that it is IND-CPA secure. However, it is easy to turn our IND-CPA secure CL-PKE scheme into an IND-CCA secure CL-PKE scheme using the technique proposed by Fujisaki and Okamoto **9**, as did in **3**.

- Setup( $\kappa$ ) Given a security parameter  $\kappa$ , a RSA group  $\langle n, p, q, e, d, g \rangle$  is generated, where p', q' are  $\kappa$ -bit prime numbers,  $p = 2p' + 1, q = 2q' + 1, n = pq, e \langle \phi(n), \gcd(e, \phi(n)) = 1, ed \equiv 1 \pmod{\phi(n)}$ , and  $\phi$  denotes the Euler totient function. Chooses two cryptographic hash functions  $H : \{0, 1\}^* \rightarrow \mathbb{Z}_n^*, H_2 : \mathbb{Z}_n^* \rightarrow \{0, 1\}^l$ , where l is the length of the plaintext message. The master secret key is defined as msk = d. The common parameter is  $params = (n, e, H, H_2)$ .
- PartialKeyExtract(*params*, *msk*, ID) Given *params*, *msk* = d and an identity ID  $\in \{0, 1\}^*$ , outputs the partial private key

$$d_{\mathsf{ID}} = H(\mathsf{ID})^d.$$

SetSecretValue(*params*, ID) Given *params* and an identity ID, randomly chooses  $x_{ID} \in \mathbb{Z}_n^*$  and outputs

$$s_{\mathsf{ID}} = x_{\mathsf{ID}}.$$

SetPrivateKey( $params, d_{ID}, s_{ID}$ ) Given *params*, the partial private key  $d_{ID}$  and the secret value  $s_{ID} = x_{ID}$  of an identity ID, outputs

$$SK_{ID} = x_{ID}$$
.

SetPublicKey(*params*,  $d_{ID}$ ,  $s_{ID}$ ) Given *params*, the partial private key  $d_{ID} = H(ID)^d$  and the secret value  $s_{ID} = x_{ID}$  of an identity ID, outputs

$$\mathsf{PK}_{\mathsf{ID}} = H(\mathsf{ID})^{d+x_{\mathsf{ID}}}.$$

 $\mathsf{Enc}(params, m, \mathsf{ID}, \mathsf{PK}_{\mathsf{ID}})$  Given *params*, a message *m* and the public key  $\mathsf{PK}_{\mathsf{ID}}$  of an identity  $\mathsf{ID}$ , randomly chooses  $r \in \mathbb{Z}_n^*$  and computes

$$c_1 = H(\mathsf{ID})^{er}, c_2 = H_2(\mathsf{PK}_{\mathsf{ID}}^{er}H(\mathsf{ID})^{-r}) \oplus m,$$

then outputs  $c = (c_1, c_2)$ .

 $Dec(params, SK_{ID}, c)$  Given *params*, the private key  $SK_{ID}$  of an identity ID, and a ciphertext  $c = (c_1, c_2)$ , outputs

$$m = H_2(c_1^{\mathsf{SK}_{\mathsf{ID}}}) \oplus c_2.$$

It can be easily seen that the above decryption algorithm is consistent, i. e.,

$$\begin{aligned} \mathsf{PK}_{\mathsf{ID}}^{er} H(\mathsf{ID})^{-r} &= H(\mathsf{ID})^{(d+x_{\mathsf{ID}})er} H(\mathsf{ID})^{-r} \\ &= H(\mathsf{ID})^r H(\mathsf{ID})^{erx_{\mathsf{ID}}} H(\mathsf{ID})^{-r} \\ &= c_1^{x_{\mathsf{ID}}} = c_1^{\mathsf{SK}_{\mathsf{ID}}}. \end{aligned}$$

We now prove the security of the scheme by two theorems.

**Theorem 1.** Assume the hash functions  $H, H_2$  are random oracles and the RSA problem is  $(t, \epsilon)$ -intractable. Then, the above CL-PKE scheme is  $(t', q_{pub}, q_{par}, q_{prv}, \epsilon')$ IND-CPA secure against Type I adversary  $\mathcal{A}_I$  for

$$t > t' + t_{ex}(q_H + q_{pub}), \epsilon > \frac{2\epsilon'}{q_{H_2}\tau(q_{par} + q_{prv} + 1)},$$

where  $t_{ex}$  denotes the time for computing exponentiation in  $\mathbb{Z}_n^*$ ,  $\tau$  denotes the base of the natural logarithm and  $q_H$  (resp.  $q_{H_2}$ ) denotes the number of H (resp.  $H_2$ ) queries by the adversary.

*Proof.* Let  $\mathcal{A}_I$  be a Type I adversary that  $(t', q_{pub}, q_{par}, q_{prv}, \epsilon')$ -breaks the IND-CPA security of the certificateless public key encryption scheme described above. We construct an algorithm  $\mathcal{B}$ , that solves the RSA problem, as follows.  $\mathcal{B}$  is given an instance of the RSA problem, which consists of (n, e, z).  $\mathcal{B}$ 's goal is to find  $y \in \mathbb{Z}_n^*$  such that  $y^e = z$ . It interacts with  $\mathcal{A}_I$  as follows.

- Setup  $\mathcal{B}$  maintains three lists H-List, H<sub>2</sub>-List and KeyList. Initially the lists are empty. The common parameter params = (n, e) is sent to  $\mathcal{A}_I$ . The master secret key msk = d, where  $ed \equiv 1 \pmod{\phi(n)}$ , is unknown to  $\mathcal{B}$ .
- Query phase 1  $A_I$  adaptively issues H, H<sub>2</sub>, Public-Key-Request, Partial-Key-Extract, Private-Key-Request and Public-Key-Replace queries.  $\mathcal{B}$  answers them as follows:
  - H query on ID: If a record (ID,  $h_{\text{ID}}$ ,  $f_{\text{ID}}$ , coin) appears in the H-List, sends  $h_{\text{ID}}$  to  $\mathcal{A}_I$ ; otherwise,  $\mathcal{B}$  picks  $coin \in \{0, 1\}$  at random such that  $\Pr[coin = 0] = \rho$ . ( $\rho$  will be determined later.) Then, randomly chooses  $f_{\text{ID}} \in \mathbb{Z}_n^*$ . Finally, the record (ID,  $h_{\text{ID}} = z^{coin} \cdot f_{\text{ID}}^e$ ,  $f_{\text{ID}}$ , coin) is added to the H-List and  $h_{\text{ID}}$  is sent to  $\mathcal{A}_I$ .

- $H_2$  query on  $\omega$ : If a record  $(\omega, k)$  appears in the  $H_2$ -List, sends k to  $\mathcal{A}_I$ ; otherwise, picks  $k \in \{0, 1\}^l$  at random, adds the record  $(\omega, k)$  to  $H_2$ -List and sends k to  $\mathcal{A}_I$ .
- **Public-Key-Request** query on ID: Randomly chooses  $x_{\mathsf{ID}} \in \mathbb{Z}_n^*$  and searches H-List for a record (ID,  $h_{\mathsf{ID}}$ ,  $f_{\mathsf{ID}}$ , coin). Then, adds the record (ID,  $\mathsf{PK}_{\mathsf{ID}} = f_{\mathsf{ID}}h_{\mathsf{ID}}^{x_{\mathsf{ID}}}$ ,  $\mathsf{SK}_{\mathsf{ID}} = x_{\mathsf{ID}}$ , coin) to KeyList and sends  $\mathsf{PK}_{\mathsf{ID}}$  to  $\mathcal{A}_I$ .
- Partial-Key-Extract query on ID: Searches H-List for a record (ID,  $h_{\text{ID}}$ ,  $f_{\text{ID}}$ , *coin*). If *coin* = 0, sends  $f_{\text{ID}}$  to  $\mathcal{A}_I$ ; otherwise, aborts and terminates.
- **Private-Key-Extract** query on ID: Searches KeyList for a record (ID,  $PK_{ID}$ ,  $SK_{ID}$ , coin). If coin = 0, sends  $SK_{ID}$  to  $A_I$ ; otherwise, aborts and terminates.
- **Public-Key-Replace** query on (ID,  $\mathsf{PK}'_{\mathsf{ID}}$ ): Replaces  $\mathsf{PK}_{\mathsf{ID}}$  with  $\mathsf{PK}'_{\mathsf{ID}}$ .
- **Challenge**  $\mathcal{A}_I$  submits two messages  $m_0, m_1$  and an identity  $\mathsf{ID}^*$  with the public key  $\mathsf{PK}_{\mathsf{ID}^*}$ .  $\mathcal{B}$  searches H-List for a record  $(\mathsf{ID}^*, h_{\mathsf{ID}^*}, f_{\mathsf{ID}^*}, coin)$ . If coin = 0, it aborts and terminates; otherwise,  $\mathcal{B}$  picks  $r \in \mathbb{Z}_n^*$  at random. Let  $r^* = d + r$ , which is unknown to  $\mathcal{B}$ . Then  $\mathcal{B}$  randomly chooses  $c_2^* \in \{0, 1\}^l$  and computes

$$c_1^* = H(\mathsf{ID}^*)^{er^*} = H(\mathsf{ID}^*)^{e(d+r)} = h_{\mathsf{ID}^*}^{1+er}.$$

Finally, it sends  $c^* = (c_1^*, c_2^*)$  to  $\mathcal{A}_I$ .

- Query phase 2  $\mathcal{A}_I$  makes a new sequence of queries, and  $\mathcal{B}$  responds as in Query phase 1.
- **Guess:** Finally, the adversary  $\mathcal{A}_I$  outputs a bit  $\beta'$ .  $\mathcal{B}$  picks a tuple  $(\omega, k)$  from H<sub>2</sub>-List at random and outputs  $\frac{\mathsf{PK}_{1}^{1+er}}{\omega h_{10^*}^r f_{10^*}}$  as the solution to the RSA problem.

**Probability Analysis:** Let  $AskH_2^*$  denotes the event that  $\mathsf{PK}_{\mathsf{ID}^*}^{er^*}H(\mathsf{ID}^*)^{-r^*}$  has been queried to  $H_2$ . Note that,

$$\begin{aligned} \mathsf{PK}_{\mathsf{ID}^*}^{er^*} H(\mathsf{ID}^*)^{-r^*} &= \mathsf{PK}_{\mathsf{ID}^*}^{e(d+r)} H(\mathsf{ID}^*)^{-d-r} \\ &= \mathsf{PK}_{\mathsf{ID}^*}^{1+er} h_{\mathsf{ID}^*}^{-d} h_{\mathsf{ID}^*}^{-r} \\ &= \mathsf{PK}_{\mathsf{ID}^*}^{1+er} (zf_{\mathsf{ID}^*}^e)^{-d} h_{\mathsf{ID}^*}^{-r} \\ &= \mathsf{PK}_{\mathsf{ID}^*}^{1+er} z^{-d} f_{\mathsf{ID}^*}^{-1} h_{\mathsf{ID}^*}^{-r}. \end{aligned}$$

If the event  $\mathsf{AskH}_2^*$  happens, then  $\mathcal{B}$  will be able to solve the RSA problem by choosing a tuple  $(\omega, k)$  from the H<sub>2</sub>-List and computing  $\frac{\mathsf{PK}_{D^*}^{1+er}}{\omega h_{D^*}^r f_{D^*}}$  with the probability at least  $\frac{1}{q_{H_2}}$ , where  $q_{H_2}$  is the number of  $H_2$  queries by the adversary. If the event  $\mathsf{AskH}_2^*$  does not happen,  $\mathcal{B}$ 's simulations are perfect and are identically distributed as the real one from the construction.

We observe that the probability that  $\mathcal{B}$  does not abort during the simulation is given by  $\rho^{q_{par}+q_{prv}}(1-\rho)$  which is maximized at  $\rho = 1 - 1/(q_{par} + q_{prv} + 1)$ . Hence the probability that  $\mathcal{B}$  does not abort is at most  $\frac{1}{\tau(q_{par}+q_{prv}+1)}$ , where  $\tau$  denotes the base of the natural logarithm.

Now, the event  $AskH_2^*|\neg Abort$  denoted by Good, where Abort denotes the event that  $\mathcal{B}$  aborts during the simulation. If Good dose not happen, it is clear that

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the adversary does not gain any advantage greater than 1/2 to guess  $\beta$ . Namely, we have  $\Pr[\beta' = \beta | \neg \text{Good}] \leq 1/2$ . Hence, by splitting  $\Pr[\beta' = \beta]$ , we obtain  $|\Pr[\beta' = \beta] - \frac{1}{2}| \leq \frac{1}{2}\Pr[\text{Good}]$ . To sum up, we have  $\epsilon > \frac{2\epsilon'}{q_{H_2}\tau(q_{par}+q_{prv}+1)}$ .

**Time Complexity.** In the simulation,  $\mathcal{B}$ 's overhead is dominated by the exponentiation computation in response to  $\mathcal{A}_I$ 's H and **Public-Key-Request** queries. So, we have  $t > t' + t_{ex}(q_H + q_{pub})$ , where  $t_{ex}$  denotes the time for computing exponentiation in  $\mathbb{Z}_n^*$ .

This concludes the proof of Theorem 1.

**Theorem 2.** Assume the hash functions H and  $H_2$  are random oracles and the CDH problem is  $(t, \epsilon)$ -intractable. Then, the above CL-PKE scheme is  $(t', q_{pub}, q_{par}, q_{prv}, \epsilon')$ IND-CPA secure against Type II adversary  $A_{II}$  for

$$t > t' + t_{ex}(q_H + q_{pub}), \epsilon > \frac{2\epsilon'}{q_{H_2}\tau(q_{prv} + 1)},$$

where  $t_{ex}$  denotes the time for computing exponentiation in  $\mathbb{Z}_n^*$ ,  $\tau$  denotes the base of the natural logarithm and  $q_H$  (resp.  $q_{H_2}$ ) denotes the number of H (resp.  $H_2$ ) queries by the adversary.

*Proof.* Let  $\mathcal{A}_{II}$  be a Type II adversary that  $(t', q_{pub}, q_{par}, q_{prv}, \epsilon')$ -breaks the IND-CPA security of the CL-PKE scheme described above. We construct an algorithm  $\mathcal{B}$ , that solves the CDH problem, as follows.  $\mathcal{B}$  is given an instance of the CDH problem, which consists of  $(n, p, q, g, g^a, g^b)$ .  $\mathcal{B}$ 's goal is to compute  $g^{ab}$ . It interacts with  $\mathcal{A}_{II}$  as follows.

- Setup  $\mathcal{B}$  maintains three lists H-List, H<sub>2</sub>-List and KeyList. Initially the lists are empty. Then  $\mathcal{B}$  picks  $e < \phi(n), \gcd(e, \phi(n)) = 1$  at random and computes dsuch that  $ed \equiv 1 \pmod{\phi(n)}$ , where  $\phi$  denotes the Euler totient function. (It can be computed by p, q.) Finally, it sends the common parameter params = (n, e) and the master secret key msk = d to  $\mathcal{A}_{II}$ .
- Query phase 1  $A_{II}$  adaptively issues H, H<sub>2</sub>, Public-Key-Request and Private-Key-Request queries. B answers them in the following way:
  - H query on ID: If a record (ID,  $h_{\text{ID}}, t_{\text{ID}}$ ) appears in the H-List,  $\mathcal{B}$  sends  $h_{\text{ID}}$  to  $\mathcal{A}_I$ ; otherwise,  $\mathcal{B}$  randomly chooses  $t_{\text{ID}}$  such that  $t_{\text{ID}} < \phi(n)$ ,  $gcd(t_{\text{ID}}, \phi(n)) = 1$ , adds the record (ID,  $h_{\text{ID}} = g^{t_{\text{ID}}}, t_{\text{ID}}$ ) to H-List and sends  $h_{\text{ID}}$  to  $\mathcal{A}_{II}$ .
  - $H_2$  query on  $\omega$ : If a record  $(\omega, k)$  appears in the  $H_2$ -List,  $\mathcal{B}$  sends k to  $\mathcal{A}_I$ : otherwise,  $\mathcal{B}$  picks  $k \in \{0, 1\}^l$  at random, adds the record  $(\omega, k)$  to  $H_2$ -List and sends k to  $\mathcal{A}_{II}$ .
  - **Public-Key-Request** query on ID:  $\mathcal{B}$  searches H-List for a record (ID,  $h_{\text{ID}}, t_{\text{ID}}$ ). Then, it picks  $coin \in \{0, 1\}$  at random such that  $\Pr[coin = 0] = \rho$  ( $\rho$  will be determined later). Finally, it randomly chooses  $x_{\text{ID}} \in \mathbb{Z}_n^*$ , adds the record (ID,  $\mathsf{PK}_{\mathsf{ID}} = h_{\mathsf{ID}}^{d+x_{\mathsf{ID}}} \cdot (g^a)^{t_{\mathsf{ID}} \cdot coin} = h_{\mathsf{ID}}^{d+a \cdot coin+x_{\mathsf{ID}}}$ ,  $\mathsf{SK}_{\mathsf{ID}} = x_{\mathsf{ID}}$ , coin) to KeyList and sends  $\mathsf{PK}_{\mathsf{ID}}$  to  $\mathcal{A}_{II}$ .

- **Private-Key-Extract** query on ID:  $\mathcal{B}$  searches KeyList for a record (ID, PK<sub>ID</sub>, SK<sub>ID</sub>, *coin*). If *coin* = 0, it sends the SK<sub>ID</sub> to  $\mathcal{A}_{II}$ ; otherwise, it aborts and terminates.
- **Challenge**  $\mathcal{A}_{II}$  submits two messages  $m_0, m_1$  and an identity  $\mathsf{ID}^*$  with the public key  $\mathsf{PK}_{\mathsf{ID}^*}$ .  $\mathcal{B}$  searches H-List for a record  $(\mathsf{ID}^*, h_{\mathsf{ID}^*}, t_{\mathsf{ID}^*})$  and KeyList for a record  $(\mathsf{ID}^*, \mathsf{PK}_{\mathsf{ID}^*}, \mathsf{SK}_{\mathsf{ID}^*} = x_{\mathsf{ID}^*}, coin)$ . If coin = 0, it aborts and terminates; otherwise,  $\mathcal{B}$  randomly chooses  $c_2^* \in \{0, 1\}^l$ . Let  $r^* = b$ , which is unknown to  $\mathcal{B}$ . Then  $\mathcal{B}$  computes

$$c_1^* = (g^b)^{et_{\mathsf{ID}^*}} = (g^{t_{\mathsf{ID}^*}})^{er^*} = h_{\mathsf{ID}^*}^{er^*} = H(\mathsf{ID}^*)^{er^*}.$$

and sends  $c^* = (c_1^*, c_2^*)$  to  $\mathcal{A}_{II}$ .

- **Query phase 2**  $A_{II}$  makes a new sequence of queries, and B responds as in Query phase 1.
- **Guess** Finally, the adversary  $\mathcal{A}_{II}$  outputs a bit  $\beta'$ .  $\mathcal{B}$  picks a tuple  $(\omega, k)$  from H<sub>2</sub>-List at random and outputs  $\frac{\omega^{dt_{\mathrm{ID}^*}}}{(g^b)^{\varpi_{\mathrm{ID}^*}}}$  as the solution to the CDH problem. Note that,  $\mathcal{B}$  knows p, q, so  $t_{\mathrm{ID}^*}^{-1}$  can be computed.

**Probability Analysis:** Let  $AskH_2^*$  denotes the event that  $PK_{ID^*}^{er^*}H(ID^*)^{-r^*}$  has been queried to  $H_2$ . Note that,

$$\begin{aligned} \mathsf{PK}_{\mathsf{ID}^*}^{er^*} H(\mathsf{ID}^*)^{-r^*} &= \mathsf{PK}_{\mathsf{ID}^*}^{eb} H(\mathsf{ID}^*)^{-b} \\ &= h_{\mathsf{ID}^*}^{eb(d+a+x_{\mathsf{ID}^*})} h_{\mathsf{ID}^*}^{-b} \\ &= h_{\mathsf{ID}^*}^{eb(a+x_{\mathsf{ID}^*})} \\ &= (g^{ab})^{et_{\mathsf{ID}^*}} (g^b)^{et_{\mathsf{ID}^*} x_{\mathsf{ID}^*}} \end{aligned}$$

If the event  $\mathsf{AskH}_2^*$  happens, then  $\mathcal{B}$  will be able to solve the CDH problem by choosing a tuple  $(\omega, k)$  from the H<sub>2</sub>-List and computing  $\frac{\omega^{dt_{\mathsf{ID}}^*}}{(g^b)^{x_{\mathsf{ID}}*}}$  with the probability at least  $\frac{1}{q_{H_2}}$ , where  $q_{H_2}$  is the number of  $H_2$  queries by the adversary. If the event  $\mathsf{AskH}_2^*$  does not happen,  $\mathcal{B}$ 's simulations are perfect and are identically distributed as the real one form the construction.

We observe that the probability that  $\mathcal{B}$  does not abort during the simulation is given by  $\rho^{q_{prv}}(1-\rho)$  which is maximized at  $\rho = 1 - 1/(q_{prv} + 1)$ . Hence, the probability that  $\mathcal{B}$  does not abort is at most  $\frac{1}{\tau(q_{prv}+1)}$ , where  $\tau$  denotes the base of the natural logarithm.

Now, the event  $\mathsf{AskH}_2^*|\neg\mathsf{Abort}$  denoted by Good, where Abort denotes the event that  $\mathcal{B}$  aborts during the simulation. If Good dose not happen, it is clear that the adversary does not gain any advantage greater than 1/2 to guess  $\beta$ . Namely, we have  $\Pr[\beta' = \beta | \neg \mathsf{Good}] \leq 1/2$ . Hence, by splitting  $\Pr[\beta' = \beta]$ , we obtain  $|\Pr[\beta' = \beta] - \frac{1}{2}| \leq \frac{1}{2}\Pr[\mathsf{Good}]$ . To sum up, we have  $\epsilon > \frac{2\epsilon'}{q_{H_2}\tau(q_{prv}+1)}$ .

Time Complexity. In the simulation,  $\mathcal{B}$ 's overhead is dominated by the exponentiation computation in response to  $\mathcal{A}_{II}$ 's H and Public-Key-Request

query. So, we have  $t > t' + t_{ex}(q_H + q_{pub})$ , where  $t_{ex}$  denotes the time for computing exponentiation in  $\mathbb{Z}_n^*$ .

This concludes the proof of Theorem 2.

## 4 Conclusion

We have presented a new practical CL-PKE scheme that does not depend on the paring. We have proven that our scheme is, in the random oracle model, secure under the assumption that the RSA problem is intractable.

However, the model of our scheme is slightly weaker than the original model II. It is still an open problem to design a CL-PKE scheme without paring in the original model II that is IND-CCA secure, even relies on the random oracles.

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# Strongly Unforgeable ID-Based Signatures without Random Oracles

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Abstract. In this paper, we construct a strongly unforgeable ID-based signature scheme without random oracles. The signature size of our scheme is smaller than that of other schemes based on varieties of the Diffie–Hellman problem or the discrete logarithm problem. The security of the scheme relies on the difficulty to solve three problems related to the Diffie–Hellman problem and a one-way isomorphism.

**Keywords:** Digital signatures, ID-based signatures, Strong unforgeability, Standard models.

## 1 Introduction

In 1984, Shamir **[19]** introduced the concept of ID-based cryptosystems, in which the private key of an entity was generated from his identity information (e.g. an e-mail address, a telephone number, etc.) and a master key of a trusted third party called a Private Key Generator (PKG). The advantage of this cryptosystem is that certificates as used in a traditional public key infrastructure can be eliminated. The first ID-based signature (IBS) scheme was proposed by Shamir **[19]**. Later, many IBS schemes were presented in **[7] [2] [15] [18]**.

For (ID-based) signatures [3,5]6[8]11116[21]20] or ID-based encryptions [2,20], constructing schemes whose security can be proved without random oracles is one of the most important themes of study, since commonly used hash functions such as MD5 or SHA-1 are not random oracles.

It is known that strongly unforgeable IBS schemes can be constructed with the approach of attaching certificates to strongly unforgeable (non-ID-based) signatures. This approach is mentioned in passing within several papers [9]1110. We can construct strongly unforgeable IBS schemes without random oracles by applying the approach to strongly unforgeable signature schemes without random oracles such as the Boneh–Boyen [3], the Zhang–Chen–Susilo–Mu [21], the

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Camenisch-Lysyanskaya **6**, the Okamoto **14** or the Boneh-Shen-Waters **5**. However, these constructions need at least six signature parameters to include a public key of the signer and two ordinary signatures.

Also, Huang–Wong–Zhao **13** proposed a general method to transform (weakly) unforgeable IBS schemes into strongly unforgeable ones by attaching strong one-time signatures. Therefore, this enables us to construct strongly unforgeable IBS schemes without random oracles by applying it on them to any unforgeable ones such as the Paterson–Schuldt **16**. However, in this transformation, signature sizes of the IBS scheme depend on the public key size and signature size of the underlying strong one-time signature scheme. Almost all the current one-time signature schemes suffer from a drawback that these signature scheme (in the sense of Definition 2 in **13**) is also a strong one-time signature scheme (Definition 3 in **13**). However, by using a strongly unforgeable signature, these constructions also need at least six signature parameters.

In this paper, we propose a strongly unforgeable IBS scheme without random oracles, with five signature parameters. The security of the scheme relies on the difficulty to solve three problems related to the Diffie–Hellman (DH) problem, and a one-way isomorphism that no PPT adversary can find the inverse one. The signature size of our scheme is smaller than that of other schemes based on varieties of the DH problem or the discrete logarithm problem. One of the reasons why the number of parameters can be reduced from six to five is that our scheme is directly constructed without applying **9**(**1**,**1**0) or **1**(**3**).

The paper is organized in the following way. In Section 2, we prepare for the construction of our scheme, along with its proof of security. In Section 3, we will provide two new assumptions related to the DH problem, and make a proposal for our ID-based signature scheme. We prove our scheme satisfying security of strong unforgeability in Section 4, and discuss efficiency in Section 5. We provide conclusions in Section 6.

## 2 Preliminaries

The aim of this section is to define a one-way isomorphism, a bilinear map, the co-Diffie-Hellman (co-DH) problem, an IBS scheme and the strong unforgeability.

### 2.1 One-Way Isomorphism and Bilinear Map

The following definitions are due to 17.4. We assume that

<sup>&</sup>lt;sup>1</sup> Currently, the most practical strongly unforgeable signature schemes [11]S without random oracles are constructed based on the Strong RSA assumption. It is known that each component in the parameters of the signature and the public key generated by these schemes needs to be at least 1024-bits in size. On the other hand, it is sufficient to be 160-bits in size for signature schemes constructed based on the discrete logarithm problem (including varieties of the DH problem) over elliptic curves. In this paper, we only consider such schemes.

- $\mathbb{G}_1, \mathbb{G}_2$  and  $\mathbb{G}_T$  are multiplicative cyclic groups of prime order p;
- $-g_2$  is a generator of  $\mathbb{G}_2$ ;
- $f: \mathbb{G}_2 \to \mathbb{G}_1$  is a one-way isomorphism satisfying  $f(g_2^x) = g_1^x$ , where  $x \in \mathbb{Z}_p$ and  $g_1$  is a generator of  $\mathbb{G}_1$ ;
- $-e: \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$  is the cryptographic *bilinear map* satisfying the following properties:

**Bilinearity**:  $e(u^a, v^b) = e(u, v)^{ab}$  for any  $u \in \mathbb{G}_1$ ,  $v \in \mathbb{G}_2$  and any  $a, b \in \mathbb{Z}$ . Non-degenerate:  $e(g_1, g_2) \neq 1_{\mathbb{G}_T}$  for  $\langle g_1 \rangle = \mathbb{G}_1$  and  $\langle g_2 \rangle = \mathbb{G}_2$ .

**Computable**: There is an efficient algorithm to compute e(u, v) for any  $u \in \mathbb{G}_1$  and any  $v \in \mathbb{G}_2$ .

### 2.2 The Co-Diffie–Hellman Problem

We provide the co-DH problem in  $(\mathbb{G}_2, \mathbb{G}_1)$  as follows. Given

$$(g_1, g_2, g_2^x, g_2^y)$$

as input for random generators  $g_1 \in_{\mathbf{R}} \mathbb{G}_1, g_2 \in_{\mathbf{R}} \mathbb{G}_2$  and random numbers  $x, y \in_{\mathbf{R}} \mathbb{Z}_p^*$ , compute  $g_1^{xy}$ . We say that algorithm  $\mathcal{A}$  has an advantage  $\varepsilon$  in solving the co-DH problem in  $(\mathbb{G}_2, \mathbb{G}_1)$  if

$$\Pr\left[\mathcal{A}\left(g_1, g_2, g_2^x, g_2^y\right) = g_1^{xy}\right] \ge \varepsilon ,$$

where the probability is over the choice  $g_1 \in_{\mathbf{R}} \mathbb{G}_1$ ,  $g_2 \in_{\mathbf{R}} \mathbb{G}_2$ ,  $x, y \in_{\mathbf{R}} \mathbb{Z}_p^*$  and the random bits of  $\mathcal{A}$ .

**Assumption 1.** The  $(t, \varepsilon)$ -co-Diffie-Hellman (co-DH) Assumption holds in  $(\mathbb{G}_2, \mathbb{G}_1)$  if no t-time adversary has an advantage of at least  $\varepsilon$  in solving the co-DH problem in  $(\mathbb{G}_2, \mathbb{G}_1)$ .

Notice that, if we set  $g_1 := f(g_2) \in \mathbb{G}_1$  for the one-way isomorphism  $f : \mathbb{G}_2 \to \mathbb{G}_1$ and the random generator  $g_2 \in_{\mathbb{R}} \mathbb{G}_2$ , then the generator  $g_1$  is not random.

#### 2.3 ID-Based Signature Schemes

The definition of the IBS scheme in this section is due to 16.

An IBS scheme consists of four phases: *Setup, Extract, Sign* and *Verify* as follows.

- Setup: A security parameter is taken as input and returns params (system parameters) and master-key. The system parameters include a decision of a finite message space  $\mathcal{M}$ , and a decision of a finite signature space  $\mathcal{S}$ . Intuitively, the system parameters will be publicly known, while the master-key will be known only to the Private Key Generator (PKG).
- **Extract:** The output from Setup (params, master-key) is taken along with an arbitrary  $ID \in \{0, 1\}^*$  as input, and returns a private key d. Here ID is an arbitrary string that will be used as a public key, and d is the corresponding private sign key. The Extract phase extracts a private key from the given public key, and is performed by the PKG.

- Sign: A message  $M \in \mathcal{M}$ , a private key d and params are taken as input. It returns a signature  $\sigma \in S$ .
- **Verify:** A message  $M \in \mathcal{M}, \sigma \in \mathcal{S}$ , ID and params are taken as input. It returns valid or invalid.

The parameters in Sign and Verify are used in a different order later on. These four phases must satisfy the standard consistency constraint, namely when d is the private key generated by phase *Extract* when it is given ID as the public key, then

 $\forall M \in \mathcal{M}, \forall \sigma := Sign(params, d, M) : \Pr[Verify(params, ID, M, \sigma) = valid] = 1.$ 

### 2.4 Strong Unforgeability

The definition of the strong unforgeability in this section is due to [3,5,16]. In particular, Paterson–Schuldt [16] defined the unforgeability and the strong unforgeability. However, their construction of the IBS scheme satisfied only the unforgeability.

Strong unforgeability is defined using the following game between a challenger  $\mathcal{B}$  and an adversary  $\mathcal{A}$ :

- **Setup:** The challenger  $\mathcal{B}$  takes a security parameter k and runs the Setup phase of the IBS scheme. It gives the adversary  $\mathcal{A}$  the resulting system parameters params. It keeps the master-key to itself.
- **Queries:** The adversary  $\mathcal{A}$  adaptively makes a number of different queries to the challenger  $\mathcal{B}$ . Each query can be one of the following.
- Extract Queries  $(ID_i)$ : The challenger  $\mathcal{B}$  responds by running phase Extract to generate the private key  $d_i$  corresponding to the public key  $ID_i$  issued by  $\mathcal{A}$ . It sends  $d_i$  to the adversary  $\mathcal{A}$ .
- Signature Queries  $(ID_i, M_{i,j})$ : For each query  $(ID_i, M_{i,j})$  issued by  $\mathcal{A}$  the challenger  $\mathcal{B}$  responds by first running Extract to obtain the private key  $d_i$  of  $ID_i$ , and then running Sign to generate a signature  $\sigma_{i,j}$  of  $(ID_i, M_{i,j})$ , and sending  $\sigma_{i,j}$  to  $\mathcal{A}$ .
- **Output:** Finally  $\mathcal{A}$  outputs a pair (ID<sub>\*</sub>,  $M_*, \sigma_*$ ). If  $\sigma_*$  is a valid signature of (ID<sub>\*</sub>,  $M_*$ ) according to Verify, ID<sub>\*</sub>  $\notin$  {ID<sub>i</sub>} for Extract Queries and (ID<sub>\*</sub>,  $M_*, \sigma_*) \notin$  {(ID<sub>i</sub>,  $M_{i,j}, \sigma_{i,j}$ )} for Signature Queries, then  $\mathcal{A}$  wins.

We define  $\operatorname{AdvSig}_{\mathcal{A}}$  to be the probability that  $\mathcal{A}$  wins the above game, taken over the coin tosses made by  $\mathcal{B}$  and  $\mathcal{A}$ .

**Definition 1.** An adversary  $\mathcal{A}$  ( $q_e, q_s, t, \varepsilon$ )-breaks an ID-based signature (IBS) scheme if  $\mathcal{A}$  runs in a time of at most t,  $\mathcal{A}$  makes at most  $q_e$  Extract Queries, at most  $q_s$  Signature Queries, and  $AdvSig_{\mathcal{A}}$  is at least  $\varepsilon$ . An IBS scheme is ( $q_e, q_s, t, \varepsilon$ )-strongly existential unforgeable under an adaptive chosen message attack, strongly unforgeable, if no adversary ( $q_e, q_s, t, \varepsilon$ )-breaks it.

## 3 Our Scheme

In this section, we provide two new assumptions and propose an IBS scheme.

### 3.1 Underlying Proposed Problems

We provide Assumptions 2 and 3 related to the DH problem.

The first problem is defined as follows. Given

$$\left(g_1, g_2, g_1^x, g_2^{r_i}, g_2^{x+1/r_i} \middle| i = 1, \dots, q\right)$$

as input for random generators  $g_1 \in_{\mathcal{R}} \mathbb{G}_1$ ,  $g_2 \in_{\mathcal{R}} \mathbb{G}_2$  and random numbers x,  $r_1, \ldots, r_q \in_{\mathcal{R}} \mathbb{Z}_p^*$ , compute  $\left(g_1^{r_*}, g_2^{x+1/r_*}\right)$  for some  $r_* \in \mathbb{Z}_p^*$  and  $r_* \notin \{r_1, \ldots, r_q\}$ . Note that the index  $x + 1/r_i$  means  $x + (1/r_i)$ . We say that algorithm  $\mathcal{A}$  has an advantage  $\varepsilon$  in solving the first problem if

$$\Pr\left[\mathcal{A}\left(g_{1}, g_{2}, g_{1}^{x}, g_{2}^{r_{i}}, g_{2}^{x+1/r_{i}} \middle| i = 1, \dots, q\right) = \left(g_{1}^{r_{*}}, g_{2}^{x+1/r_{*}}\right)\right] \geq \varepsilon ,$$

where the probability is over the choice  $g_1 \in_{\mathbf{R}} \mathbb{G}_1, g_2 \in_{\mathbf{R}} \mathbb{G}_2, x, r_1, \ldots, r_q \in_{\mathbf{R}} \mathbb{Z}_p^*$ , some  $r_* \in \mathbb{Z}_p^*, r_* \notin \{r_1, \ldots, r_q\}$  and the random bits of  $\mathcal{A}$ .

**Assumption 2.** A  $(q, t, \varepsilon)$ -Assumption II holds if no t-time adversary has an advantage of at least  $\varepsilon$  in solving the first problem.

The second problem is defined as follows. Given

$$\left(g_1, g_2, g_1^x, g_2^{1/x}, g_2^{r_i}, g_2^{xr_i}, g_2^{x+1/r_i} \middle| i = 1, \dots, q\right)$$

as input for random generators  $g_1 \in_{\mathbb{R}} \mathbb{G}_1$ ,  $g_2 \in_{\mathbb{R}} \mathbb{G}_2$  and random numbers  $x, r_1, \ldots, r_q \in_{\mathbb{R}} \mathbb{Z}_p^*$ , compute  $\left(g_2^{r_*}, g_2^{xr_*}, g_2^{x+1/r_*}\right)$  for some  $r_* \in \mathbb{Z}_p^*$  and  $r_* \notin \{r_1, \ldots, r_q\}$ . We say that algorithm  $\mathcal{A}$  has an advantage  $\varepsilon$  in solving the second problem if

$$\Pr\left[\mathcal{A}\left(g_1, g_2, g_1^x, g_2^{1/x}, g_2^{r_i}, g_2^{xr_i}, g_2^{x+1/r_i} \middle| i = 1, \dots, q\right) = \left(g_2^{r_*}, g_2^{xr_*}, g_2^{x+1/r_*}\right)\right] \ge \varepsilon$$

where the probability is over the choice  $g_1 \in_{\mathbf{R}} \mathbb{G}_1, g_2 \in_{\mathbf{R}} \mathbb{G}_2, x, r_1, \ldots, r_q \in_{\mathbf{R}} \mathbb{Z}_p^*$ , some  $r_* \in \mathbb{Z}_p^*, r_* \notin \{r_1, \ldots, r_q\}$  and the random bits of  $\mathcal{A}$ .

**Assumption 3.** A  $(q, t, \varepsilon)$ -Assumption III holds if no t-time adversary has an advantage of at least  $\varepsilon$  in solving the second problem.

If we set  $g_1 := f(g_2) \in \mathbb{G}_1$  for the one-way isomorphism  $f : \mathbb{G}_2 \to \mathbb{G}_1$  and the random generator  $g_2 \in_{\mathbb{R}} \mathbb{G}_2$ , then the generator  $g_1$  is not random in the two assumptions. The existence of f was proved by Saito–Hoshino–Uchiyama– Kobayashi [17], on multiplicative cyclic groups constructed on non-supersingular elliptic curves. Security of our scheme is essentially based on the co-DH assumption, our proposed two assumptions, and the isomorphism f. In particular, our proposed two assumptions which are defined in a rigorous manner contribute to prove the security of strong unforgeability for our scheme.

### 3.2 Scheme

We shall give an IBS scheme. This scheme consists of four phases: Setup, Extract, Sign and Verify. For the moment we shall assume that the identity ID are elements in  $\{0, 1\}^{n_1}$ , but the domain can be extended to all of  $\{0, 1\}^*$  using a collision-resistant hash function  $\mathcal{H} : \{0, 1\}^* \to \{0, 1\}^{n_1}$ . Similarly, we shall assume that the signature message M to be signed are elements in  $\{0, 1\}^{n_2}$ .

**Setup:** The PKG chooses multiplicative cyclic groups  $\mathbb{G}_1, \mathbb{G}_2$  and  $\mathbb{G}_T$  of sufficiently large prime order p, a random generator  $g_2$  of  $\mathbb{G}_2$ , the one-way isomorphism  $f : \mathbb{G}_2 \to \mathbb{G}_1$  with  $g_1 := f(g_2)$ , and the cryptographic bilinear map  $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ . He generates  $MK := g_2^{\alpha} \in \mathbb{G}_2$  from a random number  $\alpha \in_{\mathbb{R}} \mathbb{Z}_p^*$ , and calculates  $A_1 := f(MK) \ (= g_1^{\alpha}) \in \mathbb{G}_1$ .

$$\mathbb{Z}_p^* \longrightarrow \mathbb{G}_2 \xrightarrow{f} \mathbb{G}_1 \alpha \longmapsto MK := g_2^\alpha \longmapsto A_1 := f(MK) \ (= g_1^\alpha)$$

Also he generates  $u' := g_2^{x'} \in \mathbb{G}_2, U = (u_1, \ldots, u_{n_1}) := (g_2^{x_1}, \ldots, g_2^{x_{n_1}}) \in \mathbb{G}_2^{n_1}, v' := g_2^{y'} \in \mathbb{G}_2$ , and  $V = (v_1, \ldots, v_{n_2}) := (g_2^{y_1}, \ldots, g_2^{y_{n_2}}) \in \mathbb{G}_2^{n_2}$  for random numbers  $x', x_1, \ldots, x_{n_1}, y', y_1, \ldots, y_{n_2} \in_{\mathbb{R}} \mathbb{Z}_p^*$ . The master secret master-key is MK and the public parameter are

$$extsf{params} := (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, f, g_1, g_2, A_1, u', U, v', V)$$
 .

**Extract:** Let ID be an  $n_1$ -bit identity and  $\mathrm{id}_k$   $(k = 1, \ldots, n_1)$  denote the kth bit of ID. To generate a private key  $d_{\mathrm{ID}}$  for  $\mathrm{ID} \in \{0, 1\}^{n_1}$ , the PKG picks a random number  $s \in_{\mathrm{R}} \mathbb{Z}_p^*$ , and computes

$$d_{\rm ID} = (d_1, d_2) := \left( g_2^s, g_2^{\alpha} \cdot \left( u' \prod_{k=1}^{n_1} u_k^{\rm id_k} \right)^{1/s} \right) \in \mathbb{G}_2^2 .$$

**Sign:** Let *M* be an  $n_2$ -bit signature message to be signed and  $m_k$   $(k = 1, ..., n_2)$  denote the *k*th bit of *M*. A signature  $\sigma := (\sigma_1, ..., \sigma_5)$  of (ID, *M*) is generated as follows.

$$(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5) := \left( f(d_1), g_2^r, d_1^r, d_2, d_1 \cdot \left( v' \prod_{k=1}^{n_2} v_k^{m_k} \right)^{1/r} \right)$$
$$= \left( g_1^s, g_2^r, g_2^{sr}, g_2^{\alpha} \cdot \left( u' \prod_{k=1}^{n_1} u_k^{\mathrm{id}_k} \right)^{1/s}, g_2^s \cdot \left( v' \prod_{k=1}^{n_2} v_k^{m_k} \right)^{1/r} \right)$$

for a random number  $r \in \mathbb{Z}_p^*$ .

**Verify:** Given params, (ID, M) and  $\sigma = (\sigma_1, \dots, \sigma_5)$ , verify

$$e(\sigma_1, \sigma_2) = e(g_1, \sigma_3) ,$$
  

$$e\left(A_1^{-1} \cdot f(\sigma_4), \sigma_3\right) = e\left(f(\sigma_2), u' \prod_{k=1}^{n_1} u_k^{\mathrm{id}_k}\right) \text{ and }$$
  

$$e\left(\sigma_1^{-1} \cdot f(\sigma_5), \sigma_2\right) = e\left(g_1, v' \prod_{k=1}^{n_2} v_k^{m_k}\right) .$$

If the equalities hold the result is valid; otherwise the result is invalid.

If an entity with identity ID constructs a signature  $\sigma = (\sigma_1, \ldots, \sigma_5)$  on a message M as described in the Sign phase above, it is easy to see that  $\sigma$  will be accepted by a verifier. Thus the scheme is correct.

### 4 Security Proof

**Theorem 1.** Suppose that the  $(t_0, \varepsilon_0)$ -co-DH Assumption in  $(\mathbb{G}_2, \mathbb{G}_1)$ ,  $(q_1, t_1, \varepsilon_1)$ -Assumption II and  $(q_2, t_2, \varepsilon_2)$ -Assumption III hold with  $g_1 := f(g_2)$ . Then the proposed ID-based signature scheme is  $(q_e, q_s, t, \varepsilon)$ -strongly unforgeable, provided that  $q_e \leq q_1$ ,  $q_s \leq q_2$ ,  $t \leq \min(t_0, t_1, t_2) - O((q_e + q_s)T)$  and  $\varepsilon (1 - 2(q_e + q_s)/p) \geq \varepsilon_0 + \varepsilon_1 + \varepsilon_2$ , where T is the maximum time for an exponentiation in  $\mathbb{G}_2$ .

An outline of our proof is as follows. Suppose that there exists an adversary,  $\mathcal{A}$ , who breaks our IBS scheme in Section 3, and a challenger,  $\mathcal{B}$ , takes the Assumption II challenge. After  $\mathcal{A}$  and  $\mathcal{B}$  execute the strongly unforgeable game,  $\mathcal{A}$  outputs a valid tuple for an identity, a message and a signature. Then  $\mathcal{B}$  will compute the Assumption II response which is valid. The tuple from  $\mathcal{A}$  must not contradict the co-DH assumption and the Assumption III.

*Proof.* Suppose that there exists an adversary,  $\mathcal{A}$ , who  $(q_{\rm e}, q_{\rm s}, t, \varepsilon)$ -breaks our IBS scheme. We construct a simulator,  $\mathcal{B}$ , to play the Assumption II game. The simulator  $\mathcal{B}$  will take the Assumption II challenge

$$\left(g_1, g_2, g_1^{\alpha}, g_2^{s_i}, g_2^{\alpha+1/s_i} \middle| i = 1, \dots, q_1\right)$$

for  $\alpha, s_1, \ldots, s_{q_1} \in_{\mathbb{R}} \mathbb{Z}_p^*$ , and run  $\mathcal{A}$  executing the following steps.

#### 4.1 Simulator Description

**Setup:** The simulator  $\mathcal{B}$  generates  $u' := g_2^{x'} \in \mathbb{G}_2, U = (u_1, \dots, u_{n_1}) := (g_2^{x_1}, \dots, g_2^{x_{n_1}}) \in \mathbb{G}_2^{n_1}, v' := g_2^{y'} \in \mathbb{G}_2$ , and  $V = (v_1, \dots, v_{n_2}) := (g_2^{y_1}, \dots, g_2^{y_{n_2}}) \in \mathbb{G}_2^{n_2}$  for random numbers  $x', x_1, \dots, x_{n_1}, y', y_1, \dots, y_{n_2} \in_{\mathbb{R}} \mathbb{Z}_p^*$ , and sends

$$(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, f, g_1, g_2, g_1^{\alpha}, u', U, v', V)$$

to  $\mathcal{A}$ .

**Queries:** The adversary  $\mathcal{A}$  adaptively makes a number of different queries to the challenger  $\mathcal{B}$ .

Assume that  $\mathcal{U}_{e}$  is the subscript set of identities in Extract Queries,  $\mathcal{U}_{s}$  is that of identities in Signature Queries,  $\mathcal{U} := \mathcal{U}_{e} \cup \mathcal{U}_{s}$ , and  $\mathcal{M}_{s}^{i}$  is that of messages in Signature Queries for the ID<sub>i</sub>  $(i \in \mathcal{U}_{s})$ .

Each query can be one of the following.

- Extract Queries: The adversary  $\mathcal{A}$  adaptively issues Extract Queries  $ID_i$  ( $i \in \mathcal{U}_e$ ). Assume that

$$X_i := x' + \sum_{k=1}^{n_1} \operatorname{id}_{i,k} x_k , \qquad (1)$$

where  $ID_i := (id_{i,1}, \dots, id_{i,n_1}) \in \{0, 1\}^{n_1}$ .

(4.1-E1) If  $X_i \equiv 0 \pmod{p}$ ,  $\mathcal{B}$  aborts this game.

(4.1-E2) Otherwise (i.e.  $X_i \not\equiv 0 \pmod{p}$ ),  $\mathcal{B}$  does not abort the game, and generates  $d_i = (d_{i,1}, d_{i,2})$  of ID<sub>i</sub>:

$$(d_{i,1}, d_{i,2}) := \left( (g_2^{s_i})^{X_i}, g_2^{\alpha + 1/s_i} \right) \tag{2}$$

$$= \left(g_2^{\overline{s_i}}, g_2^{\alpha} \cdot \left(u' \prod_{k=1}^{n_1} u_k^{\mathrm{id}_{i,k}}\right)^{1/s_i}\right)$$
(3)

and sends it to  $\mathcal{A}$ . Here  $\overline{s_i} := s_i X_i \mod p$   $(i \in \mathcal{U}_e)$ . (Notice that, by eliminating all  $s_i \in_{\mathbf{R}} \mathbb{Z}_p^*$  in (2), we can regard all  $\overline{s_i} \in_{\mathbf{R}} \mathbb{Z}_p^*$  as random numbers in (3).)

- Signature Queries: The adversary  $\mathcal{A}$  adaptively issues Signature Queries  $(ID_i, M_{i,j})$   $(i \in \mathcal{U}_s, j \in \mathcal{M}_s^i)$ . Assume that  $X_i$  is from (II) for  $i \in \mathcal{U}_s$  and

$$Y_{i,j} := y' + \sum_{k=1}^{n_2} m_{i,j,k} \ y_k \ , \tag{4}$$

where  $M_{i,j} := (m_{i,j,1}, \dots, m_{i,j,n_2}) \in \{0,1\}^{n_2}$ .

(4.1-S1) If  $X_i \equiv 0 \pmod{p}$  or  $Y_{i,j} \equiv 0 \pmod{p}$ ,  $\mathcal{B}$  aborts this game.

(4.1-S2) Otherwise (i.e.  $X_i \not\equiv 0 \pmod{p}$  and  $Y_{i,j} \not\equiv 0 \pmod{p}$ ),  $\mathcal{B}$  does not abort the game, and generates  $\sigma_{i,j} = (\sigma_{i,j,1}, \ldots, \sigma_{i,j,5})$  of  $(\mathrm{ID}_i, M_{i,j})$ :

$$\begin{cases} \sigma_{i,j,1} \coloneqq (g_1^{s_i})^{X_i} = g_1^{\overline{s_i}} \\ \sigma_{i,j,2} \coloneqq (g_2)^{r_{i,j}Y_{i,j}/X_i} = g_2^{\overline{r_{i,j}}} \\ \sigma_{i,j,3} \coloneqq (g_2^{s_i})^{r_{i,j}Y_{i,j}} = g_2^{\overline{s_i} \overline{r_{i,j}}} \\ \sigma_{i,j,4} \coloneqq g_2^{\alpha+1/s_i} = g_2^{\alpha} \cdot \left(g_2^{X_i}\right)^{1/\overline{s_i}} = g_2^{\alpha} \cdot \left(u' \prod_{k=1}^{n_1} u_k^{\mathrm{id}_{i,k}}\right)^{1/\overline{s_i}} \\ \sigma_{i,j,5} \coloneqq \left(g_2^{s_i+1/r_{i,j}}\right)^{X_i} = g_2^{\overline{s_i}} \cdot \left(g_2^{Y_{i,j}}\right)^{1/\overline{r_{i,j}}} = g_2^{\overline{s_i}} \cdot \left(v' \prod_{k=1}^{n_2} v_k^{m_{i,j,k}}\right)^{1/\overline{r_{i,j}}} \end{cases}$$

and sends it to  $\mathcal{A}$ . Here  $\overline{s_i} := s_i X_i \mod p$   $(i \in \mathcal{U}_s)$  and  $\overline{r_{i,j}} := r_{i,j} Y_{i,j} / X_i \mod p$  $(i \in \mathcal{U}_s, j \in \mathcal{M}_s^i)$ . (Notice that, by eliminating all  $s_i, r_{i,j} \in_{\mathbb{R}} \mathbb{Z}_p^*$ , we can regard all  $\overline{s_i}, \overline{r_{i,j}} \in_{\mathbb{R}} \mathbb{Z}_p^*$  as random numbers.)

**Output:** The adversary  $\mathcal{A}$  outputs  $(\mathrm{ID}_*, M_*, \sigma_*)$  such that  $\sigma_* = (\sigma_{*,1}, \ldots, \sigma_{*,5}) \in \mathbb{G}_2^5$  is a valid signature of  $(\mathrm{ID}_*, M_*)$ ,  $\mathrm{ID}_* \notin \{\mathrm{ID}_i \mid i \in \mathcal{U}_e\}$  and  $(\mathrm{ID}_*, M_*, \sigma_*) \notin \{(\mathrm{ID}_i, M_{i,j}, \sigma_{i,j}) \mid i \in \mathcal{U}_s, j \in \mathcal{M}_s^i\}.$ 

Artificial Abort: Assume that

$$X_* := x' + \sum_{k=1}^{n_1} \operatorname{id}_{*,k} x_k \text{ and } Y_* := y' + \sum_{k=1}^{n_2} m_{*,k} y_k , \qquad (5)$$

where  $\mathrm{ID}_* := (\mathrm{id}_{*,1}, \ldots, \mathrm{id}_{*,n_1}) \in \{0,1\}^{n_1}$  and  $M_* := (m_{*,1}, \ldots, m_{*,n_2}) \in \{0,1\}^{n_2}$ . If  $\mathrm{ID}_* \neq \mathrm{ID}_i$  and  $X_* \equiv X_i \pmod{p}$  for some  $i \in \mathcal{U}$ , or if  $M_* \neq M_{i,j}$  and  $Y_* \equiv Y_{i,j} \pmod{p}$  for some  $i \in \mathcal{U}_s$  and  $j \in \mathcal{M}_s^i$ , then  $\mathcal{B}$  aborts this game.

#### 4.2 Analysis

The adversary  $\mathcal{A}$  cannot distinguish the above game from Simulator Description with the abort when  $X_i \equiv 0 \pmod{p}$  and  $Y_{i,j} \not\equiv 0 \pmod{p}$  or  $X_i \not\equiv 0 \pmod{p}$ and  $Y_{i,j} \equiv 0 \pmod{p}$ , and the strongly unforgeable game without this abort, since

$$\Pr\left[\bigcup_{i\in\mathcal{U}}X_i\equiv 0 \pmod{p}\cup\bigcup_{i\in\mathcal{U}_{\mathrm{s},i}\atop j\in\mathcal{M}_{\mathrm{s}}^i}Y_{i,j}\equiv 0 \pmod{p}\right]\leq \frac{q_{\mathrm{e}}+q_{\mathrm{s}}}{p}$$

and this probability is negligible when  $q_e + q_s \ll p$ . Thus we shall consider only the game from Simulator Description.

Since  $\sigma_*$  is valid, we assume that

$$\begin{cases} \sigma_{*,1} := g_{1}^{\overline{s_{*}}} \\ \sigma_{*,2} := g_{2}^{\overline{s_{*}}} \\ \sigma_{*,3} := g_{2}^{\overline{s_{*}}} \overline{r_{*}} \\ \\ \sigma_{*,4} := g_{2}^{\alpha} \cdot \left( u' \prod_{k=1}^{n_{1}} u_{k}^{\mathrm{id}_{*,k}} \right)^{1/\overline{s_{*}}} = g_{2}^{\alpha + X_{*}/\overline{s_{*}}} \\ \\ \sigma_{*,5} := g_{2}^{\overline{s_{*}}} \cdot \left( v' \prod_{k=1}^{n_{2}} v_{k}^{m_{*,k}} \right)^{1/\overline{r_{*}}} = g_{2}^{\overline{s_{*}} + Y_{*}/\overline{r_{*}}} \end{cases}$$

where  $\overline{s_*}, \overline{r_*} \in \mathbb{Z}_p^*$ .

(4.2-1) If  $X_* \equiv 0 \pmod{p}$ ,  $\sigma_{*,4} = g_2^{\alpha}$ . Then  $\mathcal{B}$  generates  $\left(g_1^{s_*}, g_2^{\alpha+1/s_*}\right)$  for some  $s_* \in \mathbb{Z}_p^*$  and  $s_* \notin \{s_1, \ldots, s_q\}$ , which is a valid output of the Assumption II challenge.

(4.2-2) Otherwise (i.e.  $X_* \not\equiv 0 \pmod{p}$ ).

(4.2-2.1) Suppose that  $ID_* \notin \{ID_i \mid i \in \mathcal{U}_e\}$  (which is an assumption of the strong unforgeability) and  $(ID_*, \overline{s_*}) \notin \{(ID_i, \overline{s_i}) \mid i \in \mathcal{U}_s\}$ . Then, it is sufficient to consider

$$\Pr\left[\mathcal{A}\left(g_{1}, g_{2}, g_{1}^{\alpha}, g_{2}^{x'}, g_{2}^{x_{1}}, \dots, g_{2}^{x_{n_{1}}}, g_{2}^{y'}, g_{2}^{y_{1}}, \dots, g_{2}^{y_{n_{2}}}, g_{2}^{X_{i}}, g_{2}^{\overline{s_{i}}}, g_{2}^{\overline{s_{i}}}, g_{2}^{\overline{s_{i}}}, g_{2}^{\overline{s_{i}}, \overline{s_{i}}}, g_{2}^{\overline{s_{i}}+Y_{i,j}/\overline{r_{i,j}}}\right| i \in \mathcal{U}_{s}, j \in \mathcal{M}_{s}^{i}\right) \\
= \left(g_{2}^{X_{*}}, g_{2}^{Y_{*}}, g_{1}^{\overline{s_{*}}}, g_{2}^{\overline{s_{*}}}, g_{2}^{\overline{s_{*}}, \overline{r_{*}}}, g_{2}^{\alpha+X_{*}/\overline{s_{*}}}, g_{2}^{\overline{s_{*}}+Y_{*}/\overline{r_{*}}}\right)\right],$$
(6)

in the case that  $\mathcal{A}$  knows all  $g_2^{\overline{s_i}}$  (=  $d_{i,1}$ ). This means that  $\mathcal{U} = \mathcal{U}_{e} = \mathcal{U}_{s}$ . Suppose that the probability (6)  $\geq \varepsilon'$  for some  $\varepsilon' \leq \varepsilon$ .

Then the probability (6) can be reduced to a contradiction of either the co-DH assumption or Assumption II.

(4.2-2.2) Otherwise (i.e.  $ID_* \notin \{ID_i \mid i \in U_e\}$  and  $(ID_*, \overline{s_*}) = (ID_l, \overline{s_l})$  for some  $l \in U_s$ ), then  $X_* = X_l$ . It is sufficient to consider

$$\Pr\left[\mathcal{A}\left(g_{1}, g_{2}, g_{2}^{y'}, g_{2}^{y_{1}}, \dots, g_{2}^{y_{n_{2}}}, g_{2}^{Y_{l,j}}, g_{1}^{\overline{s_{l}}}, g_{2}^{1/\overline{s_{l}}}, g_{2}^{\overline{r_{l,j}}}, g_{2}^{\overline{s_{l}} \overline{r_{l,j}}}, g_{2}^{\overline{s_{l}} + Y_{l,j}/\overline{r_{l,j}}} \middle| j \in \mathcal{M}_{s}^{l}\right) \\ = \left(g_{2}^{Y_{*}}, g_{2}^{\overline{r_{*}}}, g_{2}^{\overline{s_{l}} \overline{r_{*}}}, g_{2}^{\overline{s_{l}} + Y_{*}/\overline{r_{*}}}\right)\right]$$
(7)

in the case that  $\mathcal{A}$  knows  $x', x_1, \ldots, x_{n_1}$  and  $g_2^{\alpha}$ . Suppose that the probability  $(\square) \geq \varepsilon''$  for  $\varepsilon' + \varepsilon'' = \varepsilon$ .

Then the probability (7) can be reduced to a contradiction of either the co-DH assumption or Assumption III.

We omit the proof of the limited range of values  $(q_e, q_s, t, \varepsilon)$ . Therefore, we proved Theorem [].

### 5 Efficiency

In this section, we consider efficiency of strongly unforgeable IBS schemes without random oracles.

Huang-Wong-Zhao **13** proposed a general method to transform unforgeable IBS schemes into strongly unforgeable ones by attaching strong one-time signatures. Table **1** shows efficiency of IBS schemes from the Huang-Wong-Zhao **13**. For  $(x_r, y_r)$  of the row in the table,  $x_r$  represents the number of signature parameters and  $y_r$  that of the bilinear maps. For  $(x_c; y_c)$  of the column,  $x_c$  the

 Table 1. Efficiency of IBS schemes from the transformation in Huang–Wong–Zhao

Strong one-time signatures Unforgeable IBS schemes	(4;1)			[14] (4;2)	
Paterson–Schuldt $16$ $(3,3)$	7/4	7/5	8/7	7/5	7/5

$\begin{array}{c} \text{Signature schemes } \mathcal{S}'\\ \text{Certificates (signature) schemes } \mathcal{S} \end{array}$	[3] (4;1)	(4;2)	<b>6</b> (5;4)	[14] (4; 2)	(4;2)
Boneh–Boyen $(2,1)$	6/2	6/3	7/5	6/3	6/3
Zhang–Chen–Susilo–Mu [21] (2,2)	6/3	6/4	7/6	6/4	6/4
Camenisch–Lysyanskaya ver. A $[6]$ $(3,4)$	7/5	7/6	8/8	7/6	7/6
Okamoto $\boxed{14}(3,2)$	7/3	7/4	8/6	7/4	7/4
Boneh–Shen–Waters $[5]$ $(3,2)$	7/3	7/4	8/6	7/4	7/4

Table 2. Efficiency of IBS schemes from the transformation in 1910

number of signature parameters and public keys, and  $y_c$  that of the bilinear maps. For  $x_t/y_t$  in the table,  $x_t (= x_c + x_r)$  the number of signature parameters for each strongly unforgeable IBS scheme; and  $y_t (= y_c + y_r)$  that of the bilinear maps. Notice that we count the number of the signature parameters to be small. However, these constructions need at least six signature parameters.

Also, it is known that strongly unforgeable IBS schemes can be constructed with the approach of attaching certificates to strongly unforgeable (non-ID-based) signatures. Table 2 shows efficiency of IBS schemes from this construction in 1910.

All these constructions need at least six signature parameters. In our scheme of Section 3.2, it is sufficient to be five. On the other hand, our scheme is inefficient since the bilinear map is used six times during one iteration of verification in the scheme.

# 6 Conclusions

In this paper, we proposed a strongly unforgeable IBS scheme without random oracles, with five signature parameters, based on three problems related to the DH problem and a one-way isomorphism. However, our scheme is inefficient since the bilinear map (the pairing) is used six times during one iteration of verification in the scheme. Our next step is to propose more efficient schemes with the same security (or we have a possibility that the six times have not been a problem by a future study of the computation process rate of the bilinear map).

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# On the Security of a Certificate-Based Signature Scheme and Its Improvement with Pairings

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Abstract. In traditional public key signature, the public key of a signer is essentially a random string selected from a given set. It is infeasible to prove that a party is indeed the signer for a given signature. In general, the public key of a user needs a management authority to authenticate it. It results in that traditional public key cryptosystem (PKC) requires high maintenance cost for certificate management. Although, identity based cryptosystem (IBC) reduces the overhead of management, it suffers from the drawback of key escrow. Certificate-based cryptosystem combines the advantage of both PKC and IBC as it avoids the usage of certificates and does not suffer from key escrow. Recently, Liu et.al proposed an efficient Certificate-based signature and showed that the scheme was secure in the random oracles. Unfortunately, this paper shows that the scheme is insecure and discusses the flaws in their security proof. Then the corresponding attacks are given. To overcome the flaws, an improved scheme is proposed and the result shows that the scheme is provable secure against two game attacks of certificate-based signature in the random oracle model. The security is closely related to the computational Diffie-Hellman problem.

**Keywords:** Security analysis, attack, improved scheme, the CDH problem, certificate-based signature.

# 1 Introduction

In traditional public key signature (PKS) $\square$ , a user Alice signs a message using her private key. A verifier Bob verifies the signature using Alice's public key. However, generally speaking, Alice's public key is a random string selected from a given set. Therefore, it is infeasible to prove that a party is indeed the signer for a given signature. This problem can be solved by incorporating a certificate generated a trusted party called the Certification Authority (CA) which provides an unforgeable and trusted link between a public key and the identity of a signer. This hierarchical framework is referred to as the Public Key Infrastructure (PKI). In general, the signer registers its own public key with its identity in certificate server and anyone wishing to obtain the signer's public key needs to request it by sending the server the identity of the signer and gets it. When

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verifying a signature using the signer's public key, a verifier must obtain the signer's certification status of public key, hence in general the verifier makes a query on the signer's certificate status to the CA. In the point of view of a verifier, it takes two verification steps to verify a signature on a message. This approach seems to be inefficient, in particular, when the number of users is very large. And it also increases burden of the CA's certificate management.

To simplify certificate management of traditional PKI, Shamir **[13]** introduced Identity-based cryptology (IBC). It can solve the aforementioned problem by using Alice's identity or IP address as her public key while the corresponding private key is a result of some mathematical operation that takes as input the user's identity and the master secret key of a trusted authority, referred to as **Private Key Generator** (PKG). The main practical benefit of IBC lies in greatly reduction of need for public key certification. The PKG can generate the private key of all its users, so *Private Key Escrow* becomes an inherent problem in IBC. It results in unconditional trust to PKG for a user. Furthermore, the produced private key must be sent over secure channel, which makes secret key distribution a daunting task **[2]**. Certificate is implicitly provided in IBC and the explicit authentication of public key is no longer required.

To fill the gap between traditional cryptology and identity-based cryptology, Gentry 2 introduced the notion of certificate-based encryption (CBE), which combines public-key encryption (PKE) and IBE while preserving their merits. In CBE, each user generates his own private key and public key and request a certificate from the CA while the CA uses the key generation algorithm of an identity based encryption (IBE) scheme to the certificate. In this way, the certificate is implicitly used as the private key of the users as the signing key. Here CA is equivalent to PKG in the IBC. Although the CA knows the certificate of a user, it also cannot produce a signature in the name of user, since it does not have the user's private key. In addition to CBE, the notion of certificate based signature (CBS) was first suggested by Kang *et.al* . Unfortunately, one of their schemes was found insecure against key replacement attack 11 which was pointed our by Li *et.al* **S**. And Li *et.al* also gave a novel certificate-based signature scheme. In ISPEC 2007, Au et.al 11 proposed a certificate-based ring signature scheme. In 2007, Wang et. al 14 proposed a certificate-based proxy cryptosystem with revocable proxy decryption power. Recently, J.K.Liu et. al 10 proposed an efficient certificate-based signature scheme as the scheme did not require any pairing operations, which is regards costly operations compared to other operation. And they claimed that the scheme was provably secure in the random oracle model and can be against two attack games of certificate-based signature.

**Our contribution:** In this paper, we first analyze the security of the first one of Liu et.al's schemes which were proven secure against two attack games of certificate-based signature in  $\square$ . Then we show that the scheme is insecure and it cannot be against either attack game of certificate-based signature. Furthermore, we give the corresponding two attacks on the schemes and pinpoint

the reason accounting for the insecurity . Finally, to overcome the above attacks, we give an improved schemes and prove that the scheme is secure in the random oracle model.

The rest of this paper is organized as follows. We describe security and adversarial model of CBS in section 2. After we review Liu *et.al*'s signature scheme in section 3, we give the corresponding attack in section 4. In section 5, an improved scheme is given and the security of the improved scheme is analyzed. Finally, we conclude the paper in section 6.

# 2 Security Model

The certificate-based signature is a compromise between ID-based signature and PKI-based signature. It inherits merits of ID-based signature and PKI-based signature.

In the following, we reviews the definition of a certificate-based signature scheme. It consists of the six algorithms :

- Setup is a probabilistic algorithm taking a security parameter k as input. It outputs the certifier's master key msk and public parameters param. The algorithm is run by private key generator (PKG).
- UserKeyGen is probabilistic algorithm which takes as input param and is run by the user. It outputs a public key PK and a secret key usk.
- Certify is probabilistic algorithm that takes as input master secret msk, public parameters *param*, the user's public key PK and identity ID and a time period t. It returns  $Cert'_t$  to user. It is run by PKG. The aim is to issue a certificate in the phase.
- Consolidate is deterministic certificate consolidation algorithm taking as input  $(param, t, Cert'_t)$  and optionally  $Cert_{t-1}$ . It outputs certificate  $Cert_t$ which is used by a user in time period t.
- Sign is a probabilistic algorithm which takes as input  $(param, t, Cert_t, usk, m)$  and outputs a signature  $\delta$ .
- Verify is deterministic algorithm which takes as input  $(param, t, PK, ID, \delta)$ . If  $\delta$  is valid, it outputs 1. Otherwise, it outputs  $\perp$ .

Note that in a concrete certificate-based signature scheme, it may not involve certificate consolidate phase. In such situation, *Consolidate* algorithm will output  $Cert_t = Cert'_t$ . For simplicity, we will omit *Conslidate* and the time t in the rest of the paper.

### 2.1 Unforgeability

Unforgeability is a primitive property of a signature scheme. Intuitively, an adversary should not be able to forge a valid signature in a secure certificate-based signature (CBS) scheme which can be against adaptively chosen message attack [7]. Certificate-based signature is a combination of public-key signature and ID-based signature, where the signer needs both its personal secret key and

a certificate from PKG. Thus, the security of a certificate-based signature requires that the signer can generate a valid signature under the public key PKand identity ID if and only if he has Cert and usk. Only with one of those, the signer cannot generate a valid signature.

According to an adversary is in possession of the secret key of which entity, we divide the adversary into two types. One type is an uncertified entity which can randomly choose a pair key (PK, usk) which is not certified by PKG or replace a user's public key PK to produce a forgery. This attack scenarios is looked upon a malicious user. The other type is an adversary which is be in possession of the master key msk to attack a fixed entity's public key. Such attack scenarios is looked upon a malicious PKG. In the following, we use the enhanced model by Li *et.al* [8] which captures key replacement attack in the security of the first attack type.

**CBS Game 1 Existential Unforgeability**. The challenger runs *Setup*, gives *param* to the adversary  $\mathcal{A}$  and keeps *msk* secret. The adversary  $\mathcal{A}$  issues the following queries:

- On user-key-gen query (ID), if ID has already been created, then  $PK_{ID}$  is returned. Otherwise, the challenger runs the algorithm **User-Key-Gen** to produce a secret/public key pair ( $usk_{ID}, PK_{ID}$ ) and added ( $ID, usk_{ID}, PK_{ID}$ ) to the list L. Then  $PK_{ID}$  is returned.
- On corruption query (ID). The challenger checks the list L and returns the secret key  $usk_{ID}$ .
- On certification query (ID). The challenger run Certify algorithm and returns the certificate  $Cert_{ID}$  to  $\mathcal{A}$ . Note that  $Cert_{ID}$  is the certificate of the pair  $(ID, PK_{ID})$  where  $PK_{ID}$  is the public key returned from **User-Key-Gen** query.
- On signing query  $(ID, PK_{ID}, m)$ . The challenger runs the **Sign** algorithm and returns the signature  $\delta$ .

Here, we can replace any user's public key with his own choice, but once it has replaced the public key, it cannot obtain the certificate of the false public key from the challenger.

- **Output.** Finally,  $\mathcal{A}$  outputs a forge  $(m^*, \delta^*, ID^*, PK^*)$ . We say  $\mathcal{A}$  wins the game if
  - $\delta^*$  is a valid signature on the message  $m^*$  under the public key  $PK_{ID^*}$  with identity  $ID^*$ . Here  $PK_{ID^*}$  is chosen by  $\mathcal{A}$  and might not be returned from the **User-Key-Gen** oracle.
  - $ID^*$  has never been submitted as one of Certify queries.
  - $(ID^*, m^*)$  has never been submitted as one of Sign queries.

We define  $\mathcal{A}$ 's advantage in this game to be  $Adv(\mathcal{A}) = Pr[\mathcal{A} \ wins]$ .

In the second attack, the adversary  $\mathcal{A}$  is in possession of the master key of PKG and wants to generate a valid signature under the public key  $PK^*$  without

the knowledge of the corresponding secret key. The security of this type attack is denied by the following game between  $\mathcal{A}$  and the challenger  $\mathcal{C}$ .

**CBS Game 2 Existential Unforgeability.** The challenger runs *Setup*, gives *param* and *msk* to the adversary  $\mathcal{A}$ . The adversary  $\mathcal{A}$  can adaptively submit queries to User-Key-Gen oracle, corruption oracle and signing oracle as Game 1. But different from Game 1,the adversary is not allowed to replace any user's public key. At last,  $\mathcal{A}$  outputs a signature  $\delta^*$  on message  $m^*$  under public key  $PK_{ID^*}$  and identity  $ID^*$ . The adversary win the game if

- $-\delta^*$  is a valid signature on message  $m^*$  under public key  $PK^*$  and the PKG's public key msk where  $PK_{ID^*}$  is the public key output from the User-Key-Gen query  $ID^*$ .
- $-ID^*$  has never been submitted to corruption oracle.
- $-(m^*, PK^*, ID^*)$  has never been submitted as one of Sign queries.

The adversary's advantage in the game is defined as  $Adv(\mathcal{A})=P[\mathcal{A} \ wins]$ 

# 3 Reviews of Liu *et.al*'s Certificate-Based Signature Scheme without Pairings

**[Setup of System Parameters:]** Let  $\mathbb{G}$  be a multiplicative group with order q. The PKG randomly chooses a generator  $g \in \mathbb{G}$  and  $x \in Z_q$  to compute  $X = g^x$  as public key. Let  $H : \{0,1\}^* \to Z_q^*$  be a cryptographic hash function. The system parameters param and master secret key msk are as follows:

$$\mathbf{param} = (\mathbb{G}, q, g, X, H), msk = x$$

**[UserKeyGen:]** A user picks  $u \in Z_q$  as his secret key usk at random, and computes the corresponding public key PK as  $(g^u, X^u, \tau_u)$  where  $\tau_u$  is the following non-interactive proof-of-knowledge (PoK)

$$PK\{(u): U_1 = g^u \wedge U_2 = X^u\}$$

[Certify:] Let  $\tilde{h} = H(PK, ID)$  for user with public key PK and binary string ID which is used to identity the user. To produce a certificate for the user the CA randomly selects  $r \in_R Z_q^*$ , compute

$$R = g^r \qquad s = r^{-1}(\tilde{h} - xR) \mod q$$

The certificate is (R, s). Note that a correctly generated certificate should fulfill the following equality:

$$R^s X^R = g^{\tilde{h}} \tag{1}$$

**[Sign:]** To produce a signature on message  $m \in \{0, 1\}^*$ , the signer with public key PK, the certificate (R, s) and the secret key u computes as follows:

1. randomly choose  $y \in_R Z_q^*$ .

- 2. compute  $Y = R^{-y}$ , h = H(Y, R, m) and  $z = y + hsu \mod q$
- 3. the resultant signature is  $\delta = (Y, R, z)$ .

**[Verify:]** Given a signature  $\delta = (Y, R, z)$  on message m under the public key PK, a verifier first checks whether  $\tau_u$  is a valid PoK. If not, output  $\perp$ . Otherwise, compute h = H(Y, R, m) and  $\tilde{h} = H(PK, ID)$ , then checks whether

$$(g^u)^{h\tilde{h}} \stackrel{?}{=} R^z Y(X^u)^{hR} \tag{2}$$

Output valid if it holds, otherwise, output  $\perp$ .

# 4 Attack on Liu *et.al*'s Certificate-Based Signature Scheme without Pairings

In [10], Liu *et.al* claimed that their scheme is secure against the certifier's attack in which an adversary in possession of the master key msk attacks a fixed entity's public key and a uncertified entity' attack in which an adversary can replace an user's public key while the master key is unknown for him. Then they give the corresponding security proof of the scheme in the random oracle models, respectively. Unfortunately, we show that the scheme is insecure by analyzing security of the scheme.

### 4.1 Attack 1

Given a signature  $\delta = (Y, R, z, \tau_u)$  under the public key PK and the identity ID, a certifier with the master key x can attack as follows:

- 1. First, compute h' = H(PK, ID).
- 2. then, compute  $R' = x^{-1}\tilde{h}'$ , where x is master secret key.
- 3. randomly choose  $r \in Z_q^*$  to set z' = r and compute  $Y = (R'^{z'})^{-1}$ .
- 4. the forged signature on message m' is  $\delta' = (\tau_u = \tau'_u, R', z', Y')$ .

For an user, after certificated, the user's public key PK is fixed. Thus, the corresponding non-interactive proof-of-knowledge  $\tau_u$  of public key is fixed. Obviously, the above forging  $\delta' = (\tau_u = \tau'_u, R', z', Y')$  can be accepted. Since

$$R'^{z'}Y'(X^{u})^{hR'} = R'^{z'}(R'^{z'})^{-1}(X^{u})^{hR'}$$
  
=  $(X^{u})^{hR'}$   
=  $(X^{u})^{hx^{-1}\tilde{h}'}$   
=  $(g^{u})^{h\tilde{h}'}$   
 $\tilde{h}' = H(PK, ID)$ 

According to the above verification procedure, we know the above equations hold; Hence, it indicates that our attack is successful. And in the whole forgery process, we find that, the signed message is not used in R', z' and Y'. It means that the forgery attack is valid for any message-signature.

### 4.2 Attack on Security Proof of the Scheme

In the following, we show that security proof of Liu *et.al*'s signature scheme in the **unforgeability against Game 2 Adversary** has the flaws. In the signing quering phase of security proof, when  $\mathcal{A}$  queries the signing oracle for a message m and a public key  $(g^u, X^u, \tau_u)$  with identity ID, the authors thought that the challenger  $\mathcal{C}$  responded as follows:

- 1. first check whether proof-of-knowledge for  $(g^u, X^u)$  is valid.
- 2. then check whether PK, ID, has been queried for the H() random oracle or extraction oracle before. If yes, it retrieves (R, s, H(PK, ID), PK, ID). Let
  - $\tilde{h} = H(PK, ID)$  and randomly chooses  $h, z \in_R Z_q$  to set  $Y = \frac{(g^u)^{h\tilde{h}}}{R^z(X^u)^{hR}}$
- 3. Finally, let h = H(Y, R, m) as the response to the random oracle H(Y, R, m).

But, in fact, the challenger  $\mathcal{C}$  can respond not like the above steps, but as follow:

- 1. First, compute  $\tilde{h} = H(PK, ID)$ .
- 2. then, compute  $R = x^{-1}\tilde{h}'$ , where x is master secret key, it is known to the challenger C.
- 3. randomly choose  $r \in \mathbb{Z}_q^*$  to set z = r and compute  $Y = (\mathbb{R}^z)^{-1}$ .
- 4. set h = H(R, Y, m)
- 5. the simulated signature on message m is  $\delta = (\tau_u, R, z, Y)$ .

After such signing query response is used, the challenger C cannot apply rewind technique to the point it just queries H(Y, R, m). Hash function H() cannot play a role of random oracle. Thus, the discrete logarithm problem in the security proof can be solved. It means that the security proof exits the flaws.

### 4.3 Attack3

In the following, we show that the scheme is not against an uncertified entity attack. That is to say, an uncertified entity can produce a forgeable signature without a certificate which is issued by CA. The attack is as follows:

- 1. randomly choose  $r \in Z_q$  to compute  $R = g^r$
- 2. randomly choose  $a \in Z_q$  to compute  $Y = X^{-aR}$ .
- 3. randomly select a signed message m to compute h = H(R, Y, m).
- 4. compute  $u = \frac{a}{h}$  as the secret key of an user, and set  $PK = (g^u, X^u)$  as the public key of the user.
- 5. compute  $\tilde{h} = H(PK, ID)$  to set  $z = \frac{a\tilde{h}}{r}$ , where ID is the identity of the user.
- 6. produce a non-interactive proof-of-knowledge  $\pi_u$  by the secret key u.

$$\pi_u = PK\{(u) : U_1 = g^u \land U_2 = X^u\}$$

The forged signature on message m under the public key  $PK = (g^u, X^u)$  and identity ID is  $\delta = (R, Y, z)$ .

To verify the validity of the forged signature, we only check whether the forged signature  $\delta = (R, Y, z)$  satisfies verification equation.

$$R^{z}Y(X^{u})^{hR} = R^{z}X^{-aR}(X^{u})^{hR}$$
$$= R^{z}X^{-aR}(X)^{\frac{a}{h}hR}$$
$$= (g^{r})^{\frac{a\tilde{h}}{r}}$$
$$= g^{a\tilde{h}} = (g^{\frac{a}{h}})^{h\tilde{h}}$$
$$= (g^{u})^{h\tilde{h}}$$

According to the above verification, we find that the forged signature  $\delta$  on message *m* satisfies verification equation. It denotes that our attack is successful. And it show that the scheme is not against an uncertified entity attack.

# 5 Its Improved Scheme

In the section, to overcome the attacks above, we give an improved certificatebased signature scheme. To clarify our scheme, we first review preliminaries.

# 5.1 Bilinear Maps

In the following, we recall the notations related to bilinear groups [45]. Let  $\mathbb{G}_1, \mathbb{G}_2$  be bilinear groups as follows:

- 1.  $\mathbb{G}_1$  and  $\mathbb{G}_2$  are two cyclic multiplicative groups of prime order p, where possible  $\mathbb{G}_1 = \mathbb{G}_2$
- 2.  $g_1$  is a generator of  $\mathbb{G}_1$  and  $g_2$  is a generator of  $\mathbb{G}_2$ .
- 3. *e* is a non-degenerate bilinear map  $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ , where  $|\mathbb{G}_1| = |\mathbb{G}_2| = |\mathbb{G}_T| = p$ 
  - (a) Bilinear: for all  $u, v \in \mathbb{G}_1$  and  $a, b \in \mathbb{Z}_p$ ,  $e(u^a, v^b) = e(u, v)^{ab}$ ;
  - (b) Non-degenerate:  $e(g, g) \neq 1$ ;
- 4. e and the group action in  $\mathbb{G}_1, \mathbb{G}_2$  can been computed efficiently.

## 5.2 Security Assumption

Here we first review the definition of the strong Diffie-Hellman (SDH) assumption introduced in [4], on which the security of our signature is based, and then extend it into a new security assumption, the extended strong Diffie-Hellman assumption, on which the security of a variant of our signature scheme is based.

**Strong Diffie-Hellman Assumption:** Let  $\mathbb{G}_1, \mathbb{G}_2$  be bilinear groups as shown the above section. The q-SDH problem in  $(\mathbb{G}_1, \mathbb{G}_2)$  is defined as follows: given  $g_1 \in \mathbb{G}_1$ , and the (q + 1)-tuple  $(g_2, g_2^x, \dots, g_2^{x^q}) \in \mathbb{G}_2^{q+1}$  as input, output a pair  $(g_1^{\frac{1}{x+c}}, c)$  where  $c \in Z_p$ . Algorithm  $\mathcal{A}$  has advantage,  $Adv_{SDH}(q)$ , in solving q - SDH in  $(\mathbb{G}_1, \mathbb{G}_2)$  if

$$Adv_{SDH}(q) \to Pr[\mathcal{A}(g_1, g_2, g_2^x, \cdots, g_2^{x^q})] = (g_1^{\frac{1}{x+c}}, c)$$

Where the probability is taken over the random choice of  $g_2 \in \mathbb{G}_2$ ,  $x \in \mathbb{Z}_p^*$ , and the coin tosses of  $\mathcal{A}$ .

**Definition 1.** Adversary  $\mathcal{A}(t, \epsilon)$ -breaks the q-SDH problem if  $\mathcal{A}$  runs in time at most t and  $Adv_{SDH}(q)$  is at leat  $\epsilon$ . The  $(q, t, \epsilon)$ -SDH assumption holds if no adversary  $\mathcal{A}(t, \epsilon)$ -breaks the q-SDH problem.

**Definition 2 (Computational Diffie-Hellman (CDH) Assumption).** . Let  $\mathcal{G}$  be a CDH parameter generator. We say an algorithm  $\mathcal{A}$  has advantage  $\epsilon(k)$  in solving the CDH problem for  $\mathbb{G}_1$  if for a sufficiently large k,

$$Adv_{\mathcal{G},\mathcal{A}}(t) = Pr[\mathcal{A}(p,\mathbb{G}_1,g^x,g^y) = g^{xy} \mid (p,\mathbb{G}_1) \leftarrow \mathcal{G}^k, P \leftarrow \mathbb{G}_1, x, y \leftarrow \mathcal{Z}_p]$$

We say that  $\mathbb{G}_1$  satisfies the CDH assumption if for any randomized polynomial time in t algorithm  $\mathcal{A}$  we have the  $Adv_{\mathbb{G}_1,\mathcal{A}}(t)$  is negligible function.

q-**SDH+CDH Assumption:** The q-SDH+CDH problem in  $\mathbb{G}_1$  is defined as follows: give q + 1-tuple  $(g_1, g_1^{\alpha}, \dots, g_1^{\alpha^q})$  and a random pair  $(g_1, g_1^r)$  of group  $\mathbb{G}_1$  as inputs, output  $(\rho \leftarrow (g_1)^{\frac{r}{(\alpha+c)}}, c)$ , where  $g_1$  is a generator of group  $\mathbb{G}_1$ ,  $c, r \in_R Z_p^*$ . Note that  $\alpha$  and r are unknown numbers. Algorithm  $\mathcal{A}$  has advantage,  $\operatorname{Adv}_{SDH}(q)$ , in solving q-SDH+CDH in  $\mathbb{G}_1$  if

$$Adv_{q-SDH+CDH}(q) \leftarrow Pr[\mathcal{A}(g_1, g_1^{\alpha}, \cdots, g_1^{\alpha^q}, g_1^r) = (g_1^{\overline{(\alpha+c)}}, c)]$$

Where the probability is taken over the random choices of  $g_1 \in \mathbb{G}_1$ ,  $\alpha, r \in Z_p^*$ , and the coin tosses of  $\mathcal{A}$ .

**Definition 3.** Adversary  $\mathcal{A}(t, \epsilon)$ -breaks the q-SDH+CDH problem if  $\mathcal{A}$  runs in time at most t and  $Adv_{q-SDH+CDH}(q)$  is at least  $\epsilon$ . The  $(q, t, \epsilon)$ -SDH+CDH assumption holds if no adversary  $\mathcal{A}(t, \epsilon)$ -breaks the q-SDH+CDH problem.

**Definition 4.** (Discrete Logarithm Assumption). Given a group  $\mathcal{G}$  of prime order q with generator g and elements  $A \in \mathcal{G}$ , the discrete logarithm problem in  $\mathcal{G}$ is to output  $x \in Z_q$  such that  $A = g^x$ .

An adversary  $\mathcal{B}$  has at least an  $\epsilon$  advantage if

$$\Pr[\mathcal{B}(g,A) = x | A = g^x] \ge \epsilon$$

We say that the  $(\epsilon, t)$ -DL assumption holds in a group  $\mathcal{G}$  if no algorithm running in time at most t can solve that DL problem in  $\mathcal{G}$  with advantage at least  $\epsilon$ .

### 5.3 Our Improved Scheme

In the following, our improved scheme is proposed, the detail steps are shown as follows:

**[Setup of System Parameters:]** Let  $\mathbb{G}$  be a multiplicative group with order q,  $g \in \mathbb{G}$  is a generator of group  $\mathbb{G}$ . The PKG randomly chooses  $\alpha \in Z_q$  as master secret key and compute  $X = g^{\alpha}$  as public key. Note that PKG is responsible for issuing certificate.  $h_1, h_2 \in \mathbb{G}_1$  were randomly chosen and  $H(\cdot), H_1(\cdot), H_0(\cdot)$ 

are three one-way hash functions which satisfy  $H : \mathbb{G}_1^2 \times \{0,1\}^* \to Z_q, H_1 : \mathbb{G}_1 \times \{0,1\}^* \to Z_q$  and  $H_0 : \mathbb{G}_1^2 \times \{0,1\}^* \to \mathbb{G}_1$ . The system parameters param and master secret key msk are as follows:

**param** =  $(g, h_1, h_2, e, H(\cdot), H_0(\cdot), H_1(\cdot), X), msk = x$ 

**[UserKeyGen:]** A user picks  $u \in Z_q$  as his secret key usk at random, and computes the corresponding public key PK as  $(g^u, \tau_u)$  where  $\tau_u$  is the following non-interactive proof-of-knowledge (PoK)

$$PK\{(u): U_1 = g^u\}$$

[Certify:]To certify for user with public key PK and binary string ID which is used to identity the user. The PKG computes  $h_0 = H_0(X, ID, U_1)$  and

$$d = (h_0)^{\frac{1}{\alpha - H_1(U_1, ID)}}$$

The certificate is d. Note that a correctly generated certificate should fulfill the following equality:

$$e(d, X \cdot g^{-H_1(U_1, ID)}) = e(h_0, g)$$

**[Sign:]** To produce a signature on message  $M \in \{0, 1\}^*$ , the signer with public key PK, the certificate d and the secret key u computes as follows:

- 1. randomly choose  $s \in_R Z_q^*$ .
- 2. compute  $\delta = (\delta_1, \delta_2)$ , where

$$\delta_1 = d^u \cdot (h_2 h_1^{H_1(ID,U_1)})^{sm}, \delta_2 = (Xg^{H_1(ID,U_1)})^s$$

and  $m = H(\delta_2, M, U_1)$ .

3. the resultant signature on message M is  $\delta = (\delta_1, \delta_2)$ .

According to the above signing phase, we know that the size of our signature is more shorter than that of Liu *et.al*'s signature.

**[Verify:]** Upon receiving the signature  $\delta = (\delta_1, \delta_2)$  on message M, a verifier computes as follows:

- 1. compute  $m = H(\delta_2, M, U_1)$  and  $h_0 = H_0(X, ID, U_1)$ ;
- 2. verify

$$e(Xg^{-H_1(ID,U_1)},\delta_1) = e(h_0,U_1)e(\delta_2,(h_2h_1^{H_1(ID,U_1)})^m)$$

**Correctness.** It is easy to see that the improved signature scheme is correct. It is shown as follows: If a signature  $\delta = (\delta_1, \delta_2)$  is valid, then the signature  $\delta$  must pass the verification equation. Because

$$\begin{split} &e(Xg^{-H_1(ID,U_1)},\delta_1)\\ &= e(Xg^{-H_1(ID,U_1)},d^u\cdot(h_2h_1^{H_1(ID,U_1)})^{sm})\\ &= e(Xg^{-H_1(ID,U_1)},(h_0^u)^{\frac{1}{\alpha-H_1(ID,U_1)}}\cdot(h_2h_1^{H_1(ID,U_1)})^{sm})\\ &= e(U_1,h_0)e(Xg^{-H_1(ID,U_1)},(h_2h_1^{H_1(ID,U_1)})^{sm})\\ &= e(U_1,h_0)e(\delta_2,(h_2h_1^{H_1(ID,U_1)})^m) \end{split}$$

where  $m = H(\delta_2, M, U_1)$  and  $h_0 = H_0(, U_1, ID)$ .

# 6 Security Analysis

To demonstrate that our improved scheme is secure, we must prove that the scheme can be against game 1 adversary and game 2 adversary which are discussed.

**Theorem 1 (Unforgeability against Game 1 Adversary).** If there exists a game1 adversary  $\mathcal{A}$  which breaks our improved scheme with advantage at most  $\epsilon$  and runs in times at most t, then the q-SDH+CDH problem can be  $(\epsilon', t')$ -solved.

**Theorem 2** (Unforgeability against Game 2 Adversary). If there exists a game 2 adversary  $\mathcal{A}$  which breaks our improved scheme with advantage at most  $\epsilon$  and runs in times at most t, then the Computational Diffie-Hellman problem in group  $\mathbb{G}$  can be  $(\epsilon', t')$ -solved.

# 7 Conclusion

Certificate-based cryptosystem is an important public key cryptology system. Recently, Liu *et.al* proposed an efficient Certificate-based signature scheme and showed that the scheme was secure against two game attack of certificate-based signature scheme in the random oracles. Unfortunately, Our analysis show that the first one of the certificate-based signature schemes proposed by Liu *et.al* is insecure, namely, a malicious PKG can forge a signature on arbitrary message in name of any user's identity and a uncertified user is also able to forge a messagesignature. And we discuss the flaw in their proof and the corresponding attack ways. To overcome the flaws, an improved scheme is proposed and the scheme is proven to secure against two game attacks of certificate-based signature. The security is closely related to the computational diffie-hellman problem.

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# An Empirical Investigation into the Security of Phone Features in SIP-Based VoIP Systems

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Abstract. Phone features, e.g., 911 call, voicemail, and Do Not Disturb, are critical and necessary for all deployed VoIP systems. In this paper, we empirically investigate the security of these phone features. We have implemented a number of attacks and experimented with VoIP services by leading VoIP service providers Vonage, AT&T and Gizmo. Our experimental results demonstrate that a man-in-the-middle or remote attacker could transparently 1) hijack selected E911 calls and impersonate the Public Safety Answering Point (PSAP); and 2) spoof the voicemail servers of both the caller and the callee of selected VoIP calls; and 3) make spam calls to VoIP subscribers even if Do Not Disturb is enabled. These empirical results confirm that leading deployed SIP-based VoIP systems have serious security vulnerabilities.

**Keywords:** VoIP security, SIP, voicemail fraud, 911 hijacking, voice spam.

# 1 Introduction

In addition to the basic function of making and receiving a call, VoIP systems generally offer many phone features, e.g., *911 call, voicemail*, and *Do Not Disturb*. Phone features are critical and necessary for all deployed Public Switched Telephone Network (PSTN) and VoIP systems.

Among all phone features, 911 emergency call is perhaps the most critical one. Recognizing that proper function of 911 call could impact the "life or death for millions of customers that subscribe to VoIP service" [1], the Federal Communications Commission (FCC) requires that all the interconnected VoIP services must support Enhanced 911 (E911) call and automatically report the caller ID and the registered location of E911 calls. On the other hand, voicemail is one of the most frequently used features of VoIP service. It is estimated [2] that  $60 \sim 70\%$ of phone calls will be answered by voicemail rather than human. Given that a voice message often contains personal and sensitive information, any compromise of voicemail would violate the privacy of both the sender and the receiver of the voice message. *Do Not Disturb* allows VoIP users to temporarily block all

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incoming phone calls and have some quiet time. When *Do Not Disturb* fails to work, spam phone calls might continually annoy VoIP users.

Signaling channel and voice channel are two most important components of VoIP systems. In current deployed systems, the Session Initiation Protocol (SIP) and the Real Time Transport Protocol (RTP) are the most dominant signaling and voice transport protocol, and being widely used.

Although the VoIP features are intuitively expected as trustworthy and reliable as those in the traditional PSTN, the open architecture of VoIP has enabled many attacks on voice communication that were not possible in the traditional PSTN. In this paper, we empirically investigate and evaluate the security of phone features in currently deployed SIP-based VoIP systems in the U.S., e.g., Vonage **5**, AT&T's CallVantage **6** and Gizmo **7**. Specifically, we focus on the trustworthiness of *E911 call, voicemail*, and *Do Not Disturb*.

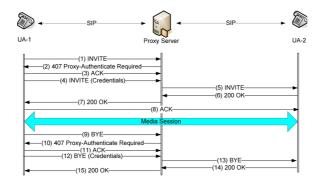
Assuming there exists a man-in-the-middle (MITM) in between the VoIP phones and the VoIP servers or a remote attacker on the Internet, we implement a number of MITM and remote attacks on the investigated voice services. Specifically, the MITM can transparently hijack selected 911 emergence calls and impersonate the PSAP. The MITM could also launch various voicemail fraud attacks against selected VoIP subscribers. When a VoIP phone makes a call to a PSTN phone or receives a call from a PSTN phone, the MITM can impersonate the callee's voicemail server and ask the caller to leave a voice message or call another phone number. In addition, the MITM can impersonate the voicemail server when the caller accesses voicemail. This would allow the attacker to generate arbitrary fake voice messages to the caller. Finally, a remote attacker,not necessarily a MITM, can circumvent *Do Not Disturb* and make arbitrary calls to Vonage and AT&T VoIP phones. Our experiments confirm that currently deployed VoIP systems are far from secure and trustworthy.

The rest of this paper is organized as follows. Section 2 briefly overviews SIP and SIP security mechanisms. Section 3 introduces our exploitation methodology. Section 4 describes our experiments on E911, voicemail and *Do Not Disturb*. Section 5 discusses the root causes of the vulnerabilities in deployed VoIP systems, and proposes some approaches to mitigate potential threats. Section 6 introduces related work. Finally, section 7 concludes the paper.

# 2 SIP Overview

Session Initiation Protocol (SIP) [3], is a HTTP-like, application layer signaling protocol used for creating, modifying, and terminating multimedia sessions (e.g. VoIP calls) among Internet endpoints. SIP signaling involves various components: user agents (UA), proxy servers, redirect servers, registrar servers, location servers. An UA represents an endpoint of the communication (i.e., a SIP phone). The proxy server is the intermediate server that acts on behalf of UA to forward the SIP messages to its destination.

The SIP specification [3] recommends using TLS or IPSec to protect SIP signaling messages. It also suggests using S/MIME to protect the integrity and



**Fig. 1.** An Example of Message Flow of SIP Authentication for INVITE and BYE Messages

confidentiality of SIP message bodies. However, most deployed SIP VoIP systems only utilize SIP authentication to protect SIP messages. Fig. I shows the typical SIP authentication of call setup and termination.

The SIP authentication of currently deployed VoIP systems has the following weaknesses **3**8:

- It only protects a few important SIP messages.
- It only protects a few SIP fields.
- It only authenticates SIP messages from SIP phones to the SIP servers.

#### 3 Investigation Approach

Our approach of the security investigation is from the perspective of VoIP customers rather than VoIP service providers. Consequently, we leave aside the attacks on the VoIP service provider's servers and instead focus on those attacks that directly target the VoIP users. We choose to experiment with the residential VoIP services by Vonage, AT&T, who are the No.1 and No.2 [9] in U.S. VoIP market share. In addition, we experiment with Gizmo's popular softphone that has been used by millions of people. Note all the attacks we experiment are against our own accounts and phones only.

The key technique used in our empirical investigation is the MITM who can monitor, modify and forge VoIP traffic to or from selected VoIP users. Since most VoIP phones are many hops away from the VoIP servers, attackers have many opportunities to play MITM attack on existing deployed VoIP services. In fact, Zhang et al III have shown that a remote attacker from anywhere on the Internet, who is not originally in between the targeted VoIP phone and its VoIP servers, can become a MITM within a few minutes by exploiting some implementation flaws in the VoIP phone.

The MITM is used to verify the weaknesses in E911 and voicemail. To circumvent *Do Not Disturb*, the MITM is not required. A remote attacker can successfully make spam calls to defeat this feature.

## 4 Experiments with Deployed VoIP Services

To empirically investigate the security of deployed VoIP systems, we build a testbed that consists of the MITM, Vonage, AT&T SIP phones and Gizmo's softphone. Fig. 2 illustrates the network setup of our testbed. All the SIP phones are within a private network 192.168.1.x, where the Gizmo softphone runs on a Windows XP virtual machine. The MITM runs on a FreeBSD 5.4 virtual machine, which acts as the Network Address Translation (NAT) router for the private network. Specifically, the natd runs on the external interface lnc0 of the FreeBSD virtual machine, and our MITM intercepts, manipulates the network traffic at the internal interface lnc1 via divert socket. Note the MITM does not need to be directly connected to the VoIP phones, and it could be at anywhere along the path of VoIP traffic.

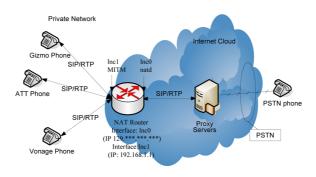


Fig. 2. Testbed Setup of MITM Attacks

#### 4.1 911 Call Hijacking

When a VoIP user dials 911, the call is supposed to be routed to a geographically appropriate PSAP by the VoIP service provider. However, our experiments show that the MITM could hijack the selected 911 call from either Vonage or AT&T SIP phone, divert it to any third party and let the third party impersonate the PSAP. In this case, the 911 call is never routed to the VoIP service provider, and yet it appears to the 911 caller that the 911 call is successfully connected to the appropriate PSAP.

Fig. 3 illustrates the message flows of the 911 call hijacking experiments. Specifically, the left and the right parts show the message flows of 911 calls from the Vonage phone and the AT&T phone respectively. Depending on the service provider's implementation, the signaling path and the RTP stream path could be different. All SIP and RTP packets are transferred on UDP. We use SIP/RTP server(s) to denote the SIP server and the RTP server which handle the signaling messages and the RTP streams respectively.

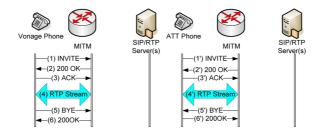


Fig. 3. Message Flow of Hijacking 911 Calls

When we dial 911 from the Vonage (or the AT&T) SIP phone<sup>1</sup>, the caller's SIP phone sends an INVITE sip:911 message to the SIP server in step (1) or (1'). Our MITM intercepts the INVITE message, pretends to be the SIP server and responds with a spoofed 200 OK message in step (2) or (2'). In the spoofed 200 OK message, the MITM sets its own IP address and port number (e.g., 12345) as the RTP stream termination point, which asks the caller's SIP phone to establish the voice stream to the MITM instead of the service provider's server. Although Vonage and AT&T's SIP server normally challenges the INVITE message with 407 proxy-authentication required as shown in Fig. 1. we find that both Vonage and AT&T SIP phones actually accept the spoofed 200 OK message directly, and they respond with an ACK message to the SIP server in step (3) or (3'). The MITM intercepts the ACK message so that it won't reach the service provider's SIP server. At this point, the three way handshake for the VoIP call setup is finished, and the 911 call between the caller and the MITM has been established. Then at step (4) or (4'), the caller talks to the MITM, who pretends to be the PSAP.

Traditionally, 911 calls can only be terminated by the PSAP. We find that the Vonage SIP phone actually allows the 911 caller to terminate the 911 call. Specifically, the Vonage SIP phone sends out a BYE message to the Vonage's SIP server once the 911 caller hangs up in step (5). Then the MITM simply responds with a fake 200 OK message. On the other hand, the AT&T SIP phone prevents the 911 caller from terminating the call until it is reset. Specifically, if the 911 caller hangs up the AT&T SIP phone, it starts and keeps ringing until the handset is picked up or the phone adapter is reset. This behavior conforms to the specification of traditional 911 call in the PSTN. At step (5'), the MITM pretends the PSAP and sends the AT&T SIP phone a fake BYE message. The AT&T SIP phone responds with a 200 OK message which would terminate the 911 call completely.

Despite the 911 call implementation differences between Vonage and AT&T SIP phones, our MITM is able to transparently hijack selected 911 calls and pretend to be the PSAP. Our experiments demonstrate that 911 calls from existing deployed Vonage and AT&T VoIP services are not trustworthy.

<sup>&</sup>lt;sup>1</sup> Note we block our experimental 911 calls at our border router so that they will not interfere with any real 911 calls.

#### 4.2 Fake Voicemail Attacks

Voicemail is intended for the caller to leave a voice message when the callee is not available. Once the caller hears the voicemail prompt after dialing the callee's phone number, he would expect it is the callee's voicemail. In addition, when someone wants to check his voicemail and dials his voicemail access number, he would expect to reach his own voicemail. In this section, we show that the MITM can compromise the trust of voicemail by spoofing both the caller's voicemail and callee's voicemail.

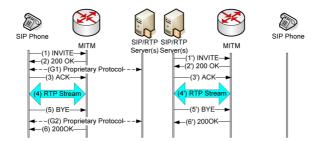


Fig. 4. Message Flows of Fake Callee's Voicemail at Caller's and Callee's Sides

Fake Callee's Voicemail Attack. When the caller places a call, the MITM intercepts the call and spoofs the callee's voicemail to prompt the caller to leave a voice message or call some other phone number. In this case, the caller is tricked into believing that the callee is not available even if the callee is actually available. Note the fake callee's voicemail attack can be launched by the MITM at either the caller's or callee's side. Therefore, the fake callee's voicemail attack is possible even if one communication party (caller or callee) uses a PSTN phone.

Fake callee's voicemail at the caller's side. We use our Vonage and AT&T SIP phones to call a PSTN phone. The left part of Fig. 4 illustrates the corresponding message flow. After dialing the callee's phone number, the caller's SIP phone sends the corresponding INVITE message to the SIP server of its service provider. The MITM intercepts the INVITE message in step (1), and replies with a fake 200 OK message in step (2). The caller's SIP phone is tricked by the fake 200 OK message and sends back an ACK message in step (3). This establishes the voice RTP session between the caller's SIP phone and the MITM. In step (4), the MITM sends the crafted RTP stream to the caller's SIP phone and the caller hears bogus voice greeting "xxx can not take your call, please leave a brief message or call 1-xxx-xxx". Then the MITM starts to record the RTP stream from the caller's SIP phone. Once the caller hangs up, his SIP phone sends a BYE message to the MITM in step (5), the MITM replies with a 200 OK message to the caller's SIP phone in step (6), which terminates the call.

When we use the Gizmo softphone to call the PSTN phone, the Gizmo softphone does not respond with an ACK message to the MITM after receiving the

200 OK message from the MITM in step (2). After further investigation, we find that the Gizmo phone does not follow the SIP specification 3 exactly:

- The IP address and the port number of the RTP server are specified by the Gizmo softphone in the INVITE message in step (1), rather than by the SIP server in the 200 OK message in step (3).
- Gizmo softphone uses some proprietary protocol, running on TCP port 443, to exchange some signaling information with Gizmo RTP server in step (G1) and (G2). Step (G1) is necessary for establishing the SIP call, and if packets in step (G1) are dropped by the MITM, the Gizmo softphone will not proceed to step (3). Once the MITM allows the traffic in step (G1), the the Gizmo softphone proceeds to establish the call with its SIP server. Step (G2) is used to terminate the SIP call.

After implementing these special handling, the MITM is able to spoof the callee's voicemail to calls from the Gizmo softphone.

Fake callee's voicemail at the callee's side. In this experiment, we use a PSTN phone to call our Vonage and AT&T SIP phones. The right part of Fig. I illustrates the corresponding message flow. After the caller dials the phone number of a SIP phone from a PSTN phone, the SIP server sends an INVITE message to the SIP phone in step (1'). The MITM intercept the INVITE message and responds with a 200 OK message in step (2'). The SIP server thinks it is from the SIP phone and sends an ACK message in step (3'). Now the voice RTP session between the SIP server and the MITM is established, and the MITM sends the RTP server crafted RTP stream in step (4'). As a result, the caller from the PSTN phone hears the bogus voice greeting "XXX can not take your call, please leave a brief message or call 1-xxx-xxx-xxx". Then the MITM starts to record the RTP stream from the RTP server. Once the caller hangs up, the SIP server sends a BYE message to the SIP server in step (6'), which terminates the call.

Fake Caller's Voicemail Attack. When a caller wants to check his voicemail, he usually dials some voicemail access number (e.g., \*123 for Vonage, \*\*\* for AT&T, 611 for Gizmo) and authenticates himself with a voicemail PIN. Again, the MITM can intercept the call and spoof the caller's voicemail. This not only allows the attacker to trick the caller with bogus voicemail messages but also let the attacker capture the caller's voicemail PIN.

Table 1. Differences between the Voicemail Services by Vonage, AT&T and Gizmo

Service Provider	Voicemail Access Number	Codec for voice	Payload Type for Event
Vonage	*123	G.711 $PCMU(0)$	101
AT&T	***	G.721(2)	100
Gizmo	611	iLBC(102)	106

We have experimented the fake caller's voicemail attack with Vonage, AT&T and Gizmo SIP phones. The SIP message flow is similar to the left part of Fig. The MITM first spoofs the SIP server and establishes a RTP voice session with the voicemail caller. Then the MITM spoofs the voicemail service and interacts with the voicemail caller. In our implementation, we record the RTP stream from the real voicemail server and replay the recorded RTP stream to the voicemail caller. Therefore, what the voicemail caller hears is exactly the same as that from the real voicemail server. After the MITM prompts the voicemail caller, it waits for responses (e.g., PIN, functional choice) from the caller and responds accordingly.

We find that Vonage, AT&T and Gizmo use different codecs and RTP payload types for their voicemail traffic. Table []] summarizes their differences in the codec and RTP payload type used. Specifically, the codecs for the RTP voice are G.711 PCMU, G.721 and iLBC respectively, and the payload types for the RTP event are 101, 100 and 106 respectively. Despite these differences in the voicemail implementation by Vonage, AT&T and Gizmo, our MITM is able to spoof the voicemail for all of them.

In summary, the MITM can spoof both the caller's and the callee's voicemail even if one side of the voice call uses a PSTN phone. Eventually, the caller is deceived to leave a voicemail or call another number while the callee does not even know he has been called. Meanwhile, the MITM can read the voicemail left by the caller and trick the voicemail caller with bogus voicemail messages. Furthermore, the MITM could obtain the voicemail PIN entered by the caller.

#### 4.3 Circumventing Do Not Disturb

Do Not Disturb enables VoIP users to block all incoming phone calls during a short time, e.g., 30 minutes. When we used a PSTN phone to call a Vonage or an AT&T VoIP phone with Do Not Disturb enabled, the call was forwarded to the voicemail system. This indicates *Do Not Disturb* is effective if the call goes through SIP servers. However, we can circumvent *Do Not Disturb* by calling the Vonage or the AT&T phone directly.

Fig. 5 shows the network setup of circumventing *Do Not Disturb*. Note unlike previous experiment, this experiment only requires a remote attacker. Fig. 6 depicts the message flow of circumventing *Do Not Disturb* attacks on the Vonage phone and AT&T phone.

In step (1) and (1'), the remote attacker sends an INVITE message to the SIP phone. Note to make the INVITE message accepted by the SIP phone, the IP address should be spoofed as that of a real SIP server. Otherwise, the SIP phone would omit this INVITE message. In the INVITE message, the IP address and port number for the RTP stream on the caller side are set to the remote attacker's IP address and 12345 respectively. Since the SIP specification does not require to authenticate SIP messages from SIP servers, the SIP phone will accept this INVITE message. Then in step (2-3) or (2'-3'), the SIP phone sends back Trying, Ringing messages to the real SIP server, and begins to ring. At

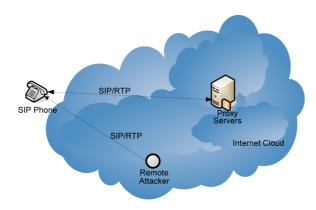


Fig. 5. Testbed Setup of Circumventing Do Not Disturb

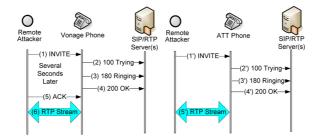


Fig. 6. Message Flow of Circumventing Do Not Disturb

this point, the called party begins to hear annoying ring tone. Consequently, *Do Not Disturb* is bypassed.

Actually, the attacker can even successfully establish a SIP call with these phones and send voice spam by exploiting some implementation flaws. After the receiver picks up the phone, the SIP phone responds with a 200 OK message to its real SIP server. According to the SIP specification, to establish a SIP call, the remote attacker needs to send an ACK message to complete an INVITE/2000K/ACK three-way handshake. Since the 200 OK message is sent to the real SIP server, the remote attacker does not know the time when the receiver picks up the phone. To solve this problem, The remote attacker can guess the time interval between the INVITE message and the 200 OK message, e.g., 5 seconds. Then 5 seconds after sending the INVITE message, the remote attacker sends an ACK message with a spoofed source IP address to the SIP phone. In addition, to ensure the correctness, the callee should check whether the Tag value in the To field of the ACK message is the exactly same as that in the 200 OK message. Since the remote attacker can not see the 200 OK sent to the SIP server, the remote attacker can not craft an ACK SIP message with an correct Tag value. After further investigation, we find:

- The Vonage phone does not check the Tag value, and accepts an ACK message with a random Tag value.
- The AT&T phone does not need a complete INVITE/2000K/ACK three-way handshake before sending RTP stream. As soon as the receiver picks up the phone, the AT&T phone begins to send RTP stream to the remote attacker.

Consequently, the remote attacker can exploit these implementation flaws to talk to both the Vonage and AT&T phone.

#### 5 Discussion

We have demonstrated that a MITM or remote attacker, could successfully launch many attacks on phone features in deployed SIP-based VoIP systems. These attacks exploit the inherent vulnerabilities in current deployed VoIP systems.

- Only SIP authentication mechanism is employed to protect SIP messages. Due to the weaknesses in SIP authentication, an attacker can freely craft unprotected SIP messages (e.g., Trying, Ringing, 200 OK, ACK), and SIP messages from a SIP server to a SIP phone. In addition, the attacker can also modify unprotected SIP fields (e.g., SIP message body, From, To).
- Since RTP voice stream is unencrypted and unauthenticated, the attack can easily capture voice traffic, generate bogus RTP stream and talk to VoIP phones.

Leveraging these weaknesses in SIP and RTP, the attacker can seamless spoof the SIP and RTP server when talking to the caller, and seamless spoof the callee when talking to the SIP server and RTP server. Accordingly, the attacker can readily hijack 911 calls, spoof the caller's and the callee's voicemail, and make a call to the selected phone number. Note during an attack, both a VoIP phone and the PSTN or cell phone communicating with the VoIP phone are potential victims. In addition to the features that we have investigated in this paper, we believe other phone features, e.g., 411, all have similar exploitable vulnerabilities.

Since our attacks target VoIP end users, intrusion defense measures deployed on VoIP servers side are ineffective to mitigate the attacks. To defeat or minimize these attacks, the best solution is to provide privacy, integrity and authentication protection for SIP signaling messages and RTP voice streams. For example, SIP over TLS and Secure Real-time Transport Protocol (SRTP) could be deployed to protect SIP messages and RTP voice streams. Unfortunately, we still have not widely seen such a deployment. Considering Vonage and AT&T Callvangtage's VoIP phones are being utilized to replace traditional PSTN phones, and millions of VoIP subscribers are using their VoIP phones to perform many security critical activities, e.g, phone banking and 911 calls, the current state of VoIP security is not optimistic.

#### 6 Related Work

Most previous work is on the defense side. Arkko et al [11] proposed a scheme to negotiate the security mechanism used between a SIP phone and and its next-hop SIP entity. Baugher et al [12] proposed SRTP to protect the RTP traffic. However, none of them are widely being used. Reynolds et al [13] proposed multi-protocol protection against flood-based DoS attacks on VoIP networks. Wu et al [14] described a cross protocol intrusion detection architecture for VoIP environments. Sengar et al [15] proposed an intrusion detection system based on interactive protocol state machines. The above intrusion detection systems or methods are deployed on VoIP servers side, and ineffective to defend against the attacks we proposed.

Mintz-Habib et al **16** proposed a VoIP emergence service architecture and developed a prototype system. Zhang et al **8 10** implemented four billing attacks on deployed VoIP systems, and demonstrated a remote attacker can become a MITM by exploiting some implementation flaws in a VoIP phone. Wang et al **17** systematically investigated the trust issue of SIP-based VoIP and identified the voice pharming attack on VoIP systems. McGann and Sicker **18** analyzed detection capability of several VoIP security tools: SiVuS,PROTOCOS **19**, SIP Forum Test Framework **20**, and some commercial products. They showed that there exists a large gap between known VoIP security vulnerabilities and the tool's detection capability.

#### 7 Conclusion

In this paper, we empirically investigated the trustworthiness of phone features in leading deployed SIP-based VoIP systems. We demonstrated that the MITM could transparently hijack E911 calls and spoof the PSAP. In addition, it can spoof voicemail and use it for many potential voicemail related frauds. Finally, a remote attacker can circumvent *Do Not Disturb* and make annoying phone calls.

We hope our work can raise the awareness of millions of VoIP subscribers that the currently deployed VoIP phone features are not as trustworthy and reliable as expected. Before VoIP service providers employ SIP over SSL and SRTP to protect SIP signaling messages and RTP voice streams, a VoIP subscriber should be aware of the risk associated with current VoIP services.

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## Reconstructing a Packed DLL Binary for Static Analysis<sup>\*</sup>

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Abstract. DLLs (Dynamic Link Libraries) are usually protected by various anti-reversing engineering techniques. One technique commonly used is code packing as packed DLLs hinder static code analysis such as disassembly. In this paper, we propose a technique to reconstruct a binary file for static analysis by loading a DLL and triggering and monitoring the execution of the entry-point function and exported functions of packed DLLs. By monitoring all memory operations and control transfer instructions, our approach extracts the original hidden code which is written into the memory at run-time and constructs a binary based on the original DLL, the codes extracted and the records of control transfers. To demonstrate its effectiveness, we implemented our prototype ReconPD based on QEMU. The experiments show that ReconPD is able to analyze the packed DLLs, yet practical in terms of performance. Moreover, the reconstructed binary files can be successfully analyzed by static analysis tools, such as IDA Pro.

**Keywords:** Security Analysis, Malware Analysis, Binary Reconstructing, Dynamic Analysis, Static Analysis.

#### 1 Introduction

Reverse engineering becomes a prevalent technique to analyze malware. To make the analysis more difficult, malware writers usually protect their programs including executables and DLLs by various anti-reverse engineering techniques. One of the most popular anti-reverse engineering methods is binary code packing [18]20]. It is impossible to do static analysis on packed malware because of the inaccuracy of disassembler. This paper focuses on identifying and extracting the hidden code generated using binary code packing and reconstructing a binary for static analysis.

To identify and extract the hidden code in packed binary programs various tools have been developed. However, most of commonly known tools such as PEiD [4]

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are only valid for some special known packing algorithms applied on the packed executables; what is more, they are usually unable to analyze packed DLLs.

DLL is commonly used in the malware. Windows makes certain features available only to DLLs. For example, you can install certain hooks (set using **SetWindowsHookEx** and **SetWinEventHook**) only if the hook notification function is contained in a DLL. As a number of critical technologies and codes are packed in DLL, it is becoming one important target of anti-reverse engineering protection.

By now, the difficulty of statically analyzing a packed binary program is how to correctly disassemble them. One of approaches to solve this problem is to reversely analyze the packing algorithm and develop specific automatic unpacking tools. But this method is only valid for known packing tools(such as UPX). In fact, it is easy for malware writers to implement new anti-reverse engineering algorithms and tricks. Dynamic analysis is a promising solution to the problem of hidden code extraction because it does not depend on signatures. The original code or its equivalent must eventually be restored in memory and get executed at some point at run-time regardless of what packing algorithms might be applied [20]. By taking advantage of this intrinsic nature of packed binary programs, we could potentially extract the original hidden binary code or its equivalent and reconstruct a binary for static analysis by patching the extracted code on the original packed binary program. Most of the work to date on identification and extraction of the original hidden code has focused on executables. This paper, by contrast, focuses on the DLL.

In this paper, we present a fully dynamic approach for automatically identifying packed DLLs, and extracting the original hidden code which is dynamically generated and executed at run-time and additional information useful for reconstructing of the packed binary. Finally we reconstruct a binary which can be directly analyzed by static analysis tools, such as IDA Pro. This method is valid for unknown packing algorithms without the complex reverse analysis. In summary, the contributions of this paper are as follows:

- 1. Propose a general and fully dynamic approach for identifying and extracting the original hidden code of packed DLLs: Previous work relies on either heuristics of known packing tools or the accuracy of the disassembler and is only valid for packed executables. In this paper, we present a binary extraction technique which is applicable to packed DLLs. This method is fully dynamic and thus does not depend on the program disassembly or the known signatures of packing algorithms. It triggers the execution of entry-point function and exported functions of a DLL, monitors the execution and records related memory operations by shadow memory and extracts the content of the memory which is dynamically generated and executed.
- 2. Provide a method to reconstruct a DLL binary file based on the execution trace: By patching the extracted code on an original packed DLL binary file, we reconstruct a binary file which can be directly disassembled.
- 3. Implement and evaluate ReconPD, an automated framework for extracting hidden code and reconstructing binary files. Applying

our proposed technique, we build a framework for automatically examining DLL binaries, extracting their original hidden code and reconstructing a binary file based on the extracted code and additional information. The reconstructed binary file can be successfully analyzed by static techniques. Based on the prototype, we have successfully done a series of experiments on the analyses of packed DLL binary files. We also present the evaluation results of ReconPD, demonstrating that it is applicable to analyze packed DLL binary.

The rest of the paper is organized as follows: In Section 2 we review the related work. Section 3 presents the problem definition and describes the proposed method. In Section 4 we provide details of the technique and the prototype implementation. Section 5 shows the experimental results. Finally, Section 6 concludes the paper.

#### 2 Related Work

Static analysis is one of the most popular software analysis methods nowadays, it is widely used in software reverse engineering [1] and software security analysis [2][3]. Static analysis tools can analyze intermediate binary code that is unable to run, since they don't need to really execute the code at all. And they have a good global view sight of the whole binary code and can figure out the entire program logic before running it.

To protect the privacy of software and prevent reverse engineering, software writers usually have their programs heavy-armored with various anti-reverse engineering techniques [7].8]. Such techniques include SMC (Self-Modifying Code) [9]10, encryption [11]12, binary and source code obfuscation [14]15, control-flow obfuscation [13], instruction virtualization [5]6, and binary code packing [18]. The development of anti-reverse technology makes static analysis more and more difficult. Although dynamic analysis can compensate static analysis for its effectiveness and accuracy to a certain extent, static analysis has some advantages that dynamic analysis doesn't have such as the more comprehensive view of a program's behavior.

Currently, extracting and re-building the original program from a protected binary has been one of the major challenges for software reverse engineers and security community.

#### Identifying and extracting the original hidden code

PolyUnpack **[18]** is a general approach for extracting the original hidden code without any heuristic assumptions. As a pre-analysis step, PolyUnpack disassembles the packed executable to partition it into the code and data sections, and then it executes the binary instruction by instruction, checking whether the instructions are in the code section or not. However, in terms of performance, disassembling binary executables as being done in **[18]** and **[19]** is an arduous task and correctly disassembling a binary program is challenging and error-prone, as demonstrated in **[19]**. In Addition, PolyUnpack can not extract the hidden code inside the DLL.

Min Gyung Kang et al. proposed a fully dynamic method Renovo [20] which monitors currently-executed instructions and memory writes at run-time. This approach inserts a kernel module into the emulated system and registers several call-back functions, by observing the program execution. Compared with it, our approach doesn't need touch anything in GusetOS and is completely transparent to analysis target; moreover, our method is applicable to a packed DLL binary and we can reconstruct a binary file for static analysis by monitoring the control transfers.

#### Re-building the original program

PEiD [4] is a tool for identifying compressed Windows PE binaries. Using the database of the signatures for known compression and encryption techniques, it identifies the packing method employed and suggests which unpacker tool can be applied. However, despite their ability to perfectly restore the original program, executables packed with unknown or modified methods are beyond the scope of this approach.

Christodorescu et al. proposed several normalization techniques that transform obfuscated malware into a normalized form to help malware analysis [16]. Their unpacking normalization is similar to our approach. Its basic idea is to detect the execution of newly-generated code by monitoring memory writes after the program starts. Our approach goes further; we identify and monitor memory modification and execution in all code, data, stack and heaps, and also our approach can tackle dynamic code generation inside the packed DLL binary.

#### 3 Problem Statement

One of the most popular anti-reverse engineering methods is binary code packing which is usually used to protect binary program against reverse engineering and other attacks. Code packing transforms a program into a packed program by encrypting or compressing the original code and data into packed data and associating it with a restoration routine [20]. It is impossible to analyze the packed binary program using static techniques, thus, the target of our work is to identify them and extract the original hidden code in packed binary files. We generate a binary file which can be successfully analyzed by static techniques by patching a packed binary file with extracted code and restoring its control transfers.

The protecting procedure can be expressed as below:

$$dummy_i = HR_i(target_i, key_i)$$

A hiding routine  $HR_i$  is a set of instructions which camouflage a target instruction  $target_i$  with a dummy instruction  $dummy_i$  based on encrypt key  $key_i$ . The key may be null or an invariant if the hiding routine is just a transformer.

A restoring routine  $RR_i$  is a set of instructions which un-camouflage an original target instruction hidden by a dummy instruction based on the decrypt key  $key_i$ .  $RR_i$  can be viewed as a set of instructions that replaces  $dummy_i$  with  $target_i$  at run-time  $\Omega$ . Fig.1 shows how a packed program works 20.

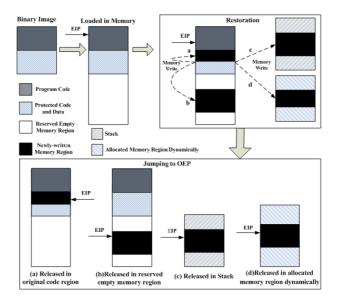


Fig. 1. How a packed program works

The restoring procedure during execution can be described as below:

$$target_i = RR_i(dummy_i, key_i)$$

 $RR_i$  is a reverse procedure of  $HR_i$ . In an analysis,  $HR_i$  is unavailable usually, so we can only analyze  $RR_i$ . But it's difficult and time-consuming to analyze  $RR_i$  directly, so we observe the execution of  $RR_i$  in order to identify and extract the hidden code it releases.

In the following discussion, we assume there is a valid key, and also we assume that at least a part of dynamically released code will be executed at run-time.

 $RR_i$  may be only used to release the code, or used to release both code and data. In the procedure of reconstructing a binary, we must disable the  $RR_i$  code generation but retain its data generation. Otherwise, it may disturb the reconstructed binary's execution.

In the procedure of restoring code, the memory used to store code may be used to store one block of code or blocks of code iteratively. That is, there may be several  $dummy_i$ . We suppose that one of them is  $dummy_i$  whose address is  $A_d^i$ , and the related code released is  $target_i$ , whose address is  $A_t^i$ . There are two different models:

1. for any i, j > 0, and  $A_d^i \neq A_d^j$ , there is  $A_t^i \neq A_t^j$ ;

2. there exist 0 < i < j, making  $A_t^i = A_t^j$ , but  $A_d^i \neq A_d^j$ ;

In the first model, each restored code block is stored in different memory regions, that is, they do not overlap with each other. In the second, there are at least two released blocks stored in the same memory region, resulting in some newlygenerated code overlapping with each other. For the second model, we need to transform the addresses because of the address confliction when we reconstruct the binary. The details will be discussed in Section 4.

## 4 System Design and Implementation

To demonstrate the effectiveness of our approach, we design and implement a system, ReconPD, to automatically identify packed DLLs and extract their original hidden code. Fig.2 depicts an overview of ReconPD. The prototype ReconPD is a subsystem of WooKoo, a malware dynamic binary analysis platform, which was developed by us based on QEMU [22]. We modified QEMU to monitor the dynamic execution of a target executed in the Guest OS.

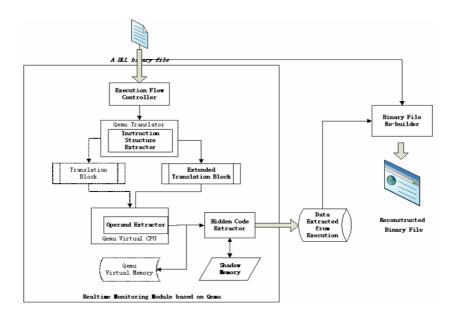


Fig. 2. The Framework of ReconPD

ReconPD extended QEMU Translator by adding the module of Instruction Structure Extractor and extended QEMU Virtual CPU by adding the module of Operand Extractor. It also extended the data structure of TB (Translation Block) to store the information of the instruction structure and added a new data structure of shadow memory. When analyzing a DLL, we run it in an *emulated environment. Execution Flow Controller* loads a DLL and triggers the entry-point function and exported functions. *Hidden Code Extractor* observes its execution, consults the *shadow memory*, and determines if any hidden code is currently executed. If so, it extracts the hidden code and other data. Finally, *Binary File Re-builder* reconstructs a binary file based on the original DLL and the data collected by Hidden Code Extractor. We will present these components and provide some implementation details in turn, and discuss the limitations of the current implementation of ReconPD.

#### 4.1 Recognizing and Extracting the Hidden Code

No matter what packing methods are applied and where the hidden code is restored, the original program code and data should eventually be present in memory to be executed, and also the instruction pointer should jump to the OEP of the restored program code which has been written in memory at runtime [20]. Taking advantage of this inevitable nature of packed DLLs, we propose a technique to dynamically extract the hidden original code from the packed DLLs by examining whether the current instruction is newly-generated after the DLL is loaded.

**Execution Flow Controller.** Before an application (or another DLL) can call functions in a DLL, the DLL's file image must be mapped into the calling process' address space. Normally, when a DLL is mapped into a process' address space, the system calls a special function in the DLL (we call it the entry-point function, usually **DllMain**) which is used to initialize the DLL. Usually the restoration of the hidden code and data is a part of the initialization process for packed DLL.

Windows is unable to launch DLL directly, so in this paper we make use of a DLL loader named LoadDLL.exe for loading a DLL by calling Windows API **LoadLibraryEx**. In order to monitor the execution of the entry-point function of a DLL, we set *dwFlags* to **DONT\_RESOLVE\_DLL\_REFERENCES** to prevent the operation system from calling the entry-point function to initialize the DLL. To obtain a comprehensive analysis of the DLL as the entry-point function is executed, we control the execution flow and trigger all exported functions of the DLL. As the return instruction of the entry-point function is "RET 0C", we take it as a sign of the end of the execution of the entry-point function. An exported function is typically allowed to run until it exits normally or crashes.

Considering that determining whether a program has hidden code or not is an *un-decidable* problem [18], we introduce a configurable time-out parameter into the system. We terminate the extraction procedure if we do not observe any hidden code being executed within this time-out. In the experiments, we set this parameter to be 30 minutes.

**Shadow Memory.** Based on the above inevitable instinct nature of packed DLLs, shadow memory is introduced to record every memory modification. According to whether the written content is executed or not, we can determinate whether it is code or data. Thus, we need to record the following information for each byte of memory:

1.  $EIP_M$ : The instruction modifying this byte.

- 2.  $EIP_C$ : The instruction referring this byte, such as directly jumping to this memory by JMP and JNZ, or calling the function whose entry point is this memory.
- 3. *State*: The state of this byte, there are three possible states, *clean*, *dirty*, and *executed*. Fig.3 shows how the states transfer.

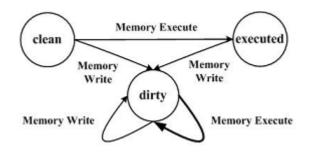


Fig. 3. How a memory state transfers

We just need to pay attention to the state *dirty* and *clean* in order to recognize and extract the original hidden code. In ReconPD, we assign a block of shadow memory for each monitored memory region (including static region and dynamic region such as stack and heaps allocated during execution) of the DLL-calling process (LoadDll.exe).

In order to reconstruct the control flow, we record *control transfer instructions*, such as JNZ and CALL, which will change the execution flow. During the reconstruction of a binary file, we modify their destination address if the target code is reassigned.

**Single-Step Monitoring.** Our monitoring on the execution needs to intercept each instruction to get the real values of operands, so we make the monitored code execute in single-step mode and each TB contain one instruction in QEMU.

In QEMU, there is a flag of virtual CPU for single-step execution mode. If the flag is set, the code of both operation system and applications is executed in single-step mode. Furthermore, the flag is perceptible for applications in the GuestOS, and the analyzed object may detect it and destroy itself or stay inactive if it found the flag is set. In ReconPD, an additional flag, *TSingleStep*, was introduced, which is transparent for applications. When *TSingleStep* is set, the translator makes sure that there is only one instruction in each TB of the target process, and instructions of other process are executed as normal.

By the method above, we not only keep the analysis transparent for the target process but also improve the analysis efficiency by avoiding the whole system running in single-step mode.

**Instruction Interception.** During monitoring the execution, we intercept the following two classes of instructions:

**Memory Modifications:** including MOV, MOVS, STOSB and so on. These operations are used to write dynamic code into memory. The stack operations such as POP are not included, which are rarely used to generate code.

**Control Transfers:** including direct jumps/calls, indirect jumps/calls, and return instructions, which determine the control flow of the execution.

ReconPD firstly records the structure of instruction during the translation and extracts the real values of operands during run-time.

#### 4.2 Reconstructing Binary Files

In this phrase, we patch the hidden code exacted on the packed DLL binary file and restore the control transfers to generate a binary file for static analysis. There are two parts of work during this phrase.

## Patch the original hidden code extracted on the packed DLL binary file

- 1. The code is released in code region and covers the original protected code just as showed in Fig2(a). We write the dynamic code into the binary file where this block is mapped from the binary file.
- 2. The code is released in reserved empty memory of code region without covering the original code just as showed in Fig2(b), we set SizeOfRawData of the code section to VirtualSize, then we write the dynamic code into the revised binary file where this block is mapped from the binary file.
- 3. The code is released in data region, such as stack and heap just as showed in Fig2(c,d), we expand a new section into the PE binary and the code is written into the expanded section. It's a mature technique to expand the PE binaries, thus, we do not discuss it in this paper.
- 4. There are two or blocks written into the same memory being covered each other, the first block write into the binary file where this block is mapped from, and the rest are written into the expanded section in the binary file.

In the restoration process, we should modify *dest* of the control transfer to the correct address where the instruction is reassigned in a binary file. New value of *dest* can be easily got by address mapping. But sometimes, we can not directly update it. For example, some branch instructions like "jmp ebx;" may transfer control to different dynamic generated code blocks at different conditions. For this case, we make the branch instruction transfer control to a dispatch routine, which will mimic the original branch conditions to call different dynamic generated code blocks at different time. By this way, though we can not completely rebuild the original logic of the execution, we can still provide all possible control transfers, which is much helpful for static analysis.

**Disable the restoration, preventing it from modifying the patched code.** When releaser's code generation behavior is disabled, we should consider whether the releaser dynamically modifies code only or both code and data. If a releaser only modifies code, we could disable it simply by replacing it with NOP instruction. Or else, to keep the original program logic, we have to reserve its function of modifying data and prevent it from modifying code. So we replace the releaser with a unconditional jump, which transfers control to a judge function in the new code section, and the judge function is an encapsulation of the releaser, which behaves just like the releaser if the modification's target is data but does nothing if the target is code.

## 5 Evaluation

The experiments are finished on the WooKoo Platform, based on Fedora Core 3 Linux 2.6.9-1.667smp and QEMU 0.8.2. The experiments are all finished with GuestOS of Windows XP Professional SP2. In this section, we describe two experiments and present the evaluation results, demonstrating that ReconPD is an accurate and practical solution for extracting the original hidden code of packed DLLs and the reconstructed binaries can be successfully analyzed by static analysis tools such as IDA pro.

#### 5.1 Synthetic Samples

Because of the difficulty of collecting wild samples, we have to develop some samples. To verify that ReconPD generates accurate results, we have tested ReconPD against the synthetic sample programs generated by using a packing tool developed by us. The packing tool applies different packing techniques as well as encryption, code obfuscation to thwart reverse engineering.

We use some DLLs belong to Microsoft Windows Operating System as the original binary to generate synthetic packed program samples. With the knowledge that the packing tool usually restore and execute the original binary instructions at run-time, we could verify the correctness of our extraction technique by comparing the extracted hidden code regions with the .text section of the original binary.

Files	Size of Packed Files(KB)	Momory (KB)	Time with- out moni- tor(Seconds)	der monitor	Reconstructed Binary
rasapi32.dll	234	2928	54	62	Loadable
jscript.dll	444	5376	38	45	Loadable
mfc42.dll	1008	12240	32	39	Loadable
msi.dll	2792	33744	22	25	Loadable
shell32.dll	8112	97728	674	681	Loadable

Table 1. The Experimental Results of Packed DLLs

As shown in Table 1, ReconPD fully extracted the original binaries processed by our packing tools. And the reconstructed DLL binary files could be loaded and disassembled by IDA Pro properly. From the experimental results, we found the size of shadow memory is almost 12 times of DLLs.

#### 5.2 Performance Overhead

We evaluated the performance by running the samples with and without monitor in single step mode. From the results shown in Table 1, we found that the monitor decreased the performance about 18.9%. Without the monitor, we also compared the performance of running in single step mode with that of running in normal mode. We found that the performance decreased greatly by single step mode even we have optimized it by only keeping the target process in single-step mode but not the whole system as QEMU. Considering that ReconPD is aiming to provide hidden code extraction environment for malware analysis which usually takes several hours to days, this degree of slowdown in initial execution time is tolerable.

#### 5.3 Limitations

Just as discussed above, there are some limitations of ReconPD.

Some exported functions crash at run-time because of the improper parameters provided by us. We are able to analyze the number of parameters of exported functions but not recognize the data type of parameters. We may patch both the data and code to reconstruct complete binary while there may be no comprehensive solution to this problem. We have intercepted typical memory write instructions. However, there is a method to evade the monitor by replacing some instructions. For example, "XOR EAX,EAX; ADD EAX,EBX" can replace the instruction "MOV EAX,EBX;". In the future, we will make it support much more instructions.

Finally, some of the reconstructed binary are unable to be loaded because ReconPD does not patch the data. It just monitors and restores both the original hidden code and control transfers.

#### 6 Conclusion

We proposed a technique to reconstruct a binary file for static analysis by monitoring the executions of packed DLLs. This method is valid for unknown protection algorithms without the complex reverse analysis. To demonstrate the idea, we have implemented ReconPD based on QEMU. By triggering the execution of entry-point function and exported functions of a DLL, and monitoring all the memory operations and control transfer instructions, it identifies and extracts the code which is written into the memory during execution and constructs a binary file based on the original binary, the codes extracted and the records of control transfers. We show that ReconPD can successfully analyze packed DLLs and the reconstructed binary can be successfully analyzed by static analysis tools, such as IDA Pro.

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## Static Analysis of a Class of Memory Leaks in TrustedBSD MAC Framework<sup>\*</sup>

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**Abstract.** Security labels of subjects and objects are crucial for some security policies and are an essential part of the TrustedBSD MAC framework. We find that security labels not being destroyed properly will result in memory leaks. This paper analyzes the security labels management of the TrustedBSD MAC framework and presents a path-sensitive static analysis approach to detect potential memory leaks caused by the security label management. This approach verifies complete destruction of security labels through compiler-integrated checking rules at compile-time. It achieves complete coverage of execution paths and has low false positive rate.

**Keywords:** static analysis, memory leak, TrustedBSD MAC framework, security label, mygcc.

#### 1 Introduction

The TrustedBSD MAC (Mandatory Access Control) framework (hereinafter referred to as the MAC framework) is a group of hooks inserted in OS (Operating System) kernel to support different security policies [1]. While some security policies employ existing subject and object information (such as UNIX credential data and file permissions) most of the mandatory access control policies require additional information. For example, the BLP [2] confidentiality security policy makes decision based on subject and object sensitivity labels: subjects are assigned clearances, and objects are assigned classifications. Consequently, the MAC framework includes a number of hooks to manage security labels of subjects and objects.

At present, the hooks of the MAC framework are manually inserted into the OS kernel. Although the developers of the MAC framework are highly skilled and experienced hackers, errors are unavoidable when they deal with the highly complex OS kernel (millions of code lines) manually. For example, Figure 2 depicts an improper placement of the security label hooks found in our work on implementing a FreeBSD based trusted operating system. Consequently, the correctness of the hooks placement

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needs to be carefully analyzed. Furthermore, most of the OS kernels are still in the course of development, and more components are being added to them. So, the MAC framework needs to add new hooks or/and revise existing hooks to address the newly added components. This is an error-prone process because the framework developer may not fully understand the impact of the newly added components upon the OS kernel and the MAC framework. Therefore, we need an automatic approach to analyze the correctness of the hooks placement.

The first study on the correctness of the hooks placement is taken by the researchers of the IBM T.J.Waston Research Center. They analyze the authorization hooks placement of the LSM (Linux Security Module) framework [3] using both the dynamic approach and the static approach and successfully find some placement errors [4] [5]. While the efficiency of the dynamic approaches is higher than that of the static approaches, the execution paths coverage rate of the dynamic approach is poor. So, we decide to use the static approach. Zhang et al use the CQUAL [6] to implement the static analysis of hooks placement. However, the CQUAL is flow-insensitive, and cannot implement path-sensitive analysis. Therefore, using the CQUAL cannot find the error in Figure 1. This paper presents a path-sensitive static analysis approach to the correctness verification of the hooks placement related to security labels. This approach is based on Mygcc [7] which can execute user-defined checking rules at compile-time automatically.

The rest of the paper is organized as follows. Section 2 introduces the security label management of the MAC framework. Section 3 describes the extensions to the Mygcc. Section 4 gives result of the static analysis of the MAC framework. Section 5 discusses the related work, and Section 6 concludes the paper.

## 2 Security Label Management

First of all, we give the definition of controlled objects and controlled operations. Then, we analyze the security label management of the MAC framework and depict the memory leak problem resulting from the security label management.

#### 2.1 Controlled Objects and Controlled Operations

A controlled object, also called a protected object, is the object that needs the protection provided by the MAC framework, such as *file*, *inode*, *task*, *socket* and *shared memory* etc.

A controlled operation is the system operation that performs on the controlled objects and may result in security-sensitive events, such as the read operation and the write operation. Though some operations perform on the controlled objects, they are not the controlled operation if they do not cause an information-flow, such as the lock operation and unlock operation of the *inode*.

#### 2.2 Hooks Related to Security Labels

The MAC framework supports security labels through a policy-agnostic labeling primitive, which permits security policies to tag controlled objects with security labels

for decision-making. Security labels are initialized, allocated, and destroyed along with their objects [8]:

Security label initialization occurs when the data structure for a controlled object is initialized, which permits security policies to allocate and initialize memory for the object.

Security label association or creation occurs when an initialized kernel structure is associated with an actual controlled object, such as a file or a process.

Security label destruction occurs when the controlled object is released by the kernel service implementing it. The MAC framework is given the opportunity to release storage for the security label, which permits security policies to free any allocated storage or references associated with that security label.

The security label structure stored in kernel data structures is maintained by the MAC framework: based on the life cycle of the data structure, the MAC framework provides per-object hooks for memory initialization, object allocation, and object destruction, as shown in Figure 1.

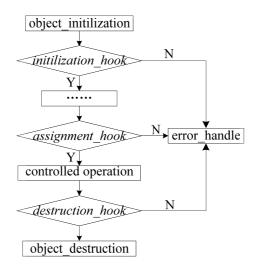


Fig. 1. Hooks related to security labels

#### 2.3 Complete Destruction of Security Labels

As shown in Figure 1, the memory space of a security label must be freed before its host object is freed. Otherwise, there exists a potential memory leak. In the OS kernel, for a specific controlled object there may be many paths to its destruction point. Therefore, we need to ensure that at each path to the controlled object destruction point there is a security label destruction hook. We call this the complete destruction problem. For example, in Figure 2, if the execution follows the path from Line 201 to 227, the *inp* will be freed without its security label being freed. This violates the complete destruction rule of security labels and results in a memory leak.

```
/*$FreeBSD:src/sys/netinet/in_pcb.c,v1.197 $*/
175
     int
176
     in pcballoc(struct socket *so, ...)
177
     {
. . .
189
     #ifdef MAC
190
     error = mac_inpcb_init(inp, M_NOWAIT);
191
     if (error != 0)
192
         goto out;
193
     SOCK_LOCK(so);
194
     mac inpcb create(so, inp);
195
     SOCK_UNLOCK(so);
196
     #endif
197
198
     #ifdef IPSEC
199
     error = ipsec_init_policy(so, &inp->inp_sp);
200
     if (error != 0)
201
        goto out;
. . .
224
     #if defined(IPSEC) || defined(MAC)
225
     out:
226
     if (error != 0)
227
        uma zfree(pcbinfo->ipi zone, inp);
228
     #endif
229
     return (error);
230
     }
```

Fig. 2. Example of a memory leak

## 3 Extensions to Mygcc

The original Mygcc mainly focuses on the general errors in the OS kernel, so we need to extend it to perform the static analysis of the memory leaks related to security labels. First, we introduce the features of Mygcc. Then, we depict the extensions to Mygcc.

#### 3.1 Features of Mygcc

After evaluating various analysis tools that may be suitable for the complete destruction analysis of the MAC framework, we choose the GCC compiler based Mygcc as our basic tool for further extension. Because Mygcc has the following features:

(1) Mygcc supports path-sensitive checking rules.

The complete destruction analysis depends on the path information. For example, though there exists the complete destruction problem in the code fragment in Figure 2, a path-insensitive tool can't find this error.

(2) Mygcc is an intra-procedure analysis tool.

In the OS kernels equipped with the MAC framework, most hooks and controlled operations are in the same function, which is suitable for Mygcc.

(3) Mygcc uses a configuration file to guide checking.

Tools like the CQUAL and SPLINT [9] are based on code annotation, which gives heavy burden to the developers. Furthermore, the annotation process may introduce new errors. However, Mygcc uses a single configuration file to guide checking, and the checking is executed in the compile process.

However, the original Mygcc mainly focuses on the general errors in the OS kernel, such as deadlock and null pointer reference etc. So, we need to extend it to address specific requirements of the complete destruction analysis.

#### 3.2 Extensions to Mygcc

Based on the analysis of the MAC framework, we extend Mygcc to address three requirements: to execute checks from the function entry, to match the types of place-holders and to relate one placeholder to another one.

(1) Executing checks from the function entry

The original Mygcc must have a specific matching node as the starting point, and then traverse the abstract syntax tree until it encounters the *avoid* pattern (success), or another controlled operation (warning). However, the MAC framework needs only to match one controlled operation. So, we extend Mygcc to implement executing checks from the function entry.

The algorithm of the function entry checking is as follows:

```
proc check entry(CFG, condate)
 substs \leftarrow \phi:
 foreach node t \in CFG do
   global_store \leftarrow \phi;
  if condate.from.format spec = "entry"
  then if match(t, condate.to)
    then
      substs \leftarrow substs \cup \{global\_store\};
        fi // match(t, condate.to)
  fi //condate.from.format spec = "entry"
 end //for
 <u>for</u> subst \leftarrow substs <u>do</u>
  global\_store \leftarrow subst;
  list \leftarrow [];
  <u>if</u> condate.from.format_spec = "entry"
  then
       <u>foreach</u> node t \in CFG.entry.succs do
         list \leftarrow [t|list];
        end
  <u>fi</u> //condate.from.format_spec = "entry"
        Do depth first check starting from the element of list
        along the edges of control flow edge;
 end //for
end //proc
```

The basic analysis procedure is shown in Figure 3.

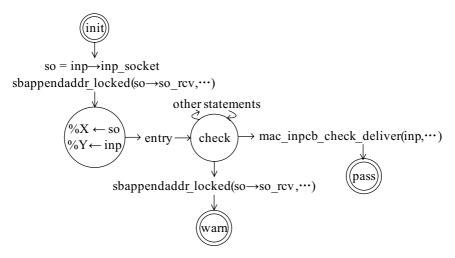


Fig. 3. Basic analysis procedure

(2) Matching the type of the placeholder

The original Mygcc doesn't match the type of the placeholder. However, in the MAC framework, static analysis needs to match the types. For example, in Figure 2, as to the function *uma\_zfree*, identifying the controlled operation depends on the second parameter. Namely, if the type of the second parameter is a structure pointer of type *inpcb*, it matches the controlled operations related to *inpcb*. The algorithm is as follows:

```
proc match(t,pattern)
 arglist \leftarrow \phi
 parmlist \leftarrow \phi
 <u>foreach</u> arg \in t.args <u>do</u>
   arglist \leftarrow [arg|arglist]
 end //for
 foreach parm ∈ pattern.parms do
   arglist = [arg|rest]
   arglist \leftarrow rest
   if parm.type = arg.type
   <u>then</u> global_store \leftarrow [global_store|parm\leftarrowarg]
   else goto fail;
   fi
 end //for
 goto out;
 fail:
   global store \leftarrow \phi;
 out:
end //proc
```

(3) Relating one placeholder to another

In the MAC framework, the parameters of some access authorization are not the controlled objects themselves. Therefore, in order to match access authorization contexts and controlled objects, we need to create the relationship between them. The following is the depth-first checking algorithm based on placeholders matching.

```
proc check_match(CFG, condate)
 for subst \leftarrow substs do
   global store \leftarrow subst
  match\_store \leftarrow global\_to\_match[subst]
  list \leftarrow construct the depth first check starting node list
  while list = [t|rest] do
    list \leftarrow rest
    if \neg visited(t)
    then
      visited(t) \leftarrow true
      if matched(t,condate.to, global_store)
      then warning "reached t"
      else foreach edge \ e = t \leftarrow t' do
        if ¬ matched(e,avoid, match store)
        then list \leftarrow [t'|list]
        fi
       end//for
      <u>fi</u> // matched(t,condate.to, global_store)
    fi // \neg visited(t)
  end //while
 end //for
end //proc
```

Besides the extensions described above, we also accomplish some other extensions to Mygcc to satisfy the static analysis requirements of the MAC framework, and we have issued them as a patch of Mygcc- $1.0.0^{1}$ .

## 4 Static Analysis of Memory Leaks

In addition to extending the Mygcc, another important work is to write checking rules of the static analysis. First, we describe some typical checking rules of the static analysis of the MAC framework. Then, we give the analysis results.

#### 4.1 Rules of Static Analysis

To enforce static analysis of the MAC framework, we write 32 checking rules based on the extended Mygcc *condate*, and they cover the major modules of the FreeBSD kernel. Here, we give two examples of the *condate* rules.

<sup>&</sup>lt;sup>1</sup> http://wiki.freebsd.org/ZhouyiZHOU?action=AttachFile&do=get&target=mygcc.patch

Figure 4 gives the checking rule of the *inpcb* destruction. Line 1 is the name of the rule, Line 2 matches the type of the placeholder and Line 3 indicates the depth-first traverse starting from the function entry. Line 4 gives conditions of warnings and indicates to execute function *uma\_zfree* for the controlled destruction operation. Line 5 gives the conditions that the rule doesn't throw warnings as follows:

• In the execution path, the controlled operation *uma\_zfree* is ahead of the destruction hook *mac\_inpcb\_destroy*.

• The initialization hook of the MAC framework fails. Because the security label hasn't been allocated, it needs not to be destroyed.

```
1 condate inpcb_destory {
2 types "%X struct_inpcb *"
3 from "entry"
4 to "uma_zfree(%_,%X)"
5 avoid "mac_inpcb_destroy(%X)" or
6 +"%w = mac_inpcb_init (%X,%_);%w != 0"
7 } warning("mac_inpcb_destory needed");
```



Figure 5 gives the checking rule of the security label initialization. After the controlled operation  $m\_gethdr$  or  $m\_getcl$  creates an *mbuf* (Line 3 and 4), on the execution path there is a security label initialization hook *mac\_inpcb\_create\_mbuf* (Line 7) ahead of a controlled operation on this *mbuf* (Line 5 and 6).

```
condate mbuf_create {
1
2
   types "%X struct mbuf *"
3
   from "%X = m_gethdr(%_,%_)" or
4
   "%X = m_getcl(%_,%_,%_)"
5
   to "ip_output(%X,%_,%_,%_,%_,%_)" or
6
   "ip6_output(%X,%_,%_,%_,%_,%_)"
7
   avoid "mac_inpcb_create_mbuf(%_,%X)"
8
   }warning("uninitialized mbuf send");
```

Fig. 5. Example of initialization hook checking rules

#### 4.2 Result of Static Analysis

We have analyzed 320 C files based on the standard kernel configuration, and the tool throws 27 warnings. After these warnings are evaluated manually and discussed in the TrustedBSD mail list, 8 of them are confirmed by the FreeBSD community and the other 19 are false positive. 5 of the confirmed warnings are related to *NFS* and *FI-FOFS*, and they may be used to guide the improvement of these subsystems. 3 of the confirmed warnings are related to *IPv4* and *IPv6* network protocol stack, and they have been corrected in the FreeBSD CVS.

Our experiment shows the false positive rate of the verification is 70.4%, which partially results from the imperfect of the checking rules and the algorithms. However, compared to other tools (e.g. in [10] it is 95%), the false positive rate is relatively low.

#### 5 Related Work

[4] proposes a static analysis approach, which leads off the researches on the static analysis of hooks placement. At first, they use the modified GCC to output the locations of controlled operations. Then they use a perl script to annotate the kernel source code with the type information needed by CQUAL. At last, they use CQUAL to perform inter-procedural analysis. Because CQUAL is flow-insensitive, they cannot enforce path-sensitive analysis.

[5] presents a run-time verification tool, which is simpler than that in [4], to verify LSM hooks placement. Their approach begins with logging system call entry/exit/arguments, function entry/exits, controlled operations and authorizations. Then they filter the logging results using filtering rules defined by a rule language. Finally they manually examine authorization graph generated from the filtered results. [5] is not suitable for complete destruction verification because its coverage rate is low and cannot enforce verification automatically.

[11] puts forward a novel approach to automatically inserting authorization hooks into the Linux kernel. Their approach is not suitable for analyzing the MAC framework because of its path-insensitive nature.

There are also a number of general purpose static analysis tools for C programs [12]. After thorough investigation, we find that Mygcc is more suitable for the analysis of the MAC Framework, because it is open-source, path-sensitive and has parsing power of an integrated compiler.

#### 6 Conclusion

We analyze a class of memory leaks resulting from the security label management in the MAC framework. In order to detect potential memory leaks in the MAC framework, we design and implement a static analysis approach based on the extended Mygcc. This approach can cover all execution paths of a specific controlled object to check whether the destruction hook is properly placed. Our work can help the framework developers to improve the correctness of the hooks placement.

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## Efficient Concurrent $n^{poly(\log n)}$ -Simulatable Argument of Knowledge\*

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**Abstract.** In **[16**], Pass generalized the definition of zero knowledge proof and defined  $n^{O(\sigma(n))}$ -simulatable proof which can be simulated by a simulator in  $n^{O(\sigma(n))}$  time. Assuming the existence of one-way permutation secure against sub-exponential circuits and 2-round perfect hiding commitment scheme, an efficient 4-round perfect  $n^{poly(\log n)}$ -simulatable argument of knowledge was presented there.

In this paper, we construct an efficient concurrent  $n^{poly(\log n)}$ simulatable argument of knowledge under more general assumption. The new scheme is 5-round and is based on the existence of one-way permutation secure against sub-exponential circuits. However, for the scheme in [16], if using ordinary  $\Sigma$ -protocol for the corresponding statement as sub-protocol, instead of  $\Sigma$ -protocol with honest verifier perfect zero knowledge, the resulting protocol is not necessarily closed under concurrent composition.

**Keywords:** straight-line  $n^{poly(\log n)}$ -simulatable, argument of knowledge,  $\Sigma$ -protocol.

#### 1 Introduction

Zero knowledge (short for ZK) proof, brought out by Goldwasser, Micali and Rackoff [11], is a protocol between two parties: prover and verifier, in which prover can convince verifier the validity of a statement but reveals nothing. Since its invention, ZK proof has attracted much attention and a great deal of work has been done on it. One of the most fundamental results achieved is that every language in  $\mathcal{NP}$  has a ZK proof [12]. Nowadays, ZK proof has played a central role in study of cryptographic protocols. For example, it can be used in multi-party computation [S]9] to have the parties prove the correctness of their computations.

The definition of ZK proof is formalized through simulation paradigm. That is, for every probabilistic polynomial time verifier, there exists a probabilistic

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polynomial time simulator such that what the simulator outputs is indistinguishable from the real interaction. Although ZK proof is very useful in the design of increasingly complex cryptographic tasks, many limitations are known **IOI13.14**. As a result, general protocols, whose security requirements are formalized by simulation paradigm of ZK proof, inherit these limitations. For example, recently several limitations have been shown concerning the concurrent composition **[6]** of secure two-party and multi-party protocols **II4.15**. In order to overcome some of these limitations, some relaxed notions which are good enough for applications are brought out. A first step in this direction is the definition of witness indistinguishability **[7]**. Recently, Pass **[16]** defined another relaxed notion— $n^{O(\sigma(n))}$ -simulatable proof (argument). It was pointed out that  $n^{O(\sigma(n))}$ -simulatable argument can be used as a sub-protocol to construct universal composable secure protocols.

 $n^{O(\sigma(n))}$ -simulatable argument is that for every probabilistic polynomial time verifier, there exists a simulator with running time  $n^{O(\sigma(n))}$  which can simulate the real conversation. From definition of ZK argument, it is concluded that ZK argument is  $n^{O(1)}$ -simulatable argument. Therefore,  $n^{O(\sigma(n))}$ -simulatable argument is a generalization of definition of ZK argument. Furthermore,  $n^{O(\sigma(n))}$ -simulatable argument is witness indistinguishable. Therefore,  $n^{O(\sigma(n))}$ -simulatable argument is a notion between ZK and witness indistinguishability.

In **[16**], based on the assumption that there exist one-way permutation secure against sub-exponential circuits and 2-round perfect hiding commitment scheme, an efficient 4-round concurrent perfect  $n^{poly(\log n)}$ -simulatable argument of knowledge was constructed. Honest verifier perfect ZK property of  $\Sigma$ -protocol contributes to the "perfect" property of the construction, which in turn results in its closeness under concurrent composition. However, if using ordinary  $\Sigma$ protocol instead of  $\Sigma$ -protocol with honest verifier perfect ZK property as a tool, the resulting scheme is not necessarily closed under concurrent composition.

In this paper, using ordinary  $\Sigma$ -protocol as a tool, we give an efficient concurrent  $n^{poly(\log n)}$ -simulatable argument of knowledge under more general assumption. The scheme is 5-round and is based on the existence of one-way permutation secure against sub-exponential circuits.

It is organized as follows. In section 2, some related notions and definitions are given. In section 3, we present the 5-round concurrent  $n^{poly(\log n)}$ -simulatable argument of knowledge and gives the corresponding proofs.

## 2 Preliminaries

A function f(n) is called negligible, if for every positive polynomial q(n), there exists N such that for all  $n \ge N$ , we have  $f(n) \le 1/q(n)$ .

**Definition 1.** (one-way function secure against sub-exponential circuits [16])  $f: \{0,1\}^* \to \{0,1\}^*$  is called one-way function secure against  $2^{n^k}$  adversary, if the following conditions hold:

- (1) There exists a polynomial time algorithm A, such that A(x) = f(x);
- (2) For any probabilistic algorithm B with running time  $2^{n^k}$ , any positive polynomial  $p(\cdot)$  and all sufficiently large n, any auxiliary input  $z \in \{0,1\}^{poly(n)}$ , we have

$$Pr[B(f(U_n), z) \in f^{-1}(f(U_n)] < 2^{-n^{\kappa}};.$$
(1)

where  $U_n$  is a random variable uniformly distributed in  $\{0,1\}^n$ .

f is called one-way function secure against sub-exponential circuits iff there exists a constant 0 < k < 1 such that f is secure against  $2^{n^k}$  adversary.

**Definition 2.** (Argument of knowledge  $[\underline{\Omega}]$ ) Let R be a binary relation, probabilistic polynomial time machine V is called knowledge verifier for language L with relation R, if the following conditions hold:

- (1) (non-triviality) There exists a probabilistic polynomial time machine P, such that for every  $(x, y) \in R$  all possible interaction of P with V on common input x and auxiliary input y is accepting;
- (2) (knowledge soundness) There exists a probabilistic polynomial time oracle machine K such that for every polynomial time machine  $P^*$  and every  $x, y, r \in \{0, 1\}^*$ , machine K satisfies the following condition: Let p(x, y, r) be the probability that  $P^*_{x,y,r}$  convinces V. If p(x, y, r) > u(|x|) where u(|x|) is a negligible function, then machine K can output  $s \in R(x)$  with probability at least 1 - u(|x|) in polynomial time.

If V is a knowledge verifier for language L with relation R and P is a machine satisfying non-triviality condition, (P, V) is called argument of knowledge for language L. K is called knowledge extractor.

**Definition 3.** (straight-line  $n^{O(\sigma(n))}$ -simulatable argument of knowledge [16]) Let (P, V) be an argument of knowledge for language  $L \in \mathcal{NP}$  with relation R. (P, V) is called straight-line  $n^{O(\sigma(n))}$ -simulatable argument of knowledge, if for every probabilistic polynomial time machine  $V^*$ , there exists a simulator S with running time  $n^{O(\sigma(n))}$  such that the following two ensembles are computational indistinguishable:

(a)  $\{ < P(y), V^{\star}(z) > (x) \}_{z \in \{0,1\}^{\star}, x \in L}$ , where  $y \in R(x)$ ; (b)  $\{ < S, V^{\star}(z) > (x) \}_{z \in \{0,1\}^{\star}, x \in L}$ .

**Definition 4.** (straight-line concurrent  $n^{O(\sigma(n))}$ )-simulatable argument of knowledge [16]) Let (P, V) be an argument of knowledge for language  $L \in \mathcal{NP}$ with relation R. (P, V) is called straight-line concurrent  $n^{O(\sigma(n))}$ -simulatable argument of knowledge, if for every probabilistic polynomial time oracle machine Athat is not allowed to rewind the oracle it has access to, for any positive polynomial g(n), there exists a simulator S(i, x) with running time  $n^{O(\sigma(n))}$  such that the following two ensembles are computational indistinguishable:  $(a) \{A^{P(x_1,y_1),\dots,P(x_{g(n)},y_{g(n)})}(z,x_1,\dots,x_{g(n)})\}_{z\in\{0,1\}^{\star},x_1,\dots,x_{g(n)}\in L}, where \ y_i \in R(x_i) \ for \ i=1,\dots,g(n);$ 

(b)  $\{A^{S(1,x_1),\dots,S(g(n),x_{g(n)})}(z,x_1,\dots,x_{g(n)})\}_{z\in\{0,1\}^*,x_1,\dots,x_{g(n)}\in L}$ 

**\Sigma-Protocol ([5]):**  $\Sigma$ -protocol is a special 3-move witness indistinguishable proof (argument) of knowledge. Let the conversation in the interaction be (a, e, z).  $\Sigma$ -protocol for language  $L \in \mathcal{NP}$  with relation R is required to satisfy the following conditions:

- (a) (special soundness) For common input x, from any pair of accepting conversations (a, e, z) and (a, e', z') where  $e \neq e'$ , one can efficiently compute w such that  $(x, w) \in R$ ;
- (b) (special honest verifier computational zero knowledge) There exists a simulator M such that on input x and the challenge e, M can output an conversation which is computational indistinguishable from the real interaction.

**Commitment Schemes:** A Commitment scheme is a two-party interactive protocol consisting of two phases. S and R are polynomial time machines. In the first phase, sender S computes a commitment c to value m and sends c to receiver R; In the second phase, S reveals the value m along with the random coins used to R and R checks the validity of the commitment c. A commitment scheme is required to satisfy the following property:

- (a) Hiding: two commitments to different values are computational indistinguishable; If the two commitments to different values are identically distributed, the commitment scheme is perfectly hiding;
- (b) Binding: Once having sent the commitment c to m, except for negligible probability, S cannot reveal a different value  $m' \neq m$  such that c is also a valid commitment to m'.

If for  $m' \neq m$ , the two sets Com(m) and Com(m') is disjoint, it is called perfectly binding, where Com(m) and Com(m') are sets of all possible commitments to m and m' respectively.

# 3 Concurrent $n^{poly(\log n)}$ -Simulatable Argument of Knowledge

In this section, using ordinary  $\Sigma$ -protocol as a tool, an efficient 5-round  $n^{poly(\log n)}$ -simulatable argument of knowledge which is closed under concurrent composition is presented.

It was pointed out that  $n^{poly(\log n)}$ -simulatable argument of knowledge is not necessarily closed under concurrent composition [16]. In the construction of [16], if using ordinary  $\Sigma$ -protocol as a sub-protocol, instead of  $\Sigma$ -protocol with honest verifier perfect ZK property, the reduction used to prove concurrency fails. The reason is that when using hybrid argument to reduce concurrency to stand-alone circumstances, some oracle queries to the simulator are required to be answered by polynomial time algorithm, not by oracles. However, the simulator can only compute these answers in  $n^{poly(\log n)}$  time. To solve this problem, the new scheme works in the following way: first, V executes a  $\Sigma$ -protocol to prove statement  $s \in L_1 \lor t \in L_2$ . Then, P executes another  $\Sigma$ -protocol to prove  $x \in L \lor (s, t) \in L_3$ . Because witness for  $t \in L_2$  is also a witness for  $(t, c) \in L_3$ , by special soundness of  $\Sigma$ -protocol, we can extract a witness for  $t \in L_2$  from  $V^*$ . With the witness, the queries can be computed efficiently. Thus, the reduction works.

Let  $f : \phi : \{0,1\}^* \to \{0,1\}^*$  be one-way permutation secure against  $2^{n^k}$ adversary, where the constant 0 < k < 1. Suppose  $Com(\cdot)$  be a perfectly binding commitment scheme and G be a pseudo-random generator stretching strings of length n to 2n. Three Languages are defined respectively as follows:  $L_1 = \{s : \exists m, a, s.t.s = Com(m, a)\}, L_2 = \{t : \exists r, s.t.t = f(r)\}, L_3 = \{(t, c) : \exists r, s.t.t = f(r) \land c = G(r)\}.$ 

#### Construction of Protocol $\Pi$

Common Input:  $x \in L$ , where the length of x is n. Auxiliary Input of Prover  $P: y \in R(x)$ .

- (P1): P randomly chooses m, then he computes s = Com(m) and sends s to V;
- (V1): V randomly picks  $r \in \{0, 1\}^{(\log n)^m}$ , computes t = f(r) and sends t to P, where  $m = 1 + \frac{1}{k}$ . At the same time, V uses  $\Sigma$ -protocol to prove statement  $s \in L_1 \lor t \in L_2$  and sends the first round message  $a_1$  to P;
- (P2): P produces a random challenge  $e_1$  for message  $a_1$  and sends it to V. Also, he randomly picks  $c \in \{0, 1\}^{2(\log n)^m}$  and uses  $\Sigma$ -protocol to prove statement  $x \in L \lor (t, c) \in L_3$ . He sends c and the first round message  $a_2$ of  $\Sigma$ -protocol to V;
- (V2): V produces a random challenge  $e_2$  for message  $a_2$  and computes message  $z_1$  to answer challenge  $e_1$ . He sends  $e_2, z_1$  to P;
- (P3): If  $(a_1, e_1, z_1)$  is an accepting conversation of statement  $s \in L_1 \lor t \in L_2$ , P produces message  $z_2$  and sends it to V to answer challenge  $e_2$ ; Otherwise, P aborts;
- (V3): If  $(a_2, e_2, z_2)$  is an accepting conversation of statement  $x \in L \lor (t, c) \in L_3$ , V output 1. Otherwise, he outputs 0.

**Lemma 1.** If there exists one-way permutation secure against sub-exponential circuits, then protocol  $\Pi$  is straight-line  $n^{poly(\log n)}$ -simulatable argument of knowledge.

*Proof.* First, *Completeness* of protocol  $\Pi$  comes from completeness of  $\Sigma$ -protocol for statement  $x \in L \lor (t, c) \in L_3$ . For  $x \in L$ , P can use  $y \in R(x)$  to prove the statement  $x \in L \lor (t, c) \in L_3$ .

Second, we prove knowledge soundness of protocol  $\Pi$ . For any probabilistic polynomial time machine  $P^*$  satisfying that  $Pr[< P^*, V > (x)] = p$  is nonnegligible in |x|, there exists a probabilistic polynomial time oracle machine K such that K can extract witness for statement  $x \in L \lor (t, c) \in L_3$  with probability at least 1 - u(|x|), where u(|x|) is a negligible function. For these accepting transcripts produced by  $P^*$ , assume that there exists non-negligible fraction of (s, t), such that K can extract witnesses of  $(t, c) \in L_3$  with nonnegligible probability from the transcripts containing (s, t). For such (s, t), we can construct a malicious verifier  $V^{\star\star}$  by incorporating algorithm  $P^{\star}$  such that  $V^{\star\star}$  can distinguish the witnesses for statement  $s \in L_1 \lor t \in L_2$ .

Algorithm  $V^{\star\star}$  is constructed as follows: On input (s, t) and auxiliary input  $x, V^{\star\star}$  runs algorithm  $P^{\star}$  in the following way:

- (1) For statement  $s \in L_1 \lor t \in L_2$ ,  $V^{\star\star}$  interacts with prover externally to prove the statement. When receiving messages,  $V^{\star\star}$  gives the messages to  $P^{\star}$ ; When is required to send messages, he reads messages from the communication-tape of  $P^{\star}$  and sends them to prover.
- (2) For statement  $x \in L \lor (t, c) \in L_3$ ,  $V^{\star\star}$  acts as honest verifier to interact with  $P^{\star}$  internally. Then,  $V^{\star\star}$  runs knowledge extractor for statement  $x \in L \lor (t, c) \in L_3$  to output a witness w. If w is a witness for  $x \in L$ ,  $V^{\star\star}$  outputs 0. If w is a witness for  $(t, c) \in L_3$ ,  $V^{\star\star}$  outputs 1.

From the construction of  $V^{\star\star}$ , when prover uses a witness for  $t \in L_2$  to interact with  $V^{\star\star}$ , the probability that  $V^{\star\star}$  outputs 1 is non-negligible. When prover uses a witness for  $s \in L_1$  to interact with  $V^{\star\star}$ , the view of  $V^{\star\star}$  is independent of witness of  $(t, c) \in L_3$ . So in this case the probability that  $V^{\star\star}$  outputs 1 is negligible. Hence,  $V^{\star\star}$  can distinguish the witnesses for statement  $s \in L_1 \vee$  $t \in L_2$ , which contradicts with witness indistinguishability. Therefore, except for negligible probability, what K extracts from an accepting conversation is a witness for  $x \in L$ . That is, knowledge soundness of protocol  $\Pi$  holds.

At last, we prove that  $\Pi$  is straight-line  $n^{poly(\log n)}$ -simulatable. For any probabilistic polynomial time verifier  $V^*$ , simulator S is constructed as follows: First, he randonly chooses a value m and computes s = Com(m). Then, on receiving message  $(t, a_1)$  from  $V^*$ , S performs exhaustive search to find  $r \in \{0, 1\}^{(\log n)^m}$ such that f(r) = t. Then, S uses r to compute c = G(r) and takes r as a witness for statement  $x \in L \lor (t, c) \in L_3$  to interact with  $V^*$ . The running time of S is  $poly(2^{(\log n)^m}) = poly(n^{(\log n)^{\frac{1}{k}}})$ . By witness indistinguishability of  $\Sigma$ -protocol, the distribution produced by simulator is computational indistinguishable from the real interaction of P and  $V^*$ .

From above, we can conclude that protocol  $\Pi$  is straight-line  $n^{poly(\log n)}$ -simulatable argument of knowledge.

**Lemma 2.** In protocol  $\Pi$ , for any probabilistic polynomial time  $V^*$ , if he can convince P the statement  $s \in L_1 \lor t \in L_2$  with non-negligible probability, then except for negligible probability, what the extractor  $K_1$  extracts from  $V^*$  is a witness for  $t \in L_2$ .

*Proof.* Assume that there exists a probabilistic polynomial time  $V^*$  who can convince P the statement  $s \in L_1 \lor t \in L_2$  with non-negligible probability, the probability that  $K_1$  outputs a witness for  $s \in L_1$  is non-negligible. we can construct a polynomial time algorithm B to break the hiding property of commitment scheme.

Algorithm B can be constructed as follows: On input  $s_b, m_0, m_1$ , where  $s_b$  is a commitment to  $m_0$  or  $m_1$ , B runs the knowledge extractor  $K_1$  for  $s_b \in L_1 \lor t \in$ 

 $L_2$ . When  $K_1$  fails or outputs a witness for  $t \in L_2$ , B aborts. Otherwise, suppose  $K_1$  output m. If  $m = m_0$ , B outputs 0, else B outputs 1.

Hence, B can distinguish the commitment of  $m_0$  and  $m_1$  with non-negligible probability, which contradicts with the hiding property of commitment scheme. Therefore, except for negligible probability, what  $K_1$  extracts is a witness for  $t \in L_2$ .

**Lemma 3.** If there exists one-way permutation secure against sub-exponential circuits, then protocol  $\Pi$  is straight-line concurrent  $n^{poly(\log n)}$ -simulatable.

*Proof.* For any probabilistic polynomial time oracle machine A that is not allowed to rewind the oracle it has access to, for any positive polynomial g(n), let S(i,x) = S(x) where S(x) is the simulator constructed in Lemma 1. It is sufficient to prove the following two ensembles are computational indistinguishable:

(a) 
$$\{A^{P(x_1,y_1),\dots,P(x_{g(n)},y_{g(n)})}(z,x_1,\dots,x_{g(n)})\}_{z\in\{0,1\}^*,x_1,\dots,x_{g(n)}\in L}$$
, where  $y_i \in R(x_i)$  for  $i=1,\dots,g(n)$ ;

(b) 
$$\{A^{S(1,x_1),\dots,S(g(n),x_{g(n)})}(z,x_1,\dots,x_{g(n)})\}_{z\in\{0,1\}^*,x_1,\dots,x_{g(n)}\in L}$$

We use hybrid argument to prove it. Assume that there exists an polynomial time oracle machine A and  $z \in \{0,1\}^*$ , such that for infinitely many n, the above two ensembles are distinguishable by a probabilistic polynomial time distinguisher D. The distinguishing gap is  $\frac{1}{q(n)}$ , where q(n) is a positive polynomial. For  $i = 0, 1, 2, \dots, g$ , experiment  $H_i$  is defined in the following way: For the first i sessions,  $P(x_i, y_i)$  acts as the oracle to answer queries of A. For the subsequent sessions, S(j, x) is taken as the oracle to return answers to A's queries, where  $j = i + 1, \dots, g$ . It is seen that experiment  $H_0$  corresponds to ensemble (b) and experiment  $H_g$  corresponds to ensemble (a). Then, there must exist  $i \in \{0, 1, \dots, g\}$  such that D can distinguish  $H_i$  and  $H_{i+1}$  with distinguishing gap  $\frac{1}{g(n)g(n)}$ .

Using algorithm A, we can construct an probabilistic polynomial time oracle machine  $A_1$ : On input  $x_{i+1}$  and auxiliary input  $z' = z \circ x_1 \circ \cdots \circ x_i \circ x_{i+2} \circ \cdots \circ x_{g(n)} \circ y_1 \circ \cdots \circ y_i \circ y_{i+2} \circ \cdots \circ y_{g(n)}$ ,  $A_1$  runs algorithm  $A(x_1, \cdots, x_{g(n)}, z)$  in the following way:

- (1) For  $j = 1, 2, \dots, i$ , when A queries for the *j*-th session,  $A_1$  runs algorithm  $P(x_j, y_j)$  to provide answers to A;
- (2) For  $j = i + 2, \dots, g$ , when A queries for the j-th session,  $A_1$  invokes the extractor  $K_1$  for statement  $s_j \in L_1 \lor t_j \in L_2$  to get a witness  $r_j$  for statement  $t_j \in L_2$  (by lemma 2). Then,  $A_1$  uses  $r_j$  as a witness to compute messages to answer A;
- (3) For the i + 1-th session,  $A_1$  provides answers to A by access to oracle  $P(x_{i+1}, y_{i+1})$  or  $S(i+1, x_{i+1})$ . At last,  $A_1$  outputs what A outputs.

From the construction of algorithm  $A_1$ , it is seen that D can distinguish  $A_1^{P(x_{i+1},y_{i+1})}$  and  $A_1^{S(i+1,x_{i+1})}$ , which contradicts with the fact that  $\Pi$  is straight-line  $n^{poly(\log n)}$ -simulatable. Therefore, ensembles (a) and (b) are computational indistinguishable. That is, protocol  $\Pi$  is straight-line concurrent  $n^{poly(\log n)}$ -simulatable.

From above three lemmas, we conclude that  $\Pi$  is straight-line concurrent  $n^{poly(\log n)}$ -simulatable arguments of knowledge. Therefore, we have

**Theorem 1.** If there exists one-way permutation secure against sub-exponential circuits, then every language  $L \in \mathcal{NP}$  has an efficient 5-round straight-line concurrent  $n^{poly(\log n)}$ -simulatable argument of knowledge.

## 4 Conclusion

 $n^{O(\sigma(n))}$ -simulatable argument of knowledge is a generalization of definition of ZK argument of knowledge and is very useful in constructing multi-party secure computation schemes and universal composable secure protocols. Under the assumption that there exists one-way permutation secure against sub-exponential circuits, using ordinary  $\Sigma$ -protocol as a tool, we construct an efficient 5-round  $n^{poly(\log n)}$ -simulatable argument of knowledge. The new scheme is closed under concurrent composition.

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# New Constructions for Reusable, Non-erasure and Universally Composable Commitments

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Abstract. This paper proposes a novel construction of reusable and non-erasure commitment schemes in the common reference string model. We show that our implementation is secure in the universally composable paradigm assuming that the decisional Diffie-Hellman problem over a squared composite modulus of the form N = pq is hard. Our methodology relies on state-of-the-art double trap-door public-key encryption protocols so that a simulator in charge of a common reference string can extract messages of cipher-text rather than the equivocability of underlying cryptographic systems. As a result, our method differs from those presented in [2] and [7]. The double trap-door mechanism is of great benefit to an ideal-world simulator since no modifications will be charged to unopened commitments in case that the participants who generated these commitments are corrupted, and thus enables us to implement efficient simulators in the ideal-world.

Keywords: Non-erasure, reusability, simulator, universally composable commitment.

### 1 Introduction

Universally composable (UC) commitments guarantee that commitment protocols behave like an ideal commitment service, even when concurrently composed with an arbitrary set of protocols. A commitment protocol with UC-security implies that it is non-malleable not only respect to other copies of the same protocol but even with respect to other protocols. To prove security of a commitment scheme in the UC model, one must construct an ideal-world adversary such that an adversary's view of a real-life execution of a commitment protocol can be simulated given just the data the adversary is entitled to.

Considering a scenario where a commitment protocol execution between a corrupted committer  $P_i$  and an honest receiver  $P_j$  is performed, and a dummy adversary is assumed to merely send messages that are generated by an environment  $\mathcal{Z}$ , and relay to the messages sent to  $P_i$ . Assume that the environment  $\mathcal{Z}$  secretly picks a random bit  $b \in \{0, 1\}$  at the beginning of the commitment protocol execution and generates the messages for  $P_i$  by running the protocol of the honest committer for b and  $P_j$ 's answers. To simulate this adversarial behavior, an ideal world simulator  $\mathcal{S}$  must first extract input bit b from the messages generated by  $\mathcal{Z}$  on the tape of the ideal-world uncorrupted party  $\tilde{P}_i$  and then copies

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this extracted bit b to the functionality  $\mathcal{F}_{\text{com}}$ . If the commitment scheme allows the simulator to successfully extract the committed bit, then the commitment is not secure since the successful simulator must come up with the true bit b at the commitment step which contradicts the security of a commitment protocol.

To simulate the adversarial behavior above, Canetti and Fischlin [2] have proposed two methods sketched below for constructing an ideal world simulator: 1) one-time-usable common reference string model: a fresh common reference string  $\sigma$  must be generated for each committed bit in this model. The common reference string  $\sigma$  is constructed from Crescenzo, Ishai and Ostrovsky's protocol with a specified trap-door pseudo-random generator which guarantees the required equivocability and thus allows a simulator to adapt committed values of unopened commitment in case that the participants who generated these commitments are corrupted. We remark that the computation complexity of their basic construction is heavy since the commitment scheme uses one-time common reference string, and each generation of a common reference string costs at least 3n computations of the underlying one-way permutation, where n is a security parameter for the underlying pseudo-random generator stretching n-bit inputs to 4n-bit outputs.

2) reusable common reference string model: a common reference string in this model is constructed from claw-free trap-door permutation pairs and a public key encryption scheme to guarantee that a simulator can produce a correct cipher-text and a fake random string suggesting that the cipher-text has been obtained by the oblivious sampling procedure. This common reference string can be reused for committing multi-bit. To prove the security of the commitment schemes in this model, Canetti and Fischlin first assume that a committer generates a cipher-text of a committer's identity and then erases the randomness used for the encryption of  $(0^n, P_i)$  before the commitment message is sent. Assuming the existence of obliviously sample encryption schemes, Canetti and Fischlin further proposed an improved construction such that the unpleasant assumption of erasure of randomness used for the encryption is eliminated.

The security proof of commitment schemes in the one-time-usable common reference string model relies on the equivocable bit commitment of Crescenzo, Ishai and Ostrovsky [8] which in turn is a modification of Naor's commitment scheme [10]. The security proof of commitment schemes in the reusable common reference string model relies on oblivious sample property of the underlying encryption scheme (constructed from Cramer and Shoup's encryption scheme [5]) such that one cipher-text is open while the remaining cipher-text is never opened.

#### 1.1 Recent Work

At Crypto 2008, Dodis, Shoup and Walfish [7] proposed an efficient constructions of composable commitments based on  $\Omega$  protocols. An  $\Omega$ -protocol first introduced and formalized by Garay, MacKenzie and Yang [9] in the context of zero-knowledge proof system, is similar with the notion of a  $\Sigma$ -protocol, with an extra property that one can generate a public parameter  $\sigma$  of a system together with a trap-door information  $\tau$ , such that the knowledge of  $\tau$  allows one to extract the witness from any valid conversation between a prover and sender. They use the same technique of Canetti and Fischlin [2] in the sense that the proof of security relies on the equivocability of the underlying cryptographic systems (e.g., Crescenzo, Ishai and Ostrovsky's bit commitment scheme, Canetti and Fischlin's equivocable encryption scheme, and Garay, MacKenzie and Yang's  $\Omega$ -protocol) such that a simulator can extract inputs of corrupted parties.

### 1.2 This Work

In this paper, we propose a novel construction of reusable, non-erasure commitment protocols and show that:

**Theorem.** Our protocol (described in Section 4) is universally composable in the common reference string model assuming that the decisional Diffie-Hellman problem over a squared composite modulus of the form N = pq is hard.

The technique: The novelty of this paper is that an ideal-world simulator does not rely on the equivocability of underlying cryptographic systems but on the existence of double trapdoor public-key encryption schemes, (the existence of semantically secure double trap-door encryption scheme is not a problem, see Section 3 for more details). That is, a simulator in our model is allowed to generate a public-key/secret key pair  $(pk_S, sk_S)$  while a simulated participant  $P_i$  is allowed to generate its own public-key  $(pk_i, sk_i)$ . The double trap-door mechanism ensures that any cipher-text generated by the encryption scheme  $E_{pk_i}(m, r_m)$  can be correctly decrypted with the help of trap-door string  $sk_S$ , where  $m \in \mathcal{M}_{pk_i}$  and  $\mathcal{M}_{pk_i}$  is the message space specified by the public-key  $pk_i$ . Notice that if Bresson, Catalano and Pointcheval's public-key encryption scheme is applied, then  $\mathcal{M}_{pk_i} = \mathcal{M}_{pk_S}$  (throughout the paper, we simply abbreviate the notation  $\mathcal{M}_{pk_i}$  as  $\mathcal{M}$ ). Furthermore, we are able to show that the proposed commitment scheme enjoys the following nice features:

1) Simulatability: In our model, a public key  $pk_S$  is a common reference string that will be used for constructing universally composable commitments. To simulate the scenario where the environment  $\mathcal{Z}$  secretly picks a random element  $m \in \mathcal{M}$  at the beginning of the protocol execution and generates the messages for  $P_i$  by running the protocol of the honest committer for m and  $P_j$ 's answers, we simply allow a simulator  $\mathcal{S}$  to decrypt the delivered cipher-text  $E_{pk_i}(m, r_m)$  and obtain the exact value  $m \in \mathcal{M}$ ; The simulator  $\mathcal{S}$  then instructs  $\tilde{P}_i$  to forward the extracted string  $m \in \mathcal{M}$  the functionality. Consequently, the simulator  $\mathcal{S}$  works if it holds the auxiliary string  $sk_S$ .

**Non-erasure**: A protocol is called non-erasure if all random strings used for generating cipher-text by a corrupted participant is revealed to the adversary. It will be clear that any participant in our model is forced to provide its randomness to the adversary once it is corrupted (see Section 4 for more details).

**Reusability**: Any public-key and secret-key pair  $(pk_i, sk_i)$  (including the common reference string  $pk_S$ ) of a double trap-door public-key encryption scheme

generated by a real-world participant  $P_i$  is reusable in our model (see Section 4 for more details).

#### 1.3 Why Our Methodology Is Interesting?

It is important to evident why our methodology is interesting. Two evidences are due to the following observations:

- efficient constructions of simulators: consider a scenario where a simulated real-world adversary  $\mathcal{A}$  demands to corrupt a real-world party  $P_i$ . To simulate this situation, an ideal-world simulator  $\mathcal{S}$  must corrupt the corresponding party  $\tilde{P}_i$  (a dummy party of  $P_i$ ) in the ideal model and learns all internal information of the party. To deal with such an active corruption, the technique presented in [2] and [7] must instruct  $\mathcal{S}$  to modify all decommitment information about the unopened commitment of this party  $P_i$  to match the received data and hands this modified internal information to  $\mathcal{A}$ . A simulator  $\mathcal{S}$  in our model, however needs not to adapt possible decommitment of this party since the trap-door information  $sk_p$  allows to  $\mathcal{S}$  to decrypt a valid cipher-text generated by  $P_i$ . As a result, the double-trap-door public-key encryption based construction allows us to implement an efficient ideal-world simulator  $\mathcal{S}$  corresponding to the real-world adversary  $\mathcal{A}$ .
- concise of common reference string: a common reference string used in our commitment scheme is a public key  $pk_S$  of a simulator ( $pk_S = N$ , if Bresson, Catalano and Pointcheval's public-key encryption scheme is applied). The common reference strings used in [2] consists of two parts: a string describing two claw-free permutations and a string describing a public key encryption scheme; The common reference string used in [7] consists of two parts: a string describing an identity of a participant and a string describing  $\Omega$ -protocol. As a result, the common reference string used in this paper is much more concise than that used in [2] and [7].

**Road map:** The rest of this paper is organized as follows: In Section 2, preliminaries including the notions of UC security and notions of functionalities are briefly sketched; In Section 3, a double trap-door encryption scheme due to Bresson, Catalano and Pointcheval is sketched. The proposed commitment protocol is described and analyzed in Section 4. We conclude our work in Section 5.

### 2 Preliminaries

We work in the standard universally composable framework, where all participants are modeled as probabilistic polynomial time (PPT) Turing machines. Security of protocols is defined by comparing the protocol execution to an ideal process for carrying out the desired task. Namely, the process of executing a protocol in the presence of an adversary and in a given computational environment is first formalized. Next an ideal processing for carrying out the task at hand is formalized. In the ideal processing the parties do not communicate with each other; instead they have access to an ideal functionality which is essentially an incorruptible trust party that is programmed to capture the desired requirements from the task at hand. A protocol is said to securely realize a task if the processing of running the protocol emulates the ideal process of that task. We assume the reader is familiar with the standard notion of UC security. The detailed descriptions of the executions, and definitions of  $IDEAL_{\mathcal{F},\mathcal{S},\mathcal{Z}}$  and  $REAL_{\pi,\mathcal{A},\mathcal{Z}}$  are omitted and refer to the reader [3] for more details.

### 2.1 The Common Reference String Model

The functionality of common reference strings (crs) model described in Fig.1. assumes that all participants have access to a common string that is drawn from some specified distribution  $\mathcal{D}$ .

## Functionality $\mathcal{F}_{crs}^{\mathcal{D}}$

 $\mathcal{F}^{\mathcal{D}}_{crs}$  proceeds as follows, when parameterized by a distribution  $\mathcal{D}$ .

- when receiving a message (sid,  $P_i$ ,  $P_j$ ) from  $P_i$ , let  $\mathsf{crs} \leftarrow \mathcal{D}(1^n)$  and send (sid,  $\mathsf{crs}$ ) to  $P_i$ , and send ( $\mathsf{crs}$ , sid,  $P_i$ ,  $P_j$ ) to the adversary, where sid is a session identity. Next when receiving (sid,  $P_i$ ,  $P_j$ ) from  $P_j$  (and only from  $P_j$ ), send (sid,  $\mathsf{crs}$ ) to  $p_j$  and to the adversary, and halt.

Fig. 1. Functionality  $\mathcal{F}_{crs}^{\mathcal{D}}$  (due to 2)

The common reference string model  $\mathcal{F}_{crs}$  produces a string with a distribution that can be sampled by a PPT algorithm  $\mathcal{D}$ .

### 2.2 The Commitment Functionalities

The ideal functionality of a commitment scheme is described in Fig.2.

To capture the notion of reusability, we must define the functionality of multi commitment, de-commitment processes (see Fig.3. for more details).

**Definition 1.** Let  $\mathcal{F}_{com}$  be a functionality. A protocol  $\pi$  is said to universally composable realize  $\mathcal{F}_{com}$  if for any adversary  $\mathcal{A}$ , there exists a simulator  $\mathcal{S}$  such that for all environments  $\mathcal{Z}$ , the ensemble  $IDEAL_{\mathcal{F}_{com},\mathcal{S},\mathcal{Z}}$  is computationally indistinguishable with the ensemble  $REAL_{\pi,\mathcal{A},\mathcal{Z}}$ .

**Definition 2.** If the protocol securely realizes the functionality  $\mathcal{F}_{mcom}$ , then it is called a reusable common reference string and universally composable commitment protocol.

#### Functionality $\mathcal{F}_{com}$

 $\mathcal{F}_{com}$  proceeds as follows, running with parties  $P_1, \ldots, P_n$  and an adversary  $\mathcal{S}$ 

- Upon receiving a value (commit, sid,  $P_i$ ,  $P_j$ ,  $m \in \mathcal{M}$ ), record the value m, send the message (receipt, sid,  $P_i$ ,  $P_j$ ) to  $P_j$  and S, and ignore any sequent commit messages, where sid is a session identity.
- Upon receiving a value (open, sid,  $P_i$ ,  $P_j$ ) from  $P_j$ , proceed the following computations: if some value m was previously recorded, then send the message (open, sid,  $P_i$ ,  $P_j$ , m) to  $P_j$  and S and halt; otherwise halt

Fig. 2. Functionality  $\mathcal{F}_{com}$  (due to 2)

#### Functionality $\mathcal{F}_{mcom}$

 $\mathcal{F}_{mcom}$  proceeds as follows, running with parties  $P_1, \ldots, P_n$  and an adversary  $\mathcal{S}$ 

- Upon receiving a value (commit, sid, cid,  $P_i$ ,  $P_j$ ,  $m \in \mathcal{M}$ , record the tuple (cid,  $P_i, P_j, m$ ), send the message (receipt, sid, cid,  $P_i, P_j$ ) to  $P_j$  and S, and ignore any subsequent (commit, sid, cid,  $P_i, P_j, \ldots$ ) messages, where sid is a session identity, and cid is a commitment identity.
- Upon receiving a value (open, sid, cid,  $P_i$ ,  $P_j$ ) from  $P_j$ , proceed the following computations: if the tuple (sid, cid,  $P_i$ ,  $P_j$ ,  $m \in \mathcal{M}$ ) was previously recorded, then send the message (open, sid, cid,  $P_i$ ,  $P_j$ ,  $m \in \mathcal{M}$ ) to  $P_j$  and  $\mathcal{S}$  and halt; otherwise halt

Fig. 3. Functionality  $\mathcal{F}_{mcom}$  (due to 2)

### 3 Building Blocks

Paillier's encryption scheme and its variations will be used as building blocks for implementing reusable, non-erasure and universally composable commitment schemes.

#### 3.1 Paillier's Encryption Scheme

Paillier investigated a novel computational problem called the composite residuosity class problem (CRS), and its applications to public key cryptography in **11**.

**Decisional composite residuosity class problems:** Let N = pq, where p and q are two large safe prime numbers. A number z is said to be a N-th residue modulo  $N^2$ , if there exists a number  $y \in Z_{N^2}^*$  such that  $z = y^N \mod N^2$ . The decisional composite residuosity class problem states the following thing: given

 $z \in_r Z_{N^2}^*$  deciding whether z is N-th residue or non N-th residue. The decisional composite residuosity class assumption means that there exists no polynomial time distinguisher for N-th residues modulo  $N^2$ .

**Paillier's encryption scheme:** The public key is a 2k-bit RSA modulus N=pq, where p, q are two large safe primes with length k and the secret key is (p, q). The plain-text space is  $Z_N$  and the cipher-text space is  $Z_{N^2}^*$ . To encrypt a message  $m \in Z_N$ , one chooses  $r \in Z_N^*$  uniformly at random and computes the cipher-text as  $E_{PK}(m,r) = g^m r^N \mod N^2$ , where g = (1+N) has order N in  $Z_{N^2}^*$ . The private key is (p,q). It is straightforward to verify that given  $c = (1+N)^m r^N \mod N^2$ , and the trapdoor information (p,q), one can first compute  $c_1=c \mod N$ , and then compute r from the equation  $r=c_1^{N^{-1} \mod \phi(N)} \mod N$ ; Finally, one can compute m from the equation  $cr^{-N} \mod N^2 = 1 + mN$ . The encryption function is homomorphic, i.e.,  $E_{PK}(m_1, r_1) \times E_{PK}(m_2, r_2) \mod N^2 = E_{PK}(m_1 + m_2 \mod N, r_1 \times r_2 \mod N)$ . Paillier's scheme is semantically secure if the decisional composite residuosity class problem is hard.

### 3.2 Bresson-Catalano-Pointcheval Cryptosystem

At Asiacrypt'03, Bresson, Catalano and Pointcheval  $\blacksquare$  presented a double encryption scheme based on the hardness assumption that inverting RSA[N, N] (i.e., RSA with public exponent set to N, a variant of Paillier's encryption scheme  $\blacksquare$ ) which is sketched below:

- Key generation algorithm KG: On input a security parameter n, KG produces composite modulus of the form N = pq that is a product of two safe primes p and q, and g that is an element of order  $\lambda(N)$  in  $Z_{N^2}^*$ , where  $\lambda(\cdot)$  is Carmichael function. KG randomly chooses  $z \in [0, N^2/2]$  and sets  $h = g^z \mod N^2$  (we remark that g can be an element of order p'q' as well, see [4] for more details). The outputs KG is a pair of public/secret keys (pk, sk), where pk = (N, g, h), sk = (z, p, q).
- Encryption algorithm E: to encrypt a message  $m \in Z_N$ , one chooses a random value  $r \in [0, N/4]$ , and computes the cipher-text (u, v) where  $u = g^r \mod N^2$  and  $v = h^r (1+N)^m \mod N^2$ .
- Decryption algorithm D: to decrypt a cipher-text (u, v), two methods are possible: 1) computing (1+mN) as  $v/u^z$  with the help of the trapdoor string z; 2) computing  $v^{\lambda(N)} = 1 + m\lambda(N)N$  with the help of the trap-door string (p,q).

Assuming that the decisional Diffie-Hellman problem is hard over a squared composite modulus of the form N = pq, Bresson-Catalano-Pointcheval cryptosystem is semantically secure (see  $\Pi$  for more details).

## 4 The Protocols

In this section, we propose an implementation of reusable, non-erasure and universally composable commitment schemes described in Fig.4. The idea behind of

our construction is that an ideal-world simulator is allowed to generate a publickey/secret key pair  $(pk_S, sk_S)$  while a simulated participant  $P_i$  is allowed to generate its own public-key  $(pk_i, sk_i)$  such that any valid cipher-text  $E_{pk_i}(m, r_m)$ generated by the underlying public-key encryption scheme can be correctly decrypted with the help of trap-door string  $sk_S$ . To prevent a corrupted committer from copying a commitment generated by an uncorrupted party, a session identity sid and a commitment identity cid must be included in the encryption scheme.

#### The description of protocol $\pi$

**Common reference string generator**: On input a security parameter k, a common reference string generator  $\mathcal{G}_{crs}$  produces a composite modulus of the form N = pq that is a product of two safe primes p and q, where |p| = q = k, p = 2p' + 1, q = 2q' + 1. The common reference string  $\sigma$  is N.

**Public key/secret key generation algorithm:** Let  $L \subseteq Z_{N^2}^*$  be a cyclic of order 2NN', and G be a subgroup of Nth powers of L, where N=pq, N'=p'q', p=2p'+1 and q=2q'+1. It is clear that  $G \subseteq L$  is a cyclic group of order 2N'.  $P_i$  chooses  $z_i \in Z_{N^2}^*$  uniformly at random, and computes  $g_i = -z_i^{4N} \mod N^2$ . As a result,  $\langle g_i \rangle$  is a subgroup of G such that  $|\langle g_i \rangle| = N'$  with overwhelming probability [4].  $P_i$  then randomly chooses a secret key  $x_i \in [0, N^2/2]$  and sets  $h_i = g_i^{x_i} \mod N^2$ . The public key is  $(N, g_i, h_i)$  and the secret key is  $x_i$ .

**Commitment**: Commitment for a random string  $m \in Z_N$  with a session id sid and a commitment cid:

- computing  $c = E_{pk_i}(\text{sid}, \text{cid}, P_i, P_j, m, r_m)$ , where  $E_{pk_i}(\cdot)$  is Bresson, Catalano and Pointcheval's cryptosystem defined over  $G_i, G_i = \langle g_i \rangle$ ;
- sending (commit, sid, cid,  $P_i$ ,  $P_j$ , c) to the receiver  $P_j$ .

**De-commitment** De-commitment for (sid, cid,  $P_i$ ,  $P_j$ , c)

- The committer  $P_i$  sends (sid, cid,  $P_i$ ,  $P_j$ , m,  $r_m$ ) to  $P_j$ ;
- $P_j$  checks the valid of the equation  $c = E_{pk_i}$  (sid, cid,  $P_i$ ,  $P_j$ , m,  $r_m$ ). If the check is valid, then output accept, otherwise output reject;

Fig. 4. Reusable, Non-erasure and Universally Composable Commitments

#### 4.1 The Proof of Security

**Theorem 1.** The protocol described above is reusable, non-erasure and universally composable in the common reference string model assuming that the decisional Diffie-Hellman problem over a squared composite modulus of the form N = pq is hard.

*Proof.* Given a real-world adversary  $\mathcal{A}$ , we describe an ideal-world adversary  $\mathcal{S}$  corresponding to  $\mathcal{A}$  such that for all environments  $\mathcal{Z}$ , the ensemble

IDEAL<sub> $\mathcal{F}_{mcom}, \mathcal{S}, \mathcal{Z}$ </sub> is computationally indistinguishable with the ensemble REAL<sub> $\pi, \mathcal{A}, \mathcal{Z}$ </sub>. Simulator  $\mathcal{S}$  produces a composite modulus of the form N = pq that is a product of two safe primes p and q; Let  $pk_S = N$ , and  $sk_S = (p, q)$ , and let N be a common reference string; For the given common reference string N,  $P_i$  chooses  $z_i \in \mathbb{Z}_{N^2}^*$  uniformly at random and sets  $g_i = -z_i^{4N} \mod N^2$ .  $P_i$  randomly chooses a secret key  $x_i \in [0, N^2/2]$  and sets  $h_i = g_i^{x_i} \mod N^2$ . The public key is  $(N, g_i, h_i)$ ; the secret key is  $x_i$ . We then consider five cases below:

- Case 1: at some point in the protocol execution Z secretly picks a random string  $m \in Z_N$  and generates a cipher-text c for a corrupted party  $P_i$  by running the protocol of the honest committer for m and  $P_j$ 's answers, then instructs a dummy adversary  $\mathcal{A}$  to inform the corrupted party  $P_i$  sending cto  $P_j$ ; To simulate this case, S first decrypts c with the help of trap-door information  $sk_S$  to obtain the exact value  $m \in Z_N$ , then instructs  $\widetilde{P}_i$  to send the message (commit, sid, cid,  $P_i, P_j, m$ ) to  $\mathcal{F}_{mcom}$ . The commitment functionality  $\mathcal{F}_{mcom}$  sends a reply message (receipt, sid, cid,  $P_i, P_j$ ) to  $P_j$  and S, and ignores any subsequent (commit, sid, cid,  $P_i, P_j$ ) messages.
- Case 2: at some point in the protocol execution  $\mathcal{Z}$  instructs an uncorrupted party  $P_i$  to send a commitment c of a message  $m \in Z_N$  to  $P_j$ . To simulate this case,  $\mathcal{S}$  first decrypts c with the help of trap-door information  $sk_S$  to obtain the exact value  $m \in Z_N$ , then instructs  $\tilde{P}_i$  sending the message (commit, sid, cid,  $P_i$ ,  $P_j$ , m) to  $\mathcal{F}_{mcom}$ . The functionality  $\mathcal{F}_{mcom}$  sends the message (receipt, sid, cid,  $P_i$ ,  $P_j$ ) to  $P_j$  and  $\mathcal{S}$ , and ignores any subsequent (commit, sid, cid,  $P_i$ ,  $P_j$ ) messages.
- Case 3: at some point in the execution  $\mathcal{A}$  instructs a corrupted party  $P_i$  to send a commitment of a message m to  $P_j$ . To simulate this case,  $\mathcal{S}$  obtains mand instructs  $\tilde{P}_i$  to send the message (commit, sid, cid,  $P_i, P_j, m$ ) to  $\mathcal{F}_{mcom}$ . The functionality  $\mathcal{F}_{mcom}$  sends the message (receipt, sid, cid,  $P_i, P_j$ ) to  $P_j$ and  $\mathcal{S}$  if  $m \in \mathbb{Z}_N$ , otherwise,  $\mathcal{F}_{mcom}$  sends  $\perp$  to  $P_i$  and  $\mathcal{S}$ .  $\mathcal{F}_{mcom}$  ignores any subsequent (commit, sid, cid,  $P_i, P_j$ ) messages.
- Case 4: at some point in the protocol execution  $\mathcal{A}$  tells a corrupted party  $P_i$  to open a valid commitment c correctly with the string  $m \in Z_N$ . To simulate this case,  $\mathcal{S}$  compares m with the previously extracted bit and stops if they differ; otherwise  $\mathcal{S}$  sends (open, sid, cid,  $P_i$ ,  $P_j$ ) in the name of the party of the functionality  $\mathcal{F}_{mcom}$ .
- Case 5: Whenever the simulated  $\mathcal{A}$  demands to corrupt a party,  $\mathcal{S}$  corrupts this party in the ideal model and learns all internal information of the party. Notice that  $\mathcal{S}$  needs not to adapt possible decommitment information about a previous given but yet unopened commitment of this party since the trap-door information  $sk_S$  allows to S to decrypt a valid cipher-text generated by  $P_i$ .

According to the simulator described above, we know that  $pk_S$  is the common reference string that serves as a global security parameter of the proposed commitment scheme and  $sk_S$  is only used for decryption of a submitted cipher-text c. It follows that all commitments/decommitments whether submitted by corrupted parities or honest parties can be simulated by S; Notice that individual commitment c for a string  $m \in Z_N$  is computationally hiding and statistically binding assuming that the underlying Bresson, Catalano and Pointcheval publickey encryption scheme is semantically secure which is true assuming that the decisional Diffie-Hellman problem over a squared composite modulus of the form N = pq is hard (i.e., a semantically secure public-key encryption scheme is a computationally hiding and statistically binding commitment, but the UC-security is not claimed at all). Consequently, the environment's output in the real-life model is indistinguishable from its output in the ideal-process.

### 5 Conclusion

In this paper, an simulation efficient reusable and non-erasure commitment schemes has been proposed and analyzed. Under the assumption that the decisional Diffie-Hellman problem over a squared composite modulus N is hard, we have shown that the proposed commitment scheme is secure in the universally composable model within the common reference model.

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## **Certificateless Hybrid Signcryption**

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Abstract. Signcryption is a cryptographic primitive that fulfills both the functions of digital signature and public key encryption simultaneously, at a cost significantly lower than that required by the traditional signature-then-encryption approach. In this paper, we address a question whether it is possible to construct a hybrid signcryption scheme in the certificateless setting. This question seems to have never been addressed in the literature. We answer the question positively in this paper. In particular, we extend the concept of signcryption tag-KEM to the certificateless setting. We show how to construct a certificateless signcryption scheme using certificateless signcryption tag-KEM. We also give an example of certificateless signcryption tag-KEM.

**Keywords:** Certificateless signcryption, hybrid signcryption, signcryption tag-KEM, DEM.

### 1 Introduction

Confidentiality, integrity, non-repudiation and authentication are the important requirements for many cryptographic applications. A traditional approach to achieve these requirements is to sign-then-encrypt the message. Signcryption, first proposed by Zheng [13], is a cryptographic primitive that fulfills both the functions of digital signature and public key encryption simultaneously, at a cost significantly lower than that required by the traditional signature-then-encryption approach. In Zheng's scheme, the public key of a user is essentially a random bit string picked from a given set. So, the signcryption does not provide the authorization of the user by itself. This problem can be solved via a certificate which provides an unforgeable and trusted link between the public key and the identity of the user by the signature of a certificate authority (CA), and there is a hierarchical framework that is called public key infrastructure (PKI) to issue and manage certificates. However, the certificates management including revocation, storage, distribution and the computational cost of certificates verification is the main difficulty against traditional PKI.

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To simplify key management procedures of traditional PKI, Shamir  $\square$  proposed the concept of identity-based cryptography (IBC) in 1984. The idea of IBC is to get rid of certificates by allowing the user's public key to be any binary string that uniquely identifies the user. Examples of such strings include email addresses and IP addresses. The main practical benefit of IBC is in greatly reducing the need for the public key certificates. But IBC uses a trusted third party called private key generator (PKG). The PKG generates the secret keys of all of its users, so a user can decrypt only if the PKG has given a secret key to it (so, certification is implicit), hence reduces the amount of storage and computation. On the other hand, the dependence on the PKG who can generate all users' private keys inevitably causes the key escrow problem to the IBC.

To solve the key escrow problem in the IBC, Al-Riyami and Paterson [2] introduced a new paradigm called certificateless cryptography. The certificateless cryptography does not require the use of certificates and yet does not have the built-in key escrow feature of IBC. It is a model for the use of public key cryptography that is intermediate between traditional PKI and IBC. A certificateless system still makes use of a trusted third party which is called the key generating center (KGC). By way of contrast to the PKG in the IBC, the KGC does not have access to the user's private key. Instead, the KGC supplies a user with a partial private key that the KGC computes from the user's identity and a master key. The user then combines the partial private key with some secret information to generate the actual private key. The system is not identity-based, because the public key is no longer computable from a user's identity. When Alice wants to send a message to Bob in a certificateless system, she must obtain Bob's public key. However, no authentication of Bob's public key is necessary and no certificate is required. In 2008, Barbosa and Farshim 4 introduced the notion of certificateless signeryption (CLSC) and proposed an efficient scheme.

The practical way to perform secrecy communication for large messages is to use hybrid encryption that separates the encryption into two parts: one part uses public key techniques to encrypt a one-time symmetric key; the other part uses the symmetric key to encrypt the actual message. In such a construction, the public key part of the algorithm is known as the key encapsulation mechanism (KEM) while the symmetric key part is known as the data encapsulation mechanism (DEM). A formal treatment of this paradigm originates in the work of Cramer and Shoup [7]. The resulting KEM-DEM hybrid encryption paradigm has received much attention in recent years. It is very attractive as it gives a clear separation between the various parts of the cipher allowing for modular design. In [1], Abe, Gennaro, and Kurosawa introduced tag-KEM which takes as input a tag in KEM. Bentahar et al. [5] extended KEM into identity-based and certificateless settings and gave generic constructions of identity-based KEM (IB-KEM) and certificateless KEM (CL-KEM).

The use of hybrid techniques to build signcryption schemes has been studied by Dent [8]. He generalized KEM to signcryption KEM which includes an authentication in KEM. However, he only consider the insider security for authenticity. That is, if the sender's private key is exposed, an attacker is able to recover the key generated by signcryption KEM. The full insider security [3] means that (a) if the sender's private key is exposed, an attacker is still not able to recover the message from the ciphertext and (b) if the receiver's private key is exposed, an attacker is still not able to forge a ciphertext. In 2006, Bjørstad and Dent [6] showed how to built signcryption schemes using tag-KEM. However, they also only consider the insider security for authenticity and not for confidentiality. In 2008, Tan [12] proposed full insider secure signcryption KEM and tag-KEM in the standard model. Tan's schemes are insider secure for both authenticity and confidentiality. Note that the using of tag-KEM yields simpler scheme descriptions and better generic security reductions.

All the above hybrid signcryption schemes **[6,8,9,12]** are not in the certificateless setting. In this paper, we address a question whether it is possible to construct a hybrid signcryption scheme in the certificateless setting. This question seems to have never been addressed in the literature. We answer the question positively in this paper. In particular, we extend the concept of signcryption tag-KEM to the certificateless setting. We show that a CLSC scheme can be constructed by using a certificateless signcryption tag-KEM (CLSC-TKEM) and a DEM. We also give an example of CLSC-TKEM. Our scheme is insider secure for both authenticity and confidentiality.

### 2 Preliminaries

#### 2.1 Certificateless Signcryption (CLSC)

A generic CLSC scheme consists of the following six algorithms.

- Setup: This algorithm takes as input the security parameter  $1^k$  and returns the KGC's master secret key msk and system parameters params including a master public key mpk and descriptions of message space  $\mathcal{M}$ , ciphertext space  $\mathcal{C}$  and randomness space  $\mathcal{R}$ . This algorithm is executed by the KGC, which publishes params.
- Extract-Partial-Private-Key: This algorithm takes as input params, msk and a user's identity  $ID \in \{0,1\}^*$ , and returns a partial private key  $D_{ID}$ . This algorithm is run by the KGC, after verifying the user's identity.
- Generate-User-Keys: This algorithm takes as input params and an identity ID, and outputs a secret value  $x_{ID}$  and a public key  $PK_{ID}$ . This algorithm is run by a user to obtain a public key and a secret value which can be used to construct a full private key. The public key is published without certification.
- Set-Private-Key: This algorithm takes as input a partial private key  $D_{ID}$  and a secret value  $x_{ID}$ , and returns the full private key  $S_{ID}$ . Again, this algorithm is run by a user to construct the full private key.
- Signcrypt: This algorithm takes as input *params*, a plaintext message  $m \in \mathcal{M}$ , the sender's full private key  $S_{ID_s}$ , identity  $ID_s$  and public key  $PK_{ID_s}$ , and the receiver's identity  $ID_r$  and public key  $PK_{ID_r}$ , and outputs a ciphertext  $\sigma \in \mathcal{C}$ .

- Unsigncrypt: This algorithm takes as input params, a ciphertext  $\sigma$ , the sender's identity  $ID_s$  and public key  $PK_{ID_s}$ , and the receiver's full private key  $S_{ID_r}$ , identity  $ID_r$  and public key  $PK_{ID_r}$ , and outputs a plaintext m or a failure symbol  $\perp$  if  $\sigma$  is an invalid ciphertext.

Barbosa and Farshim [4] defines the security notions for CLSC schemes. A CLSC scheme should satisfy confidentiality (indistinguishability against adaptive chosen ciphertext attacks (IND-CCA2)) and unforgeability (existential unforgeability against adaptive chosen messages attacks (UF-CMA)). For the stronger notion of insider security, we use the notion of strong existential unforgeability (sUF-CMA). The strong existential unforgeability means that an adversary wins if it outputs a valid message/signcryption pair  $(m, \sigma)$  for identities  $ID_s$  and  $ID_r$  and the signcryption  $\sigma$  was not returned by the signcryption oracle when queried on the message m. As in [4], we do not consider attacks targeting signcryptions where the identities of the sender and receiver are the same. That is, we disallow such queries to relevant oracles and do not accept this type of signcryption as a valid forgery. Please see the full version of this paper for details [10].

#### 2.2 Date Encapsulation Mechanism (DEM)

A DEM is a symmetric encryption scheme which consists of the following two algorithms.

- Enc: This algorithm takes as input  $1^k$ , a key K and a message  $m \in \{0, 1\}^*$ , and outputs a ciphertext  $c \in \{0, 1\}^*$ , where  $K \in \mathcal{K}_{\text{DEM}}$  is a key in the given key space, and m is a bit string of arbitrary length. We denote this as  $c \leftarrow \text{Enc}(K, m)$ .
- Dec: This algorithm takes as input a key K and a ciphertext c, and outputs the message  $m \in \{0,1\}^*$  or a symbol  $\perp$  to indicate that the ciphertext is invalid.

For the purposes of this paper, it is only required that a DEM is secure with respect to indistinguishability against passive attackers (IND-PA).

## 3 Certificateless Signcryption Tag-KEM (CLSC-TKEM)

In this section, we extend the concept of signcryption tag-KEM to the certificateless setting. We give the formal definition for certificateless signcryption tag-KEM (CLSC-TKEM).

### 3.1 Generic Scheme

A generic CLSC-TKEM consists of the following seven algorithms.

- Setup: Same to CLSC described in Section 2
- Partial-Private-Key-Extract: Same to CLSC described in Section 2
- Generate-User-Keys: Same to CLSC described in Section 2

- Set-Private-Key: Same to CLSC described in Section 2
- Sym: This is symmetric key generation algorithm which takes as input the params, the sender's full private key  $S_{ID_s}$ , identity  $ID_s$  and public key  $PK_{ID_s}$ , the receiver's identity  $ID_r$  and public key  $PK_{ID_r}$ , and outputs a symmetric key K together with internal state information  $\omega$ . Here  $K \in \mathcal{K}_{\text{CLSC-TKEM}}$  is a key in the space of possible session keys at a given security level. We denote this as  $(K, \omega) \leftarrow$  $\text{Sym}(params, S_{ID_s}, ID_s, PK_{ID_s}, ID_r, PK_{ID_r}).$
- Encap: This is key encapsulation algorithm which takes as input the state information  $\omega$  and an arbitrary tag  $\tau$ , and returns an encapsulation  $\psi \in \mathcal{E}_{\text{CLSC-TKEM}}$ . We denote this as  $\psi \leftarrow \text{Encap}(\omega, \tau)$ .
- Decap: This is decapsulation algorithm which takes as input the *params*, an encapsulation  $\psi$ , a tag  $\tau$ , the sender's identity  $ID_s$  and public key  $PK_{ID_s}$ , the receiver's full private key  $S_{ID_r}$ , identity  $ID_r$  and public key  $PK_{ID_r}$ , and outputs a key K or a special symbol  $\perp$  indicating invalid encapsulation. We denote this as  $K \leftarrow \text{Decap}(params, \psi, \tau, ID_s, PK_{ID_s}, S_{ID_r}, ID_r, PK_{ID_r})$ .

We make the consistency constraint that if

 $(K, \omega) \leftarrow \operatorname{Sym}(params, S_{ID_s}, ID_s, PK_{ID_s}, ID_r, PK_{ID_r}) \text{ and } \psi \leftarrow \operatorname{Encap}(\omega, \tau),$ 

then

 $K \leftarrow \text{Decap}(params, \psi, \tau, ID_s, PK_{ID_s}, S_{ID_r}, ID_r, PK_{ID_r}).$ 

### 3.2 Security Notions

A CLSC-TKEM should satisfy confidentiality and unforgeability. To define the security notions for CLSC-TKEM, we simply adapt the security notions of CLSC into the TKEM framework. There are two types of adversary against a CLSC-TKEM: Type I and Type II. A Type I adversary models an attacker which is a common user of the system and is not in possession of the KGC's master secret key. But it is able to adaptively replace users'public keys with (valid) public keys of its choice. A Type II adversary models an honest-but-curious KGC who knows the KGC's master secret key. But it cannot replace users' public keys.

For confidentiality, we consider two games "IND-CCA2-I" and "IND-CCA2-II" where a Type I adversary  $\mathcal{A}_I$  and a Type II adversary  $\mathcal{A}_{II}$  interact with their "challenger" in these two games, respectively. Note that the challenger keeps a history of "query-answer" while interacting with the attackers. Now we describe the two games.

IND-CCA2-I: This is the game in which  $\mathcal{A}_I$  interacts with the "challenger":

Initial: The challenger runs  $(params, msk) \leftarrow \text{Setup}(1^k)$  and gives params to  $\mathcal{A}_I$ . The challenger keeps master secret key msk to itself.

**Phase 1:** The adversary  $\mathcal{A}_I$  can perform a polynomially bounded number of queries in an adaptive manner.

- Extract partial private key:  $\mathcal{A}_I$  chooses an identity *ID*. The challenger computes  $D_{ID} \leftarrow \texttt{Extract-Partial-Private-Key}(params, msk, ID)$  and sends  $D_{ID}$  to  $\mathcal{A}_I$ .

- Extract private key:  $\mathcal{A}_I$  chooses an identity ID. The challenger first computes  $D_{ID} \leftarrow \texttt{Extract-Partial-Private-Key}(params, msk, ID)$  and then  $(x_{ID}, PK_{ID}) \leftarrow \texttt{Generate-User-Key}(params, ID)$ . Finally, it sends the result of  $S_{ID} \leftarrow \texttt{Set-Private-Key}(x_{ID}, D_{ID})$  to  $\mathcal{A}_I$ . The adversary is not allowed to query any identity for which the corresponding public key has been replaced. This restriction is imposed due to the fact that it is unreasonable to expect that the challenger is able to provide a full private key for a user for which it does not know the secret value.
- Request public key:  $\mathcal{A}_I$  chooses an identity *ID*. The challenger computes  $(x_{ID}, PK_{ID}) \leftarrow \text{Generate-User-Keys}(params, ID)$  and sends  $PK_{ID}$  to  $\mathcal{A}_I$ .
- Replace public key:  $A_I$  may replace a public key  $PK_{ID}$  with a value chosen by it.
- Symmetric key generation queries:  $\mathcal{A}_I$  chooses a sender's identity  $ID_s$  and a receiver's identity  $ID_r$ . The challenger finds  $S_{ID_s}$  from its "query-answer" list and runs  $(K, \omega) \leftarrow \text{Sym}(params, S_{ID_s}, ID_s, PK_{ID_s}, ID_r, PK_{ID_r})$ . The challenger then stores the value  $\omega$  (hidden from the view of the adversary, and overwriting any previously stored values), and sends the symmetric key K to  $\mathcal{A}_I$ . Note that, it is possible that the challenger is not aware of the sender's secret value, if the associated public key has been replaced. In this case, we require the adversary to provide it. We disallow queries where  $ID_s = ID_r$ .
- Key encapsulation queries:  $\mathcal{A}_I$  produces an arbitrary tag  $\tau$ . The challenger checks whether there exists a stored value  $\omega$ . If not, it returns  $\perp$  and terminates. Otherwise it erases the value from storage and returns  $\psi \leftarrow \texttt{Encap}(\omega, \tau)$  to  $\mathcal{A}_I$ .
- Key decapsulation queries: The adversary  $\mathcal{A}_I$  chooses a sender's identity  $ID_s$ , a receiver's identity  $ID_r$ , an encapsulation  $\psi$ , and a tag  $\tau$ . The challenger finds  $S_{ID_r}$  from its "query-answer" list and sends the result of  $\mathsf{Decap}(params, \psi, \tau, ID_s, PK_{ID_s}, S_{ID_r}, ID_r, PK_{ID_r})$  to  $\mathcal{A}_I$ . Note that, it is possible that the challenger is not aware of the receiver's secret value, if the associated public key has been replaced. In this case, we require the adversary to provide it. We also disallow queries where  $ID_s = ID_r$ .

Challenge: The adversary  $\mathcal{A}_I$  decides when Phase 1 ends.  $\mathcal{A}_I$  generates a sender's identity  $ID_s^*$  and a receiver's identity  $ID_r^*$  on which it wishes to be challenged. Note that  $ID_r^*$  should not be queried to extract a private key in Phase 1. Note also that  $ID_r^*$  cannot be equal to an identity for which both the public key has been replaced and the partial private key has been extracted. The challenger computes  $(K_1, \omega^*) \leftarrow \text{Sym}(params, S_{ID_s^*}, ID_s^*, PK_{ID_s^*}, ID_r^*, PK_{ID_r^*})$ . Then the challenger chooses  $K_0 \leftarrow \mathcal{K}_{\text{CLSC}-\text{TKEM}}$  and a bit  $b \in \{0, 1\}$  randomly, and sends  $K_b$  to  $\mathcal{A}_I$ . When  $\mathcal{A}_I$  receives  $K_b$ , it may ask the same queries as previously. Then  $\mathcal{A}_I$  generates a tag  $\tau^*$ . The challenger computes  $\psi^* \leftarrow \text{Encap}(\omega^*, \tau^*)$  and sends it to  $\mathcal{A}_I$  as a challenge encapsulation.

**Phase 2:** The adversary  $\mathcal{A}_I$  can ask a polynomially bounded number of queries adaptively again as in Phase 1. The same rule is applied here:  $\mathcal{A}_I$  cannot extract the private key for  $ID_r^*$ .  $\mathcal{A}_I$  cannot extract the partial private key for  $ID_r^*$  if the public key of this identity has been replaced before the challenge

phase. In addition,  $\mathcal{A}_I$  cannot make a decapsulation query on  $(K_b, \psi^*)$  under  $ID_s^*$  and  $ID_r^*$ , unless the public key  $PK_{ID_s^*}$  or  $PK_{ID_r^*}$  has been replaced after the challenge phase.

**Guess:** The adversary  $\mathcal{A}_I$  produces a bit b' and wins the game if b' = b. The advantage of  $\mathcal{A}_I$  is defined to be

$$\operatorname{Adv}_{\operatorname{CLSC-TKEM}}^{\operatorname{IND-CCA2-I}}(\mathcal{A}_I) = |2\operatorname{Pr}[b'=b] - 1|,$$

where  $\Pr[b' = b]$  denotes the probability that b' = b.

IND-CCA2-II: This is the game in which  $\mathcal{A}_{II}$  interacts with the "challenger": Initial: The challenger runs  $(params, msk) \leftarrow \text{Setup}(1^k)$  and gives both params and msk to  $\mathcal{A}_{II}$ .

Phase 1: The adversary  $\mathcal{A}_{II}$  can perform a polynomially bounded number of queries in an adaptive manner. Note that we do not need Extract partial private key since  $\mathcal{A}_{II}$  can compute partial private keys by itself.

- Extract private key: Same to CLSC-TKEM's IND-CCA2-I game.
- Request public key: Same to CLSC-TKEM's IND-CCA2-I game.
- Symmetric key generation queries: Same to CLSC-TKEM's IND-CCA2-I game.
- Key encapsulation queries: Same to CLSC-TKEM's IND-CCA2-I game.
- Key decapsulation queries: Same to CLSC-TKEM's IND-CCA2-I game.

**Challenge:** The adversary  $\mathcal{A}_{II}$  decides when Phase 1 ends.  $\mathcal{A}_{II}$  generates a sender's identity  $ID_s^*$  and a receiver's identity  $ID_r^*$  on which it wishes to be challenged. Note that  $ID_r^*$  should not be queried to extract a private key in Phase 1. The challenger runs  $(K_1, \omega^*) \leftarrow \text{Sym}(params, S_{ID_s^*}, ID_s^*, PK_{ID_s^*}, ID_r^*, PK_{ID_r^*})$ . Then the challenger chooses  $K_0 \leftarrow \mathcal{K}_{\text{CLSC}-\text{TKEM}}$  and a bit  $b \in \{0, 1\}$  randomly, and sends  $K_b$  to  $\mathcal{A}_I$ . When  $\mathcal{A}_{II}$  receives  $K_b$ , it may ask the same queries as previously. Then  $\mathcal{A}_{II}$  generates a tag  $\tau^*$ . The challenger computes  $\psi^* \leftarrow \text{Encap}(\omega^*, \tau^*)$  and sends it to  $\mathcal{A}_{II}$  as a challenge encapsulation.

**Phase 2:** The adversary  $\mathcal{A}_{II}$  can ask a polynomially bounded number of queries adaptively again as in Phase 1.  $\mathcal{A}_{II}$  cannot extract the private key for  $ID_r^*$ . In addition,  $\mathcal{A}_{II}$  cannot make a decapsulation query on  $(K_b, \psi^*)$  under  $ID_s^*$  and  $ID_r^*$ , unless the public key  $PK_{ID_s^*}$  or  $PK_{ID_r^*}$  has been replaced after the challenge phase.

**Guess:** The adversary  $\mathcal{A}_{II}$  produces a bit b' and wins the game if b' = b. The adversary of  $\mathcal{A}_{II}$  is defined to be

The advantage of  $\mathcal{A}_{II}$  is defined to be

$$\mathrm{Adv}_{\mathrm{CLSC-TKEM}}^{\mathrm{IND-CCA2-II}}(\mathcal{A}_{II}) = |2\mathrm{Pr}[b'=b] - 1|,$$

where  $\Pr[b' = b]$  denotes the probability that b' = b.

**Definition 1.** A CLSC-TKEM scheme is said to be IND-CCA2-I secure (resp. IND-CCA2-II secure) if there is no PPT adversary  $\mathcal{A}_I$  (resp.  $\mathcal{A}_{II}$ ) which wins IND-CCA2-I (resp. IND-CCA2-II) with non-negligible advantage. A CLSC-TKEM scheme is said to be IND-CCA2 secure if it is both IND-CCA2-I secure and IND-CCA2-II secure.

Notice that the adversary is allowed to extract the private key of  $ID_s^*$  in the IND-CCA2-I and IND-CCA2-II games. This condition corresponds to the stringent requirement of insider security for confidentiality of signcryption 3.

For the strong existential unforgeability, we consider two games "sUF-CMA-I" and "sUF-CMA-II" where a Type I adversary  $\mathcal{F}_I$  and a Type II adversary  $\mathcal{F}_{II}$ interact with their "challenger" in these two games, respectively. Note that the challenger keeps a history of "query-answer" while interacting with the attackers. Now we describe the two games.

sUF-CMA-I: This is the game in which  $\mathcal{F}_I$  interacts with the "challenger":

Initial: The challenger runs  $(params, msk) \leftarrow \text{Setup}(1^k)$  and gives params to  $\mathcal{F}_I$ . The challenger keeps master secret key msk to itself.

Attack: The adversary  $\mathcal{F}_I$  performs a polynomially bounded number of queries just like in the CLSC-TKEM's IND-CCA2-I game.

Forgery:  $\mathcal{F}_I$  produces a quadruple  $(\tau^*, \psi^*, ID_s^*, ID_r^*)$ . Note that  $ID_s^*$  should not be queried to extract a private key. Note also that  $ID_s^*$  cannot be equal to an identity for which both the public key has been replaced and the partial private key has been extracted. In addition,  $\psi^*$  was not returned by the key encapsulation oracle on the input  $(\tau^*, ID_s^*, ID_r^*)$  during Attack stage.  $\mathcal{F}_I$  wins the game if the result of  $\mathsf{Decap}(params, \psi^*, \tau^*, ID_s^*, PK_{ID_s^*}, S_{ID_r^*}, ID_r^*, PK_{ID_r^*})$ is not the  $\bot$  symbol.

The advantage of  $\mathcal{F}_I$  is defined as the probability that it wins.

sUF-CMA-II: This is the game in which  $\mathcal{F}_{II}$  interacts with the "challenger": Initial: The challenger runs  $(params, msk) \leftarrow \text{Setup}(1^k)$  and gives both params and msk to  $\mathcal{F}_{II}$ .

Attack: The adversary  $\mathcal{F}_{II}$  performs a polynomially bounded number of queries just like in the CLSC-TKEM's IND-CCA2-II game.

Forgery:  $\mathcal{F}_{II}$  produces a quadruple  $(\tau^*, \psi^*, ID_s^*, ID_r^*)$ .  $ID_s^*$  should not be queried to extract a private key. In addition,  $\psi^*$  was not returned by the key encapsulation oracle on the input  $(\tau^*, ID_s^*, ID_r^*)$  during Attack stage.  $\mathcal{F}_{II}$  wins the game if the result of  $\mathsf{Decap}(params, \psi^*, \tau^*, ID_s^*, PK_{ID_s^*}, S_{ID_r^*}, ID_r^*, PK_{ID_r^*})$  is not the  $\bot$  symbol.

The advantage of  $\mathcal{F}_{II}$  is defined as the probability that it wins.

**Definition 2.** A CLSC-TKEM scheme is said to be sUF-CMA-I secure (resp. sUF-CMA-II secure) if there is no PPT adversary  $\mathcal{F}_I$  (resp.  $\mathcal{F}_{II}$ ) which wins sUF-CMA-I (resp. sUF-CMA-II) with non-negligible advantage. A CLSC-TKEM scheme is said to be sUF-CMA secure if it is both sUF-CMA-I secure and sUF-CMA-II secure.

Note that the adversary is allowed to extract the private key of  $ID_r^*$  in the above definition. Again, this condition corresponds to the stringent requirement of insider security for signcryption [3].

### 4 Certificateless Hybrid Signcryption

We can combine a CLSC-TKEM with a DEM to form a CLSC scheme. We describe it in Figure 1. Note that the tag is the ciphertext output by the DEM.

Such construction yields simpler scheme descriptions and better generic security reductions.

We give the security results for such construction in Theorems  $\blacksquare$  and 2

**Theorem 1.** Let CLSC be a certificateless hybrid signcryption scheme constructed from a CLSC-TKEM and a DEM. If the CLSC-TKEM is IND-CCA2 secure and the DEM is IND-PA secure, then CLSC is IND-CCA2 secure. In particular, we have

$$\operatorname{Adv}_{\operatorname{CLSC}}^{\operatorname{IND-CCA2-i}}(\mathcal{A}) \leq 2\operatorname{Adv}_{\operatorname{CLSC-TKEM}}^{\operatorname{IND-CCA2-i}}(\mathcal{B}_1) + \operatorname{Adv}_{\operatorname{DEM}}^{\operatorname{IND-PA}}(\mathcal{B}_2),$$

where  $i \in \{I, II\}$ ,  $\operatorname{Adv}_{\operatorname{CLSC}}^{\operatorname{IND-CCA2-i}}(\mathcal{A})$  is the advantage of the IND-CCA2 adversary against CLSC,  $\operatorname{Adv}_{\operatorname{CLSC-TKEM}}^{\operatorname{IND-CCA2-i}}(\mathcal{B}_1)$  is the advantage of the IND-CCA2 adversary against CLSC-TKEM, and  $\operatorname{Adv}_{\operatorname{DEM}}^{\operatorname{IND-PA}}(\mathcal{B}_2)$  is the advantage of the IND-PA adversary against DEM.

*Proof.* See the full version of this paper 10.

**Theorem 2.** Let CLSC be a certificateless hybrid signcryption scheme constructed from a CLSC-TKEM and a DEM. If the CLSC-TKEM is sUF-CMA secure, then CLSC is sUF-CMA secure. In particular, we have

$$\operatorname{Adv}_{\operatorname{CLSC}}^{\operatorname{sUF-CMA-i}}(\mathcal{F}) \leq \operatorname{Adv}_{\operatorname{CLSC-TKEM}}^{\operatorname{sUF-CMA-i}}(\mathcal{B}),$$

where  $i \in \{I, II\}$ ,  $\operatorname{Adv}_{\operatorname{CLSC}}^{\operatorname{sUF-CMA-i}}(\mathcal{F})$  is the advantage of the sUF-CMA adversary against CLSC, and  $\operatorname{Adv}_{\operatorname{CLSC-TKEM}}^{\operatorname{sUF-CMA-i}}(\mathcal{B})$  is the advantage of the resulting sUF-CMA adversary against CLSC-TKEM.

*Proof.* See the full version of this paper  $\boxed{10}$ .

### 5 An Example of CLSC-TKEM

The Barbosa-Farshim CLSC scheme [4] fits the new generic framework. Here we give an example of CLSC-TKEM based on the Barbosa-Farshim scheme. If we combine the CLSC-TKEM with a DEM as Figure 1, we can get a scheme that is very similar to the Barbosa-Farshim scheme.

The CLSC-TKEM consists of the following seven algorithms.

- Setup: Let  $G_1$  be a cyclic additive group generated by P, whose order is a prime q, and  $G_2$  be a cyclic multiplicative group of the same order q.  $\hat{e} : G_1 \times G_1 \to G_2$  is a pairing. Let  $H_1, H_2, H_3$ , and  $H_4$  be four cryptographic hash functions where  $H_1 : \{0,1\}^* \to G_1, H_2 : \{0,1\}^* \to$  $\{0,1\}^n, H_3 : \{0,1\}^* \to G_1$ , and  $H_4 : \{0,1\}^* \to G_1$ . Here n is the key length of a DEM. The PKG chooses a master secret key  $s \in Z_q^*$  randomly and computes  $P_{pub} \leftarrow sP$ . The PKG publishes system parameters  $\{G_1, G_2, n, \hat{e}, P, P_{pub}, H_1, H_2, H_3, H_4\}$  and keeps the master key s secret.

CLSC.Setup: On input  $1^k$ : 1.  $(params, msk) \leftarrow CLSC-TKEM.Setup(1^k)$ 2. Output the system parameters params and the master secret key mskCLSC.Partial-Private-Key-Extract: On input the params, msk, and an identity  $ID \in \{0, 1\}^*$ : 1.  $D_{ID} \leftarrow \text{CLSC-TKEM.Partial-Private-Key-Extract}(params, msk, ID)$ 2. Output the partial private key  $D_{ID}$  of the identity IDCLSC.Generate-User-Keys: On input the params and an identity  $ID \in \{0,1\}^*$ : 1.  $(x_{ID}, PK_{ID}) \leftarrow \texttt{CLSC-TKEM}.\texttt{Generate-User-Keys}(params, ID)$ 2. Output the secret value  $x_{ID}$  and the public key  $PK_{ID}$  of the identity IDCLSC.Set-Private-Key: On input the partial private key  $D_{ID}$  and the secret value  $x_{ID}$ : 1.  $S_{ID} \leftarrow \text{CLSC-TKEM.Set-Private-Key}(D_{ID}, x_{ID})$ 2. Output the full private key  $S_{ID}$ CLSC.Signcrypt: On input the params, a message  $m \in \{0, 1\}^*$ , the sender's full private key  $S_{ID_s}$ , identity  $ID_s$  and public key  $PK_{ID_s}$ , the receiver's identity  $ID_r$  and public key  $PK_{ID_r}$ : 1.  $(K, \omega) \leftarrow \text{CLSC-TKEM.Sym}(params, S_{ID_s}, ID_s, PK_{ID_s}, ID_r, PK_{ID_r})$ 2.  $c \leftarrow \text{DEM.Enc}(K, m)$ 3.  $\psi \leftarrow \texttt{CLSC-TKEM}.\texttt{Encap}(\omega, c)$ 4. Output the ciphertext  $\sigma \leftarrow (\psi, c)$ CLSC.Unsigncrypt: On input the *params*, a ciphertext  $\sigma$ , the sender's identity  $ID_s$  and public key  $PK_{ID_s}$ , the receiver's full private key  $S_{ID_r}$ , identity  $ID_r$  and public key  $PK_{ID_r}$ : 1.  $K \leftarrow \text{CLSC-TKEM.Decap}(params, \psi, c, ID_s, PK_{ID_s}, S_{ID_r}, ID_r, PK_{ID_r})$ 2. If  $K = \bot$ , then output  $\bot$  and stop 3.  $m \leftarrow \texttt{DEM.Dec}(K, c)$ 

4. Output the message m

Fig. 1. Certificateless hybrid signcryption

- Partial-Private-Key-Extract: Given an identity  $ID \in \{0, 1\}^*$ , the PKG computes  $Q_{ID} \leftarrow H_1(ID)$  and returns the partial private key  $D_{ID} \leftarrow sQ_{ID}$ .
- Generate-User-Keys: A user with identity ID chooses a random element  $x_{ID}$  from  $Z_q$  as the secret value, and sets  $PK_{ID} \leftarrow x_{ID}P$  as the public key.
- Set-Private-Key: Given a partial private key  $D_{ID}$  and a secret value  $x_{ID}$ , this algorithm returns the full private key  $S_{ID} \leftarrow (x_{ID}, D_{ID})$ .
- Sym: Given the sender's full private key  $S_{ID_s}$ , identity  $ID_s$  and public key  $PK_{ID_s}$ , the receiver's identity  $ID_r$  and public key  $PK_{ID_r}$ , this algorithm works as follows.
  - 1. Choose  $r \in Z_q^*$  randomly.
  - 2. Compute U = rP and  $T \leftarrow \hat{e}(P_{pub}, Q_{ID_r})^r$ .
  - 3. Compute  $K \leftarrow H_2(U, T, rPK_{ID_r}, ID_r, PK_{ID_r})$ .
  - 4. Output K and set  $\omega \leftarrow (r, U, S_{ID_s}, ID_s, PK_{ID_s}, ID_r, PK_{ID_r})$ .
- Encap: Given the state information  $\omega$  and an arbitrary tag  $\tau$ , this algorithm works as follows.
  - 1. Compute  $H \leftarrow H_3(U, \tau, ID_s, PK_{ID_s})$ .
  - 2. Compute  $H' \leftarrow H_4(U, \tau, ID_s, PK_{ID_s})$ .
  - 3. Compute  $W \leftarrow D_{ID_s} + rH + x_{ID_s}H'$ .
  - 4. Output  $\psi \leftarrow (U, W)$ .
- Decap: Given the the sender's identity  $ID_s$  and public key  $PK_{ID_s}$ , the receiver's full private key  $S_{ID_r}$ , identity  $ID_r$  and public key  $PK_{ID_r}$ , an encapsulation  $\psi$  and a tag  $\tau$ , this algorithm works as follows.
  - 1. Compute  $H \leftarrow H_3(U, \tau, ID_s, PK_{ID_s})$ .
  - 2. Compute  $H' \leftarrow H_4(U, \tau, ID_s, PK_{ID_s})$ .
  - 3. If  $\hat{e}(P_{pub}, Q_{ID_s})\hat{e}(U, H)\hat{e}(PK_{ID_s}, H') = \hat{e}(P, W)$ , compute  $T = \hat{e}(D_{ID_r}, U)$  and output the  $K \leftarrow H_2(U, T, x_{ID_r}U, ID_r, PK_{ID_r})$ . Otherwise, output symbol  $\perp$ .

We give the security results for the CLSC-TKEM in Theorems 3 and 4.

**Theorem 3.** In the random oracle model, the above CLSC-TKEM is IND-CCA2 secure under the assumption that the gap bilinear Diffie-Hellman problem is intractable.

*Proof.* See the full version of this paper 10.

**Theorem 4.** In the random oracle model, the above CLSC-TKEM is sUF-CMA secure under the assumption that the GDH' problem is intractable.

*Proof.* See the full version of this paper 10.

## 6 Conclusions

In this paper, we extended the concept of signcryption tag-KEM to the certificateless setting. We showed that a certificateless signcryption scheme can be constructed by combining a certificateless signcryption tag-KEM with a DEM. To show that our framework is reasonable, we also gave an example of certificateless signcryption tag-KEM based on the Barbosa-Farshim certificateless signcryption scheme.

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## **On Non-representable Secret Sharing Matroids**

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Abstract. The characterization of the access structures of ideal secret sharing schemes is one of the main open problems in secret sharing and has important connections with matroid theory. Because of its difficulty, it has been studied for several particular families of access structures. Multipartite access structures, in which the set of participants is divided into several parts and all participants in the same part play an equivalent role, have been studied in seminal works on secret sharing by Shamir, Simmons, and Brickell, and also recently by several authors.. In the EUROCRYPT'07, Farras made a important contribution to this work: By using discrete polymatroids, they obtained a necessary condition and a sufficient condition for a multipartite access structure to be ideal respectively. In particular, they further gave a very difficult open problem, that is, characterizing the representable discrete polymatroids, i.e., which discrete polymatroids are representable and which ones are non-representable. In this paper, by dealing with a family of matroids derived from the Vamos matroid, which was the first matroid that was proved to be non-representable, we obtain a family of non-representable matroids. As a consequence, we extend it to the general case and obtain a sufficient condition for a discrete polymatroid to be non-representable, which is a new contribution to the open problem given by Farras.

**Keywords:** Ideal secret sharing schemes, Ideal access structures, Multipartite access structures, Discrete polymatroids, Vamos matroid.

### 1 Introduction

Secret sharing schemes were introduced independently by Shamir [2] and Blakley [1] in 1979. In a secret sharing scheme, every participant receives a share of a secret value. Only the qualified sets of participants, which form the access structure of the scheme, can recover the secret value from their shares. This paper deals exclusively with unconditionally secure perfect secret sharing schemes, that is, the shares of the participants in a non-qualified set do not provide any information about the secret value.

The length of the shares is the main measure of the complexity of secret sharing schemes. In general, the shares must be much larger than the secret. An access structure is said to be ideal if it admits an ideal secret sharing scheme. The characterization

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of ideal access structures is one of the main open problems in secret sharing and has important connections with matroid theory.

A necessary condition for an access structure to be ideal was given by Brickell and Davenport [4], who proved that every ideal access structure is matroid-related. Matroids that are obtained from ideal secret sharing schemes are said to be secret sharing representable (or ss-representable for short). Vamos matroid was the first matroid that was proved to be non-ss-representable. Nevertheless, as a consequence of the results in [3], all representable matroids (that is, matroid that can be represented by a matrix over some finite field) are ss-representable. This implies a sufficient condition for an access structure to be ideal. Namely, an access structure is ideal if it is related to a representable matroid.

Due to the difficulty of finding general results, the characterization of ideal access structures has been studied for several particular classes of access structures. Multipartite access structure, informally, is that the set of participants can be divided into several parts in such a way that all participants in the same part play an equivalent role in the structure. Since we can always consider as many parts as participants, every access structure is multipartite. More accurately, we can consider in any access structure the partition that is derived from a suitable equivalence relation on the set of participants. Because of its practical interest, secret sharing for multipartite access structures has been studied by several authors[2,3,5,6,7,8].

Recently, in the EUROCRYPT'07, Farras[9] made a important contribution to this work by using discrete polymatroids. In particular, for solving the main open problems in secret sharing, they further gave a very difficult open problem, that is, characterizing the representable discrete polymatroids, i.e., which discrete polymatroids are representable and which ones are non-representable. In this paper, by dealing with a family of matroids derived from the Vamos matroid, which was the first matroid that was proved to be non-representable, we obtain a family of non-representable matroids. As a consequence, we extend it to the general case and obtain a sufficient condition for a discrete polymatroid to be non-representable, which is a new contribution to the open problem given by Farras.

### 2 Definitions and Preliminaries

In this section we review some basic definitions and notations that will be used through the paper.

#### 2.1 Matroids and Ideal Secret Sharing

The reader is referred to [12] for an introduction to secret sharing and to [10, 11] for general references on Matroid Theory.

A matroid  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  is formed by a finite set  $\mathcal{Q}$  together with a family  $\mathcal{I} \subseteq \mathcal{P}(\mathcal{Q})$  ( $\mathcal{P}(\mathcal{Q})$ ) is the power set of the set  $\mathcal{Q}$ .) such that 1.  $\phi \in \mathcal{I}$ , and 2. if  $I_1 \in \mathcal{I}$  and  $I_2 \subseteq I_1$ , then  $I_2 \in \mathcal{I}$ , and 3. if  $I_1, I_2 \in \mathcal{I}$  and  $|I_1| \leq |I_2|$ , then there exists  $x \in I_2 - I_1$  such that  $I_1 \cup \{x\} \in \mathcal{I}$ .

The set  $\mathcal{Q}$  is the ground set of the matroid  $\mathcal{M}$  and the elements of  $\mathcal{I}$  are called the independent sets of  $\mathcal{M}$ . The bases of the matroid are the maximally independent sets. The family  $\mathcal{B}$  of the bases determines the matroid. Moreover, by [10, Theorem 1.2.5],  $\mathcal{B} \subseteq \mathcal{P}(\mathcal{Q})$  is the family of bases of a matroid on  $\mathcal{Q}$  if and only if

- 1.  $\mathcal{B}$  is nonempty, and
- 2. for every  $B_1, B_2 \in \mathcal{B}$  and  $x \in B_1 B_2$ , there exists  $y \in B_2 B_1$  such that  $(B_1 \{x\}) \cup \{y\}$  is in  $\mathcal{B}$ .

All bases have the same number of elements, which is the rank of  $\mathcal{M}$  and is denoted  $r(\mathcal{M})$ . The dependent sets are those that are not independent. A circuit is a minimally dependent subset. A matroid is said to be connected if, for every two points  $x, y \in \mathcal{Q}$ , there exists a circuit C with  $x, y \in C$ . The rank of  $X \subseteq \mathcal{Q}$ , which is denoted r(X), is the maximum cardinality of the subsets of X that are independent. Observe that the rank of  $\mathcal{Q}$  is the rank of the matroid  $\mathcal{M}$  that was defined before. The rank function  $r: \mathcal{P}(\mathcal{Q}) \to \mathbb{Z}$  of a matroid satisfies

- 1.  $0 \le r(X) \le |X|$  for every  $X \subseteq \mathcal{Q}$ , and
- 2. *r* is monotone increasing: if  $X \subseteq Y \subseteq Q$ , then  $r(X) \leq r(Y)$ , and
- 3. r is submodular:  $r(X \cap Y) + r(X \cup Y) \le r(X) + r(Y)$  for every  $X, Y \subseteq Q$ .

Let  $\mathbb{K}$  be a field. A matroid  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  is  $\mathbb{K}$ -representable (or representable for short) if there exists a matrix M over  $\mathbb{K}$  whose columns are indexed by the elements of  $\mathcal{Q}$  such that a subset  $I = \{i_1, ..., i_k\} \subseteq \mathcal{Q}$  is independent if and only if the corresponding columns of M are independent. In this situation, we say that the matrix M is a  $\mathbb{K}$ -representation of the matroid  $\mathcal{M}$ .

Let  $\mathbb{K}$  be a finite field and let  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  be a  $\mathbb{K}$ -representable matroid. Let  $p_0 \in \mathcal{Q}$  be special participant called dealer.and  $\mathcal{Q} = P \bigcup \{p_0\}$ . For every  $k \times (n+1)$  matrix M representing  $\mathcal{M}$  over  $\mathbb{K}$ , let E be a vector space of finite dimention dim E = k over  $\mathbb{K}$ . For every  $i \in \mathcal{Q}$ , we define a surjective linear mapping:  $\pi_i : E \to \mathbb{K}$ , and the i-th column of M corresponds to the linear form  $\pi_i$ . In that situation, for every random choice of an element  $x \in E$ , we can obtain  $s_i = \pi_i(x) \in \mathbb{K}$  is the share of the participant  $i \in P$  and  $s = \pi_{p_0}(x) \in \mathbb{K}$  is the shared secret value. Hence, by the columns of M, we define an ideal secret sharing

scheme with access structure  $\Gamma_{p_0}(\mathcal{M})$ . Therefore, the access structures that are related to representable matroids are ideal.

#### 2.2 Multipartite Access Structures and Multipartite Matroids

We write  $\mathcal{P}(P)$  for the power set of the set P. An m-partition  $\Pi = \{P_1, ..., P_m\}$ of a set P is a disjoint family of m nonempty subsets of P with  $P = P_1 \cup ... \cup P_m$ . Let  $\Lambda \subseteq \mathcal{P}(P)$  be a family of subsets of P. For a permutation  $\sigma$  on P, we define  $\sigma(\Lambda) = \{\sigma(A) : A \in \Lambda\} \subseteq \mathcal{P}(P)$ . A family of subsets  $\Lambda \subseteq \mathcal{P}(P)$  is said to be  $\Pi$ -partite if  $\sigma(\Lambda) = \Lambda$  for every permutation  $\sigma$  such that  $\sigma(P_i) = P_i$  for every  $P_i \in \Pi$ . We say that  $\Lambda$  is m-partite if it is  $\Pi$ -partite for some m-partition  $\Pi$ . These concepts can be applied to access structures, which are actually families of subsets, and they can be applied as well to the family of independent sets of a matroid. A matroid  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  is  $\Pi$ -partite if  $\mathcal{I} \subseteq \mathcal{P}(\mathcal{Q})$  is  $\Pi$ -partite.

Let  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  be a connected matroid and, for a point  $p_0 \in \mathcal{Q}$ , let  $\Pi = \{P_1, ..., P_m\}$  and  $\Pi_0 = \{\{p_0\}, P_1, ..., P_m\}$  be partitions of the sets  $P = \mathcal{Q} - \{p_0\}$  and  $\mathcal{Q}$  respectively. Then the access structure  $\Gamma = \Gamma_{p_0}(\mathcal{M})$  is  $\Pi$ -partite if and only if the matroid  $\mathcal{M}$  is  $\Pi_0$ -partite.

The partition  $\Pi'$  is a refinement of the partition  $\Pi$  if every set in  $\Pi'$  is a subset of some set in  $\Pi$ . Clearly, if  $\Lambda \subseteq \mathcal{P}(P)$  is  $\Pi$ -partite and  $\Pi'$  is a refinement of  $\Pi$ , then  $\Lambda$  is  $\Pi'$ -partite. Among all partitions  $\Pi$  for which a family of subsets  $\Lambda \subseteq \mathcal{P}(P)$  is  $\Pi$ -partite, there exists a partition  $\Pi_{\Lambda}$  that is not a refinement of any other such partition. Following [13], we consider the following equivalence relation: two elements  $p, q \in P$  are said to be equivalent according to  $\Lambda$  if the transposition  $\tau_{pq}$  satisfies  $\tau_{pq}(\Lambda) = \Lambda$ . The partition  $\Pi_{\Lambda}$  is the one defined by this equivalence relation. It is not difficult to check that  $\Lambda$  is  $\Pi$ -partite if and only if  $\Pi$  is a refinement of  $\Pi_{\Lambda}$ .

For every integer  $m \ge 1$ , we consider the set  $J_m = \{1, ..., m\}$ . Let  $\mathbb{Z}_+^m$  denote the set of vectors  $u = (u_1, ..., u_m) \in \mathbb{Z}^m$  with  $u_i \ge 0$  for every  $i \in J_m$ . For a partition  $\Pi = \{P_1, ..., P_m\}$  of a set P and for every  $A \subseteq P$  and  $i \in J_m$ , we define  $\Pi_i(A) = A \cap P_i$ . Then the partition  $\Pi$  defines a mapping  $\Pi : \mathcal{P}(P) \to \mathbb{Z}_+^m$  by considering  $\Pi(A) = (\Pi_1(A), ..., \Pi_m(A))$ . If  $\Lambda \subseteq \mathcal{P}(P)$  is  $\Pi$ -partite, then

 $A \in \Lambda$  if and only if  $\Pi(A) \in \Pi(\Lambda)$ . That is,  $\Lambda$  is completely determined by the partition  $\Pi$  and the set of vectors  $\Pi(\Lambda) \subset \mathbb{Z}_{+}^{m}$ .

Discrete polymatroids, a combinatorial object introduced by Herzog and Hibi [13], are closely related to multipartite matroids and, because of that, they play an important role in the characterization of ideal multipartite access structures. Before giving the definition of discrete polymatroid, we need to introduce some notation. If  $u, v \in \mathbb{Z}_+^m$ , we write  $u \leq v$  if  $u_i \leq v_i$  for every  $i \in J_m$ , and we write u < v if  $u \leq v$  and  $u \neq v$ . The vector  $w = u \lor v$  is defined by  $w_i = \max(u_i, v_i)$ . The modulus of a vector  $u \in \mathbb{Z}_+^m$  is  $|u| = u_1 + \dots + u_m$ . For every subset  $X \subseteq J_m$ , we write  $u(X) = (u_i)_{i \in X} \in \mathbb{Z}_+^{|X|}$  and  $|u(X)| = \sum_{i \in X} u_i$ 

A discrete polymatroid on the ground set  $J_m$  is a nonempty finite set of vectors  $D \subset \mathbb{Z}_+^m$  satisfying:

- 1. if  $u \in D$  and  $v \in \mathbb{Z}_+^m$  is such that  $v \leq u$ , then  $v \in D$ , and
- 2. for every pair of vectors  $u, v \in D$  with |u| < |v|, there exists  $w \in D$  with  $u < w \le u \lor v$ .

The next proposition, which is easily proved from the axioms of the independent sets of a matroid, shows the relation between multipartite matroids and discrete polymatroids.

**Proposition 2.1.** Let  $\Pi$  be a partition of a set  $\mathcal{Q}$  and let  $\mathcal{I} \subseteq \mathcal{P}(\mathcal{Q})$  be a  $\Pi$ -partite family of subsets. Then  $\mathcal{I}$  is the family of the independent sets of a  $\Pi$ -partite matroid  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  if and only if  $\Pi(\mathcal{I}) \subset \mathbb{Z}_+^m$  is a discrete polymatroid.

A basis of a discrete polymatroid D is a maximal element in D, that is, a vector  $u \in D$  such that there does not exist any  $v \in D$  with u < v. Similarly to matroids, a discrete polymatroid is determined by its bases. Specifically, the following result is proved in [13, Theorem 2.3].

**Proposition 2.2.** A nonempty subset  $\mathcal{B} \subset \mathbb{Z}_+^m$  is the family of bases of a discrete polymatroid if and only if it satisfies:

- 1. all elements in  $\mathcal{B}$  have the same modulus, and
- 2. for every  $u \in \mathcal{B}$  and  $v \in \mathcal{B}$  with  $u_i > v_i$ , there exists  $j \in J_m$  such that  $u_j < v_j$  and  $u e_i + e_j \in \mathcal{B}$ , where  $e_i$  denotes the *i*-th vector of the canonical basis of  $\mathbb{Z}^m$ .

The rank function of a discrete polymatroid D with ground set  $J_m$  is the function  $h: \mathcal{P}(J_m) \to \mathbb{Z}$  defined by  $h(X) = \max \{ |u(X)| : u \in D \}$ . The next proposition is a consequence of [13, Theorem 3.4].

**Proposition 2.3.** A function  $h: \mathcal{P}(J_m) \to \mathbb{Z}$  is the rank function of a discrete polymatroid with ground set  $J_m$  if and only if it satisfies

- 1.  $h(\phi) = 0$ , and
- 2. *h* is monotone increasing: if  $X \subseteq Y \subseteq J_m$ , then  $h(X) \le h(Y)$ , and
- 3. *h* is submodular: if  $X, Y \subseteq J_m$ , then  $h(X \bigcup Y) + h(X \cap Y) \le h(X) + h(Y)$ .

Moreover, a polymatroid D is completely determined by its rank function. Specifically,  $D = \left\{ u \in \mathbb{Z}_{+}^{m} : |u(X)| \leq h(X) \text{ for all } X \subseteq J_{m} \right\}.$ 

For a discrete polymatroid D with ground set  $J_m$  and for every  $X \subseteq J_m$ , we define the discrete polymatroid D(X) with ground set X by  $D(X) = \{u(X) : u \in D\} \subset \mathbb{Z}_+^{|X|}$ . This concept will be very useful in this paper.

Let  $\mathbb{K}$  be a field, E a  $\mathbb{K}$ -vector space, and  $V_1, ..., V_m$  subspaces of E. It is not difficult to check that the mapping  $h: \mathcal{P}(J_m) \to \mathbb{Z}$  defined by  $h(X) = \dim(\sum_{i \in X} V_i)$  is the rank function of a discrete polymatroid  $D \subset \mathbb{Z}_+^m$ . In this situation, we say that D is  $\mathbb{K}$ -representable and the subspaces  $V_1, ..., V_m$  are a  $\mathbb{K}$ -representation of D. The next proposition is proved in [9, Theorem 7.1].

**Proposition 2.4.** Let  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$  be a  $\Pi$ -partite matroid and let  $D = \Pi(\mathcal{I})$  be its associated discrete polymatroid. If  $\mathcal{M}$  is  $\mathbb{K}$ -representable, then so is D. In addition, if D is  $\mathbb{K}$ -representable, then  $\mathcal{M}$  is representable over some finite extension of  $\mathbb{K}$ .

#### **3** A Family of Non-representable Secret Sharing Matroids

In this section, we give a family of non-representable matroids derived from the Vamos matroid. Firstly, we introduce the Vamos matroid and give the proof of Vamos matroid being a non-representable multipartite matroid. Afterwards, through combining the partition of the ground set of Vamos matroid, we construct three "matroids", which are proved to be non-representable. However, according to the definition of matroid, we obtain these three "matroids" are pseudo matroids. Finally, from the concept of D(X) defined above, a family of non-representable matroids derived from the Vamos matroid is obtained, which we call Vamos Family.

#### 3.1 Vamos Matroid

The definition of Vamos matroid is as follows:

**Definition 3.1.** The Vamos matroid is defined on  $\mathcal{Q} = \{1, 2, 3, 4, 5, 6, 7, 8\}$  with bases all 4-sets except the five 4-sets which are:  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 5, 6\}$ ,  $\{1, 2, 7, 8\}$ ,  $\{3, 4, 5, 6\}$ ,  $\{3, 4, 7, 8\}$ .

The following proposition gives a new proof of the Vamos matroid being a non-representable multipartite matroid.

Proposition 3.1. The Vamos matroid is non-representable.

 $\Pi_0 = \{P_1, P_2, P_3, P_4\}$ Proof. For а partition  $(P_1 = \{1, 2\}, P_2 = \{3, 4\}, P_3 = \{5, 6, \}, P_4 = \{7, 8\})$  of the ground set  $\mathcal{Q}$ , the partition  $\Pi_0$  defines a mapping  $\Pi_0: \mathcal{P}(\mathcal{Q}) \to \mathbb{Z}_+^4$ . For every non-basis 4-set A, we compute  $\Pi_0(A)$  and obtain (2, 2, 0, 0), (2, 0, 2, 0), (2, 0, 0, 2), (0, 2, 2, 0), (0, 2, 0, 2). every basis B, we compute  $\Pi_0(B)$ Similarly. for and obtain (1,1,1,1),(1,1,2,0),(1,1,0,2),(1,2,1,0),(1,2,0,1),(1,0,1,2),(1, 0, 2, 1),(0,1,1,2), (0,1,2,1), (0,2,1,1), (2,1,0,1), (2,1,1,0), (2,0,1,1), (0,0,2,2). We can verify that for every 3-set C, there must exist a basis B such that  $\Pi_0(C) < \Pi_0(B)$ and  $C \subset B$ . Therefore, all 3-sets are independent.

Suppose that over some finite field  $\mathbb{K}$  there exists a matrix M which is a representation of the Vamos matroid, and every element  $i \in \mathcal{Q}$  correspond to the column vector  $v_i$  of M. Apparently, all vectors of M are non-zero vectors. Arbitrary four column vectors of M are linearly independent except  $(v_1, v_2, v_3, v_4)$ ,  $(v_1, v_2, v_5, v_6)$ ,  $(v_1, v_2, v_7, v_8)$ ,  $(v_3, v_4, v_5, v_6)$ ,  $(v_3, v_4, v_7, v_8)$ . Because all 3-sets are independent, for every one of these five vector groups, its rank is 3 and every vector in it can be uniquely represented by the other three vectors over  $\mathbb{K}$ . The following operations are over the finite field  $\mathbb{K}$ :

For the vector group 
$$(v_1, v_2, v_7, v_8)$$
, let  $v_8 = a_1v_1 + a_2v_2 + a_7v_7$  (1)

For the vector group 
$$(v_3, v_4, v_7, v_8)$$
, let  $v_8 = a_3 v_3 + a_4 v_4 + a_7' v_7$ . (2)

where  $a_1, a_2, a_7, a_3, a_4, a_7 \in \mathbb{K}$  and  $a_1, a_2, a_7, a_3, a_4, a_7 \neq 0$ . Simultaneous equations (1)(2), then

$$(a_7' - a_7)v_7 = a_1v_1 + a_2v_2 - a_3v_3 - a_4v_4$$
(3)

For the vector group 
$$(v_1, v_2, v_3, v_4)$$
, let  $v_4 = b_1 v_1 + b_2 v_2 + b_3 v_3$  (4)

where  $b_1, b_2, b_3 \in \mathbb{K}$  and  $b_1, b_2, b_3 \neq 0$ .

If  $(a_7 - a_7) \neq 0$ , then simultaneous equations (3)(4) and we obtain  $(v_1, v_2, v_3, v_7)$  are linearly dependent. Since  $\{1, 2, 3, 7\}$  is a basis of the Vamos matroid, a contradiction. Hence, there must be  $(a_7 - a_7) = 0$ , that is  $a_7 = a_7$ , then from equation (3) we obtain:

$$a_1 v_1 + a_2 v_2 = a_3 v_3 + a_4 v_4 \tag{5}$$

For the vector group  $(v_1, v_2, v_5, v_6)$ , let  $v_6 = c_1 v_1 + c_2 v_2 + c_5 v_5$  (6)

For the vector group  $(v_3, v_4, v_5, v_6)$ , let  $v_6 = c_3 v_3 + c_4 v_4 + c_5' v_5$  (7)

Similarly, we can obtain  $c_5 = c_5'$  and  $c_1v_1 + c_2v_2 = c_3v_3 + c_4v_4$  (8)

Computing equation  $c_1(5) - a_1(8)$ , then:

$$(a_2c_1 - a_1c_2)v_2 = (a_3c_1 - a_1c_3)v_3 + (a_4c_1 - a_1c_4)v_4$$
(9)

Because  $(v_2, v_3, v_4)$  are linearly independent, then  $a_2c_1 - a_1c_2 = 0$  (10) Computing equation  $c_1(1) - a_1(6)$ , then:

$$c_1 v_8 - a_1 v_6 = (a_2 c_1 - a_1 c_2) v_2 + a_7 c_1 v_7 - a_1 c_5 v_5$$
(11)

Simultaneous equations (10)(11), then  $c_1v_8 - a_1v_6 = a_7c_1v_7 - a_1c_5v_5$  (12)

Due to  $a_1, c_1, a_7, c_5 \neq 0$ , from equation (12) we can obtain  $(v_5, v_6, v_7, v_8)$  are linearly dependent. Since  $\{5, 6, 7, 8\}$  is a basis of the Vamos matroid, a contradiction. Therefore, it is impossible that there exists a matrix M over some finite field  $\mathbb{K}$  which is a representation of the Vamos matroid, that is, the Vamos matroid is non-representable.

#### 3.2 Three Non-representable Pseudo Matroids

Through combining the partition of the ground set of Vamos matroid, we construct three "matroids" as follow:

**Definition 3.2.** The Pseudo-1 matroid is defined on  $\mathcal{Q} = \{1, 2, 3, 4, 5, 6, 7, 8\}$  with bases all 4-sets except the thirteen 4-sets which are:  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 5, 6\}$ ,  $\{1, 2, 5, 7\}$ ,  $\{1, 2, 5, 8\}$ ,  $\{1, 2, 6, 7\}$ ,  $\{1, 2, 6, 8\}$ ,  $\{1, 2, 7, 8\}$ ,  $\{3, 4, 5, 6\}$ ,  $\{3, 4, 5, 7\}$ ,  $\{3, 4, 5, 8\}$ ,  $\{3, 4, 6, 7\}$ ,  $\{3, 4, 6, 8\}$ ,  $\{3, 4, 7, 8\}$ .

**Definition 3.3.** The Pseudo-2 matroid is defined on  $\mathcal{Q} = \{1, 2, 3, 4, 5, 6, 7, 8\}$  with bases all 4-sets except the thirteen 4-sets which are:  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 5, 6\}$ ,  $\{1, 3, 5, 6\}$ ,  $\{1, 4, 5, 6\}$ ,  $\{2, 3, 5, 6\}$ ,  $\{2, 4, 5, 6\}$ ,  $\{3, 4, 5, 6\}$ ,  $\{1, 2, 7, 8\}$ ,  $\{1, 3, 7, 8\}$ ,  $\{1, 4, 7, 8\}$ ,  $\{2, 3, 7, 8\}$ ,  $\{2, 4, 7, 8\}$ ,  $\{3, 4, 7, 8\}$ .

**Definition 3.4.** The Pseudo-3 matroid is defined on  $\mathcal{Q} = \{1, 2, 3, 4, 5, 6, 7, 8\}$  with bases all 4-sets except the thirty-seven 4-sets which are:  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 5, 6\}$ ,  $\{1, 3, 5, 6\}$ ,  $\{1, 4, 5, 6\}$ ,  $\{2, 3, 5, 6\}$ ,  $\{2, 4, 5, 6\}$ ,  $\{3, 4, 5, 6\}$ ,  $\{1, 2, 5, 7\}$ ,  $\{1, 3, 5, 7\}$ ,  $\{1, 4, 5, 7\}$ ,  $\{2, 3, 5, 7\}$ ,  $\{2, 4, 5, 7\}$ ,  $\{3, 4, 5, 7\}$ ,  $\{1, 2, 5, 8\}$ ,  $\{1, 3, 5, 8\}$ ,  $\{1, 4, 5, 8\}$ ,  $\{2, 3, 5, 8\}$ ,  $\{2, 4, 5, 8\}$ ,  $\{3, 4, 5, 8\}$ ,  $\{1, 2, 6, 7\}$ ,  $\{1, 3, 6, 7\}$ ,  $\{1, 4, 6, 7\}$ ,  $\{2, 3, 6, 7\}$ ,  $\{2, 4, 6, 8\}$ ,  $\{3, 4, 6, 7\}$ ,  $\{1, 2, 6, 8\}$ ,  $\{1, 3, 6, 8\}$ ,  $\{1, 4, 7, 8\}$ ,  $\{2, 3, 7, 8\}$ ,  $\{2, 4, 7, 8\}$ ,  $\{3, 4, 7, 8\}$ .

In the following propositions, we prove that these three "matroids" stated above are all non-representable.

Proposition 3.2. The Pseudo-1 matroid is non-representable.

**Proof.** For a partition  $\Pi_1 = \{P_1, P_2, P_3\}$  ( $P_1 = \{1, 2\}, P_2 = \{3, 4\}, P_3 = \{5, 6, 7, 8\}$ ) of the ground set  $\mathcal{Q}$ , the partition  $\Pi_1$  defines a mapping  $\Pi_1 : \mathcal{P}(\mathcal{Q}) \to \mathbb{Z}_+^3$ . For every non-basis 4-set A, we compute  $\Pi_1(A)$  and obtain (2, 2, 0), (2, 0, 2), (0, 2, 2). Similarly, for every basis B, we compute  $\Pi_1(B)$  and obtain (1, 1, 2), (1, 2, 1), (1, 0, 3), (0, 1, 3), (2, 1, 1), (0, 0, 4). We can verify that for every 3-set C, there must exist a basis B such that  $\Pi_1(C) < \Pi_1(B)$  and  $C \subset B$ . Therefore, all 3-sets are independent.

Suppose that over some finite field  $\mathbb{K}$  there exists a matrix M which is a representation of the Pseudo-1 matroid, and every element  $i \in \mathcal{Q}$  correspond to the column vector  $v_i$  of M. Apparently, all vectors of M are non-zero vectors. Arbitrary four column vectors of M are linearly independent except  $(v_1, v_2, v_3, v_4)$ ,  $(v_1, v_2, v_5, v_6)$ ,  $(v_1, v_2, v_5, v_7)$ ,  $(v_1, v_2, v_5, v_8)$ ,  $(v_1, v_2, v_6, v_7)$ ,  $(v_1, v_2, v_5, v_6)$ ,  $(v_1, v_2, v_7, v_8)$ ,  $(v_3, v_4, v_5, v_6)$ ,  $(v_3, v_4, v_5, v_7)$ ,  $(v_3, v_4, v_5, v_8)$ ,  $(v_3, v_4, v_6, v_7)$ ,  $(v_3, v_4, v_6, v_8)$ ,  $(v_3, v_4, v_7, v_8)$ . Because all

3-sets are independent, for every one of these thirteen vector groups, its rank is 3 and every vector in it can be uniquely represented by the other three vectors over  $\mathbb{K}$ . The following proof is the same to the proof of Proposion 3.1.

Proposition 3.3. The Pseudo-2 matroid is non-representable.

**Proof.** For a partition  $\Pi_2 = \{P_1, P_2, P_3\}$   $(P_1 = \{1, 2, 3, 4\}, P_2 = \{5, 6\}, P_3 = \{7, 8\})$  of the ground set  $\mathcal{Q}$ , the partition  $\Pi_2$  defines a mapping  $\Pi_2 : \mathcal{P}(\mathcal{Q}) \to \mathbb{Z}_+^3$ . For every non-basis 4-set A, we compute  $\Pi_2(A)$  and obtain (4, 0, 0), (2, 0, 2), (2, 2, 0). Similarly, for every basis B, we compute  $\Pi_2(B)$  and obtain (2, 1, 1), (3, 1, 0), (3, 0, 1), (1, 1, 2), (1, 2, 1), (0, 2, 2). We can verify that for every 3-set C, there must exist a basis B such that  $\Pi_2(C) < \Pi_2(B)$  and  $C \subset B$ . Therefore, all 3-sets are independent. The following proof is the same to the proof of Proposion 3.1.

Proposition 3.4. The Pseudo-3 matroid is non-representable.

**Proof.** For a partition  $\Pi_3 = \{P_1, P_2\}$  ( $P_1 = \{1, 2, 3, 4\}, P_2 = \{5, 6, 7, 8\}$ ) of the ground set  $\mathcal{Q}$ , the partition  $\Pi_3$  defines a mapping  $\Pi_3 : \mathcal{P}(\mathcal{Q}) \to \mathbb{Z}_+^2$ . For every non-basis 4-set A, we compute  $\Pi_3(A)$  and obtain (4, 0), (2, 2). Similarly, for every basis B, we compute  $\Pi_3(B)$  and obtain (1, 3), (3, 1), (0, 4). We can verify that for every 3-set C, there must exist a basis B such that  $\Pi_3(C) < \Pi_3(B)$  and  $C \subset B$ . Therefore, all 3-sets are independent. The following proof is the same to the proof of Proposion 3.1.

If these three non-representable "matroids" accord with the definition of matroid, it means there exist non-representable bipartite and tripartite matroids. However, we will show these three non-representable "matroids" are pseudo matroids.

From Proposition 2.2, for every  $u \in \mathcal{B}$  and  $v \in \mathcal{B}$  with  $u_i > v_i$ , there exists  $j \in J_m$  such that  $u_j < v_j$  and  $u - e_i + e_j \in \mathcal{B}$ , where  $e_i$  denotes the *i*-th vector of the canonical basis of  $\mathbb{Z}^m$ . In Pseudo-1 matroid, for u = (2,1,1) and v = (0,0,4) with  $u_2 > v_2$ , there only exists  $u_3 < v_3$  but (2,0,2) is not a basis. Therefore, Pseudo-1 matroid is not a matroid, namely, a pseudo matroid. Similarly, in Pseudo-2 matroid, for u = (1,2,1) and v = (3,1,0) with  $u_3 > v_3$ , there only exists  $u_1 < v_1$  but (2,2,0) is not a basis. Therefore, Pseudo-2 matroid, for u = (3,1) and v = (0,4) with  $u_1 > v_1$ , there only exists  $u_2 < v_2$  but (2,2) is not a basis. Therefore, Pseudo-3 matroid, for u = (3,1) and v = (0,4) with  $u_1 > v_1$ , there only exists  $u_2 < v_2$  but (2,2) is not a basis. Therefore, Pseudo-3 matroid is a pseudo matroid. As a consequence, these three non-representable "matroids" are all pseudo matroids.

### 3.3 Vamos Family

For the Vamos matroid  $\mathcal{M} = (\mathcal{Q}, \mathcal{I})$ , there exists a partition  $\Pi_0 = \{P_1, P_2, P_3, P_4\}$   $(P_1 = \{1, 2\}, P_2 = \{3, 4\}, P_3 = \{5, 6,\}, P_4 = \{7, 8\})$  of the ground set  $\mathcal{Q}$ , and the partition  $\Pi_0$  defines a mapping  $\Pi_0 : \mathcal{P}(\mathcal{Q}) \to \mathbb{Z}_+^4$  and, hence, we obtain a discrete polymatroid  $D_V = \Pi_0(\mathcal{I})$  corresponding to the Vamos matroid.

**Proposition 3.5.** For a discrete polymatroid D with ground set  $J_m$ , if there exists  $X \subseteq J_m$ , where |X| = 4, such that  $D(X) = D_V$ , then D must be a non-representable discrete polymatroid, and hence, the multipartite matroid corresponding to D must be non-representable. All of these discrete polymatroids construct a family of non-representable matroids, that is,  $F_{D_V} = \{D \subset \mathbb{Z}^m_+ : D(X) = D_V, X \subset J_m \boxplus |X| = 4\}$ , which we call Vamos Family.

The proof of Proposition 3.5 is very simple, which is a special case of the proof of Theorem 4.1. Suppose a discrete polymatroid D in Vamos Family is representable. We will obtain the Vamos matroid is representable, contradiction. Therefore, the discrete polymatroids in Vamos Family is non-representable.

# 4 A Sufficient Condition for a Discrete Polymatroid to Be Non-representable

In this section, we extend the Vamos Family to the general case and obtain a sufficient condition for a discrete polymatroid to be non-representable.

**Theorem 4.1.** Let  $D \subset \mathbb{Z}_+^m$  be a discrete polymatroid with ground set  $J_m$ , if there exists  $X \subseteq J_m$  such that  $D(X) = \{u(X) : u \in D\} \subset \mathbb{Z}_+^{|X|}$  is a non-representable discrete polymatroid, then D must be a non-representable discrete polymatroid and, hence, the multipartite matroid corresponding to D must be non-representable.

**Proof.** Let  $D \subset \mathbb{Z}_+^m$  be a discrete polymatroid with ground set  $J_m$ . There exists  $X \subseteq J_m$  such that  $D(X) = \{u(X) : u \in D\} \subset \mathbb{Z}_+^{|X|}$  is a non-representable discrete polymatroid. Suppose D is representable over some finite field  $\mathbb{K}$ , i.e., there exists a vector space  $E = \mathbb{K}^s$  over  $\mathbb{K}$ , where  $s = h(J_m)$ , such that m subspaces  $V_1, \ldots, V_m$  of E are a  $\mathbb{K}$ -representation of D. Let  $X = \{x_1, \ldots, x_r\}$ , where |X| = r and, hence, the subspaces corresponding to the elements of  $X \subseteq J_m$  are

 $V_{x_1},...,V_{x_r}$ . Since  $D(X) = \{u(X) : u \in D\} \subset \mathbb{Z}_+^{|X|}$ , it means r subspaces  $V_{x_1},...,V_{x_r}$  of  $E = \mathbb{K}^s$  are a  $\mathbb{K}$ -representation of D(X), namely, D(X) is a  $\mathbb{K}$ -representable discrete polymatroid, contradiction. Therefore, D is a non-representable discrete polymatroid and, hence, the multipartite matroid corresponding to D must be non-representable.

As a consequence, Theorem 4.1 gives a sufficient condition for a discrete polymatroid to be non-representable.

### 5 Conclusion

In this paper, by dealing with a family of matroids derived from the Vamos matroid, which was the first matroid that was proved to be non-representable, we obtain a family of non-representable matroids. As a consequence, we extend it to the general case and obtain a sufficient condition for a discrete polymatroid to be nonrepresentable, which is a new contribution to the open problem given by Farras.

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# A Novel Adaptive Watermarking Scheme Based on Human Visual System and Particle Swarm Optimization

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**Abstract.** In this paper, we proposed a novel watermarking scheme based on adaptive quantization index modulation and singular value decomposition in the hybrid discrete wavelet transform (DWT) and discrete cosine transform (DCT). The secret watermark bits are embedded on the singular values vector of blocks within low frequency subband in host image hybrid DWT-DCT domain. To embed watermark imperceptibly, robustly and securely, we model the adaptive quantization steps by utilizing human visual system (HVS) characteristics and particle swarm optimization (PSO) algorithm. Experimental results demonstrate that the proposed scheme is robust to variety of image processing attacks. In the proposed algorithm the quantized embedding strategy is adopted, so no host image is needed for blind extraction of watermarking image.

**Keywords:** quantization index modulation, singular value decomposition, human visual system, particle swarm optimization, blind extraction.

# 1 Introduction

With the development and popularization of multimedia technology and computer network technology, various multimedia products such as image, audio, video and three-dimensional model are increasingly vulnerable to illegal possession, reproduction and dissemination. How to effectively protect the copyright and content integrity of multimedia information has been attached importance to by more and more researchers. Digital watermarking has emerged as a leading technique that could solve the fundamental problem of legal ownership and content authentication for digital multimedia data. In general, the watermarking scheme shall satisfy two properties. First, the watermark should not affect the quality of the host media and be imperceptible to human eyes. Second, if watermark is used for Internet applications such as transmitting data through a noisy channel or compressing data, the watermark must survive under those situations [1].

In accordance with embedded position, watermarking algorithm can be divided into two categories: space domain and transform domain. The transform domain algorithm robustness is better than the space domain algorithm generally. The common transform includes Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT) and Discrete Wavelet Transform (DWT) and so on. Image matrix singular value decomposition (SVD) reflects the internal image characteristics and has good stability if image processing is performed. Therefore many watermarking schemes that are combined different transforms with SVD have been proposed lately. For example, Tsai and Yang [2] proposed an approach emphasized that watermark message bit is embedded on the blocks of the DCT coefficient's singular value within an original color image. Bao and Ma [3] described a hybrid DWT-SVD watermarking algorithm that watermark bits are embedded on the singular value of the blocks within wavelet subband of the original image. However, most of these hybrid transform methods are non-adaptive or may not exploit the HVS features for more effectively embedding robust and secure watermark.

Recently, many researchers focus on adaptive determination of the quantization parameters for SVD-based watermarking. They consider to solving this problem using adopting artificial intelligence techniques or analyzing statistical model of each block in the image. Aslantas [4][5] respectively utilized the genetic algorithm (GA) and particle swarm optimization algorithm (PSO) for optimizing the quantization steps. Lai et al. [6] choose the micro-genetic algorithm (micro-GA) for optimization quantization steps. However, these methods are all non-blinding algorithm so that the security and practicality of these algorithms are not robust. Qi et al. [7] applied a grid search algorithm to find the effective pairs for a set of training images covering a variety of textures ranging from high, medium, low, to extremely low textures. The quantization steps are determined to be adaptive to the statistical model of the block, but this approach is not incorporate with the human visual system for improving the adaptive capability of the watermarking system.

In this paper, we design a novel adaptive watermarking scheme by combing the HVS and PSO. The watermark bits are embedded on singular value vector of each embedding block within low frequency subband in the hybrid DWT-DCT domain. One quantization parameter is determined by exploiting the characteristics of HVS, the other quantization parameter is optimized through PSO algorithm. These two quantization parameters are combined for ensuring the final adaptive quantization steps are optimal for all embedding blocks and reaching better trade off between the imperceptibility and robustness of the digital watermarking system. Experimental results show that the proposed scheme not only has good visual quality irrespective of the nature of the images chosen but also is robust to image processing attacks.

The rest of the paper is structured as follows. In section 2, we present the basic idea and key techniques of PSO algorithm. In section 3, the proposed watermarking scheme combining with human visual masking and particle swarm optimization algorithm in the hybrid DWT-DCT is described. The performance analysis for the proposed watermarking scheme and comparisons with other designs are presented in section 4. Finally, the paper ends with a brief conclusion.

#### 2 Particle Swarm Optimization

PSO is a population-based stochastic optimization method introduced first by Kennedy and Eberhart [8], derived by the social behavior of bird flocking and fish schooling. The population is called a swarm consisted of different particles, which have a position and a velocity and change their positions in multi-dimensional search space over time. Each particle represents a possible solution to the optimization problems in the multi-dimensional problem space. These particles start at a random initial position and search for the minimum or maximum of a given objective function by moving through the search space. After each iteration process in the search space, each particle records its own personal best solution as well as the discovered global best position. The movement of each particle depends only on its velocity and the location where good solutions have already been found by the particle itself or in neighboring particles. When a particle's neighborhood is defined as the whole swarm, the PSO is called the global version, otherwise it is called the local version [8].In the following parts global version will be discussed.

Let a swarm include different particles  $X_i$  (i = 1, 2, ..., m) and the *i*th particle in a ddimensional space be represented as  $X_i = (x_{i1}, x_{i2}, ..., x_{id})$ . The best previous position of the *i*th particle *pbest<sub>i</sub>* is denoted by  $P_i = (p_{i1}, p_{i2}, ..., p_{id})$ . The best global location is represented *pbest<sub>g</sub>* among all the particles. The velocity for the *i*th particle is represented as  $V_i = (v_{i1}, v_{i2}, ..., v_{id})$ . During each iteration process, each particle in the swarm is updated the velocity and location towards its *pbest<sub>i</sub>* and *pbest<sub>g</sub>* locations each iteration according to following two equations respectively:

$$V_i = w_i v_i + c_1 \xi \left( pbest_i - X_i \right) + c_2 \eta \left( pbest_g - X_i \right)$$
(1)

$$X_i = X_i + V_i \tag{2}$$

where  $\xi$  and  $\eta$  are random variables drawn from a uniform distribution in the range [0,1] so as to provide a stochastic weighting of the different components participating in the particle velocity definition.  $c_1$  and  $c_2$  are two acceleration constants and called as cognitive acceleration and social acceleration respectively. They are factors regulating the particle relative velocities with respect to the best global and local positions respectively. The inertia weight  $w_i$  is used to decide a tradeoff between the global and local exploration capabilities of the swarm. Typical implementations of the PSO adapt the value of linearly decreasing it from 1.0 to near 0 over the execution. In general, the inertia weight  $w_i$  is set according to the following equation [9]:

$$w_i = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter$$
(3)

where  $iter_{max}$  is the maximum number of iterations, and *iter* is the current number of iterations.

### 3 Proposed Watermarking Scheme

In our image watermarking scheme, the watermark can be embedded into the host image by three steps. First, DWT is performed on the host image. Second, the low frequency component is segmented into non-overlapping blocks and these blocks are performed on DCT. Then a set of final quantization steps are modeled both the characteristics of the DCT domain human visual masking and particle swarm optimization of each block to ensure a high perceptual quality of watermarked image and a low bit error rate of the detected watermark. Finally, watermark is embedded into the singular values vector of each block by adaptive and optimized quantization steps. A diagram of our image watermark embedding scheme is shown in Fig.1.

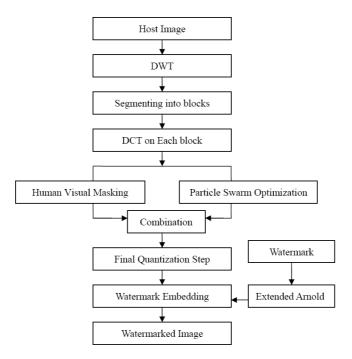


Fig. 1. Watermark embedding scheme

### 3.1 Watermarking Embedding Scheme

Suppose the host image is  $I_0$  with m×n that is decomposed in *j* levels DWT, we obtain the low frequency subband  $LL_j$  and three high frequency subbands  $HL_j$ ,  $LH_j$ ,  $HH_j$ . To take the advantage of low frequency coefficients which have a higher energy value and robustness against various signal processing, the DCT is only performed on low frequency coefficient  $LL_j$ . The embedding procedure can be described as follows steps:

Step 1: Segment the  $LL_i$  into non-overlapping blocks  $A_i$  of size  $w \times w$ , i = 1, 2, ..., M, where M is the number of the blocks.

Step 2: Each block  $A_i$  is individually transformed to a frequency coefficient using DCT transform, and then compute the singular values vector of each frequency coefficient block  $A_i$  by SVD according to the following equation.

$$DCT(A_i) = U_i \sum_i V_i^T$$
(4)

Step 3: Compute  $N_i^s = ||s_i|| + 1$  and quantize it by adaptive quantization step  $\delta_i$  that represents the quantization level for  $N_i^s$  corresponds to the frequency coefficient block  $A_i$  according to the following equation.

$$N_{i} = \left\lfloor \frac{N_{i}^{s}}{\delta_{i}} \right\rfloor (i = 1, 2, \dots M)$$
(5)

where  $s_i = (\sigma_{i1}, \sigma_{i2}, ..., \sigma_{iw})$ ,  $s_i$  denotes a vector formed by the singular values of the frequency coefficient block  $A_i$ .

Step 4:Embed each watermark bit by modifying integer number  $N_i$ , according to the following equation.

$$N_{iw} = \begin{cases} N_i + 1 & if \\ N_i + 1 & if \\ N_i & else \end{cases} \frac{\text{mod}(N_i, 2) = 1 \& w_i = 1}{\text{mod}(N_i, 2) = 0 \& w_i = 0} \\ N_i & else & otherwise \end{cases}$$
(6)

Step 5: Compute the value  $N_{iw}^{s} = \delta_i \times (N_{iw} + \frac{1}{2})$  and the modified singular values.

$$(\sigma'_{i1}, \sigma'_{i2}, ..., \sigma'_{iw}) = (\sigma_{i1}, \sigma_{i2}, ..., \sigma_{iw}) \times (\frac{N_{iw}}{N_i^s})$$
(7)

Step 6: Compute the watermarked block  $A'_i$  with modified singular values. The watermarked low frequency subband  $LL'_i$  is reshaped through  $A'_i$  performed on Inverse Discrete Cosine Transform (IDCT), then the watermarked image  $I'_0$  is obtained utilizing Inverse Discrete Wavelet Transform (IDWT).

#### 3.2 Adaptive Quantization Steps

The embedding strength is more or less proportional to the perceptual sensitivity to distortions for using adaptive quantization step size. In order to resist the normal signal processing and other different attacks, we wish the quantization step to be as high as possible. However, because the watermark directly affects the host image, it is obvious that the higher the quantization step, the lower the quality of the watermarked image will be. In other words, the robustness and the imperceptibility of the watermark are contradictory to each other. In this section, we propose a novel model for

determining the adaptive quantization steps combining with the characteristics of the HVS and PSO in order to guarantee robustness and transparency of watermark.

#### 3.2.1 Human Visual Masking

HVS is one of the most complex biological systems which include three stages: encoding, representation and interpretation [10]. Many factors cause human vision to have a limited sensitivity. For example, the surface of the cornea causes the refraction; the circular entrance of the pupil causes the diffraction; the optical lens has the chromatic aberration effects, and the mosaic of photo-receptors only does the spatial sampling process [10]. Human vision perceptive redundancy gives us a good opportunity to choose proper quantization step for embedding watermark in the image.

According to the Watson model [11] that is designed in DCT domain, the brighter the background, the higher the visibility threshold (luminance masking), the background texture more complex, the higher the visibility threshold (texture masking).We determine the luminance masking and texture masking based on the image features in DCT domain, and then combine these two masking together to get a comprehensive final masking. The main steps of the proposed HVS model can be described as follows:

Step 1: Segment the  $LL_i$  into non-overlapping blocks  $A_i$  of size  $w \times w$ , i = 1, 2, ..., M, where M is the number of the blocks.

Step 2: Calculate luminance masking  $M_i^L$ 

The reason to use luminance is that the darkness and brightness in the image is reflected by luminance. The visibility of luminance threshold in the DCT domain depends on background luminance that is expressed by the Direct Current (DC) components. We can compute a luminance masking value  $M_i^L$  for each block  $A_i$ .

$$M_i^{\ L} = D_i(0,0) \tag{8}$$

where  $D_i(0,0)$  is the DC coefficient of  $A_i$  that performed on DCT domain.

Step3:Calculate texture masking  $M_i^T$ 

As for texture masking, the simplest method is to use the block variance.

$$M_{i}^{T} = \frac{1}{w \times w} \sum_{i=1}^{w} \sum_{j=1}^{w} (D_{i}(i, j) - \overline{D}_{i})^{2}$$
(9)

$$\overline{D}_{i} = \frac{1}{w \times w} \sum_{i=1}^{w} \sum_{j=1}^{w} D_{i}\left(i, j\right)$$

$$\tag{10}$$

where  $\overline{D}_i$  is average value of each block  $A_i$  DCT coefficients.

Step 4: Calculate quantization step  $\delta_i$  for each block  $A_i \cdot \delta_{i0}$  is the basic step of each block that will be obtained by utilizing PSO for achieving optimal watermarking performance depending on both the transparency and the robustness factors.

$$\delta_{i} = \left\lfloor \log 2^{M_{i}^{L} \times M_{i}^{T}} \times 1000 \right\rfloor / 1000 + \delta_{i0}$$
<sup>(11)</sup>

#### 3.2.2 Optimal Basic Step

This section illustrates how to use PSO help search proper basic step of each block in order to optimize watermark embedding process. It is a difficult for determining the proper values of multiple basic quantization steps. In some cases, the choice of them may be based on some general assumption [12]. Therefore, an efficient and optimal algorithm is required for achieving both invisibility and robustness. Here we use PSO to automatically determine these values without making any assumption. In the application of PSO, we should consider three essential components:

(1)Solution representation and Initialization

The representation scheme determines how the problem is structured, as well as the iteration operators that can be used [6].Each particle in the swarm represents a possible solution to the problem and hence consists of a set of basic step of each block. Meanwhile we randomly generate each particle value the in the initial swarm.

(2) Fitness function

The watermarked image transparency and robustness should be measured in order to formulate a proper fitness function. We adopt the same fitness function as used in [4], that is:

$$f_{i} = \left[\frac{m}{\sum_{i=1}^{m} NC_{i}(W_{i}', W)} - NC(I_{0}', I_{0})\right]^{-1}$$
(12)

where  $NC_i$  denotes the two-dimensional normalized correlation, and *m* represents the number of attacking methods.

(3) PSO training operation

The similar PSO based watermarking algorithm proposed by Aslantas et al [5] is adopted. A diagram of PSO optimization training is shown in Fig.2.

1).Define the swarm size, particle size, cognitive acceleration and social acceleration and set the inertia weight of the swarm equation.

2).Generate the initial swarm, initial velocity, and initial fitness function value randomly.

3).Produce watermarked images utilizing the solutions in the swarm by means of embedding scheme combing with HVS.

4).Calculate the NC values between the host image and watermarked images.

5). Apply the attacks upon the watermarked images one by one.

6).Extract out watermarks from the attacked images using the extraction scheme.

7).Calculate the NC values between the watermark and the extracted ones.

8).Calculate the value of each particle, feedback optimal values to the PSO and obtain new particle value.

9).Repeat Steps 3-8 until the predefined termination criterion is satisfied.

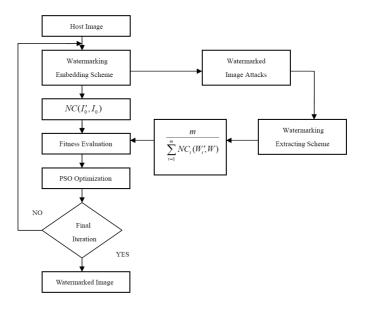


Fig. 2. Diagram of PSO training

#### 3.3 Watermarking Extracting Scheme

The watermark extracting scheme is the inverse of embedding procedure that neither needs the host image signal nor any other side information. Suppose the watermarked image is  $I'_0$  that is decomposed in *j* levels DWT, we obtain low frequency subband  $LL'_i$ . The extracting procedure is given as follows:

Step 1: Segment the  $LL'_{i}$  into non-overlapping blocks  $A'_{i}$  of size  $w \times w$  i = 1, 2, ..., M, where *M* is the number of the blocks.

Step 2: Transform each block  $A'_i$  to the frequency coefficient by DCT, then compute  $N'_i = ||s'_i|| + 1$  and quantize it by optimal final quantization step  $\delta_i$ , where  $s'_i = (\sigma'_{i1}, \sigma'_{i2}, ..., \sigma'_{iw})$ ,  $s'_i$  denotes a vector formed by the singular values of the frequency coefficient block  $A'_i$ .

$$N_i' = \begin{bmatrix} N_i'^s \\ \delta_i \end{bmatrix} (i = 1, 2, ..., M)$$
(13)

Step 3: Extract watermark bits according to the following equation.

$$w'_{i} = \begin{cases} 1 & if \mod(N'_{i}, 2) = 0\\ 0 & else \mod(N'_{i}, 2) = 1 \end{cases}$$
(14)

Step 4: Reshape the original binary watermark image by performing inverse extended Arnolded scrambling on watermark image according to the extracted watermark bits.

# 4 Experimental Results

The performance of the proposed watermarking scheme is tested on a large number of experiments. Here the results are presented for grayscale 8-bit different texture images of size  $512\times512$ . The logo used for watermark image is the binary image of size  $32\times32$ . For 1-level wavelet decomposition Haar filter coefficients are used. In the PSO training process,  $c_1$  and  $c_2$  are set as 1.2 and 1.8 respectively, the number of particles are chosen as 20 and the number of total iterations is set as 20. At the same time, the experiments compare the performance of proposed scheme with Qi's scheme that is a novel watermarking based on adaptive quantization steps determined by the statistical model of the block in the image.

Fig.3 (a) and (b) are the host image and the watermarked image. In subjective, it seems difficult to distinguish the difference between the host and the watermarked images by the human eye.



Fig. 3. Host image and watermarked image

In objective, The Peak Signal to Noise Ratio (PSNR) is used efficiently measure of visual fidelity between the host image and the watermarked image. It is found that the image quality measured by PSNR among the watermarked image is greater than 43dB. This indicates that the proposed watermarking scheme has good visual fidelity. Meanwhile Tab.1 simultaneity compares in terms of PSNR for evaluating the visual fidelity performance of our scheme with Qi's scheme.

Image	Proposed Scheme	Qi <sup>°</sup> Scheme
	PSNR(dB)	PSNR(dB)
Lena	43.94	43.67
Peppers	44.63	44.26
Baboon	44.83	44.54

Table	1.	The	values	of PSNR
I GOIC		1110	raraco	0110101

The watermarking scheme should be robust to signal processing which could be intentional or unintentional. Normalized Correlation (NC) and Bit Error Ratio (BER) that are defined as following equation are adopted for evaluating the robustness of the watermarking scheme. Without any image attacks, the NC value is 1 and the BER value is 0. In other words, the watermark can be completely extracted.

$$NC(W',W) = \frac{\sum_{i=1}^{p} \sum_{j=1}^{q} w(i,j)w'(i,j)}{\sqrt{\sum_{i=1}^{p} \sum_{j=1}^{q} w^{2}(i,j)} \sqrt{\sum_{i=1}^{p} \sum_{j=1}^{q} w'^{2}(i,j)}}$$
(15)

$$BER(W',W) = \frac{\sum_{i=1}^{p} \sum_{j=1}^{q} \sqrt{w'(i,j) \oplus w(i,j)}}{p \times q} \times 100\%$$
(16)

Tab. 2. illustrates results after various attacks on the watermarked image including noise addition, low pass filtering, scaling and cropping.

Attack	NC	BER
Gauss noise(var 0.1%)	0.9689	2.54
Pepper&Salt nosie(density 0.1%)	0.9963	0.29
Gaussian filtering	0.9804	1.56
Median filtering	0.9038	7.71
Scaling 50%	0.9766	0.98
Cropping 25%	0.8569	15.38

Table 2. Experiment results after different attacks

Then, we test the robustness by JPEG compression that is one of the most important attacks with different quality factors (QF). Related results are shown in Tab.3.for the BER versus JPEG compression. Compared with Qi's scheme, it is observed that there is higher robustness to JPEG compression with proposed scheme.

Table 3. Experiment results comparison under JPEG compression

QF	Proposed scheme's BER(%)	Qi's scheme BER(%)
40	0.18	0.2
30	0.78	1.25
20	8.49	8.98
10	43.45	45.13

# 5 Conclusion

In this paper, we propose a novel adaptive watermarking scheme based on HVS and PSO in hybrid DWT-DCT transform. After decomposing the host image by DWT and DCT, watermark bits are embedded into the singular values of the embedded blocks within low frequency coefficients subband of the host image. The quantization steps of watermarking are modeled based on HVS characteristics and PSO. For adopting

the quantization steps, the embedded information can be extracted without host image. The experimental results show that the proposed scheme preserves not only the high perceptual quality, but also is robust against many different types of attacks.

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# Defending against the Pirate Evolution Attack

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Abstract. A trace and revoke scheme is an encryption scheme for secure content distribution so that only authorized users can access the copyrighted content. When a clone device is recovered, the "trace" component detects the pirate users that have compromised the secret keys in their devices and participated in the construction of the clone device. The "revoke" component excludes the pirate users from accessing the future content. The state-of-art trace-revoke scheme is the very efficient subset difference based NNL scheme 11 which is also deployed in AACS 1. the industry new content protection standard for high definition DVDs. While its revocation and tracing are both very efficient, as pointed out by Kiayias and Pehlivanoglu from Crypto 2007, in its deployment NNL scheme may suffer from a new attack called *pirate evolution attack*. In this attack attackers reveal the compromised secret keys to the clone decoder very slowly through a number of generations of pirate decoders that will take long time to disable them all. They showed in a system with N users, the attacker can produce up to t \* logN generations of pirate decoders given t sets of keys. In AACS context, that means a pirate can produce more than 300 generations of decoders by compromising only 10 devices. If this happens, it will indeed be a nightmare.

In this paper we are interested in practical solutions that can defend well against the pirate evolution attack in practice. In particular we devise an easy and efficient approach for the subset difference based NNL scheme  $\square$  to defend well against the potential pirate evolution attack. Indeed it takes as small as 2 generations to detect and disable a traitor in a coalition. This can be achieved by only negligibly increasing the cipher text header size in an application like AACS. The simplicity, efficiency and practicality of our approach has made AACS to adopt it to defend against the pirate evolution attack.

**Keywords:** Traitor Tracing, Broadcast Encryption, Pirate Evolution Attack.

# 1 Introduction

In this paper we are concerned with content protection for copyrighted materials when distributed to large number of users (receivers or devices), for example, a system for distributing physical media, like DVDs. Broadcast encryption schemes have been used to enable a set of privileged users to access content but exclude another set of revoked users from accessing it. In a broadcast encryption system, each user is assigned a set of secret keys, sometimes termed as *device keys*. The broadcaster broadcasts *header* information and *body* to a group of users. The *header* carries information of a session key, with which the *body* is encrypted. To be more concrete, a session key (sometimes termed a *media key* K) is indirectly used to encrypt the content in the body. The header contains a structure sometimes called a MKB (media key block), which is basically the media key encrypted with secret device keys from privileged users and only privileged users. It may contain some random string (garbage) encrypted by device keys from revoked users. In this way, each privileged user can use his secret device keys to process the MKB differently but get the same valid media key. In contrast, all the revoked users will process the MKB and get garbage.

Traitor tracing is another important technology used in content protection to detect the non-compliant users (denoted as *traitors*) who are involved in pirate attacks. A particular pirate attack in a broadcast encryption scheme is when some of the legitimate users circumvent the devices and extract secret device keys out of the boxes. They collude to build a pirate decoder using the extracted device keys and sell it to illegitimate users for commercial interest. For example, pirate decoders enable illegitimate users to make permanent copies of DVD movies that they rented. When a pirate decoder is found, a traitor tracing scheme detects the traitorous devices' keys inside the clone decoder.

When future content is distributed, the license agency will construct and distribute new MKBs together with the new content. In the new MKBs, those newly compromised device keys detected by the "tracing" component will be revoked. As a result, the clone decoder who has the newly revoked device keys in it will not be able to decrypt the new MKB successfully and play back new content. The clone device and all the guilty device keys in it are therefore "revoked" by the new MKB. Figure [] shows the high level architecture of a content protection system using a trace-revoke scheme. While step 0 is only done once, step 1, 2 and 3 (trace and revoke) are repeated throughout the lifetime of a content protection system.

Note that the MKB is distributed together with the content, for example, on a DVD. This enables revocation to happen in off line mode which is essential for protecting content on consumer electronics devices.

In the literature broadcast encryption was first introduced by [3]. Both symmetric-key-based [11] and asymmetric-key-based broadcast encryption schemes [4] were studied further. Traitor tracing for the clone attack has also been studied extensively [6][8][9] since its introduction in [5]. Furthermore, trace and revoke schemes that combine these two functionalities have been studied in [7][1][12] for the pirate decoder attack. A trace-revoke scheme on another type of attack, namely the anonymous attack, has also appeared in [10].

The current state-of-art symmetric-key-based broadcast encryption scheme, the subset difference based "NNL scheme" [11], is adopted in the new industry content protection standard "Advanced Access Content System", AACS in short [1], for next generation high definition DVDs.



Fig. 1. A content protection system using a trace-revoke scheme

#### 1.1 Trace-Revoke in NNL Scheme 11 for Clone Attack

Naor, et. al  $\square$  present a broadcast encryption framework using subset covers. In this section we will briefly overview its revocation and tracing mechanism. Let  $\mathcal{D}$  be the set of devices and  $\mathcal{K}$  be the set of device keys. Every device  $d \in \mathcal{D}$ owns a subset of keys, denoted by  $\mathcal{K}_d$ . Similarly, associated with every key  $k \in \mathcal{K}$ is a set of users  $\mathcal{D}_k = \{d \in \mathcal{D} : k \in \mathcal{K}_d\}$ .

Suppose we want to broadcast some media M, which, for all intent and purpose, is a binary string. We would like to encrypt M in such a way that a set of legitimate devices  $L \subseteq \mathcal{D}$  is able to decrypt and view the media. The first step is to encrypt M with some media key K. We will use the term key without a qualifier to refer to device keys. We then find a subset of device keys C such that all legitimate devices are covered. That is, C is chosen such that  $\bigcap_{k \in C} \mathcal{D}_k = L$ . Now, for every  $k \in C$  we separately encrypt the media key, giving us  $E_k(K)$ . These items together  $\langle E_{k_1}(K), E_{k_2}(K), \ldots, E_{k_{|C|}}(K) \rangle$  are referred to as a media key block (MKB). Every device  $d \in L$  will own a key used in the MKB and every device  $r \in \mathcal{D}/L$  will own none. Hence, they cannot recover the content.

When a clone decoder built from compromised device keys is discovered and brought to the forensic lab, the goal of the NNL tracing algorithm is either identify a traitor by detecting its compromised device keys, or create a MKB that the clone device cannot decrypt (i.e., the clone decoder is disabled). In a black box tracing algorithm, the only means to diagnosis traitors is to submit tests, termed as *forensic MKBs*, to the clone and observe its response. In this context, a subset cover is also called a *Frontier*. When constructing a forensic MKB at frontier  $\mathcal{F}$ , we intentionally *enable* certain keys by using them to encrypt a valid media key and *disable* certain keys (not necessary traitorous) by encrypting a random bit string instead of the media key. The tracing agency feeds encrypted content together with a forensic MKB to the clone. Based on the keys inside the clone, the clone may or may not decrypt/play the content. If all of the keys in a clone are enabled then it will play. If none of the keys are enabled then it cannot play. If some keys are enabled and some are not, the clone will play with some probability. Suppose we construct forensic MKBs that enable a subset Tkeys and disable keys in  $\mathcal{F}/T$ . We feed this type of MKBs to the clone multiple times. Let  $p_T$  be the probability that the clone plays. Since the clone is assumed stateless, NNL treats the response of the clone as a Bernoulli random variable. If the probability of playing two different MKBs, (enabling T and T' respectively), are not equal then it must be case that the clone owns a key in the exclusive-or of T and T'. This is the essence of the *subset tracing procedure* that can identify a key in  $\mathcal{F}$  owned by the clone. The overall clone tracing relies on two things:

- 1. Bifurcation property: for every key  $k \in \mathcal{K}$  such that  $|\mathcal{D}_k| > 1$ , there exists keys  $k_1$  and  $k_2$  such that  $\mathcal{D}_{k_1} \cup \mathcal{D}_{k_2} = \mathcal{D}_k$  and  $D_{k_1} \cap D_{k_2} = \emptyset$ . With this, we can replace k with  $k_1$  and  $k_2$  and still cover the same set of devices.
- 2. subset tracing procedure: finds at least one key in current frontier  $\mathcal{F}$  owned by the clone device. This is used as a subroutine in the clone tracing algorithm.

The tracing algorithm starts from an initial frontier  $\mathcal{F}$  and proceeds by repeatedly using the subset tracing procedure to identify a compromised key  $k \in \mathcal{F}$ , removing it, and adding to  $\mathcal{F}$   $k_1$  and  $k_2$  satisfying the bifurcation property. If  $|\mathcal{D}_k| = 1$  then the single device in  $\mathcal{D}_k$  is a traitor. This process is reiterated until the detected compromised key is at the lowest level of the frontier and cannot split further, or the clone box is unable to play the MKB associated with  $\mathcal{F}$ . Figure 2 illustrates this process. The efficiency of this type of tracing is mainly measured by how many forensic MKBs are totally needed to complete tracing. That number for NNL tracing is  $O(T^3 logT)$  where T is the number of colluding traitors involved in clone attack.

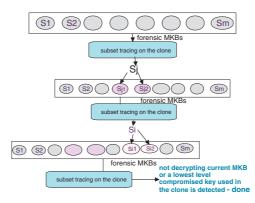


Fig. 2. Dynamic Clone Tracing algorithm using forensic MKBs (in the lab)

# 1.2 Pirate Evolution Attack

The pirate evolution attack is a newly described attack in [2] that particularly targets on trace-and-revoke schemes. In this attack, after the attacker succeeds in circumventing devices and extracts their device keys, they reveal those keys slowly to the decoders. They will build and release a first pirate decoder. After this decoder and the device keys in it are detected by the tracing agency and then disabled by the new updated MKB, they roll out a second version of the

pirate decoder. This step is repeated. Using this evolving strategy, the pirates will only evolve a new version of the decoder after the current generation of the decoder is disabled by an updated MKB.

In particular they studied the pirate evolution attack on the NNL scheme  $\square$ . The *t* attackers compromised *t* set of device keys. But each set of keys are released to clones one at a time. In other words, the keys are drizzled. Only when the revealed key gets revoked will the attacker release the next key in the next clone. In fact, it takes the entire height of the tree number of iterations (i.e., logN) to identify the hacked device (traitor) at a leaf node. So in general it could take up to t \* logN generations of clone decoders to detect and disable *t* traitors. In a system like AACS that supports billion devices, there are 30 levels in the tree. A pirate can produce more than 300 pirate decoder generations by compromising only 10 devices. It could take 30 generations (iterations)' battle with attackers before the license agency can revoke all the keys one guilty device revealed. In reality this converts to years of time to stop one hacked device. This is certainly unacceptable. It is highly desirable to be able to quickly respond to this attack.

In this paper we will present our efficient approach that can make the NNL scheme [11] defend well against the pirate evolution attack without sacrificing much of the efficiency of the scheme. We know forensic MKBs intentionally choose which keys to enable/disable; they are not operational and only serve for forensic purpose. In our approach to defend against pirate evolution attack, we introduce a special type of forensic MKBs that are also operational. We will illustrate how to construct such type of MKBs and show why this enables the trace-revoke scheme [11] to defend against pirate evolution attacks. In AACS context it will take as small as 2 generations to detect and disable traitors.

The rest of the paper is organized as follows. In Section 2 we will present our approach to defend well against pirate evolution attack using a special type of MKBs. In section 2.1, we will illustrate how to construct the special type of MKBs for the efficient subset-difference method in NNL scheme. In Section 3 we will analyze the efficiency of our approach and the overhead incurred in our approach. We will conclude in Section 4

### 2 Defending against Pirate Evolution Attack

In an evolution attack, the attackers exploit the fact that each device owns a set of device keys, one at each level, to release only one key from each compromised device at a time. A new version of the clone is not generated until its previous version (keys) are disabled. That evolving strategy is important to slow down the license agency to disable/revoke a set of traitors.

Our idea is simple. In order to shut down this attack quickly, we will disable the attackers' possibility of releasing their keys one at a time. We will force them to use their device keys at a deeper level when they create each of their evolving clone decoders. In this way we can reduce the number of clone decoder generations that the attackers can produce from the sets of device keys they compromised. In our approach, the tracing relies on two things:

- 1. dynamic clone tracing: given a clone that was generated when the lowest level frontier is F, finds at least one key k in F owned by the clone device. This is the original NNL dynamic tracing algorithm as illustrated in Figure 2.
- 2. split *i* levels down: for every key  $k \in \mathcal{K}$  such that  $|\mathcal{D}_k| > 1$ , there exists keys  $k_1, k_2, \ldots, k_{2^i}$  such that  $\mathcal{D}_{k_1} \cup \mathcal{D}_{k_2} \cdots \cup \mathcal{D}_{k_{2^i}} = \mathcal{D}_k$  and  $D_{k_1} \cap D_{k_2} \cdots \cap D_{k_{2^i}} = \emptyset$ . With this, we can replace k with  $k_1, k_2, \ldots, k_{2^i}$  at the *i*th level down and still cover the same set of devices.

Similar to the original subset-based trace-revoke scheme, our tracing algorithm also starts from an initial frontier  $\mathcal{F}$ . Given a recovered clone decoder that builds upon current frontier  $\mathcal{F}$ , the algorithm identifies a compromised key  $k \in \mathcal{F}$  in current clone. This is done at a forensic lab using forensic MKBs exactly as shown in Figure 2 Once a compromised key  $k \in \mathcal{F}$  is detected, our algorithm removes it, and adds to  $\mathcal{F} k_1, k_2, \ldots, k_{2^i}$ . If  $|\mathcal{D}_k| = 1$  then the single device in  $\mathcal{D}_k$  is a traitor. This process is reiterated with the new frontier  $\mathcal{F}$  until it reaches the leaf level and no more clone decoder can be built, or the attackers are not able to play the MKB associated with  $\mathcal{F}$ . Figure 3 illustrates this process.

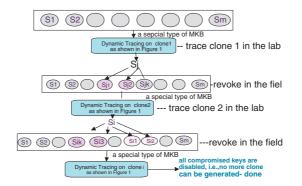


Fig. 3. Our Trace-revoke Approach for Evolution Attacks

There are some subtleties here. First of all, when a traitorous key in the current generation of the clone is detected, it must always be possible to find keys  $k_1, k_2, \ldots, k_{2^i}$  such that  $\mathcal{D}_{k_1} \cup \mathcal{D}_{k_2} \cdots \cup \mathcal{D}_{k_{2^i}} = \mathcal{D}_k$  and  $D_{k_1} \cap D_{k_2} \cdots \cap D_{k_{2^i}} = \emptyset$ . The choice of *i* depends on how quickly one wants to shut down the attack (refer to Theorem [] in Section [3]). Second, the original clone tracing algorithm shown in Figure [2] is now only a procedure of our trace-revoke approach. Keep in mind the original clone tracing occurs only when a clone decoder is found and brought to a forensic lab where forensic MKBs can be created to probe the clone. More concretely, for a frontier  $\mathcal{F}$ , a forensic MKB is constructed in a way that intentionally chooses to enable some keys in  $\mathcal{F}$  and disable other keys in  $\mathcal{F}$ . It does not necessary enable all compliant device keys and disable all non-compliant device keys. For that reason a forensic MKB is almost always non-operational. However, a pirate evolution attack is an on-going evolving attack in the field.

Our goal is to reduce the number of generations of clone decoders the attackers can build given the sets of compromised device keys. In order to achieve this goal, we need to build a type of MKB that is both operational and forensic.

Creating this type of special MKB is very different from creating a normal operational MKB. There the creation of the MKB is driven by revocations, and the algorithm generates the fewest possible subsets that cover the nodes that are not revoked. In our special MKBs that also serve the forensic purpose we intentionally want to cover some of the enabled nodes with a larger number of subsets at a much lower level, so that each subset covers far fewer devices. Note that each device needs to use one of its keys enabled in the MKB in order to process the MKB. In a normal MKB, with fewer subsets, an attacker uses a key shared by many devices. In our special MKB, the attacker is forced to use a key shared by far fewer devices, so he has to identify himself more quickly.

In order to create a special type of MKB for our tracing, we will start from an operational MKB that uses the fewest subsets to enable/disable the right sets of devices, in other words, enable all compliant devices and disable all noncompliant devices. We will then recursively perform the splitting operation to the next level based on the bifurcation property until we reach the desired *i*th level down. It is easy to see the recursive splitting operations based on bifurcation property guarantee that for a key k it is always possible to find keys  $k_1, k_2, \ldots, k_{2^i}$ such that  $\mathcal{D}_{k_1} \cup \mathcal{D}_{k_2} \cdots \cup \mathcal{D}_{k_{2^i}} = \mathcal{D}_k$  and  $D_{k_1} \cap D_{k_2} \cdots \cap D_{k_{2^i}} = \emptyset$ .

The recursive approach also guarantees that it is possible to construct MKBs that are at a much deeper level but are still operational. This works for two reasons. First of all, the split subsets after splitting operation enable the same set of devices as the original subset. Second, the splitting operations always make progress to the next level, so the recursion will eventually terminate. Therefore it is always possible to split a subset any levels down. Next we will show how to construct this special type of MKB for the subset-difference NNL scheme.

#### 2.1 Creating Special Type of MKBs for Subset-Difference Method 11

Each device in NNL scheme is associated with a leaf of the tree. Each node (representing a subset) in the tree is associated with a key. In a subset-difference NNL tree, a device is given every key in the tree except the keys between its leaf and the root. In a subset-difference method, a subset is not a simple subset, it is actually the difference between two simple subsets, thus called a subset-difference. As shown in Figure 4 a subset difference  $S_{ij}$  is defined as all the leaf nodes in the subtree rooted at node  $V_i$  but not in the subtree rooted at  $V_j$ . The subset-difference method contains a mechanism to find a subset cover given a revoked node list R. A sample subset cover is illustrated in Figure 5 The MKB for the subset-difference method represents the revocation information by the subset differences in the cover set. Each subset difference represents the set  $V_i$  minus  $V_j$ .

Furthermore, NNL does not just have a single system of keys; it has literally billions of systems of keys. In fact, every subtree in the master tree defines a completely independent system of keys. For example, one level down from the

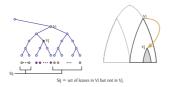


Fig. 4. Subset Difference Definition

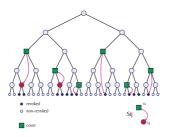


Fig. 5. Subset Cover of non-revoked devices

root of the tree, there are two subtrees, each with one billion devices in them. Each of these subtrees has its own system of keys, in addition to the system of keys in the master tree. Likewise, the next level down there are four subtrees of size one-half billion devices, and each have independent systems of keys, and so on. Eventually, at the bottom, every pair of sibling leaf devices also defines its own system of keys. A device has a whole set of device keys in every subtree that it belongs to, 31 independent subtree systems per device if there are 31 levels in the tree. Of course NNL tree needs a mechanism (refer to NNL III) so that the device does not have to store billions of keys. Now, with all these different key systems based on subtrees, NNL tree provides a mechanism to revoke all possible combinations of revoked devices. On average, there are 1.28 encryptions per revocation in NNL tree. It has the most concise expression of revocations.

In our approach, to create the special type of MKB, we must subdivide subtrees deep down but without adding any revocations. We will first show how to split a subset and go down just one level. There are three primitive splitting operations.

#### Splitting a subset without any revocation, see Figure 6

In this case, the subset difference excludes the left most leaf, but it is not a real revocation. It can be split into two mutually excluding subsets. The split subsets do not exclude the left most leaf. The original subset is replaced by two new subsets. The new subsets enable/disable the exact same set of devices.

### Splitting a subset with an excluded child, see Figure 7

The enabled child is split into two subsets, each with an excluded child. Again these two subsets are mutually excluding. The original excluded child disappears from the list of subsets, unless some previously existing subset enables it.

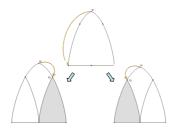


Fig. 6. Splitting a subset without any revocation

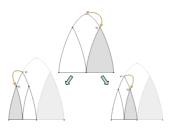


Fig. 7. Splitting a subset with an excluded child

#### Splitting a subset with an excluded descendant of a child

As illustrated in Figure 8, one subset enables the included child. The other subset enables the child with an excluded descendant. The descendant is still excluded; it may be a child in the new subset, or a descendant of a child.

Now that we have shown how to split a subset to one level down, to descend further, it is necessary to apply the same operations recursively. In general, because the special MKB has to be fully operational, we have to start with a cover set that incorporates the existing revocations, and then subdivide the existing subset differences using the above described splitting operations recursively so that the (suspect) subtree is covered by a large number of subset differences.

We will use an example to illustrate how we can expand and subdivide the existing subset differences into more subset differences. In a tree of height 22 with no revocations, Figure  $\bigcirc$  (part 1) shows how we can expand the fourth eighth of the tree with more subset differences. Using the above described primitive splitting operations, we first split the height 22 tree in two; then the first half of it in two again, then the second quarter in two. The fourth eighth of the tree will be of height 19. Again, by recursively performing splitting operations we finely subdivide the fourth eighth into  $2^{10} = 1024$  small subtrees of uniform height at 9th level. The 1024 subtrees are enlarged and shown in part 2 of the Figure  $\bigcirc$ 

During splitting operation, to enable devices, the subset differences covering the small subtrees are encoded in mutually excluding sibling pairs, as shown in Figure <sup>6</sup> and re-shown as part 3 of the Figure <sup>9</sup>. The original subset difference, which covered all the devices in the height 22 subtree, is replaced by 1027 new subset differences. Each of the 1024 subset differences will cover subtrees of

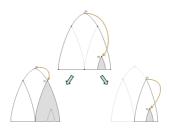


Fig. 8. Splitting a subset with an excluded descendant of a child

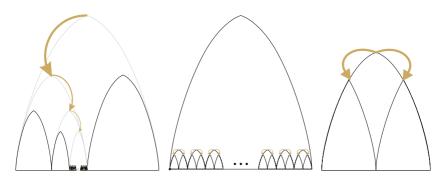


Fig. 9. Splitting a subset into multiple subsets without adding revocations

height 19 - 10 = 9, each containing only  $2^9 = 512$  devices. As a result, if the attacker reveals a key that can be used to process this MKB, it will be a key shared with only 512 devices, much more identifying than a key shared by 4 million devices with the original MKB.

# 3 Discussion

As one can imagine, in our approach expanding into more subsets will increase the MKB size which is used to measure the efficiency of the trace-revoke scheme. There is a tradeoff between the number of iterations the license agency is willing to spend and the MKB size. This tradeoff is shown in the following theorem.

**Theorem 1.** If the tracing algorithm wants to detect and disable a traitor in only b generations of pirate decoder, then the MKB size is  $O(N^{1/b})$ .

*Proof.* Suppose every time when a primitive splitting function is performed, a subset is split into j subsets. We know the devices/users are arranged as the leaf nodes. The height of the subset structure is therefore  $log_j N$ . If the tracing wants to identify a traitor at the leaf node in b generations, it means for each generation of the special MKB (clone decoder), the splitting dives  $i = \frac{\log_j N}{b}$ 

levels down on average. In other words, the primitive splitting function needs to perform  $\frac{\log_j N}{b}$  times. So a subset is expanded into  $j^{\frac{\log_j N}{b}} = N^{1/b}$  subsets. Therefore the MKB size is  $O(N^{1/b})$ .

The above theorem shows a tradeoff between immunity to evolution attack and revocation efficiency. Different applications may need different tradeoffs between these two aspects. Based on the above theorem, in order to reduce the number of generations of clone decoder to 2, our approach will dive 11 levels down each time to build and distribute a special type of MKB that is of size  $O(\sqrt{N})$ . However in practice, that MKB size does not convert to big numbers. For example, in a tree of height 22 an optimally-sized MKBs (i.e. using the fewest number of subsets based on NNL scheme) are less than 16KB. Using our approach we first create multiple subsets at 11th level then again multiple subsets at the leaf (22th) level. That dive would increase the size of an MKB by only 46KB. In AACS, disc replicators allocate 1MB space for the MKB, and even that is negligible overhead on a blue-ray disc that can store at least 25GB. There is plenty of room to store our special type of MKBs. Therefore our approach is very feasible for AACS to quickly shut down the pirate evolution attack. For this reason, AACS adopts our approach to defend against the evolution attack.

Our approach is applicable to any subset-based broadcast encryption scheme. At a high level, a subset-based scheme organizes the devices into many overlapping subsets, with each subset associated with a key. Each device belongs to many different subsets and knows the key for every subset it is a member of. In face of pirate evolution attack, from the minimal sets of subsets that cover all and only innocent devices, one needs to intentionally split a subset into multiple subsets with each subset covering fewer devices and construct a special type of MKB that is both operational and forensic. This type of MKB will force the attackers to reveal keys that are much more identifying.

### 4 Conclusion

In this paper we are concerned with the pirate evolution attack (introduced in [2]) on trace-revoke schemes where, during deployment, the pirate users reveal the compromised keys slowly into pirate decoders trying to stay ahead of the revocation. In this paper we are interested in a practical approach that can defend well against this attack in practice.

In particular, we are interested in the subset-difference-based NNL scheme that is very efficient in revocation and is deployed by AACS in practice, but was pointed out in [2] that it can suffer very badly from the evolution attack. We devise an easy and efficient approach for the NNL scheme to defend well against the evolution attack while maintaining very reasonable revocation efficiency. For example in the AACS context our approach only takes as small as 2 iterations to detect and disable a traitor. And achieving this only increases the ciphertext size by a negligible 46KB in a 25GB disc space. The simplicity, efficiency and practicality of our approach caused its adoption by AACS to use to defend against the evolution attack.

As future work we would like to formalize our approach more so as to guide the future design of trace-revoke schemes that can achieve balances between immunity to evolution attack and revocation efficiency.

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# Security Specification for Conversion Technologies of Heterogeneous DRM Systems

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Abstract. Digital Right Management (DRM) can be used to prohibit illegal reproduction, and redistribution of digital content, to protect copyrights. However, current DRM systems are incompatible and lack of interoperability which exchange of data, different platform, designed and protected by different content providers. To overcome these drawbacks, three ways of interoperability are full-formation interoperability, connected interoperability, configuration-driven interoperability, allowing consumers to use the purchased content in their equipments of choice. In this paper, we study on the security specification of configurationdriven interoperability for heterogeneous DRM systems, using the Common Criteria. Then, we study security boundary, security environment, security objectives, and rationale of an CTHDS\_PP(Conversion Technologies of Heterogeneous DRM Systems Protection Profile) to find important security features. The CTHDS\_PP gives a discussion covered the current security problems to conversion technologies and lists threats to solve those problems. Moreover, this CTHDS\_PP can be used for potential developers and system integrators, and reviewed and assessed by evaluators.

**Keywords:** Digital rights management(DRM), Common Criteria(CC), Protection Profile(PP), Interoperability.

# 1 Introduction

Nowadays, wireless mobile device not only be everywhere but also owned by the majority of the people. Business and technology trends indicate an exponential growth in the potential sales of content through mobile devices. The protection of content used to prohibit illegal reproduction or redistribution of digital content, is achieved by technology that protects the copyrights of digital content. This is called Digital Rights Management (DRM) system. Various DRM systems exist, e.g., Windows Media DRM[1], Apple iTunes' Fairplay[2], the Open Mobile Alliance's(OMA) DRM scheme[12], PachyDRM[3], Secure Digital Container(SDC)[4], etc.

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However, the incompatibility and the lack of interoperability between current DRM technologies heavily restricts the rights of end-users, such as private reproduction. For example, once any contents are made under one specific DRM system, those can not be used under the other DRM systems because system and data of different type designed and produced by a different providers.

Therefore, many researchers study interoperability technology for heterogeneous DRM systems such as Open DRM platforms [19], EXIM(EXport and IMport) [9], Networked Environment for Media Orchestration(NEMO) [18], etc. Full interoperability can be categorized into three ways [17] [16]:

- Full formation interoperability: All protected content conforms to some globally standardized format.
- *Connected interoperability*: On-line third parties are used to translate operations from one DRM regime to another.
- Configuration driven interoperability: End-user device can acquire the ability to process content protected by any DRM regime by downloading appropriate "tools".

In this paper, we focus on configuration-driven interoperability such as Motion Picture Exports Group(MPEG)[20] and EXIM[9][10]. And we will describe using EXIM approached configuration-driven interoperability. Recently, EXIM is planing the commercial service in South Korea. This paper proposes the security specification for conversion technologies of heterogeneous DRM systems, using standardized CC methodology, syntax, and notation. A security specification for configuration-driven interoperability do not exist. As a result, EXIM needs acceptable international terms and standards for security protection, such as by employing a Protection Profile. This protection profile is hereafter referred to as the CTHDS\_PP(Conversion Technologies of Heterogeneous DRM Protection Profile). The CTHDS\_PP is a stated security environment that includes assumptions, potential threats, organizational security policies, and security policies, and assumptions 5.

The remainder of this paper is organized as follows. Section 2 reviews related works, Section 3 presents the security specification for CTHDS\_PP. Section 4 provides an rationale of the CTHDS\_PP. Finally, section 5 concludes the paper.

# 2 Related Works

#### 2.1 Overview of the Common Criteria

The Common Criteria (CC) [5] is an internationally approved set of security standards, providing a clear and reliable evaluation of the security capabilities of Information Technology products. The CC is useful as a guide for the development of products or systems with IT security functions and for the procurement

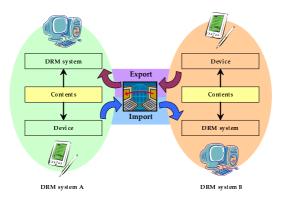


Fig. 1. Basic EXIM Architecture Model

of commercial products and systems with such functions. This is the belief behind the ISO/IEC standard 15408, CC for IT Security Evaluations **5 6 7**. It represents the outcome of efforts to develop criteria for evaluation of IT security, and is widely applicable internationally. The CC defines a set of IT requirements of known validity, which can be used in establishing security requirements for prospective products and systems **5**. The CC also defines the Protection Profile (PP) construct, which allows prospective consumers or developers to create standardized sets of security requirements. Moreover, consumers state their security functional and assurance requirements in a PP**8**.

These profiles form the basis for Common Criteria evaluation. By listing required security features for product families, the Common Criteria allows products to state conformity to a relevant PP. Follows are TOE steps to derive IT security requirements. During Common Criteria evaluation, the product is tested against a specific protection profile, providing reliable verification of the security capabilities of the product **[13]**. A PP is intended to be reusable and to define requirements which are known to be useful and effective in meeting the identified objectives **[5] [14] [15]**.

#### 2.2 EXIM Based Configuration-Driven Interoperability

DRM provides a method to prevent digital content from being copied and distributed, and manages the circulation and use of digital content **12**. However, there is no standard DRM system, though many content providers create DRM systems. These digital content providers operate in a competitive market. As a result, each content provider creates its own DRM system, and their DRM systems are not interoperable with each other, heterogeneous DRM systems. This is a common problem between provider and user. The problem is that the providers cannot sell their digital content to users who do not have a suitable device to use their content, and users can only use certain digital content because each format is different. The user needs to purchase same content two or more times if he wants to play digital content on his/her various devices. EXIM (EXport and IMport)[10] is a technique developed by the ETRI(Electronics and Telecommunications Research Institute, in South Korea), to solve the DRM systems' tack of interoperability based configuration-driven interoperability. EXIM is not a kind of DRM system, but a technique that guarantees secure translation of digital content among the different DRM systems. The architecture of EXIM is illustrated in Fig. []

For example, in Fig.  $\square$  contents in a DRM system can be freely played a device having a DRM system A. To play a content of the DRM system A to a device of DRM system B, DRM system or device in DRM system A send a converted a EXIM format content through EXIM to the device of DRM system B. After the device of the DRM system B convert the EXIM format content into a content of the DRM system B through EXIM, then can freely play the content.

# 3 The Security Specification for CTHDS\_PP

This section consists of TOE (Target Of Evaluation) description, identification of the TOE, TOE security Environment, and security object of the CTHDS\_PP. It is designed to provide potential threats, organizational security policies, assumptions, and security objectives of the CTHDS\_PP to compatible services. We regards that as now, conversion technologies for heterogeneous DRM system is only one, EXIM, and so we study CTHDS\_PP based on the EXIM.

# 3.1 TOE Description

The property for security of TOE is content, TOE and important data (security and TOE security function data) in TOE, when each other DRM device transfers content. In addition, TOE of CTHDS\_PP protects the content user rights object and user information from various threats. The Fig. 2 illustrates CTHDS scenarios for content distribution.

# **TOE** Environment

- Content Mall: This is similar to a content provider. A user connects to a content mall through source DRM agent(it is explained in a next subsection) and then selects her preferred content. After payment of chosen content by the user, the content mall sends user's Rights Object about the content to a content storage server.
- DRM Clearing House: The DRM clearing house is an entity that assigns permissions and control to DRM content, and generated Rights Objects. After the DRM clearing house receives a Right Object Token from the source DRM agent, the DRM clearing house issues a Rights Object information to

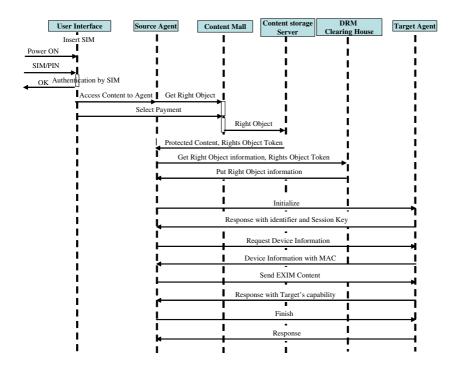


Fig. 2. CTHDS Scenarios Based on EXIM 10

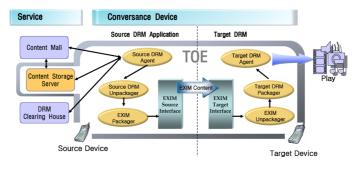


Fig. 3. TOE

the source DRM agent. The Rights Object information is the "Right status" of using the content, with an expiration date, number of times accessed, etc.

#### **TOE** Components

 DRM Agent: In source device or target device, a DRM agent can negotiate the delivery of content, services, and Rights Object with suppliers. The DRM agent may need to access to the content mall, content storage server, DRM clearing house, DRM unpackager or packager, and EXIM interface.

- DRM Unpackager or Packager: A DRM unpackager converts a packaged(encrypted) content into unpackaged(decrypted) content. The DRM packager is its opposite. The DRM unpackager sends the unpackaged DRM content to the EXIM packager or the DRM packager sends the packaged DRM to the target DRM agent.
- EXIM Packager or Unpackager: The EXIM packager converts the unpackaged DRM content into the EXIM content format, that consists of the encrypted content is metadata(identifier, title, creator, contributor,etc.), rights expression(valid date, valid times, rights permission, etc.), resource, etc. The EXIM unpackager unpacks the content in EXIM content format.
- EXIM Interface: The EXIM source interface exchanges the content of the EXIM content format with the EXIM target interface.
- Content Storage Server: The content storage server stores content, and may be located in the content mall or user's device, for example in network storage, a PC, on removable media, etc.

# 3.2 TOE Security Environment

The TOE security environment describes the security aspects of the environment in which the TOE is intended to be used and the manner in which it is expected to be employed. The TOE security environment consists of threats, organizational security politics, and assumptions of the security of the TOE environment.

**Threats.** This subsection characterizes potential threats, including accidental and malicious attempts, and any known or presumed threats to the assets against which protection will be required, either by the TOE or by its environment[II]. The potential threats of TOE are as follows, Table [] Label of Threats started with letter 'T'.

**Organizational Security Policies.** This subsection relates to the TOE. It includes Information flow control rules, Access control rules, policies regarding the use of encryption, security audit policies and procedures, and Policies regarding use of a standardized IT base. Label of Polices started with letter 'P'. Table 2 shows the organizational security policies.

Assumptions. The assumptions regarding secure usage of the TOE are made when defining the security function requirements and security assumption requirements. Label of Assumption started with letter 'A'. Table 3 shows the assumptions.

### Table 1. The Threats

Threat Lable	Description
T.P_Replace	Physical Replace: Physical replacement of any component of the TOE.
T.P_Damage	Physical Damage: Physical damage to any part of the TOE.
T.Permit	Access: An attacker may access information or services without having
	permission from the rightful owner, service mall, or rights issuer.
T.Trans	Transmission to others: After the Source User delivers content to the
	Target User, the Target User may pass the content to an unauthenti-
	cated user.
T.Residual	Residual content in reallocated resources: An authorized user may ac-
	cess content or service that he is not entitled to access, or does not have
	permission to access, through reallocation of TOE resources. This oc-
	curs when the reallocated resources contain residual content or access
	permissions from another user or process, gained through legitimate
	access.
T.Illegal_Access	Resource consumption: An authorized user may extremely consume
	resource, through legitimate access, in Content Mall or DRM Clearing
	House, which ensure other authorized users do not access information
	or services.
T.Damage	Damage to other resources: An authorized user may access information
	or services in a way that result in damage or discloses other information
	or services.
T.Install	Installation of new functionality: An authorized user may install a mali-
	cious program or new functionality into the TOE that has the capability
T.Intro	to damage or disclose TOE resources.
1.Intro	Introduction of new functionality: An authorized user may create or introduce new functionality into the TOE that has the capability to
	damage or disclose TOE resources.
T.Permit	Permission of access: An authorized user may permit access permissions
1.1 CIIII0	to others that do not have a legitimate need to access the resources.
T.Deny	User deny of service: An authorized user may send the content to a
112 0115	target user, and then the target user may deny having participated in
	the transaction.
T.Delet	Deletion of audit information: Particular information of the authorized
	user may be damaged or destroyed. Audit information should only be
	destroyed in accordance with security policy.
T.Deny_Service	Denial of illegal access: An attacker may deny authorized users legiti-
	mate access to services or information e.g. through resource exhaustion,
	or by causing legitimate users to be blocked out from access.
T.TSF_Block	Access blacked by TSF modification: An attacker may modify TOE se-
	curity functionality so that subsequently authorized users can no longer
	access services or information that they are authorized to access.
T.Disguise	Disguise of authorized user using stolen authentication data: An at-
	tacker may masquerade as a legitimate user by presenting authentica-
	tion data of an authorized user.
T.TOE_Error	Insecure TOE after system error: While a source user sends information
	or content to a target user, if the TOE security state becomes incorrect
	or inconsistent, following a system error, TOE security functionality
	may not operate correctly.

### Threat Lable Description

T.TOE_Change	Insecure TOE after TOE changes: Changes to the TOE may disable or
0	corrupt security functionality. Very few systems remain in their original,
	delivered configuration. Even small maintenance actions may disrupt or
	disable security.
T.Sys_Error	System Error in TOE: Content, information, or service protected by the
	TOE may occur due to system error such as user error, hardware errors,
	transmission error, and failure to allocate adequate external resources.
T.Env_Failure	Environmental failure: Environmental failure, such as human error or
	failure of software, hardware or power supply, may result in unexpected
	interruption of TOE operations, causing damage or loss to information
	or service.
T.Env_Stress	Stress by environment factor: Environmental factors may obstruct TOE
	services (e.g. electrical power, temperature, humidity).
T.Eavesdrop	Information eavesdropping from TOE: Content or information may be
	disclosed from the TOE, and an attacker may be able to connect to
	legal users of the network in a trusted zone.

Table 1. (Continued)

Table 2. The Organizational Se	ecurity Policies
--------------------------------	------------------

Policy Lable	Description
P.Audit	Audit : The TOE must audit recognizing, recoding, and storing infor-
	mation related to security relevant activities.
P.S_Update	Update TOE service software : When a device accesses a Content Mall,
	if EXIM software is updated for DRM service, the device must au-
	tomatically download the software. This allows the Agent to convert
	content to another DRM device.
P.Guide	Guide for User : A Content Mall must provide guide lines demonstrating
	how a user can use service.
P.Sec_Mang	Secure Management: An authenticated manager must manage the TOE
	with the safe method.
P.Auth_Crypt	Authenticated Cryptographic Algorithm : Using cryptographic algo-
	rithms, such as RSA, AES, SHA-1, and DES, must be authenticated
	by ISO.
P.Ident_Chk	Identity Check : Devices must check the identity each other through
	trusted Certification Authority(CA).

### Table 3. The Assumptions

Assumption	Description
Lable	
A.See_Ch	Secure Channels : Each component is connected through secure chan-
	nels.
A.User_SIM	User security owner of SIM Card: SIM Card is securely issued to user.
A.Data_Protec	Data Protection: User's right and content are securely encrypted and
	stored in a content mall, content storage server, and DRM clearing
	house.
A.SIM_Lose	Lose SIM Card: If a user loses the SIM Card, which is subsequently
	gained by an attacker, the attacker cannot retrieve the user's private
	information.

<b>N</b> I	
Name O.Sec_Ch	Description
O.Issu_SIM	Secure Channels: TSF must provide secure channels.
	Issue of SIM Card: SIM Card must be securely issued to users.
O.Data_Prot	Data Protection: The TOE must support cryptographic functions in a secure manner.
O.S_Udate	Software Update: The TOE must regularly update DRM service soft-
	ware.
O.Guide	Guide: The TOE must provide guidelines for users consuming the con-
	tent.
O. Auth_Fail	Authentication fail of Password: TSF must detect repeated authenti-
	cation failures in the user authentication system.
O.Auth	User Authentication: The TOE must be accessed by authenticated
	Users.
O.Data_Prot	Data Protection: The TOE must protect exposure, modification, and
	deletion of the recoded data.
O.Flow_Cont	Flow Control: The TOE must control the flow of unauthenticated data
	through security policy
O.Install	Installation: The TOE must be installed by unauthenticated software.
O.TSF_Sec	Protect of TSF data: The TOE must protect the recoded TSF data in
	the TOE from unauthenticated exposure, modification, and delete.
O.Audit	Audit: The TOE must have audit information and digital signature
	function.
O.Test	Test: TSF must prove the integrity of data(Metadata, Rights, Resource,
	etc.) in the TOE.
O.Update	Update: The TOE must be reinstalled or updated when becoming aware
	of the system error.
OE.Phy_Prot	Physical Protection: The TOE must be located in a safe environment
OE.Secu_Manag	Security Function Management: TSF must be restricted within the au-
	thenticated manager who has the capability to modify and delete se-
	curely.
OE.Sec_Maintain	Security Maintain: When the TOE or network service suffers an un-
	predicted fault, the TOE must provide identical service to the previous
	state.

#### Table 4. The Security Objectives

#### 3.3 Security Object

This section provides the security object is traced back to the Security Environment (threats, organizational security policies, assumptions). Each security object is traced back to at least one threat or organizational security policy. In other words, a threat or organizational security policy is addressed entirely by one or more security objectives. Table 4 shows the security objectives.

# 4 Rationale

This section contains the security objectives rationale. It demonstrates that the CTHDS\_PP is complete, correct, consistent, and coherent, with Assurance, Threat, and Policy, and suitable to counter the identified threats to security. In other words, this subsection proves that the security objectives uphold counter all threats, enforce all organizational security policies, and all assumptions. The Table **5** shows mapping between threat, and policy and objectives, assurance.

Objectives Threats Policies Assumptions	O.Sec_Ch	O.Issu.SIM	O.Data_Prot	O.S_Udate	O.Guide	O.Auth_Fail	O.Auth	O.Data_Prot	O.Flow_Cont_	O.Install	O.TSF_Sec	O.Audit	O.Test	O.Update	OE.Phy_Prot	OE.Sec_Maintain	OE.Secu_Manag
A.Sec_Ch	*																
A.User_SIM		*															
A.Data_Protec			*														
A.SIM_Lose			*			*	*										
P.Audit												*	*				
P.S_Update				*													
P.Guide					*												
P.sec_Mang																	*
P.Auth_Crypt			*														
P.Ident_Chk								*					*				
T.P_Replace								*									*
T.P_Damage																*	
T.Permit							*										
T.Trans							*										
T.Residual								*									
T.Illegal_Acc									*								
T.Damage									*								
T.Install										*							
T.Permit							*										
T.Intro											*						

Table 5. Mapping Security Objectives to Threats, Policies, and Assumptions

T.Deny						*			*				
T.Delet								*					
T.Deny_Service				*									
T.TSF_Block								*					
T.Disguise					*								
T.Pretend_User					*								
T.TOE_Error										*			
T.TOE_Change											*		
T.Sys_Error						*							
T.Env_Failure										*			
T.Env_Stress												*	
T.Eavesdrop	*												

Table 5. (Continued)

## 5 Conclusion

For protection of content, DRM can be used to prohibit illegal reproduction or redistribution of digital content, enabling the protection of copyrights. However, the incompatibility and lack of interoperability of the current DRM technologies heavily restricts the right of end-users, such as private reproduction. To address these problems, EXIM was developed by ETRI. In this paper, we studied the security specification for conversion technologies of heterogeneous DRM system, using the CC that meets specific consumer needs. The CTHDS\_PP is a stated security environment that includes potential threats, and organizational security policies assumptions and security objectives, in order to uphold counter potential threats, and enforce organizational security requirements to potential developers, and provides a foundation from which a Security Target (ST) can be developed and formal evaluation can be conducted. Furthermore, a PP of similar security requirements can reuse all or part of existing PP or the CTHDS\_PP.

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# Analysing Protocol Implementations<sup>\*</sup>

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Abstract. Many protocols running over the Internet are neither formalised, nor formally analysed. The amount of documentation for telecommunication protocols used in real-life applications is huge, while the available analysis methods and tools require precise and clear-cut protocol clauses. A manual formalisation of the Session Initiation Protocol (SIP) used in Voice over IP (VoIP) applications is not feasible. Therefore, by combining the information retrieved from the specification documents published by the IETF, and traces of real world SIP traffic we craft a formal specification of the protocol in addition to an implementation of the protocol. In the course of our work we detected several weaknesses, both of SIP call setup and in the Asterisk implementation of the protocol. These weaknesses could be exploited and pose as a threat for authentication and non-repudiation of VoIP calls.

### 1 Introduction

Voice over IP (VoIP) is widely used, and is about to replace the traditional, public switched telephone networks (PSTN) for two main reasons: (1) Providers and customers experience cost savings, especially for long distance calls. Since VoIP uses the Internet as carrier, the cost of setting up a phone call needs no more effort than sending an email. (2) Added functionality and flexibility. The VoIP protocols are capable of providing a number of additional services like instant messaging, presence, conferencing, events notification, video calls and other multimedia transmissions and location independence (mobility).

VoIP services cannot rely their security on the telecommunication infrastructure, dedicated lines, physically protected switches, and certified telephony equipment. VoIP services have to be secured by cryptographic techniques. Therefore, the employed protoscols and their implementations must undergo a thorough formal crypto-analysis.

A common combination in VoIP is to use the Session Initiation Protocol (SIP) [14] for signalling, e.g., setting up and tearing down calls, and specific protocols for the actual media transfer. The designers of SIP focused on functionality for

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providing specific services rather than security features **15**. However, security issues have been recognised to be an area of further investigation and improvement **16410**. Discussions about potential weaknesses and attacks on SIP have, in most cases, been kept on an informal level **16413181712**.

Our goal has been to use formal modelling of SIP in order to (1) verify whether the Asterisk implementation of SIP follow the specifications; and (2) perform attacks exploiting weaknesses in the protocol definition, and its implementations. This work is based on previous work [9] where we analysed digest access authentication in the SIP registration process. The same authentication mechanism is used in the call-setup explained in this paper.

There have been other works that analyse SIP and its security configurations formally [7]2, and the work of the AVISPA project. However, they consider the authentication and registration sub-protocol in combination with the Diameter protocol, rather than the call-setup protocol as presented here.

The rest of the paper is organised as follows: In Section 2, we give a high level overview of SIP, and the tool PROSA that we have used to analyse the call setup. In Section 3, the formal specification of digest access authentication and call setup is described. After discussing whether Asterisk implements the SIP protocol correctly (Section 3.4), we show that a vicious attack on the call setup specification can be performed using the specification obtained previously is presented (Section 4). Finally, in Section 5, we evaluate the approach and point out future extensions of the PROSA tool.

## 2 Background

SIP is an application layer signalling protocol developed by the IETF. The core functionality of SIP is specified in RFC3261 [14], additional functionality is specified in over 40 RFCs, and nearly 30 pending SIP-related drafts. SIP is used to establish, maintain and tear down multimedia sessions between two or more participants. A session can be an ordinary call between two participants, or an advanced multimedia conference session with several participants. More specific, SIP set up the session context, but does not carry multimedia content.

We illustrate the operation of SIP with a scenario where Alice calls Bob. A session including call setup and tear down is shown in Fig.  $\blacksquare$  While a session generally may traverse several SIP proxies, we restrict our scenario to only one SIP proxy. The SIP proxy has three roles: (i) it acts as registration server, (ii) handles call setup, and (iii) routes the SIP messages and in some cases the media stream. In the call setup Alice calls Bob sending an 'INVITE' message (1, 2). A receipt for each such hop is returned by the 'Trying' message (3, 4). If Bob's phone is connected then it starts to ring and propagates the 'Ringing' message back to Alice (5, 6). When Bob answers the call, he sends an 'OK' message

<sup>&</sup>lt;sup>1</sup> Automated Validation of Internet Security Protocols and Applications (AVISPA) project: http://avispa-project.org/library/sip.html

<sup>2</sup> IETF Session Initiation Protocol Charter: http://www.ietf.org/html.charters/sip-charter.html

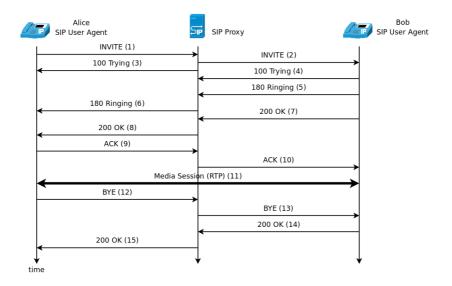


Fig. 1. Flow graph showing successful session establishment and termination using SIP

routed back to Alice through proxy S. Bob answers the phone in message (7), and the call content (voice) is transferred using the Real-time Transport Protocol (11). Alice terminates the call (12) and a 'BYE' message is sent. Before the 'INVITE' message (1) is sent, a SIP proxy may challenge Alice to authenticate, as formalised in Section 3.1.

For the purpose of this paper we restrict our work to how the widely used open source telephony platform Asterisk [11] implements SIP. Asterisk is a private branch exchanges (PBX), whose functionality is to connect phone calls. Asterisk also supports a range of other common telephony services like voice mail, conference calls and telephone menus.

#### 2.1 The Method

In order to gain initial knowledge of the behaviour of the SIP implementation in Asterisk, we record traces from real phone traffic going through the Asterisk server on a real-world Asterisk configuration. This is done by using the network monitoring tool Wireshark. Traces retrieved from Wireshark can be presented both textually and as interaction diagrams. The process of obtaining a formal specification of SIP was roughly as follows: From the core IETF standards we derived an as accurate descriptions of SIP as possible. These resulting specifications were typically incomplete, interaction diagrams showing the transmissions and message content were lacking. Traces of the call setup with real softphones were then used to supplement the incomplete specifications, with details of message

<sup>&</sup>lt;sup>3</sup> Asterisk homepage: http://www.asterisk.org/

<sup>&</sup>lt;sup>4</sup> Wireshark homepage: http://www.wireshark.org

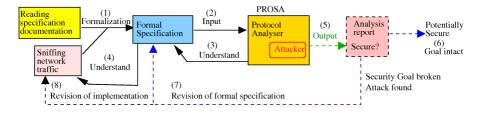


Fig. 2. Workflow for analysis of implementations

credentials. Hence, based on the Asterisk traces and SIP standard we constructed the formal models manually, and analysed the specifications in the the protocol analyser PROSA **S**.

The analysis process is depicted in Fig. 2 In this process both IETF documentation and the traces are used to obtain a formal description (1) which typically enhances the understanding of the implementation (4). The formal specification is then validated by PROSA (2). If there are any errors or unreasonable elements found at this stage, the formal specification is revised (3). A correct protocol specification will then be subject to hand-crafted or automatically generated attacks. The correctness of manually constructed attacks will then be checked by validation in a similar way as for non-compromised protocol specifications, thereafter simulated in PROSA. Finally, the output of the analysis is a report that either confirms the initial security requirements or points at weaknesses that break some security goals (5). If no attacks are found then the protocol is considered preliminary secure (6). If an attack is found then the protocol is not secure and a revision is made (7). Since the formal specifications are derived from a concrete implementation, the report gives feedback onto the implementation (8).

#### 2.2 The Protocol Analyser PROSA

PROSA 🛛 is a tool developed for the specification, static analysis and simulation of security protocols. PROSA consists of three main modules: (a) a specification language based on temporal epistemic logic; (b) a static analysis module; (c) a simulator for executing intended protocols and attacks on protocols.

The static analysis module consists of algorithms for *automated refinement* of both protocol specifications and attack descriptions. The automated refinement results in an explicit specification that contains local assumptions, i.e. pre- and postconditions, for each transmission clause. Refined specifications can then be *validated*. The validation process of a trace specification is performed in two steps in PROSA: First, a tool-supported refinement of the specification is generated. This will always give a longer specification that contains information about the agents beliefs and construction of credentials, like the generation of nonces, timestamps, assumptions about keys, and cryptographic operation like encryption, decryption and hashing. Secondly, the refined specification is validated to check whether a participant in the protocol setting possesses any beliefs that have not been legally obtained through communication or cryptography.

## 3 Formal Specification of SIP Call-Setup

SIP defines distinct functionality for registration, call setup and modification, call control and mid-call signalling. We refer to these parts as 'sub-protocols' since each of these parts requires its own sequence of message exchanges. We take a closer look at the sub-protocols *digest access authentication, call setup* and *call teardown. Call teardown* denotes the explicit event of terminating a call, specified by message 12-15 in Fig.  $\blacksquare$  These are specified in a form commonly used in the literature. A protocol clause where "agent A sends a message M to the agent B" is of the form:

 $(P) \qquad A \longrightarrow B \quad : \quad M$ 

Messages in the protocols consist of basic entities as follows:

A, B, C, D, T, S, I, I(A)	agent terms
$K_{AB}$	symmetric key shared by $A$ and $B$
$N_A$	nonce generated by $A$
$W_A^Y$	string containing the text $Y$ related to $A$
$X_A$	miscellaneous entities related to $A$

We use three composition operators in the notation: concatenation of message content denoted by "," (comma), hashing H[M], and encryption denoted by E(K : M), where K denotes a key and M a message content. The particular agent term I(A) reads that "the intruder I impersonates as A".

#### 3.1 Authentication

According to RFC3261 14, there are three ways to configure SIP authentication: plain-text authentication, weak authentication, and strong authentication. Plaintext authentication sends the authentication credentials unprotected. Weak authentication is an adaptation of the HTTP digest access authentication 5 that requires a shared secret between the two participants. Strong authentication uses certificates in the same way as web browsers and servers use them. Weak authentication using a digest is by far the most common method. A typical application of digest access authentication is given by a challenger S requesting a client Ato authenticate as described in the following protocol skeleton:

$$\begin{array}{ll} (\mathbf{D}_{1}) & S \longrightarrow A: N_{S} \\ (\mathbf{D}_{2}) & A \longrightarrow S: W_{A}^{\mathrm{uname}}, W^{\mathrm{realm}}, N_{S}, W_{A}^{\mathrm{URI}}, X_{\mathrm{nc}}, N_{A}, W^{\mathrm{qop}}, \\ & & \mathsf{H}[\mathsf{H}[W_{A}^{\mathrm{uname}}, W^{\mathrm{realm}}, K_{AS}^{\mathrm{pwd}}], N_{S}, X_{\mathrm{nc}}, N_{A}, W^{\mathrm{qop}}, \mathsf{H}[W^{\mathrm{meth}}, W_{A}^{\mathrm{URI}}]] \end{array}$$

Agents A and S share the symmetric key  $K_{AS}^{\text{pwd}}$ . Initially, the challenger S sends a nonce  $N_S$  to the client A. The client responds by sending the basic entities in plain text, except the password, and then the response itself. The entities involved in digest access authentication are as follows:

$W_A^{\text{uname}}$	username of $A$
$W^{\mathrm{realm}}$	a protective domain
$K_{AS}^{\mathrm{pwd}}$	shared password between ${\cal A}$ and ${\cal S}$
$W^{\text{meth}}$	main method of message (like HTTP)
$W_A^{\mathrm{URI}}$	Digest URI for client $A$
$N_S$	nonce of the challenger $S$
$X_{\rm nc}$	nonce counter
$N_A$	A's nonce
$W^{\mathrm{qop}}$	quality of protection

The authentication is one-way: A is authenticated to S, guaranteed by the secrecy of the shared key  $K_{AS}^{\text{pwd}}$ . Agent S can be certain that the message comes from A, since A is the only agent except S who possesses the key. Integrity of the message entities involved is provided by the fact that the hash could only be generated by A and freshness of the message is provided by the challenger nonce  $N_S$ .

### 3.2 The Teardown Sub-protocol

The trace described in Fig.  $\square$  is one out many possible traces. *Teardown* of sessions can be performed at any stage in the session. Therefore the final four messages 12-15 in Fig.  $\square$  and  $(T_{14} - T_{17})$  in Fig.  $\square$  can be considered as a **teardown** sub-protocol run by three agents. Instances of the teardown protocol might be running in parallel with the call setup protocol, which implies that a 'BYE' message received to any participant causes the host session of the call setup to be terminated. A SIP compliant specification where SIP methods are propagated correctly results in the following specification of teardown, extracted from the Wireshark protocol dump:

$$\begin{array}{ll} (\mathrm{TD}_1) & C \longrightarrow T & : \ W^{\mathrm{BYE}}, D, W^{\mathrm{DRI}}_D, W^{\mathrm{Contact}}_C, W^{\mathrm{URI}}_C, N^{\mathrm{callid}}_C \\ (\mathrm{TD}_2) & T \longrightarrow D & : \ W^{\mathrm{BYE}}, C, W^{\mathrm{URI}}_C, N^{\mathrm{callid}}_D \\ (\mathrm{TD}_3) & D \longrightarrow T & : \ W^{\mathrm{OK}}, W^{\mathrm{Contact}}_D, W^{\mathrm{URI}}_D, N^{\mathrm{callid}}_D \\ (\mathrm{TD}_4) & T \longrightarrow C & : \ W^{\mathrm{OK}}, W^{\mathrm{Contact}}_D, W^{\mathrm{URI}}_D, N^{\mathrm{callid}}_C \end{array}$$

where C denotes the role of the agent initiating the teardown, D denotes the responder agent, while T denotes the proxy server. Instances of the teardown protocol are started by the call setup protocol, while the call-setup is terminated by the teardown protocol.

### 3.3 Formalising Call Setup Permitting Arbitrary Teardown

Since we do not know which of the agents actively closes a session, we model that each agent starts a passive session of the teardown sub-protocol. In the example shown in Fig.  $\square$  Alice actively closes the session, why she plays the role of B of

$(Q_0)$	A	:	start(teardown, Role(B, C), Role(A, D), Role(S, T))
$(P_1)$	$A \longrightarrow S$	:	$W^{\text{INVITE}}, A, B, W^{\text{Contact}}_A, W^{\text{URI}}_A, N^{\text{callid}}_A$
$(Q_1)$	S	:	$\operatorname{start}(\operatorname{teardown},\operatorname{Role}(A,C),\operatorname{Role}(B,D),\operatorname{Role}(S,T))$
			$\operatorname{start}(\operatorname{teardown},\operatorname{Role}(B,C),\operatorname{Role}(A,D),\operatorname{Role}(S,T))$
$(P_2)$	$S \longrightarrow A$	:	$W^{\text{PAR}}, W^{\text{uname}}_A, W^{\text{realm}}, N_S, A, B, N^{\text{callid}}_A$
$(P_3)$	$A \longrightarrow S$	:	$W^{ACK}, B, W^{Contact}_A, W^{URI}_A, N^{callid}_A$
$(P_4)$			$W^{\text{INVITE}}, A, B, N_S, W_A^{\text{Contact}}, W_A^{\text{URI}}, X_{\text{nc}}, N'_A, W^{\text{qop}}, N_A^{\text{callid}}$
	H[H[W	A	$\mathcal{M}^{\text{ame}}, W^{\text{realm}}, K_{AS}^{\text{pwd}}], N_A', X_{\text{nc}}, N_S, W^{\text{qop}}, H[W^{\text{INVITE}}, W_B^{\text{URI}}]]$
$(P_5)$	$S \longrightarrow A$	:	$W^{\mathrm{Trying}}, W_B^{\mathrm{Contact}}, W_B^{\mathrm{URI}}, N_A^{\mathrm{callid}}$
$(P_6)$	$S \longrightarrow B$	:	$W^{\text{INVITE}}, A, B, W_A^{\text{Contact}}, W_A^{\text{URI}}, N_B^{\text{callid}}$
			$\operatorname{start}(\operatorname{teardown},\operatorname{Role}(B,D),\operatorname{Role}(A,C),\operatorname{Role}(S,T))$
$(P_7)$	$B \longrightarrow S$	:	$W^{\mathrm{Trying}}, W^{\mathrm{Contact}}_B, W^{\mathrm{URI}}_B, N^{\mathrm{callid}}_B$
$(P_8)$	$B \longrightarrow S$	:	$W^{\text{Ringing}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_B$
$(P_9)$	$S \longrightarrow A$	:	$W^{\text{Ringing}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_A$
$(P_{10})$	$B \longrightarrow S$	:	$W^{\mathrm{OK}}, W^{\mathrm{Contact}}_B, W^{\mathrm{URI}}_B, N^{\mathrm{callid}}_B$
			$W^{\mathrm{OK}}, W^{\mathrm{Contact}}_B, W^{\mathrm{URI}}_B, N^{\mathrm{callid}}_A$
$(P_{12})$	$A \longrightarrow S$	:	$W^{ACK}, W^{Contact}_A, W^{URI}_A, N^{callid}_A$
			$W^{ACK}, W^{Contact}_A, W^{URI}_A, N^{callid}_B$
	$A \longleftrightarrow B$	:	Media session

Fig. 3. Call setup with flexible teardown based on RFC 3261

the teardown sub-protocol. Consequently, the Asterisk server acts in the role of S. Generally, both Alice and Bob might actively tear down a session by starting an instance of teardown in the role of A.

Combining the Wireshark protocol dump with the SIP specification we get Fig.  $\square$  In order to model this we go beyond standard protocol notation and introduce the clauses  $(Q_0 - Q_3)$ , as seen in Fig.  $\square$  The call setup involves the additional SIP methods and primitives:

$W^{\rm INVITE}$	The INVITE method that indicates a request for phone call
$W^{ACK}$	An acknowledgement method
$W^{\mathrm{PAR}}$	Proxy Authentication Required
$W^{\mathrm{Trying}}$	100 Trying, receipt to a previous SIP message
$W^{OK}$	200 OK method notifies successful registration
$W^{\text{Ringing}}$	The responder's phone is ringing
$W^{\rm BYE}$	Tearing down session (hang up phone)
$N_A^{\text{callid}}, N_B^{\text{callid}}$	Call identifiers for $A$ (and $S$ ) and $B$ (and $S$ ) respectively

Clause  $(Q_0)$  reads "agent A locally starts a session of the teardown protocol, such that agent A plays the responder role D, agent B plays the initiator role C, and the server S plays the server role". The notation Role(A, C) means that the agent A plays the C role in the given protocol, similar to procedure calls with parameter substitution in ordinary programming languages. The first local clause  $(Q_0)$  then says that A initially starts listening for a possible 'BYE' message from B. The server propagates the 'BYE' and 'OK' methods involved in the teardown sub-protocol. There are two cases: Clause  $(Q_1)$  starts a session where the server S is waiting for a 'BYE' from A, while in  $(Q_2)$ , the server starts a similar session waiting for B sending a 'BYE'-message.

### 3.4 Deviations from the SIP Specification

The trace in Fig. 4 shows that the Asterisk implementation of SIP diverges from the specification described in Fig. 3 in three ways:

- 1. Alice's phone starts to ring (message  $T_7$ ) before Bob is authenticated to the server. The meaning of an incoming 'Ringing' message received by Alice is that Bob has received an 'INVITE' message, and she is ready to start a call if Bob answers the call. Hence, in order to follow the SIP RFCs message ( $T_9$ ) should come before message ( $T_7$ ). Hence Alice is fooled to believe that Bob's phone is ringing, which is not the case. Therefore we simulated scenarios where the responder Bob was disconnected from the network just after receiving the 'INVITE' message. Alice still received a Ringing message. This behaviour is also confirmed in experiments using soft phones.
- 2. The acknowledgement received by Bob in message  $(T_{11})$  arrives *before* Alice sends the message in clause  $(T_{13})$ . However, at this point Bob is mislead to believe that Alice has acknowledged the Ringing request from Bob.
- 3. After teardown initiated by Alice  $(T_{14})$ , the 'OK' message from S to Alice  $(T_{15})$  is sent *before* the related 'OK' message is sent from Bob. This breaks the specification, since  $(T_{15})$  would implicate that Bob has received the 'BYE' message, which is not the case in the implementation.

Fig. 4. A trace of call setup and teardown using Asterisk as server S

This implementation can lead to unexpected results: An obvious attack targets the Ringing method: An intruder I could act as an eavesdropper until clause (T<sub>7</sub>), then take over Bob's session entirely, kick Bob out of the call, and for the rest of the trace masquerade as Bob. Persons that are used to the particularly quick response (immediate ringing) from Asterisk based VoIP would not be alerted when the intruder I impersonates as Bob later in the session.

## 4 Attack on the Call Setup

An attack on the registration protocol of SIP has been found recently [9], yet in the following, we consider attacks that do not rely on successful registration

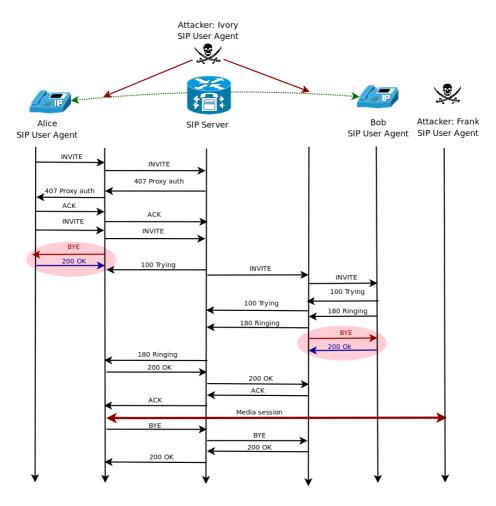


Fig. 5. Hijacking the initiator and the responder

attacks. We assume that the attacker I is as powerful as the Dolev Yao attacker [3] who controls the entire network, can intercept any message, impersonate as any other agent, and inject whatever entity it knows into SIP messages. Cryptography is assumed to be perfect, no brute force attacks on the underlying cryptographic algorithms are considered in this paper.

In the following we describe how it is possible for an attacker to hijack both the initiator and responder roles. In the initial part of the attack, described in Fig. (5), the intruder only passively listens in the authentication sub-protocol. In the protocol clauses  $(P_{1.1.a})$  through  $(P_{1.4.b})$  the intruder acts as a passive man-in-the-middle, obtaining information from plain text entities. From the knowledge gained during the initial eavesdropping, an attacker can perform a combined attack on both the caller and the callee.

In the attack, as shown in Fig. 5, the attacker Ivory (denoted *I* in Fig. 6) begins by eavesdropping the initial four messages concerned with establishing

$(P_{1.1.a}) A \longrightarrow I(S) : W^{\text{INVITE}}, A, B, W_A^{\text{Contact}}, W_A^{\text{URI}}, N_A^{\text{callid}}$
$(P_{1.1.b})  I(A) \longrightarrow S \qquad :  W^{\text{INVITE}}, A, B, W_A^{\text{Contact}}, W_A^{\text{NIR}}, N_A^{\text{Callid}}$
$(P_{1.2.a}) \xrightarrow{S \longrightarrow} I(A) \qquad : W^{\text{PAR}}, W^{\text{name}}, W^{\text{realm}}, N_S, A, B, N^{\text{callid}}_A$
$(P_{1.2.b})  I(S) \longrightarrow A \qquad :  W^{\text{PAR}}, W^{\text{uname}}, W^{\text{realm}}, N_S, A, B, N^{\text{callid}}_A$
$(P_{1.3.a})  A \longrightarrow I(S) \qquad :  W^{ACK}, B, W^{Contact}, W^{URI}, N^{callid}_A$
$(P_{1.3.b})  I(A) \longrightarrow S \qquad : \ W^{ACK}, B, W^{Contact}, W^{URI}_A, N^{Callid}_A$
$(P_{1.4.a})  A \longrightarrow I(S) \qquad : W^{\text{INVITE}}, A, B, N_S, W_A^{\text{Contact}}, W_A^{\text{URI}}, X_{\text{nc}}, N'_A, W^{\text{qop}}, N_A^{\text{callid}}$
$H[H[W_A^{\text{uname}}, W^{\text{realm}}, K_{AS}^{\text{pwd}}], N'_A, X_{\text{nc}}, N_S, W^{\text{qop}}, H[W^{\text{INVITE}}, W_B^{\text{URI}}]]$
$(P_{1.4.b})  I(A) \longrightarrow S \qquad :  W^{\text{INVITE}}, A, B, N_S, W_A^{\text{Contact}}, W_A^{\text{URI}}, X_{\text{nc}}, N'_A, W^{\text{qop}}, N_A^{\text{callid}}$
$H[H[W_A^{\text{uname}}, W^{\text{realm}}, K_{AS}^{\text{pwd}}], N'_A, X_{\text{nc}}, N_S, W^{\text{qop}}, H[W^{\text{INVITE}}, W_B^{\text{URI}}]]$
$(T_{2.3.2})$ $I(S) \longrightarrow A$ : $W^{\text{BYE}}, B, W^{\text{URI}}_B, N^{\text{callid}}_A$
$(T_{2.3.3})$ $A \longrightarrow I(S)$ : $W^{\text{OK}}, W^{\text{Contact}}_A, W^{\text{URI}}_A, N^{\text{callid}}_A$
$(P_{2.5})  S \longrightarrow I(A) \qquad : \ W^{\text{Trying}}, W_B^{\text{Contact}}, W_B^{\text{URI}}, N_A^{\text{callid}}$
$(P_{2.6.a})$ $S \longrightarrow I(B)$ : $W^{\text{INVITE}}, A, B, W^{\text{Contact}}_{A}, W^{\text{URI}}_{A}, N^{\text{Callid}}_{B}$
$(P_{2.6.b})$ $I(S) \longrightarrow B$ : $W^{\text{INVITE}}, A, B, W^{\text{Contact}}_{A}, W^{\text{URI}}_{A}, N^{\text{Callid}}_{B}$
$(P_{2.7.a})  B \longrightarrow I(S) \qquad :  W^{\text{Trying}}, W_B^{\text{Contact}}, W_B^{\text{URI}}, N_B^{\text{callid}}$
$(P_{2.7.b})$ $I(B) \longrightarrow S$ : $W^{\text{Trying}}, W_B^{\text{Contact}}, W_B^{\text{URI}}, N_B^{\text{callid}}$
$(P_{2.8.a})  B \longrightarrow I(S) \qquad :  W^{\text{Ringing}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_B$
$(P_{2.8.b})$ $I(B) \longrightarrow S$ : $W^{\text{Ringing}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_B$
$(T_{2,4,2})$ $I(S) \longrightarrow B$ : $W^{\text{BYE}}, A, W^{\text{URI}}_A, N^{\text{callid}}_B$
$(T_{2.4.3})$ $B \longrightarrow I(S)$ : $W^{\text{OK}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_B$
$(P_{2.9})  S \longrightarrow I(A) \qquad : \ W^{\text{Ringing}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_A$
$(P_{2.10})$ $I(B) \longrightarrow S$ : $W_B^{OK}, W_B^{Contact}, W_B^{URI}, N_B^{callid}$
$(P_{2.11})$ $S \longrightarrow I(A)$ : $W^{\text{OK}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_A$
$(P_{2.12})$ $I(A) \longrightarrow S$ : $W^{ACK}, W^{Contact}_A, W^{URI}_A, N^{callid}_A$
$(P_{2.13})$ $S \longrightarrow I(B)$ : $W^{ACK}, W^{Contact}_A, W^{URI}_A, N^{callid}_B$
$I(A) \longleftrightarrow F(B)$ : Media session
$(T_{2.2.1})$ $I(A) \longrightarrow S$ : $W_{BYE}^{BYE}, B, W_{BYE}^{URI}, W_{AYU}^{Contact}, W_{A}^{URI}, N_{A}^{callid}$
$(T_{2.5.2})$ $S \longrightarrow I(B)$ : $W_{A}^{BYE}, A, W_{A}^{URI}, N_{B}^{callid}$
$(T_{2.2.3})$ $I(B) \longrightarrow S$ : $W_{B}^{OK}, W_{B}^{Contact}, W_{B}^{URI}, N_{D}^{callid}$
$(T_{2.2.4})$ $S \longrightarrow I(A)$ : $W^{\text{OK}}, W^{\text{Contact}}_B, W^{\text{URI}}_B, N^{\text{callid}}_A$

Fig. 6. Formal attack when hijacking the initiator and the responder

the call and authenticating the caller Alice to the server. Then Ivory tears down Alice's session prematurely by using a 'BYE' message, and thereafter terminates Bob's session before he has entered the media session. Before the media session, the attacker has taken over the call, and can start a conversation with agent Frank (denoted F in Fig. [5].

The attacker tears down the session after pretending to be Alice. The server S cannot discover that the two local sessions at each calling party are teared down. The attack effectively breaks the authenticity of the participants, and therefore results in an identity management problem. The consequence could be that the intruder I could set up an arbitrary call, that Alice is billed for. Another consequence of this attack could be that the logs, that telephony providers are obliged to carry out by legislation, are incorrect. Hence, the attack shows that non-repudiation is broken as well.

### 5 Discussion and Conclusions

Our method reveals concrete attacks that indicate where improvements in the protocol are necessary. We discovered that the well-known SIP implementation Asterisk deviates from the SIP specification, and found a severe call hijack attack, where intruders can completely take over a phone call.

Our work provides the designers of VoIP protocols with a set of tools for protocol analysis. The PROSA language and framework can be used to formally specify and analyse protocols and their specific implementations in a rigorous way. The use of traces, and the analysis with PROSA, can help to detect differences between protocol specification and implementation. The behaviour of implementations is therefore analysed, and treated as specification for a variant of the employed protocols. Attacks could be designed and tested rapidly compared to the traditional approaches as described in **13** and **4**.

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# Measuring Anonymity<sup>\*</sup>

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**Abstract.** Some systems offer probabilistic anonymity. The degree of anonymity is considered and defined by Reiter and Rubin [I]. In this paper metrics are proposed to measure anonymity of probabilistic systems. The metric induces a topology on probabilistic applied  $\pi$  processes, which are used to model anonymous systems. The degree of anonymity is formally defined, and as an illustrating example, Crowds – an anonymous system for web transaction – is analyzed.

Keywords: Probabilistic applied  $\pi$  calculus, anonymity, metric, Crowds.

### 1 Introduction

The importance of anonymity has increased over the past few years. Various systems have been proposed to implement anonymity for various kinds of network communication, such as FOO electronic voting protocol [2], untraceable electronic cash protocol [3] and so on. Some formal methods are applied to analyze their anonymity property, for example, Chothia analyzed the MUTE anonymous file-sharing system using the  $\pi$  calculus in [4].

At the same time, some probabilistic anonymous systems, which extend nonprobabilistic anonymous systems with probability, have been proposed, such as Crowds II and some variants of DCP 5. Crowds is a system developed by Reiter and Rubin II for protecting users' anonymity on the world-wide-web. It operates by grouping users into a large group called crowd. When the web server receives a request, neither web server nor collaborating crowd members can learn the true source of this request.

Many formal methods are built to analyze the anonymity of probabilistic protocols or systems, there are two groups of frameworks for analyzing anonymity:

 Logic + Model checking. Shmatikov uses PCTL to model Crowds system and applies PRISM to automatically check the anonymity in 6; Halpern 7

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also gives the definition of anonymity in the probabilistic way, and some are corresponding to the definitions by Reiter in  $\blacksquare$ .

- Shannon's entropy. Diaz et al S give a measure system for the anonymity with Shannon's definition of entropy. However, the analysis cannot be automated and the capability of the attacker is limited.

The framework in this paper is not included in above two groups. We model anonymous systems in process calculi and use metrics to define anonymity. Process calculi provide a tool for the high-level description of interactions and communications between agents or processes. Using process calculi to analyze security protocols was first studied by Lowe in [9] with CSP. Later on the spi calculus [10] was proposed by extending the  $\pi$  calculus with cryptographic primitives, such as encryption and decryption. In [11] Abadi and Fournet introduced the applied  $\pi$  calculus, which is a simple extension of the  $\pi$  calculus with value passing, primitive functions and equations among terms. It has been used to model security protocols like Just Fast Keying [12]. Verification of authentication using the applied  $\pi$  calculus has been studied in [13][14]. It is natural and convenient to model protocols in process calculi.

In this paper, we use the probabilistic applied  $\pi$  calculus (PA $\pi$  for short) proposed by Goubault-Larrecq *et al* **15** as the framework for analyzing probabilistic anonymous systems. PA $\pi$  extends the applied  $\pi$  calculus with probabilistic choice( $\oplus$ ). Instead of bisimulation, we use *pseudo-metric* to measure the difference between two processes. A pseudo-metric is a function which defines a distance between elements of a set, and the calculation can be automatically carried out. Some pioneering work about metrics on process calculi include **16.17** and **18**. The main contributions of this paper can be summarized as follows:

- We define pseudo-metric on pairs of processes. It equals to 0 if and only if two processes are strongly bisimilar. We first replace weak bisimilarity with strong one in analyzing security protocols because we found that the protocol and its specification behave very similarly to each other. We need not consider weak bisimilarity for its complicated definition in probabilistic process calculi. More discussions can be found in Section 3
- According to the pseudo-metric, we give the formal definitions of *beyond* suspicion, probable innocence and possible innocence three degrees of anonymity from strong to weak in **1**.
- We model Crowds and analyze the anonymity degree. We study how the number of participants and the probability of forwarding influence the degree of anonymity.

The rest of the paper is organized as follows. Section 2 briefly introduces the probabilistic applied  $\pi$  calculus. Section 3 defines pseudo-metric between two probabilistic applied  $\pi$  processes, and demonstrates the correctness of the metric by showing when two processes are strongly bisimilar the metric is 0. Section 4 defines the degree of anonymity based on pseudo-metric. Section 5 analyzes Crowds system. Finally, we conclude in Section 6

## 2 Probabilistic Applied $\pi$ Calculus

This section covers the necessary background materials on  $PA\pi$ . More details are referred to **15**.

Given a signature  $\Sigma = \{(f_1, a_1), \dots, (f_n, a_n)\}$  consisting of a finite set of function symbols  $f_i$  each with an arity  $a_i$ , terms, plain processes and extended processes are defined by the following grammar:

 $\begin{array}{ll} (\text{Terms}) & M,N::=a,b,c,\cdots\mid x,y,z,\cdots\mid \mathsf{f}(M_1,\cdots,M_l)\\ (\text{Plain Process}) & P,Q::=\mathbf{0}\mid \bar{u}\langle M\rangle.P\mid u(x).P\mid P+Q\mid P\oplus_p Q\\ & \mid P\mid Q\mid !P\mid \nu \ n.P\mid \text{if }M=N \text{ then }P \text{ else }Q\\ (\text{Extended Process}) & A,B::=P\mid \{M/x\}\mid A\mid B\mid \nu \ n.A\mid \nu \ x.A \end{array}$ 

We use  $(p_1)P_1+(1-p_1)P_2$  to denote  $P_1\oplus_{p_1}P_2$ , so process  $\Sigma_{i\in I}(p_i)P_i$  represents for  $(p_1)P_1+(p_2)P_2+\cdots+(p_n)P_n$  with  $\Sigma_{i\in I}p_i=1$ .  $\prod_{i\in\{1,2,\cdots,n\}}A_i$  is abbreviation for  $A_1 | A_2 | \cdots | A_n$ , and  $\nu \tilde{l}.A$  for  $\nu l_1.\nu l_2.\cdots.\nu l_n.A$  where  $\tilde{l} = \{l_1, l_2, \cdots, l_n\}$ . We use *Act* denotes the set of all possible actions in the labeled rules.

We write fv(A), bv(A), fn(A), and bn(A) for free and bound variables, free and bound names of A. The set of names and variables that occur in A is denoted as  $n(A) \stackrel{\text{def}}{=} bn(A) \cup fn(A)$  and  $v(A) \stackrel{\text{def}}{=} bv(A) \cup fv(A)$ .

Equational theory which consists of a set of equations plays an important role in  $PA\pi$ . For every signature  $\Sigma$ , we equip it with an equational theory E. We use  $\Sigma \vdash M =_E N$  to denote terms M, N are equivalent in E. Sometimes we simply write M = N if there is no ambiguity.

A frame is an extended process built up from **0** and active substitutions by parallel composition and restriction. The frame of an extended process A, denoted by  $\phi_A$ , can be written in the form of  $\phi_A \stackrel{\text{def}}{=} \nu \tilde{n}.\{\widetilde{M}/\widetilde{x}\}$  where  $\widetilde{n} \subseteq n(\widetilde{M})$ . If  $\widetilde{M} = \langle M_1, M_2, \cdots, M_n \rangle$  and  $\widetilde{x} = \langle x_1, x_2, \cdots, x_n \rangle$ , then

$$\phi_A \equiv \nu \, \widetilde{n}.(\{M_1/x_1\} \,|\, \{M_2/x_2\} \,|\, \cdots \,|\, \{M_n/x_n\}).$$

Every extended process can be mapped to a frame. We write  $dom(\phi)$  for the domain of  $\phi$ , a set of variables which appear in active substitutions in  $\phi$  but not under a variable restriction. The domain of an extended process is the domain of its frame.

Static equivalence is an equivalence relation built on closed extended processes, which depicts the indistinguishability of their frames. The following definitions were firstly introduced in  $\square$ .

**Definition 1.** We say that two terms M and N are equal in the frame  $\phi$ , and write  $(M = N)\phi$ , if and only if  $\phi \equiv \nu \tilde{n}.\sigma, M\sigma = N\sigma$ , and  $\{\tilde{n}\} \cap (fn(M) \cup fn(N)) = \emptyset$  for some names  $\tilde{n}$  and substitution  $\sigma$ .

**Definition 2.** We say that two closed frame  $\phi$  and  $\psi$  are statically equivalent, and write  $\phi \approx_s \psi$ , when  $dom(\phi) = dom(\psi)$  and when, for all terms M and N, we have  $(M = N)\phi$  if and only if  $(M = N)\psi$ . **Definition 3 (Static equivalence).**  $(\approx_s)$  We say that two closed extended processes are statically equivalent, and write  $A \approx_s B$ , when their frames are statically equivalent.

To denote a discrete probability distribution over a set X, we use a function  $\mu$ :  $2^X \longrightarrow [0,1]$  with the following properties:  $\mu(X) = 1$  and  $\mu(\cup_i X_i) = \Sigma_i \mu(X_i)$ . We use  $\delta_x$  to denote the dirac measure on x, and we define  $\mu = \mu_1 +_p \mu_2$  as  $\mu(Y) = p \cdot \mu_1(Y) + (1-p)\mu_2(Y)$ . Bisimulation is a binary relation between state transition systems, associating systems which behave in the same way in the sense that one system simulates the other and vice-versa. It is very important in process calculi. Let A be a closed extended process, C be a set of closed extended processes, the probability of A performing action  $\alpha$  and evolving into a process in set C is defined as follows:

$$Prob_A(\alpha, \mathcal{C}) = \sum_{A' \in \mathcal{C}} \mu(A'), \quad \text{where } A \xrightarrow{\alpha} \mu$$

We now give the definition of strong bisimulation.

**Definition 4 (Strong bisimulation).** Strong bisimulation ( $\sim$ ) is the largest symmetric relation  $\mathcal{R}$  between closed extended processes with the same domain such that ARB implies:

 $-A \approx_{s} B;$ - for any  $\alpha$ , and any classes  $\mathcal{C} \in \mathcal{A}/_{\mathcal{R}}$ ,  $Prob_{A}(\alpha, \mathcal{C}) = Prob_{B}(\alpha, \mathcal{C}).$ 

## 3 Metric

As bisimulation is not a robust concept in  $PA\pi$  model, a pseudo-metric analogue of strong bisimulation is developed in this section.

## 3.1 Pseudo-metric

In mathematics, a metric or distance function is a function which defines a distance between elements of a set. A set with a metric is called a metric space. A metric induces a topology on a set but not all topologies can be generated by a metric.

**Definition 5 (Metric).** A c-bound metric on a set X is a function (called the distance function or simply distance)  $m : X \times X \rightarrow [0, c]$ . For all x, y, z in X, this function is required to satisfy the following conditions:

1.  $m(x, y) \ge 0;$ 2. m(x, y) = 0 if and only if x = y;3. m(x, y) = m(y, x);4.  $m(x, z) \le m(x, y) + m(y, z).$  In this paper, we only consider 1-bound metrics. If the second requirement is dropped, the function is called a *pseudo-metric*. Let the metric space  $X = \mathcal{A}$ , we define a pre-order on all 1-bound pseudo-metrics of the set of all closed extended processes.

**Definition 6.**  $\mathcal{M}$  is the class of 1-bounded pseudo-metrics on extended processes with the ordering:

$$m_1 \preceq m_2 \text{ if } (\forall A, B \in \mathcal{A})[m_1(A, B) \ge m_2(A, B)].$$

We use  $\top$  to denote the top element  $\forall A, B. \top (A, B) = 0$ ; the bottom element is given by  $\bot (A, B) = 1$  if  $A \neq B$  otherwise 0. One can easily get that  $(\mathcal{M}, \preceq)$  is a complete lattice.

Since the labeled transition system of  $PA\pi$  is an LCMC (labeled concurrent Markov chain), every process evolves into a distribution on processes after a probabilistic transition. We lift the metric to the set of distributions.

**Definition 7.** Suppose  $m \in \mathcal{M}$ , and  $\mu, \mu'$  be distributions on extended processes, then  $m(\mu, \mu')$  is given by the solution of the following linear program:

max:  $\Sigma_i(\mu(A_i) - \mu'(A_i))(a_i - a_K)$ <br/>subject to:  $\forall i.0 \le a_i, a_K \le 1, \forall i, j.a_i - a_i \le m(A_i, A_i)$ 

**Definition 8.** For closed extended processes A and B. The action set of A corresponding to B, denoted L(A, B), is the set:

$$\{\alpha \mid \alpha \in Act \land fv(\alpha) \subseteq dom(A) \land bn(\alpha) \cap fn(B) = \emptyset\}$$

L(A, B) is the set of actions A can perform and the bound names in these actions should be renamed in order not to clash with the free names of B. We now define a function F on  $\mathcal{M}$  that closely resembles strong bisimulation.

**Definition 9.** Given two closed extended processes A and B, a function F on  $\mathcal{M}$  is defined as follows:

 $\begin{aligned} &- if A \not\approx_s B, \ then \ F(m)(A,B) = 1; \\ &- if \ A \approx_s B, \ F(m)(A,B) = \\ &\max(\max_{\alpha \in L(A,B)} \sup_{A \xrightarrow{\alpha} \mu} \inf_{B \xrightarrow{\alpha} \mu'} m(\mu,\mu'), \ \max_{\alpha \in L(B,A)} \sup_{B \xrightarrow{\alpha} \mu'} \inf_{A \xrightarrow{\alpha} \mu} m(\mu,\mu')) \\ & where \ \inf \emptyset = 1, \ \sup \emptyset = 0. \end{aligned}$ 

We can easily get that F is monotone on  $\mathcal{M}$ , so F has a maximum fixed point

which is given by  $m_{max} = \bigsqcup_i m_i$  where  $m_0 = \top$  and  $m_{i+1} = F(m_i)$ . In the rest paper, we mean  $m_{max}(A, B)$  when we say "the metric between A, B".

We give some examples about how to calculate the metric between two processes. In each example, we use the following equational theory:

$$fst((x, y)) = x$$
$$snd((x, y)) = y$$
$$dec(enc(x, y), y) = x$$

Example 1.  $m(P \mid \{m/x\}, Q \mid \{n/x\}) = 1.$ 

Since P, Q are plain processes, and we have  $x\{m/x\} = m$  but  $x\{n/x\} \neq m$ , so the two processes are not statically equivalent.

Example 2.

$$\begin{split} A \stackrel{\text{def}}{=} & (\bar{c} \langle m \rangle + \bar{d} \langle n \rangle) \oplus_{0.5} \bar{d} \langle n \rangle \\ B \stackrel{\text{def}}{=} & \bar{c} \langle m \rangle \oplus_{0.25} \bar{d} \langle n \rangle \end{split}$$

We have  $A \approx_s B$ , therefore m(A, B) is the maximum fixed point of F(m):

$$\begin{split} F(m) &= \max(\max_{\alpha \in L(A,B)} \sup_{A \xrightarrow{\alpha} \mu} \inf_{B \xrightarrow{\alpha} \mu'} m(\mu, \mu'), \\ \max_{\alpha \in L(B,A)} \sup_{B \xrightarrow{\alpha} \mu'} \inf_{A \xrightarrow{\alpha} \mu} m(\mu, \mu')) \\ &= \max(m(\mu, \mu'), m(\mu', \mu)) \\ &= m(\mu, \mu') \end{split}$$

where  $\alpha = \tau$ ,  $\mu = (\bar{c}\langle m \rangle + \bar{d}\langle n \rangle) : 0.5, \bar{d}\langle n \rangle : 0.5$  and  $\mu' = \bar{c}\langle m \rangle : 0.25, \bar{d}\langle n \rangle : 0.75$ .  $m(\mu, \mu')$  is the solution to the following linear program:

$$\begin{aligned} \max[(0.5 - 0)(a_1 - a_K) + (0.5 - 0.75) \\ (a_2 - a_K) + (0 - 0.25)(a_3 - a_K)] \\ subject \ to: 0 \le a_1, a_2, a_3, a_K \le 1 \\ |a_1 - a_2| \le m(\bar{c}\langle m \rangle + \bar{d}\langle n \rangle, \bar{d}\langle n \rangle) = 1 \\ |a_1 - a_3| \le m(\bar{c}\langle m \rangle + \bar{d}\langle n \rangle, \bar{c}\langle m \rangle) = 1 \\ |a_2 - a_3| \le m(\bar{c}\langle m \rangle, \bar{d}\langle n \rangle) = 1 \end{aligned}$$

We can finally get that  $m_{max}(A, B) = 0.5$ .

This example gives us the institution that nondeterminism choice is different from the probabilistic one. If we apply the uniform distribution on the nondeterministic choice and get  $A' \stackrel{\text{def}}{=} (\bar{c}\langle m \rangle \oplus_{0.5} \bar{d}\langle n \rangle) \oplus_{0.5} \bar{d}\langle n \rangle$ , then m(A', B) = 0. Another important observation is that  $A \not\sim B$  and  $A' \sim B$  by the definition of strong bisimulation.

Example 3.

$$\begin{split} A &\stackrel{\text{def}}{=} \nu \, m, k.\bar{a} \langle \operatorname{enc}(m,k) \rangle.\bar{b} \langle m \rangle \\ B &\stackrel{\text{def}}{=} \nu \, m, n, k. (\bar{a} \langle \operatorname{enc}(m,k) \rangle.\bar{b} \langle n \rangle \oplus_{0.5} \bar{a} \langle \operatorname{enc}(n,k) \rangle.\bar{b} \langle n \rangle) \end{split}$$

We can get  $m_{max}(A, B) = 0$  though they look quite different.

The point of this example is that

$$\begin{split} &\nu\,k,m.\{\operatorname{enc}(m,k)/x,m/y\}\approx_s\nu\,k,m,n.\{\operatorname{enc}(m,k)/x,n/y\},\\ &\nu\,k,m.\{\operatorname{enc}(m,k)/x,m/y\}\approx_s\nu\,k,m,n.\{\operatorname{enc}(n,k)/x,n/y\}. \end{split}$$

The following theorem demonstrates the correctness of the definition of metric. **Theorem 1.**  $A \sim B$  iff  $m_{max}(A, B) = 0$ .

#### 3.2 Strong Bisimulation v.s. Weak Bisimulation

When analyzing security protocols with process calculi, we usually use observational equivalence to denote that the environment cannot distinguish two instances of a protocol (anonymity), or cannot tell the difference between the protocol and its specification (secrecy, authenticity). As observable equivalence coincides with weak bisimulation in process calculi such as  $\pi$ , Applied  $\pi$  and PA $\pi$ , weak bisimulation is used instead of observational equivalence for its convenience in proof. However, the proof of weak bisimulation is much more complicated than that of strong bisimulation. We know that strong bisimulation implies weak bisimulation, so can we replace weak bisimulation with strong one in analysis of security protocols?

To our best knowledge, the specification and the protocol itself behaves very similar in the definitions of most security properties. In 10 Abadi and Gordon define the authenticity and secrecy security protocol as follows:

Authenticity:  $Inst(M) \simeq Inst_{spec}(M), \text{ for any} M;$ Secrecy:  $Inst(M) \simeq Inst(M') \text{ if } F(M) \simeq F(M'), \text{ for any } M, M'.$ 

where  $\simeq$  is testing equivalence in Spi calculus and  $Inst_{spec}$  is the specification of Inst. In all the given example of [10], process  $Inst_{spec}$  only changes some messages of Inst. It is easy to get that  $Inst(M) \approx Inst_{spec}(M)$  if and only if  $Inst(M) \sim Inst_{speic}(M)$  in applied  $\pi$  calculus. The secrecy is similar. We find the same result in the definition of anonymity (non-probabilistic anonymity) [19]14]. So we can use strong bisimulation to eliminate the complexity caused by  $\tau$  actions in weak bisimulation.

## 4 Anonymity Degree

Anonymity is a general concept, which can be refined into more precise definitions. In [20], three types of anonymous communication properties are proposed: sender anonymity, receiver anonymity and unlinkability of sender and receiver. Sender anonymity means that the identity of the party who sent a message is hidden, while the identity of the receiver might not be. Receiver anonymity similarly means that the identity of the receiver is hidden. Unlinkability of sender and receiver means that though the sender and receiver can each be identified, but the relation between them cannot be identified.

For each type of anonymity, Reiter and Rubin defined six degrees of anonymity in **[1**]. Three of them attracted more attention. Take sender anonymity as an example:

- Beyond suspicion: if though the attacker can see evidence of a sent message, the sender appears no more likely to be the originator of that message than any other potential sender in the system.
- *Probable innocence*: if from the attacker's point of view the sender appears no more likely to be the originator than to not be the originator.

 Possible innocence: if from the attacker's point of view, there is a nontrivial probability that the real sender is someone else.

Probable innocence is weaker than beyond suspicion because the attacker may have reason to expect that the sender is more likely to be responsible than any other potential sender, but it still appears at least as likely that the sender is not responsible. Possible innocence is even weaker because the sender is almost exposed. But it is still not exactly exposed, that is to say, the probability of someone to be the actual sender is strictly less than 1.

For anonymous systems, global adversary can make the so called "intersection attack". Because not all users are active all the time, the owner of the initial message can be detected by global adversary. As a result of it, recently people add dummy traffic [21] into anonymous systems to prevent such attacks. In this paper we will not consider global attackers, but think about the local corrupted parties each of which is a local active attacker.

Suppose A is a probabilistic anonymous system, then A(s, r) denotes an instance of A that the sender is s and receiver is r. Let  $\tilde{k}_i$  denote the private knowledge of some participant i, such as the private keys, newly generated random numbers and etc.  $Chn_i$  denotes the set of all the channels related to i. For a set of corrupted participants T, we write  $A^T(s, r)$  for the instance in which the protocol A(s, r) runs with all the private knowledge of corrupted parties in T being revealed to the environment and all the private channels being under the control of the environment. In other words, these collaborated participants have become attackers. By applying the structural rules, we have  $A(s, r) \equiv \nu \tilde{n}.A'(s, r)$ where  $\tilde{n} = fn(A(s, r))$ . Let  $\tilde{K} = \bigcup_{i \in T} k_i$ ,

$$A^{T}(s,r) \stackrel{\mathsf{def}}{=} \nu \, \widetilde{n'}. (A'(s,r) \,|\, \{\widetilde{k}/\widetilde{x_{j}}\})$$

where  $\widetilde{n'} = \widetilde{n} \setminus (\bigcup_{i \in T} Chn_i)$ . Now we are ready to give the formal definition of the degree of anonymity.

Let  $d_s^T$  denotes the maximum metric between two instances with different senders and same receivers with the set of corrupted parties T, and let  $d_r^T$  means the maximum metric between two instances with same senders and different receivers with the set of corrupted parties T. Formally,

$$\begin{aligned} d_s^T &\stackrel{\text{def}}{=} \max_{r \in R} \max_{s, s' \in S \setminus T} \{ m_{max}(A^T(s, r), A^T(s', r)) \} \\ d_r^T &\stackrel{\text{def}}{=} \max_{s \in S} \max_{r, r' \in R \setminus T} \{ m_{max}(A^T(s, r), A^T(s, r')) \} \end{aligned}$$

**Definition 10.** For any anonymous system A, whose possible sender set is S, the possible receiver set is R, we say A satisfies,

- beyond suspicion sender anonymity under corrupted parties T, if  $d_s^T = 0$ ,
- probable innocence sender anonymity under corrupted parties T, if  $d_s^T < 0.5$ ,
- possible innocence sender anonymity under corrupted parties T, if  $d_s^T < 1$ ;
- Similarly, we use  $d_r^T$  to replace  $d_s^T$  in above statements to define the degree of receiver anonymity.

## 5 Analysis of Crowds

In this section, we first briefly introduce Crowds system; then we model crowds and analyze its anonymity.

### 5.1 Crowds

*Crowd* is a system developed by Reiter and Rubin [1] for protecting users' anonymity on the world-wide-web. It groups users into a large group called crowd. A user is represented in a crowd by a process on his/her computer called a *jondo*. All communication between any two jondos is encrypted using the symmetric key between them. A request will be sent to the destination with the following steps:

- When receiving the request from the browser, the proxy jondo, say I picks a jondo, say  $A_1$  (possibly himself) at random, and forwards the request to it.
- When  $A_1$  receives the request, it flips a biased coin to determine whether or not to forward the request to another jondo. The coin indicates to forward with probability  $p_f$ .
- If the result is to forward, then the jondo selects a random jondo and forward the request to it, otherwise, submits the request to the destined server.

## 5.2 Modeling

In this paper, we only consider the sender anonymity of Crowds. To simplify the analysis, we assume there is one server, denoted by r, who wants to find the identity of the real sender. Since we do not consider the global attacker, we may assume the channels are private and the messages may not be encrypted. The environment only can listen to the channels of corrupted parties.

Assume there are *n* jondos in this Crowd. We use  $J = \{1, 2, \dots, n\}$  to denote the set of jondos, and use  $c_{ij}$  with i < j to express the channel between jondo *i* and *j*. We write  $CJ = \bigcup_{i,j \in J \text{ and } i < j} \{c_{ij}\}$  for the set of channels between jondos,  $CR = \bigcup_{i \in J} \{c_{ir}\}$  for the channels between a jondo and the server *r*.  $CJ \cup CR$  is denoted by *C*. We use the process Crowds(s, r) in Figure 1 to model an instance of Crowd system where the sender is the jondo *s*.

A jondo is either an Initiator(the real sender) or a NonInitiator. The Initiator only can Forward the request, while the NonInitiator can forward the request with probability  $p_f$  or submit it with  $(1 - p_f)$ . When forwarding the request, there are three possibilities: first, the jondo chose another jondo whose index number smaller than his; second, the jondo chose another jondo whose index number larger than his; and third the jondo chose himself. Because choices are made uniformly random, so we model these three possibilities in the process Forward with uniform distribution. Jondos in Crowds can initiate and forward the requests many times, which may happen in parallel, and this is the reason why we use the replication operator ! in the processes in Figure II.

$$\begin{split} &Forward(i,req) \stackrel{\text{def}}{=} \mathcal{L}_{j \in J, i < j}(\frac{1}{n}) \overline{c_{ij}} \langle req \rangle + \mathcal{L}_{j \in J, j < i}(\frac{1}{n}) \overline{c_{ji}} \langle req \rangle + (\frac{1}{n}) SmtOrFwd(i, req) \\ &SmtOrFwd(i, req) \stackrel{\text{def}}{=} (p_f) Forward(i, req) + (1 - p_f) \overline{c_{ir}} \langle req \rangle \\ &Initator(i, s) \stackrel{\text{def}}{=} \nu req.Forward(i, req) \\ &NonInitiator(i) \stackrel{\text{def}}{=} !(\Pi_{j \in J, i < j} c_{ji}(x).SmtOrFwd(i, x) \mid \Pi_{j \in J, j < i} c_{ji}(x).SmtOrFwd(i, x)) \\ &Server(r) \stackrel{\text{def}}{=} \Pi_{i \in J} c_{ir}(x) \\ &Crowds(s, r) \stackrel{\text{def}}{=} \nu C.(Initiator(s) \mid \Pi_{i \in J} NonInitiator(i) \mid Server(r)) \end{split}$$

Fig. 1. The Crowds system in Probabilistic Applied  $\pi$  calculus

## 5.3 Analyzing

We first consider the simplest case: two jondos  $(s_1, s_2)$ . In this case the only corrupted party is the server because if one of the two jondos is an attacker, the sender must be another one. As the server is the attacker, we publicize the channels in CR:

 $Crowds^{r}(s,r) \stackrel{\text{def}}{=} \nu CJ.(Initiator(s) \mid \Pi_{i \in J} NonInitiator(i) \mid Server(r))$ 

The sender anonymity degree  $d_s^r$  equals to  $m(Crowds^r(s_1, r), Crowds^r(s_2, r))$ .

**Theorem 2.** When there are only two senders,  $d_s^r = 0$  for any  $p_f \in [0, 1]$ , which means in such situation Crowds system satisfies beyond suspicion sender anonymity.

**Theorem 3.** If all the senders are honest, then Crowds system satisfies beyond suspicion sender anonymity for any  $p_f \in [0, 1]$ .

This may go against our intuition that  $p_f$  should influence the anonymity of crowds. However, in Crowds when an initiator gets a request, he will first choose one jondo with the probability 1/n and forward to him. So it is not difficult to interpret such phenomenon.

Let us consider there are n jondos  $(s_1, \dots, s_n)$  and part of them are corrupted parties, w.l.o.g. we may assume the corrupted parties  $T = \{s_1, \dots, s_c, r\}$ . We write CK to denote the channels of  $s_1, \dots, s_c$ , let  $C' = CJ \setminus CK$ , then an instance  $Crowds^T(s, r)$  is of the following form

 $\nu C'.(Initiator(s) \mid \Pi_{i \in J} NonInitiator(i) \mid Server(r))$ 

We get some data in Table  $\blacksquare$  where n denotes the number of jondos, and c the number of corrupted parties.

We found that for any  $n \ge 3$  and  $c \ge 1$ , if  $p_f \le 0.5$ , then the degree  $d_s^T > 0.5$ , which means Crowds system satisfies possible innocence sender anonymity. If  $p_f > 0.5$  and n is large enough, it satisfies probable innocence  $(d_s^T < 0.5)$ . For the same  $p_f$  and c, the larger n is, the more anonymous Crowds system is.

Jondos $n$	Corrupted jondos $c$	Forwarding probability $p_f$	<b>Degree</b> $d_s^T$
3	1	0.5	0.833
3	1	1	0.667
6	1	0.8	0.467
10	2	0.8	0.44
20	2	0.8	0.32

Table 1.	Seder	anonymity	of	Crowds
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## 6 Conclusion

In this paper we propose a metric on pairs of processes to measure the degree of difference between them under the framework of  $PA\pi$ . It decreases to 0 when the two processes are strongly bisimilar. Based on metric, the anonymity degree is defined in  $PA\pi$  framework. We then analyze the Crowds system.

As to the future work, we will consider applying our approach to analyze some other probabilistic anonymous systems such as some electronic cash, electronic voting protocols. We would also like to develop an automatic tool to calculate metric between any two closed  $PA\pi$  processes.

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# A Hybrid E-Voting Scheme

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Abstract. There are two existing solutions to secure e-voting: homomorphic tallying and shuffling, each of which has its own advantages and disadvantages. The former supports efficient tallying but depends on costly vote validity check and does not support complex elections. The latter supports complex elections and dose not need vote validity check but depends on costly shuffling operations in the tallying operation. In this paper, the two techniques are combined to exploit their advantages and avoid their disadvantages. The resulting e-voting scheme is called hybrid e-voting, which supports complex elections, employs efficient vote validity check and only needs shuffling with a very small scale. So it is more efficient than the existing e-voting schemes, especially in complex elections.

### 1 Introduction

There are two main solutions to secure electronic voting. The first one is homomorphic tallying, which does not decrypt the encrypted votes separately but exploit homomorphism of the employed encryption algorithm to collectively open the encrypted votes using a small number of decryptions. There are two kinds of homomorphic tallying: additive homomorphic tallying and multiplicative homomorphic tallying. The former 1116171718112 employs an additive homomorphic encryption algorithm<sup>1</sup> like Paillier encryption and recovers the sum of the voters' selections. The latter **21** employs an multiplicative homomorphic encryption algorithm<sup>2</sup> like ElGamal encryption and recovers the product of the voters' selections. With homomorphic tallying, each vote must be in a special format, so that the number of every possible selection can be counted. More precisely, with additive homomorphic tallying every vote contains one or more selections (each corresponding to a candidate or a possible choice) and every selection must be one of two pre-defined integers (e.g. 0 and 1), each representing support or rejection of a candidate or choice; with multiplicative homomorphic tallying every vote contains only one selection, which corresponds to the chosen candidate or preferred choice and must be one of a few pre-defined integers (1 or e.g. small primes), each representing a candidate or choice. With such special vote formats,

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<sup>&</sup>lt;sup>1</sup> An encryption algorithm with decryption function D() is additive homomorphic if  $D(c_1) + D(c_2) = D(c_1c_2)$  for any ciphertexts  $c_1$  and  $c_2$ .

<sup>&</sup>lt;sup>2</sup> An encryption algorithm with decryption function D() is multiplicative homomorphic if  $D(c_1)D(c_2) = D(c_1c_2)$  for any ciphertexts  $c_1$  and  $c_2$ .

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usually the election rule is not complex with homomorphic tallying. When the number of candidates or possible choices is large, additive homomorphic tallying needs a large number of selections in a vote and thus compromises efficiency while the multiplicative homomorphic tallying must use large integers to represent the selections and thus the product of the selections easily overflows the multiplicative modulus. For example, in many political elections (e.g. in Australia) there are multiple candidates and each vote must indicate a complete preferential order of all the candidates. In such preferential voting applications, the existing homomorphic tallying solutions are impractical. Another drawback of homomorphic tallying is also caused by special vote format: each vote must be publicly checked to be valid (in a certain special format). Vote validity check usually depends on costly zero knowledge proof operations and corresponding verification operations.

In this paper, we are more interested in multiplicative homomorphic tallying 21, which is much less well-known than additive homomorphic tallying but has much simpler vote format. Besides simpler vote format and therefore more efficient vote sealing, multiplicative homomorphic tallying has another advantage over additive homomorphic tallying: it employs encryption algorithms depending on hardness of discrete logarithm problem like ElGamal encryption instead of the encryption algorithms depending on hardness of factorization problem. More precisely, there is no efficient method to implement key generation in a distributed way for any practical additive homomorphic encryption algorithm (e.g. Paillier encryption, all of which use factorization problem as a trapdoor) while centralized key generation for them through a trusted dealer 10 requires too strong a trust and is impractical in applications like e-voting. Note that although a modified ElGamal encryption in **18** is additive homomorphic, it is not practical in most applications as it does not support efficient decryption. There exist some distributed key generation mechanisms for RSA 3238, which is also factorization based. However, they (especially 323) are inefficient. improves efficiency to some extent by loosening the requirements on the parameters and using additional security assumptions, but is still inefficient compared to distributed key generation of DL based encryption algorithms like ElGamal 9. So they cannot provide an efficient solution to distributed key generation for additive homomorphic encryption, not to mention the relatively more efficient mechanism among them may not satisfy the parameter requirements with reasonable security assumptions when applied to additive homomorphic encryption based e-voting. In comparison, distributed key generation for multiplicative homomorphic encryption (e.g. ElGamal encryption) can be implemented very efficiently using the technique in 9.

With multiplicative homomorphic tallying, the vote space contains 1 and some small primes and each vote contains one integer in the vote space. In the tallying phase, product of all the votes are recovered and then factorized to re-construct the votes. A key point is that the product cannot overflow the multiplicative modulus, otherwise factorization fails. For example, when there are 10 possible choices in the election, even if the vote space contains 1 and the nine smallest

primes, a vote selection can still be as large as 23. In the current security standard, the multiplicative modulus is usually 1024 bits long when ElGamal encryption is employed. That means even if there are only several hundred voters the product of their votes will probably overflow the multiplicative modulus. There are two countermeasures in 21 to avoid the overflow. Firstly, the election must be simple and thus the vote space only contains very small primes. For example, the election application in **21** only support YES/NO election and thus the vote space only contains 1 and 2. Secondly, the votes are divided into multiple groups and the size of the groups are so small that the product of the votes in each group will not overflow the multiplicative modulus. A factorization is needed in each group to reconstruct the votes. There are two concerns in the existing multiplicative homomorphic tallying **[21]**. Firstly, the supported election rule is too simple and not suitable for many election applications. Secondly, its grouping mechanism weakens privacy of e-voting as each voter is known to have cast one of the recovered votes in his group. Unlike other e-voting schemes (which hide every voter's choice among all the recovered votes), the multiplicative-homomorphictallying-based e-voting scheme in **21** only achieves weaker and incomplete privacy.

Due to the drawbacks of homomorphic tallying, shuffling **[1119]14/2215/2013]** is often employed in e-voting when complex election rules like preferential election is used. A shuffling operation re-encrypts (or partially decrypts) the encrypted votes and re-orders them. Multiple shuffling operations are employed and each of them is performed by a different tallier such that the votes are untraceable if at least one tallier is honest. Finally, the repeatedly shuffled votes are decrypted to recover all the votes. Even if one of the most efficient shuffling primitives with satisfactory security **[1115]20113]** is employed, in every shuffling operation for each vote several exponentiations are needed for re-encryption and several exponentiations are needed for public proof of validity of shuffling. So the shuffling operations in e-voting are costly, especially when the number of voters is large.

In this paper, multiplicative homomorphic tallying and shuffling are modified respectively so that they can be combined to exploit their advantages and avoid their disadvantages. On one hand, the vote space of multiplicative homomorphic tallying is enlarged to hold all the possible choices for a voter even in a complex election. For example, when there are several candidates and preferential election is used, there are several hundred possible choices for a voter. In this case, we can use the 10-bit primes and with a 1024-bit multiplicative modulus each group for factorization operation can contain around 100 votes. With this modification, each vote only contains one integer even if preferential election is used and only one decryption is needed in each group to recover the votes. On the other hand, to prevent the grouping mechanism with small group size from weakening privacy a modified shuffling mechanism is employed to shuffle the groups (instead of the votes). Any existing shuffling protocol supporting ElGamal encryption can be employed on the conditions that it is properly modified to suit the new application. Firstly, it only shuffles the groups. Secondly, as shown in Section 4.1. recovery of the shuffled messages in the end of the shuffling and analysis of soundness needs adjusting. As the number of groups is much smaller than the number of votes (e.g. the former may be 1% of the latter), the modified shuffling is very efficient. In the new e-voting scheme, vote validity check is still needed to guarantee that every vote is in the vote space such that the factorization operations can recover the votes. As the group size is not very small, vote validity check is costly if existing techniques are employed to implement it. To solve this problem, a new vote validity check technique is designed for the voters to efficiently prove validity of their votes.

The resulting e-voting scheme is called hybrid e-voting, which supports simple vote format, efficient vote sealing, complex elections and efficient distributed key generation, employs efficient vote validity check and only needs shuffling with a very small scale. So it is more efficient than existing e-voting schemes, especially in complex elections. A property sometimes desired in e-voting, receipt-freeness (or called coercion-resistance in some literature), is not the focus of this paper, so is not discussed in detail. Either of the two existing solutions to receipt-freeness, deniable encryption [4] and re-encryption with untransferable zero knowledge proof of correctness by a third party (in the form of a trusted authority or a tamper-resistent hardware) [18] can be employed to achieve it if needed.

### 2 Parameters and Denotions

The symbols and notations to be used in this paper are listed as follows and in Table [].

P	the prover in a proof protocol.			
V	the verifier in a proof protocol.			
$a \in_R S$	a is a integer randomly chosen from set $S$ .			
$\lfloor x \rfloor$	the largest integer no larger than $x$ .			
$\begin{bmatrix} x \end{bmatrix}$	the smallest integer no smaller than $x$ .			
E(), RE()  and  D()	encryption, re-encryption and decryption respectively.			
$P [x_1, x_2, \ldots, x_n \mid A(x_1, x_2, \ldots, x_n)]$	] the probability of event $A$ distributed over variants			
	$x_1, x_2, \ldots, x_n.$			

Table 1. Notations

For simplicity, it is supposed that when ElGamal encryption is employed it is as follows.

- Parameter setting

A security parameter K is chosen (e.g. set to be 1024) and a K-bit prime p is chosen such that p-1 = 2q and q is a large prime. G is the cyclic subgroup in  $Z_p$  with order q and g is a generator of G.

- Key setting

The private key is an integer x in  $Z_q$ . It is generated in a distributed way using the key generation technique in  $\square$  such that it is shared by multiple parties (e.g talliers in e-voting). x can be reconstructed only if the number of cooperating share holders is over a certain threshold. The public key is  $y = g^x \mod p$ .

- Encryption

A message M is encrypted into  $E(M) = (a, b) = (g^r \mod p, My^r \mod p)$ where r is randomly chosen from  $Z_q$ .

- Re-encryption A ciphertext u = (a, b) is re-encrypted into  $RE(u) = (a', b') = (ag^{r'} \mod p, bg^{r'} \mod p)$  where r' is randomly chosen from  $Z_q$ .
- Decryption When the the number of cooperating share holders is over the threshold, given a ciphertext u = (a, b), they cooperate to calculate  $D(u) = b/a^x \mod p$ .
- Product of ciphertexts  $u_1u_2 = (a_1a_2, b_1, b_2)$  where  $u_1 = (a_1, b_1)$  and  $u_2 = (a_2, b_2)$ .

## 3 Background

Important cryptographic primitives in existing e-voting schemes are recalled in this paper. In comparison with them, more advanced and efficient designs will be employed in this paper.

### 3.1 Shuffling in E-Voting

In a shuffling protocol, a shuffling node re-encrypts and reorders multiple input ciphertexts to some output ciphertexts such that the messages encrypted in the output ciphertexts are a permutation of the messages encrypted in the input ciphertexts. The shuffling node has to publicly prove validity of its shuffling. Shuffling is usually employed to build up anonymous communication channels and its most important application is e-voting. Suppose the input ciphertexts are  $u_1, u_2, \ldots, u_n$  are inputs to a shuffling node, who outputs ciphertexts  $u'_1, u'_2, \ldots, u'_n$  where  $u'_i = RE(u_{\pi(i)})$  and  $\pi()$  is a secret permutation of  $1, 2, \ldots, n$ . The shuffling node has to publicly prove that  $D(u'_1), D(u'_2), \ldots, D(u'_n)$  is a permutation of  $D(u_1), D(u_2), \ldots, D(u_n)$  without revealing  $\pi()$ .

Even in the most computationally efficient shuffling protocols [20,13], reencryption of each ciphertext costs several exponentiations and zero knowledge proof of validity of shuffling of n ciphertexts cost O(n) exponentiations. When n is large, it is a high cost, not to mention multiple instances of shuffling are needed in e-voting.

### 3.2 Vote Validity Check

Suppose the vote space is  $S = \{p_1, p_2, \ldots, p_m\}$  and a voter submits an ElGamal ciphertext  $u = (a, b) = (g^r \mod p, My^r \mod p)$  as his encrypted vote, he has to publicly prove that D(u) is in S without revealing it. The normal technique to implement this proof is a special zero knowledge proof primitive called "proofs of partial knowledge" **6** detailed in Figure **1**.

For simplicity, suppose  $D(u) = p_m$ . 1.  $P \rightarrow V$ :  $a_i = q^{w_i} a^{c_i} \mod p$  for  $i = 1, 2, \dots, m-1$  $b_i = y^{w_i} (b/p_i)^{c_i} \mod p \text{ for } i = 1, 2, \dots, m-1$  $a_m = g^s \mod p$  $b_m = y^s \mod p$ where  $c_i \in_R Z_q$  for  $i = 1, 2, \ldots, m-1$ ,  $w_i \in_R Z_q$  for  $i = 1, 2, \ldots, m-1$  and  $s \in_R Z_q$ . 2.  $V \rightarrow P$ :  $c \in_R Z_a$ 3.  $P \rightarrow V$ :  $c_1, c_2, \ldots, c_m, \qquad w_1, w_2, \ldots, w_m$ where  $w_m = s - c_m r \mod q$  and  $c_m = c - \sum_{i=1}^{m-1} c_i \mod q$ . Verification:  $a_i = g^{w_i} a^{c_i} \mod p \text{ for } i = 1, 2, \dots, m$  $b_i = y^{w_i} (b/p_i)^{c_i} \mod p \text{ for } i = 1, 2, \dots, m$  $c = \sum_{i=1}^{m} c_i \mod q$ 

Fig. 1. ZK proof and verification of a vote encrypted with ElGamal

This solution is highly inefficient. It costs O(m) full-length exponentiations. When it is applied to vote validity check in an e-voting with n voters, the cost for vote validity check is O(nm). In a large scale election with a large number of voters and a larger number of choices for them, it is a very high cost. Especially, it is too costly for the common people to verify validity of the votes and correctness of the election result.

## 4 The New Solution

The new solution is a hybrid technique, which modifies and combines multiplicative homomorphic tallying and shuffling to exploit their advantages and avoid their disadvantages. Moreover, an efficient vote validity check mechanism is proposed to improve efficiency.

### 4.1 Combination of Multiplicative Homomorphic Tallying and Shuffling

Multiplicative homomorphic tallying and shuffling are modified and combined to design a new e-voting scheme as follows.

1. Determining the vote space

According to the election rule, the number of possible choices for a voter is calculated. For example, in a preferential election with l candidates, there are l! possible choices. When l = 5 or 6 as in many practical preferential elections, there are several hundred possible choices. Suppose there are m possible choices in the election application, the vote space  $S = \{p_1, p_2, \ldots, p_m\}$  contains 1 and the m - 1 smallest odd primes, each standing for a choice.

2. Vote submission

Suppose there are *n* voters. Each voter chooses his vote  $v_i$  from *S* and submit  $u_i = (a_i, b_i) = (g^{r_i}, v_i y^{r_i})$  where  $r_i$  is randomly chosen from  $Z_q$ . Each voter proves validity of his vote using batch proof and verification.

3. Grouping the encrypted votes

Suppose  $p_1 = 1$  and  $p_j$  is the  $j - 1^{th}$  smallest odd primes in S. The size of each group is  $\alpha = \lfloor \log_{p_m} p \rfloor$ , so that the product of the votes in any group will not overflow p. The n encrypted votes are divided into  $\beta = \lceil n/\alpha \rceil$  groups, while  $u_i$  is in the  $(\lfloor i/\alpha \rfloor + 1)^{th}$  group.

4. Shuffling the groups

The  $\beta$  groups are denoted as  $G_1, G_2, \ldots, G_\beta$ . For  $j = 1, 2, \ldots, \beta$ , the talliers calculate  $U_j = (A_j, B_j) = (\prod_{u_i \in G_j} a_i, \prod_{u_i \in G_j} b_i)$ .  $U_1, U_2, \ldots, U_\beta$  are repeatedly shuffled by multiple talliers and the last shuffling operation outputs  $U'_1, U'_2, \ldots, U'_\beta$ . Any efficient shuffling protocol supporting ElGamal encryption (e.g. [14].13) can be employed to shuffle the groups as follows.

- (a) After a shuffling tallier receives ciphertexts  $W_1, W_2, \ldots, W_\beta$ , each representing a group shuffled by the last shuffling tallier, he re-encrypts and re-orders them and output ciphertexts  $W'_1, W'_2, \ldots, W'_\beta$  such that  $W'_j = RE(W_{\pi(j)})$  for  $j = 1, 2, \ldots, \beta$  where  $\pi()$  is a permutation of  $1, 2, \ldots, \beta$  chosen by the tallier.
- (b) After  $W'_1, W'_2, \ldots, W'_\beta$  are published, random  $\theta$ -bit challenges  $t_1, t_2, \ldots, t_\beta$  are chosen (e.g. by pseudo-random functions like hash function or by multiple parties with a threshold trust on them) where  $2^{\theta} \leq q$ .
- (c) The shuffling tallier proves that

$$\prod_{j=1}^{\beta} W_j^{t_j} = \prod_{j=1}^{\beta} W_j^{t_{\pi(j)}} \mod p \tag{1}$$

and the permutation  $\pi()$  is committed and unchanged using a zero knowledge protocol (see **13** for more details).

(d) A verifier verifies the proof of (1) and is convinced that the shuffling is valid if and only if (1) is proved to be satisfied. Unfortunately, in the existing shuffling protocols employing this idea (e.g. 1413) it is not formally explained why (1) guarantees that  $D(W'_1), D(W'_2), \ldots, D(W'_{\beta})$  is a permutation of  $D(W_1), D(W_2), \ldots, D(W_{\beta})$ . When ElGamal encryption is employed like in our application, (1) actually implies that

$$\prod_{j=1}^{\beta} D(W_j)^{t_j} = \prod_{j=1}^{\beta} D(W'_j)^{t_{\pi(j)}} \mod p.$$
(2)

Although according to the formal security analysis in  $D(W'_1), D(W'_2), \dots, D(W'_{\beta})$ 20mustbe  $\mathbf{a}$ permutation of  $D(W_1), D(W_2), \dots, D(W_\beta)$  when  $\sum_{j=1}^{\beta} D(W_j) t_j = \sum_{j=1}^{\beta} D(W'_j) t_{\pi(j)}$ is satisfied, it only covers shuffling employing additive homomorphic encryption algorithms like Paillier encryption. As multiplicative homomorphic tallying is exploited in the new e-voting scheme, multiplicative homomorphic encryption algorithms like ElGamal encryption must be employed and only (2) can be guaranteed by the shuffling protocol. To the best of our knowledge, there is no existing formal analysis (in 1413) or other schemes) to guarantee that (2) implies correctness of shuffling. Actually, satisfaction of (2) cannot guarantee that  $D(W'_1), D(W'_2), \ldots, D(W'_{\beta})$  is a permutation of  $D(W_1), D(W_2), \ldots, D(W_{\beta})$ . As formally illustrated in Theorem 1, satisfaction of (2) only guarantees that  $|D(W'_1)|, |D(W'_2)|, \ldots, |D(W'_{\beta})|$ is a permutation of  $D(W_1), D(W_2), \ldots, D(W_{\beta})$  where function  $| \ |$  is defined as follows.

**Definition 1.** For an integer x in  $Z_p$ , |x| = x if x is factorized into  $\alpha$  primes in the vote space where  $\alpha$  is the group size; otherwise, |x| = p - x.

5. Decryption and factorization

The talliers cooperate to decrypt  $U'_1, U'_2, \ldots, U'_{\beta}$ . For  $j = 1, 2, \ldots, \beta$ ,  $V_j = D(U'_j)$ . Each participating tallier proves that his part of decryption is correct using zero knowledge proof of equality of discrete logarithms [5]. Then  $V_1, V_2, \ldots, V_{\beta}$  are factorized as follows.

- (a) Factorize each group represented by  $V_j$  for  $j = 1, 2, ..., \beta$ . According to Theorem 1. either  $V_j$  or  $p - V_j$  should be factorized. Suppose  $\alpha$  is the group size. If  $V_j$  is factorized into  $\alpha$  primes in the vote space,  $V_j$ is the product of the votes in a shuffled group and the factorization is accepted. Otherwise, the product of the votes in a shuffled group should be recovered as  $p - V_j$ , which is then factorized. As p is an odd prime, it is impossible that both  $V_j$  and  $p - V_j$  are factorized into only the odd primes in the vote space, so the factorization function is decisional.
- (b) The number of votes choosing  $p_2$  is counted as the number of  $p_2$  found in the  $\beta$  factorizations; the number of votes choosing  $p_3$  is counted as the number of  $p_3$  found in the  $\beta$  factorizations; .....; the number of votes choosing  $p_m$  is counted as the number of  $p_m$  found in the  $\beta$  factorizations. The number of votes choosing  $p_1$  is counted as n minus the sum of other choices.

**Theorem 1.** Suppose  $x_1, x_2, \ldots, x_\beta$  are integers in  $Z_p^*$  and  $y_1, y_2, \ldots, y_\beta$  are primes in  $Z_p^*$  and  $t_1, t_2, \ldots, t_\beta$  are  $\theta$ -bit integers. If  $\prod_{j=1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\beta} y_j^{t_{\pi(j)}} \mod p$  with a probability larger than  $2^{-\theta}$  and  $\pi()$  is a committed and unchanged permutation of  $1, 2, \ldots, \beta$ , then there is a choice of implementation of [] for each  $[x_j]$  such that  $[x_1], [x_2], \ldots, [x_\beta]$  is a permutation of  $y_1, y_2, \ldots, y_\beta$  where  $[x_j] = x_j$  or  $p - x_j$ .

*Proof.* Given any integer  $\gamma$  in  $\{1, 2, \ldots, \beta\}$ , there must exist integers  $t_1, t_2, \ldots, t_{\gamma-1}, t_{\gamma+1}, \ldots, t_n$  in  $\{0, 1, \ldots, 2^{\theta} - 1\}$  and two different integers  $t_{\gamma}$  and  $\hat{t}_{\gamma}$  in  $\{0, 1, \ldots, 2^{\theta} - 1\}$  such that the following two equations are correct.

$$\prod_{j=1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\beta} y_j^{t_{\pi(j)}} \mod p \tag{3}$$

$$\prod_{j=1}^{\gamma-1} x_j^{t_j} + x_{\gamma}^{\hat{t}_{\gamma}} + \prod_{j=\gamma+1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\gamma-1} y_j^{t_{\pi(j)}} + y_{\gamma}^{\hat{t}_{\pi(\gamma)}} + \prod_{j=\gamma+1}^{\beta} y_j^{t_{\pi(j)}} \mod p$$
(4)

Otherwise, for any combination of  $t_1, t_2, \ldots, t_{\gamma-1}, t_{\gamma+1}, \ldots, t_{\beta}$  there is at most one  $t_{\gamma}$  to satisfy equation  $\prod_{j=1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\beta} y_j^{t_{\pi(j)}} \mod p$ . This deduction implies that among the  $2^{\beta\theta}$  possible combinations of  $t_1, t_2, \ldots, t_{\beta}$ , equation  $\prod_{j=1}^{\beta} x_j^{t_j} =$  $\prod_{j=1}^{\beta} y_j^{t_{\pi(j)}} \mod p$  is correct for at most  $2^{(\beta-1)\theta}$  combinations. This conclusion leads to a contradiction: given random integers  $t_j$  from  $\{0, 1, \ldots, 2^{\theta} - 1\}$  for j = $1, 2, \ldots, \beta$  and  $\pi()$ , a committed and unchanged permutation of  $j = 1, 2, \ldots, \beta$ , equation  $\prod_{j=1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\beta} y_j^{t_{\pi(j)}} \mod p$  is correct with a probability no larger than  $2^{-\beta}$ .

(B) and (A) respectively implies

$$\prod_{j=1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\beta} y_{\pi^{-1}(j)}^{t_j} \mod p \tag{5}$$

$$\prod_{j=1}^{\gamma-1} x_j^{t_j} + x_{\gamma}^{\hat{t}_{\gamma}} + \prod_{j=\gamma+1}^{\beta} x_j^{t_j} = \prod_{j=1}^{\gamma-1} y_{\pi^{-1}(j)}^{t_j} + y_{\pi^{-1}(\gamma)}^{\hat{t}_{\gamma}} + \prod_{j=\gamma+1}^{\beta} y_{\pi^{-1}(j)}^{t_j} \mod p$$
(6)

(5) divided by (6) yields

$$x_{\gamma}^{t_{\gamma}-\hat{t}_{\gamma}} = y_{\pi^{-1}(\gamma)}^{t_{\gamma}-\hat{t}_{\gamma}} \bmod p$$

Namely

$$(x_{\gamma}/y_{\pi^{-1}(\gamma)})^{t_{\gamma}-\hat{t}_{\gamma}} = 1 \bmod p$$

As  $x_{\gamma}$  and  $y_{\pi^{-1}(\gamma)}$  are in  $Z_p^*$ ,  $x_{\gamma}/y_{\pi^{-1}(\gamma)}$  is in  $Z_p^*$  as well. As  $t_1, t_2, \ldots, t_{\beta}$  are  $\theta$ -bit integers and  $2^{\theta} \leq q$ , the absolute value of  $t_{\gamma} - \hat{t}_{\gamma}$  is smaller than q. So, as p-1 has only two dividers, 2 and q,

$$x_{\gamma}/y_{\pi^{-1}(\gamma)} = \pm 1 \mod p$$

Namely,

$$x_{\gamma} = \pm y_{\pi^{-1}(\gamma)} \mod p$$

Note that  $\gamma$  can be any integer in  $\{1, 2, \ldots, \beta\}$ . Therefore, there is a choice of implementation of [] for each  $[x_j]$  such that  $[x_1], [x_2], \ldots, [x_\beta]$  is a permutation of  $y_1, y_2, \ldots, y_\beta$ .

Theorem [] guarantees that any valid vote can be correctly recovered, while the privacy functionality of the shuffling guarantees that no matter how small the group size is the grouping mechanism does not compromise privacy of the e-voting scheme.

## 5 Properties and Comparison

In the new e-voting scheme, both homomorphic tallying and shuffling are employed. However, both techniques are optimised. In homomorphic tallying, efficiency of vote validity check is improved. When L is 40, soundness is strong enough and only a small constant number of full length exponentiations are needed. Bellare *et al* [2] have illustrated that an exponentiation with a 40-bit exponent is much more efficient than a full length exponentiation. So a satisfactory trade-off between security and efficiency is achieved. Although shuffling is employed in the new e-voting scheme, it is in a much smaller scale than in the existing application of shuffling to e-voting. For example, in a normal case the number of possible choices is no more than several hundred in an election. In this case, the primes in the vote space are no larger than a couple of thousand and with a 1024-bit multiplicative modulus each group for factorization operation can contain around 100 votes. So usually the number of groups is much smaller than the number of votes (e.g. the former may be 1% of the latter). As the cost of shuffling operation (including re-encryption and proof or validity) is linear in the number of shuffled objects, the the modified shuffling mechanism is much more efficient than the existing shuffling protocols in e-voting.

Although a grouping mechanism is employed in the new e-voting scheme, it does not compromise privacy of the election as all the groups are shuffled before the votes in them are recovered through collective decryption and factorization. If at lease one of the shuffling talliers is honest, the groups are untraceable and no vote is known to be in any group. As semantically secure encryption is used and the employed shuffling protocol and the vote validity check mechanism achieve zero knowledge privacy, privacy of the new e-voting scheme is strong. The advantages of the new e-voting scheme over the existing e-voting schemes are demonstrated in Table 2. It is clearly illustrated that the new e-voting scheme is secure, rule-flexible and more efficient.

e-voting	election rule	distributed key	vote	vote validity	shuffling
		generation	sealing	check	
additive	simple rule	inefficient	inefficient	inefficient	no need
homomorphic	only				
multiplicative	simple rule	efficient	efficient	inefficient	no need
homomorphic	only				
shuffling	supporting	can be efficient	efficient	no need	inefficient
based	complex rule				
new hybrid	supporting	efficient	efficient	efficient	efficient
scheme	complex rule				

 Table 2. Comparison of e-voting schemes

# 6 Conclusion

The new e-voting scheme proposed in this paper is a hybrid solution combining multiplicative homomorphic tallying and shuffling. It exploits advantages of the two techniques and avoids their disadvantages. As a result, it has a lot of merits. It supports simple vote format, efficient vote sealing, complex elections and efficient distributed key generation. It employs efficient vote validity check and only needs shuffling with a very small scale. So it is more efficient than existing e-voting schemes, especially in complex elections.

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# A Framework for Authenticated Key Exchange in the Standard Model<sup>\*</sup>

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Abstract. We first introduce the new notion of the so-called targetindependent smooth projective hashing (TISPHash) based on computationally-hiding commitments. Based on it and a class of pseudo-random functions (PRFs), we propose a framework for (PKI-based) authenticated key exchange protocols without random oracles and prove it to be secure in the (currently) strongest security definition, the extended Canetti-Krawczyk security definition. Our protocol is actually an abstraction of the efficient key exchange protocol of T. Okamoto. The abstracted protocol enjoys efficient instantiations from any secure encryption scheme that admits an efficient construction of TISPHash and allows a simple and intuitive understanding of its security. In some sense, our construction generalizes the design of T. Okamoto.

Keywords: authenticated key exchange, Standard Model.

## 1 Introduction

A central problem in cryptography is that of enabling parties to communicate secretly and reliably in the presence of an adversary. This is often achieved by having the parties run a protocol for generating a mutual and secret session key (*authenticated key exchange*). During the last decades, the design of 2-party authenticated key exchange(AKE) has been explored intensively. Although some efficient two-pass AKE protocols such as HMQV[1], NAXOS and CMQV[2] were presented in the literature, they usually achieved provable security in the random oracle model. In this model, all parties are assumed to have oracle access to a totally random (universal) function [3]. The common interpretation of such results is that security is likely to hold even if the random oracle is replaced by a ("reasonable") concrete function known explicitly to all parties (e.g., SHA-2). However, it has been shown that it is impossible to replace the random oracle in a generic manner with any concrete function [4]. Thus, the proofs of security of these protocols are actually heuristic in nature. Therefore designing provably secure AKE without random oracles get much attention in the current research.

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Most recently, T. Okamoto presented a new paradigm to realize AKE without random oracles under three assumptions: the decisional Diffie-Hellman (DDH) assumption, target collision resistant (TCR) hash functions and a class of pseudorandom functions (PRFs),  $\pi$ PRFs (PRFs with pairwise-independent random sources ). They proposed a (PKI-based) AKE protocol that was comparably as efficient as the existing most efficient protocols like MQV [6] and that was secure without random oracles (under these assumptions). Moreover, their protocol achieved provable security in the (currently) strongest security definition, the extended Canetti-Krawczyk (eCK) security definition introduced by LaMacchia, Lauter and Mityagin [7].

In this paper, we first introduce the new notion of the so-called targetindependent smooth projective hashing(TISPHash) based on computationallyhiding commitments. Based on it and  $\pi$ PRF, we propose a framework for (PKI-based) AKE protocols without random oracles and prove it to be secure in the (currently) strongest security definition, the extended Canetti-Krawczyk (eCK) security definition. Our protocol is actually an abstraction of the key exchange protocol of T. Okamoto **5**. The abstracted protocol enjoys efficient instantiations from any secure encryption scheme that admits an efficient construction of TISPHash and allows a simple and intuitive understanding of its security. In some sense, our construction generalizes the design of **5**. Finally, to provide the reader with an idea of how efficient target-independent smooth projective hash functions are, we give an example of TISPHash based on the El-Gamal encryption scheme. A protocol that is almost identical to that of **5** can be derived when our protocol is instantiated with it.

## 2 Extended Canetti-Krawczyk Security Definition 5

This section outlines the extended Canetti-Krawczyk (eCK) security definition for two pass PKI-based AKE protocols that was introduced by LaMacchia, Lauter and Mityagin 7, and follows the description in 2.5.

In the eCK definition, we suppose there are n parties which are modeled as probabilistic polynomial-time Turing machines. We assume that some agreement on the common parameters in the AKE protocol has been made among the parties before starting the protocol. The mechanism by which these parameters are selected is out of scope of the AKE protocol and the (eCK) security model.

Each party has a static public-private key pair together with a certificate that binds the public key to that party.  $\hat{A}(\hat{B})$  denotes the static public key A(B) of party  $\mathcal{A}(\mathcal{B})$  together with a certificate. We do not assume that the certifying authority (CA) requires parties to prove possession of their static private keys, but we require that the CA verifies that the static public key of a party belongs to the domain of public keys.

Here, two parties exchange static public keys A, B and ephemeral public keys X, Y; the session key is obtained by combining A, B, X, Y and possibly session identities. A party A can be activated to execute an instance of the protocol called a session. Activation is made via an incoming message that has one of the

following forms:  $(\hat{A}, \hat{B})$  or  $(\hat{B}, \hat{A}, X)$ . If  $\mathcal{A}$  was activated with  $(\hat{A}, \hat{B})$ , then  $\mathcal{A}$  is called the session initiator, otherwise the session responder. Session initiator  $\mathcal{A}$  creates ephemeral public-private key pair, (X, x) and sends  $(\hat{B}, \hat{A}, X)$  to session responder  $\mathcal{B}$ .  $\mathcal{B}$  then creates ephemeral public-private key pair, (Y, y) and sends  $(\hat{A}, \hat{B}, X, Y)$  to  $\mathcal{A}$ .

The session of initiator  $\mathcal{A}$  with responder  $\mathcal{B}$  is identified via session identifier  $(\hat{A}, \hat{B}, X, Y)$ , where  $\mathcal{A}$  is said the owner of the session, and  $\mathcal{B}$  the peer of the session. The session of responder  $\mathcal{B}$  with initiator  $\mathcal{A}$  is identified as  $(\hat{B}, \hat{A}, Y, X)$ , where  $\mathcal{B}$  is the owner, and  $\mathcal{A}$  is the peer. Session  $(\hat{B}, \hat{A}, Y, X)$  is said a matching session of  $(\hat{A}, \hat{B}, X, Y)$ . We say that a session is completed if its owner computes a session key.

The adversary  $\mathcal{M}$  is modeled as a probabilistic polynomial-time Turing machine and controls all communications. Parties submit outgoing messages to the adversary, who makes decisions about their delivery. The adversary presents parties with incoming messages via Send(*message*), thereby controlling the activation of sessions. In order to capture possible leakage of private information, adversary  $\mathcal{M}$  is allowed the following queries:

- EphemeralKeyReveal(sid) The adversary obtains the ephemeral private key associated with session sid.
- SessionKeyReveal(sid) The adversary obtains the session key for session sid, provided that the session holds a session key.
- StaticKeyReveal(*pid*) The adversary learns the static private key of party *pid*.
- EstablishParty(*pid*) This query allows the adversary to register a static public key on behalf of party. In this way the adversary totally controls that party.

If party *pid* is established by EstablishParty(*pid*) query issued by adversary  $\mathcal{M}$ , then we call the party dishonest. If party is not dishonest, we call the party honest.

The aim of adversary  $\mathcal{M}$  is to distinguish a session key from a random key. Formally, the adversary is allowed to make a special query  $\text{Test}(sid^*)$ , where  $sid^*$  is called the target session. The adversary is then given with equal probability either the session key held by  $sid^*$  or a random key of equal length. The adversary wins the game if he guesses correctly whether the key is random or not. To define the game, we need the notion of fresh session as follows:

**Definition 1.** (fresh session) Let sid be the session identifier of a completed session, owned by an honest party  $\mathcal{A}$  with peer  $\mathcal{B}$ , who is also honest. Let sid be the session identifier of the matching session of sid, if it exists. Define session sid to be fresh if none of the following conditions hold:

- $-\mathcal{M}$  issues a SessionKeyReveal(sid) query or a SessionKeyReveal(sid) query (if sid exists),
- $\overline{sid}$  exists and  $\mathcal{M}$  makes either of the following queries: both StaticKeyReveal( $\mathcal{A}$ ) and EphemeralKeyReveal( $\overline{sid}$ ), or both StaticKeyReveal( $\mathcal{B}$ ) and EphemeralKeyReveal( $\overline{sid}$ ),

-  $\overline{sid}$  does not exist and  $\mathcal{M}$  makes either of the following queries: both StaticKeyReveal( $\mathcal{A}$ ) and EphemeralKeyReveal(sid), or StaticKeyReveal( $\mathcal{B}$ ).

We are now ready to present the eCK security notion.

**Definition 2.** (eCK security) Let  $sk^*$  be a session key of the target session sid<sup>\*</sup> that should be "fresh", and  $b^*$  is a bit chosen uniformly at random in  $\{0, 1\}$ . As a reply to Test(sid<sup>\*</sup>) query by  $\mathcal{M}$ ,  $sk^*$  is given to  $\mathcal{M}$  if  $b^* = 0$ ; a random key of the same size is given otherwise. Finally  $\mathcal{M}$  guesses the bit  $b^*$  and outputs this guess b. We define  $AdvAKE_{\mathcal{M}}(l) = 2 |Pr[b = b^*] - \frac{1}{2}|$ .

A key exchange protocol is secure if the following conditions hold:

- If two honest parties complete matching sessions, then they both compute the same session key (or both output indication of protocol failure).
- For any probabilistic polynomial-time adversary  $\mathcal{M}$ ,  $AdvAKE_{\mathcal{M}}(l)$  is negligible in l.

This security definition is stronger than CK-security  $\square$  and it simultaneously captures all the known desirable security properties for authenticated key exchange including resistance to key-compromise impersonation attacks, weak perfect forward secrecy, and resilience to the leakage of ephemeral private keys.

## 3 Decisional Diffie-Hellman Assumption-DDH

Let G be a large cyclic group of prime order p. We consider the following two distributions:

- Given a Diffie-Hellman quadruple  $g, g^x, g^y$  and  $g^{xy}$ , where  $x, y \in Z_p$ , are random strings chosen uniformly at random;
- Given a random quadruple g,  $g^x$ ,  $g^y$  and  $g^r$ , where  $x, y, r \in Z_p$ , are random strings chosen uniformly at random.

An algorithm that solves the Decisional Diffie-Hellman problem is a statistical test that can efficiently distinguish these two distributions. Decisional Diffie-Hellman assumption means that there is no such a polynomial statistical test.

## 4 Basic Tools

The design of our protocol mainly builds on three basic tools:  $\pi PRFs$ , computationally-hiding commitments and TISPHash, where the so-called TISPHash is first introduced by us and is a specific class of smooth projective hashing recently introduced by Cramer and Shoup [9]. Our usage of these tools is to a large extent inherited from [10]. In this section we review the main definitions and results necessary for the sequel.

#### 4.1 Computationally-Hiding Commitments

The first essential component of Gennaro and Lindell's construction 10 and of our proposal are non-interactive computationally-hiding commitment schemes 11. In this section we will describe such schemes, as well as a subtle issue that arises regarding these schemes.

Roughly speaking, they should fulfill the following requirement: A commitment c to a value m is of no help in gaining any information about m for any chosen-plaintext adversary of computationally-bounded resources (i. e., *Computationally-hiding* under CPA). In this paper, we simply call it computa- tionallyhiding.

Let C be a non-interactive computationally-hiding commitment scheme as above; such schemes are known to exist in the common reference string model[11]. Any public-key encryption scheme that is semantically secure under chosen plaintext attack can be used as a non-interactive computationally-hiding scheme in the common reference string model[11].

We denote by  $C_{\rho}(m; r)$  a commitment to m using random-coins r and common reference string  $\rho$ . Such a commitment scheme is the basis for our hard problem. Let  $C_{\rho}$  denote the space of all strings that may be output by the commitment scheme C when the CRS is  $\rho$ , and let M denote the message space. We note that actually,  $C_{\rho}$  and M must be supersets of these spaces that are efficiently recognizable; the actual definition of the supersets depends on the specific commitment scheme used. Next, define the following sets:

$$-X_{\rho} = C_{\rho} \times M.$$
  
-  $L_{\rho} = \{(c, m) | \exists r \text{(called a witness): } c = C_{\rho}(m; r) \}$ 

Furthermore, define the partitioning of  $X_{\rho}$  to be by index m (i.e., consider  $X_{\rho}(m) = C_{\rho} \times m$  and  $L_{\rho}(m) = \{(c,m) | \exists r : c = C_{\rho}(m;r)\}$  (i.e.,  $L_{\rho}(m)$  is the language of all commitments to m using  $\rho$ ). The distribution  $D(L_{\rho}(m))$  is defined by choosing a random r and outputting  $(C_{\rho}(m;r),m)$ . In contrast, the distribution  $D(X_{\rho}(m) \setminus L_{\rho}(m))$  is defined by choosing a random r and outputting  $(C_{\rho}(0^{|m|};r),m)$ . Clearly, by the hiding property of C, it holds that for every m, random elements chosen from  $D(L_{\rho}(m)) \setminus L_{\rho}(m)$ . This therefore constitutes a hard partitioned subset membership problem. We refer to  $\Pi$  for more information about it.

#### 4.2 Smooth Projective Hashing

We briefly recall Gennaro and Lindell's proposal for constructing smooth projective hash families 10, given a suitable commitment scheme as above C, and then introduce the new notion of TISPHash.

We are interested in a smooth projective hash function family defined with respect to  $(X_{\rho}, L_{\rho})$ . Loosely speaking a smooth projective hash function is a function with two keys. The first key maps the entire set  $X_{\rho}(m)$  to some set G. The second key (called the projection key) is such that it can be used to correctly compute the mapping of  $L_{\rho}$  to G. However, it gives no information about the mapping of  $X_{\rho} \setminus L_{\rho}$  to G. In fact, given the projection key, the distribution over the mapping of  $X_{\rho} \setminus L_{\rho}$  to G is statistically close to uniform (or "smooth"). We now present the formal definition. A family of smooth projective hash functions  $\mathcal{HASH} = (\alpha, SmthHash, ProjHash)$  associated with  $C_{\rho}$  consists of three algorithms. Let K be the key space, and k be a key chosen at random uniformly in K. We call k a hash key. Via  $s \leftarrow \alpha(k)$ , the key projection algorithm produces projected hash keys  $s \in S$  for a hash key k, where S is the space of projected hash keys. Via  $g \leftarrow SmthHash(k, c, m)$ , the hashing algorithm computes the hash value  $g \in G$  of (c, m) using the hash key k. Via  $g \leftarrow ProjHash(s, c, m, r)$ , the projected hash key s and a witness r (i.e.  $c = C_{\rho}(m, r)$ ). Formally, the above system defines a smooth projective hash system if the following conditions are satisfied:

- For all  $(c,m) \in L_{\rho}$  and  $k \in K$ , the hash values SmthHash(k,c,m) and  $ProjHash(\alpha(k),c,m,r)$  are the same(**Projective**);
- For all  $(c,m) \in X_{\rho} \setminus L_{\rho}$  and  $k \in K$ , the hash value g = SmthHash(k, c, m) is statistically close to uniform in G and independent of the values  $(\alpha(k), c, m)$ (Smooth).

The usual smooth projective hash function actually does not suffice for our application. Now, we define a new notion— target-independent smooth projective hashing (TISPHash). We say a smooth projective hash function family is target-independent if one more condition is satisfied:

- For  $(c,m) \leftarrow D(X_{\rho} \setminus L_{\rho})$  (where an appropriate witness r is not known), no ppt adversary can find a different pair of (c',m') such that the value SmthHash(k,c,m) is dependent on the value SmthHash(k,c',m') with non-negligible probability, given the projection  $\alpha(k)$ (Target-Independent).

The so-called target-independent smooth projective hashing is just a specific smooth projective hash function. Actually, the example given in [10] based on the El-Gamal encryption scheme is smooth projective hashing but not target-independent.

## 4.3 Pseudo-Random Function (PRF)

The concept of a pseudo-random function (PRF) is defined in **12** by Goldreich, Goldwasser and Micali.

Let  $l \in N$  be a security parameter. A pseudo-random function (PRF) family **F** associated with  $\{\Sigma_l\}_{l \in N}$ ,  $\{\mathcal{D}_l\}_{l \in N}$  and  $\{\mathcal{R}_l\}_{l \in N}$  specifies a family of pseudorandom functions indexed by l, where each such function  $F_{\sigma}^l$  takes a random seed drawn by  $\sigma$  uniformly distributed over  $\Sigma_l$  and then maps an element of  $\mathcal{D}_l$ to an element of  $\mathcal{R}_l$ .

Let  $\mathcal{M}$  be a probabilistic polynomial-time machine with oracle access to  $\mathcal{O}($  output an element in  $\mathcal{R}$  with an element in  $\mathcal{D}$ ). For all l, we define two experiments,  $\operatorname{Exp}_{l}^{F}(\mathcal{M})$  and  $\operatorname{Exp}_{l}^{R}(\mathcal{M})$ , where  $\mathcal{O}$  is answered with  $F_{\sigma}^{l}$  in the first

experiment while it is answered with a truly random function  $RF : \mathcal{D}_l \to \mathcal{R}_l$  in the other experiment. **F** is a pseudo-random function (PRF) family if there is no such probabilistic polynomial time adversary  $\mathcal{M}$  that can efficiently distinguish the above two experiments.

 $\pi$ PRFs, firstly introduced in **[5]**, is a specific class of PRFs with pairwiseindependent random sources. Let  $(\sigma_0, \sigma_1, \dots, \sigma_{t(l)})$  be random variables uniformly distributed over  $\Sigma_l$  and  $\sigma_0$  be pairwisely independent from other variable, where t(l) is a polynomial of l.

Let  $\mathcal{M}$  be a probabilistic polynomial-time machine with oracle access to  $(\mathcal{O}_0, \cdots, \mathcal{O}_{t(l)})$ . For all l, we define two experiments,  $\operatorname{Exp}_l^F(\mathcal{M})$  and  $\operatorname{Exp}_l^R(\mathcal{M})$ , where  $\mathcal{O}_j$  is answered respectively with  $F_{\sigma_j}^l$  for  $j = 1, \cdots, t(l)$  in the both experiments and the only difference is that  $\mathcal{O}_0$  is answered with  $F_{\sigma_0}^l$  in the first experiment while it is answered with a truly random function  $RF : \mathcal{D}_l \to \mathcal{R}_l$  in the second experiment. **F** is a  $\pi$ PRF family if there is no such probabilistic polynomialtime adversary  $\mathcal{M}$  that can efficiently distinguish the above two experiments.

### 5 The Proposed Framework for AKE

The idea of our framework is inspired in that of the Gennaro-Lindell protocol [10], which itself is an abstraction of the password-based key exchange protocol of Katz, Ostrovsky, and Yung [13], [14]. Our protocol is also an abstraction of the AKE protocol of T. Okamoto [5]. Our protocol builds on three basic tools: PRFs including  $\pi$ PRFs, non-interactive computationally-hiding commitment scheme C and target-independent smooth projective hash family  $\mathcal{HASH}$ . The last two can be implemented in the common reference string (CRS) model.

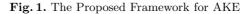
For a pictorial description of our protocol, please refer to Fig. 1. For the sake of readability, we do not explicitly refer to instances of users. The protocol description below omits many implementation details which are important for the proof of security to hold. Most important is for both parties to perform a "validity check" on the messages they receive. Let  $e \leftarrow E$  denote that e is uniformly selected from a set E. The formal description follows.

DESCRIPTION. Let  $C_{\rho}(m, r)$  denote a commitment to the message m using random coin tosses r and the CRS  $\rho$ , where m can be simply set to be 1 in our protocol. Let  $\mathcal{HASH} = (\alpha, SmthHash, ProjHash)$  be a family targetindependent smooth projective hash functions based on  $C_{\rho}$ . In particular, we assume the image of the hash functions SmthHash and ProjHash to be contained in a finite abelian group G with  $g_1$  as its generator. Recall that the key projection function  $\alpha$  is defined as a function of K. Let  $\hat{\mathbf{F}}$  and  $\tilde{\mathbf{F}}$  be PRF families and  $\mathbf{F}$  a family of  $\pi$ PRFs. The corresponding  $\Sigma$ ,  $\mathcal{D}$  and  $\mathcal{R}$  can be inferred based on the description below and thus are omitted here.  $\hat{F}$ ,  $\tilde{F}$  and F are the member functions of them respectively with respect to the security parameter l. Our protocol has a total of two rounds of communication and works as follows.

**Initialization.** Party  $\mathcal{A}$  chooses uniformly at random a value  $a \in K$  as its static private key, computes its static public key A via  $\alpha$  with a as input and

В  $\mathcal{A}$  $a \xleftarrow{U} K$  $b \xleftarrow{U} K$  $A \leftarrow \alpha(a)$  $B \leftarrow \alpha(b)$  $(x_1, x_2) \xleftarrow{U} \{0, 1\}^l \times \{0, 1\}^l$  $(x, x_3) \leftarrow \hat{F}_{x_1}(1^l) + \tilde{F}_a(x_2)$  $X \leftarrow C_{\rho}(m, x)$  $(\hat{B}, \hat{A}, X, X_3)$  $X_3 \leftarrow q_1^{x_3}$  $(y_1, y_2) \xleftarrow{U} \{0, 1\}^l \times \{0, 1\}^l$  $(y, y_3) \leftarrow \hat{F}_{y_1}(1^l) + \tilde{F}_b(y_2)$  $Y \leftarrow C_{\rho}(m, y)$  $\frac{(\hat{A},\hat{B},X,X_3,Y,Y_3)}{\sigma \leftarrow SmthHash(b,X) \times } Y_3 \leftarrow g_1^{y_3}$  $\sigma \leftarrow SmthHash(a, Y) \times$  $ProjHash(A, Y, y) \times X_3^{y_3}$  $ProjHash(B, X, x) \times Y_3^{x_3}$  $sk \leftarrow F_{\sigma}(sid)$  $sk \leftarrow F_{\sigma}(sid)$ 

Here,  $sid \leftarrow (\hat{A}, \hat{B}, X, X_3, Y, Y_3)$ , and (A, B) is confirmed indirectly through the certificates.



then register  $A = \alpha(a)$  to CA. Similarly, Party  $\mathcal{B}$  generates its static private key b and computes its static public key B and then register  $B = \alpha(b)$  to CA.

**Round 1.** Party  $\mathcal{A}$  selects an ephemeral private key  $(x_1, x_2) \stackrel{U}{\leftarrow} \{0, 1\}^l \times \{0, 1\}^l$ and computes  $(x, x_3) \leftarrow \hat{F}_{x_1}(1^l) + \tilde{F}_a(x_2), X \leftarrow C_\rho(m, x)$  and  $X_3 \leftarrow g_1^{x_3}$ . Then  $\mathcal{A}$  erase  $(x, x_3)$  and the whole computation history of the ephemeral public key and Send  $(\hat{B}, \hat{A}, X, X_3)$  to  $\mathcal{B}$ .

**Round 2.** Upon receiving the message from  $\mathcal{A}$ , Party  $\mathcal{B}$  selects an ephemeral private key  $(y_1, y_2) \xleftarrow{U} \{0, 1\}^l \times \{0, 1\}^l$  and computes  $(y, y_3) \leftarrow \hat{F}_{y_1}(1^l) + \tilde{F}_b(y_2)$ ,  $X \leftarrow C_\rho(m, y)$  and  $Y_3 \leftarrow g_1^{y_3}$ . Then  $\mathcal{B}$  erases  $(y, y_3)$  and the whole computation history of the ephemeral public key and Send  $(\hat{A}, \hat{B}, X, X_3, Y, Y_3)$  to  $\mathcal{A}$ .

**Finalization.** To compute the session key,  $\mathcal{A}(\mathcal{B})$  computes  $\sigma \leftarrow SmthHash(a, Y) \times ProjHash(B, X, x) \times Y_3^{x_3}$  (resp.  $\sigma \leftarrow SmthHash(b, X) \times ProjHash(A, Y, y) \times X_3^{y_3}$ ) and derives the session key  $sk \leftarrow F_{\sigma}(sid)$ , where  $sid = (\hat{A}, \hat{B}, X, X_3, Y, Y_3)$ .

CORRECTNESS. The correctness follows from the fact that, in an honest execution of the protocol,  $Y_3^{x_3} = X_3^{y_3}$ , SmthHash(a, Y) = ProjHash(A, Y, y), SmthHash(b, X) = ProjHash(B, X, x). It is easy to verify that both parties in the protocol will terminate by accepting and computing the same values for  $\sigma$ , sid, and  $sk = F_{\sigma}(sid)$ .

RATIONALE. One might wonder why the two parties have to construct  $X_3$ and  $Y_3$  and include them in the derivation function of  $\sigma$ . This is motivated by the following passive attack on the protocol deriving the value of  $\sigma$  simply from  $SmthHash(a, Y) \times SmthHash(b, X)$ , that allows to break the security of the protocol: The adversary eavesdrops an honest execution of the protocol between  $\mathcal{A}$  and  $\mathcal{B}$ , where a pair of matchable sessions (sid, sid) are established. When the execution is completed, the adversary queries StaticKeyReveal on the two participants, giving him the static private keys a and b, computes  $\sigma$  and thus easily learns the session key sk. Note, both the session sid and its matching session sid are still fresh since the adversary never queries EphemeralKeyReveal on either sid or sid. Therefore, the adversary is allowed to Test on sid or sid. Through this attack, the adversary could correctly guesses the bit b involved in the Test query with probability 1 and thus the protocol would not fulfill the security in Definition 2.

One might also wonder why the ephemeral private keys can not be immediately used to compute the ephemeral public key, e.g.,  $X \leftarrow C_{\rho}(m, x_1)$  and  $X_3 \leftarrow g_1^{x_2}$ . Instead they are derived from another pair of value  $(x, x_3)$  output by  $\hat{F}$  and  $\tilde{F}$ . Note that the value of  $(x, x_3)$  can only be computed in a computation process with the knowledge of ephemeral and static private keys. Reconsider the above attack where the adversary queries EphemeralKeyReveal on the two sessions *sid* and *sid* but never queries StaticKeyReveal on any of the two participants instead. He can break the security of the protocol if the ephemeral private keys are immediately used to compute the ephemeral public key in the protocol.

SECURITY. The intuition behind the security of our protocol is quite simple. We assume  $sid^*$  is the target session chosen by adversary. Let us remember the conditions of a fresh session (i.e., restrictions on  $sid^*$ ). If the adversary mounts a passive attack on  $sid^*$  in which he eavesdrops on the honest execution among the participants, i.e., there exists a matching session,  $\overline{sid^*}$  of target session  $sid^*$ ,  $\sigma^*$  involved in  $sid^*$  would be uniform and independent for him due to the hardness of the DDH problem in G. If the adversary mounts an active attacks on  $sid^*$  in which he interacts with an honest party to establish session  $sid^*$ , i.e., there exists no matching session of target session  $sid^*$ ,  $\sigma^*$  involved in  $sid^*$  would be pairwisely independent from any other  $\sigma$  involved in whichever session (not equivalent to session  $sid^*$ ) the attacker can establish with an honest party due to the target-independence of the smooth projective hashing from computationally-hiding commitment in use. As long as the security notion of PRFs and  $\pi$ PRFs in use is met, an honest party is able to implicitly authenticate his partner and safely share a session key with him. As the following theorem shows, the protocol described in Fig. 1 is a secure authenticated key exchange protocol without random oracles.

**Theorem 1.** Let  $\hat{\mathbf{F}}$  and  $\tilde{\mathbf{F}}$  be PRF families and  $\mathbf{F}$  a family of  $\pi$ PRFs, let C be a non-interactive computationally-hiding commitment scheme and  $\mathcal{HASH}$  be a family of target-independent smooth projective hash functions associated with C. Then, the proposed AKE protocol is secure (in the sense of Definition 2) if the DDH assumption holds over G.

Due to the limitation of the paper length, the complete proof is omitted here.

Note the commitment scheme in our framework is just a basis for the hard problem upon which we can build a family of target-independent smooth projective hash functions  $\mathcal{HASH}$ . Therefore, non-malleability is not needed in our

construction. As a result, a public-key encryption scheme that is semantically secure under chosen plaintext attack can be used to instantiate the commitment scheme and m can be simply set to be 1 in our protocol. The intuition behind that is quite simple. Let us assume the adversary impersonates  $\mathcal{A}$  to interact with  $\mathcal{B}$ . He sends a correct commitment X with a known x and thus knows ProjHash(B, X, x) but he can never knows SmthHash(a, Y), where a is  $\mathcal{A}$ 's static private key(the corresponding public key is A, which is included in  $\mathcal{A}$ 's certificate, and its validness can be checked in a usual way in PKI-based system) and Y is honestly generated by  $\mathcal{B}$ . So he can gain no information of the session key. This is true even when the adversary replays the message Y in another session since  $\mathcal{HASH}$  in use is a family of target-independent smooth projective hash functions.

## 6 An Example for Efficient Constructions

To provide the reader with an idea of how efficient target-independent smooth projective hash functions are, we give an example based on the El-Gamal encryption scheme 10. Note a non-interactive computationally-hiding commitment scheme can be constructed from the El-Gamal encryption encryption scheme. Our example is a little similar to that given in **5** based on the El-Gamal encryption scheme but the latter is not target-independent.

The El-Gamal scheme works as follows. Let G be a cyclic group of prime order p where p is large, and let  $g_1$  be a generator  $g_1$  of G. The key generation algorithm chooses a random  $z \in Z_p$  with  $z \neq 0$ . The secret-key is then defined to be z and the public-key is defined to be  $g_2 = g_1^z$ . To encrypt  $m \in G$ , a random  $x \in Z_p$  is chosen and the ciphertext is defined to be  $X = (X_1, X_2) = (g_1^x, g_2^x \cdot m)$ . Upon input X, decryption is carried out by computing  $m = X_2/X_1^z$ . It is well known that under the Decisional Diffie-Hellman Assumption over G, the El-Gamal scheme is semantically secure against chosen-plaintext attacks.

The smooth projective hashing for the El-Gamal encryption scheme is then defined as follows. The key space is defined by  $K = Z_p^4$  (i.e., a key is a tuple  $a = (a_1, a_2, a_3, a_4)$ , with  $a_i \in Z_p$ ). The key projection function  $\alpha$  is defined by  $\alpha(a) = A = (A_1, A_2) = (g_1^{a_1}g_2^{a_2}, g_1^{a_3}g_2^{a_4})$ . The hash function SmthHash on input (a, X, m) outputs  $\sigma = X_1^{a_1+ca_3}(X_2/m)^{a_2+ca_4}$ , and the projective hash function ProjHash on input (A, X, m, x) simply outputs  $\sigma = A_1^x A_2^{cx}$ , where c = H(X) and H is a TCR hash function. Projectiveness follows from the fact that SmthHash(a, X, m) = ProjHash(A, X, m, x) holds for all  $X \in L$ . We now prove the smoothness property. Consider  $X = (X_1, X_2) = (g_1^x, g_2^{x'} \cdot m) \neq L$ . Then this implies that  $x \neq x'$ . Then  $(A, \sigma)$  are expressed by the following three equations:

$$\log_{q_1} A_1 = a_1 + z a_2 \mod p \tag{1}$$

$$\log_{g_1} A_2 = a_3 + za_4 \mod p \tag{2}$$

$$\log_{g_1}\sigma = x(a_1 + ca_3) + zx'(a_2 + ca_4) \mod p$$
(3)

Since  $\log_{g_1} \sigma$  is linearly independent from  $\log_{g_1} A_1$  and  $\log_{g_1} A_2$  when  $x \neq x'$ , i.e.,

for every choice of A and  $\sigma$ , there exists a tuple  $a = (a_1, a_2, a_3, a_4)$  that fulfills the above equations,  $\sigma$  is uniformly distributed over G, given A. We conclude that the projective hash function is smooth.

It now remains to prove the target-independence property. Consider any such  $Y = (Y_1, Y_2) = (g_1^y, g_2^{y'} \cdot m) \neq X$ . Then  $\delta = SmthHash(a, Y, m) = Y_1^{a_1+da_3}(\frac{Y_2}{m})^{a_2+da_4}$  is expressed by the following equation:

$$\log_{g_1} \delta = y(a_1 + ca_3) + zy'(a_2 + da_4) \mod p \tag{4}$$

where d = H(Y). If  $Y \in L$ , i.e. y = y', the value of  $\sigma$  is (information theoretically) independent from  $\delta$  since, given  $\delta$  and A,  $\sigma$  is still uniformly distributed in G if  $a_2, a_4$  is chosen uniformly at random in  $Z_p^2$ . Then, we consider the case that  $Y \notin L$ , i.e.  $y \neq y'$ . We hereafter assume  $c \neq d \mod p$ . We can safely do so because H is a TCR hash function. We can show the value of  $\sigma$  is (information theoretically) independent from  $\delta$  since  $z^2(x - x')(y - y')(c - d) \neq 0 \mod p$  and thus the following matrix:

$$\begin{bmatrix} 1 & z & 0 & 0 \\ 0 & 0 & 1 & z \\ x & zx' & cx & zcx' \\ y & zy' & dy & zdy' \end{bmatrix}$$

is regular. Actually, for every choice of A,  $\sigma$  and  $\delta$ , there exists a tuple  $a = (a_1, a_2, a_3, a_4)$  that fulfills the above equations (1)-(4). Therefore,  $\sigma$  is uniformly distributed over G, given A and  $\delta$ . We conclude that the projective hash function is target-independent.

We remark that a protocol that is almost identical to that of **5** can be derived when our protocol is instantiated with the El-Gamal encryption scheme based on the DDH assumption, combined with the above construction of a targetindependent smooth projective hash function (which is thus implicit in the work of **5**). As it is the case with Gennaro and Lindell's construction **10**, our protocol enjoys efficient instantiations based on the decisional Diffie-Hellman, quadratic residuosity, and N-residuosity assumptions (see **10**).

## 7 Conclusion

We have proposed a framework for (PKI-based) authenticated key exchange protocols without random oracles and proved it to be secure in the (currently) strongest security definition. An example of efficient instantiations of our protocol also is given and and the resulting protocol is almost identical to that of T. Okamoto, which is quite efficient **5**. Our protocol is actually an abstraction of the key exchange protocol of T. Okamoto. The abstracted protocol not only inherits all its attractive features(including in efficiency) but also enjoys efficient instantiations from any secure encryption scheme that admits an efficient construction of TISPHash and allows a simple and intuitive understanding of its security. In some sense, our construction generalizes the design of T. Okamoto.

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# Secret Handshake: Strong Anonymity Definition and Construction

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Abstract. Secret handshake allows two members in the same group to authenticate each other secretly. In previous works of secret handshake schemes, two types of anonymities against the group authority (GA) of a group G are discussed: 1) Even GA cannot identify members, namely nobody can identify them (No-Traceability), 2) Only GA can identify members (Traceability). In this paper, first the necessity of tracing of the identification is shown. Second, we classify abilities of GA into the ability of identifying players and that of issuing the certificate to members. We introduce two anonymities Co-Traceability and Strong Detector Resistance. When a more strict anonymity is required ever for GA, the case 2) is unfavorable for members. Then, we introduce Co-Traceability where even if  $\mathcal{A}$  has GA's ability of identifying members or issuing the certificate,  $\mathcal{A}$  cannot trace members identification. However, if a scheme satisfies Co-Traceability, GA may be able to judge whether handshake players belong to the own group. Then, we introduce Strong Detector Resistance where even if an adversary  $\mathcal{A}$  has GA's ability of identifying members,  $\mathcal{A}$  cannot make judgments whether a handshaking player belongs to G. Additionally, we propose a secret handshake scheme which satisfies previous security requirements and our proposed anonymity requirements by using group signature scheme with message recovery.

Keywords: Secret handshake, anonymity, traceability, privacy.

# 1 Introduction

#### 1.1 Background

A secret handshake scheme (SHS), introduced by Balfanz et al. [3], allows two members of the same group to authenticate each other secretly, in the sense that each party reveals his affiliation information to the other only if the other party belongs to the same group. For example scenario: a CIA agent Alice wants to authenticate to Bob, but only if Bob is also a CIA agent. In addition, if Bob is not a CIA agent, Alice does not want to reveal her affiliation information, whether Alice is a CIA agent or not, for Bob.

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Balfanz, et al. 3 constructed a 2-party SHS by adapting the key agreement protocol of Sakai, et al. 10. Its security rests on the hardness of the Bilinear Diffie Hellman (BDH) problem. Subsequently, Castelluccia, et al. 6 developed a more efficient 2-party SHS through the use of so-called CA-oblivious encryption. It is secure under Computational Diffie Hellman (CDH) assumption.

Both previous works satisfy the basic security properties of secret handshake system; correctness, impersonator resistance, and detector resistance. Recently, the unlinkability is added to the basic security requirement. Unlinkability means that two occurrences of handshaking by the same party cannot be linked with each other by anyone. In [3,6], however, the member sends one's ID information in a handshake protocol. Thus, the construction of [3,6] does not satisfy unlinkability unless members use one-time certificate (i.e. member change IDs whenever members execute handshake protocol).

Xu and Yung  $\square$  constructed a secret handshake scheme which achieves weaker unlinkability "k-anonymity" with reusable certificate, instead of one-time certificates. Members can reuse their certificate because they always authenticate as someone among k users.

The work of  $\square$  presented the first construction of SHS with unlinkability using reusable certificate in the standard model. The scheme in  $\square$  allows each participant to specify the role and group of the other party and thus add flexibility to the authentication rules. Moreover, they achieve attribute-based secret handshakes using fuzzy identity-based-encryption.

#### 1.2 Anonymity against Group Authority

*Motivation:* Let us consider the case in which whistle-blowing the system of company is troublesome. In this case the GA is a manager of company and the members are employees of the company and the authorized persons of the company (e.g. lawyer). When the employee uses this system, he executes a secret handshake with the lawyer in order to pass on the whistle-blowing. The employee wants to tell to only his company's lawyer about exposure. The lawyer should only know the fact that the employee belongs to his company. In above scenario, a secret handshake scheme is very useful.

However, if previous secret handshake schemes are used as above whistleblowing system, there is a problem. In previous secret handshake schemes, GA can identify handshake players (player's names, IDs, etc). In this scenario, the employees, who want to exposure, want to execute a secret handshake to remain anonymous. If an employee is not guaranteed anonymity, he will not blow the whistle in the fear of the dismissal etc. However, for example in [6], a handshake player must send own ID to a handshake partner. IDs are registered to GA, when a member joins to the group. So, GA can identify handshake players by referring ID list and watching the transcript of handshake.

From the above reason, the previous schemes are not applicable to this case. So in this paper, we will define new security requirements about anonymities against GA. We introduce two new anonymities *Co-Traceability* and *Strong Detector*  Resistance. Strong Detector Resistance means that even if an adversary  $\mathcal{A}$ , who does not belong to G, has GA's ability of identifying members,  $\mathcal{A}$  cannot make judgments whether a handshaking player belongs to G. Co-Traceability means that GA alone cannot reveal members identification.

Co-Traceability can never be satisfied against GA with all of abilities, since if U belongs to G, U might be revealed identification executing a handshake protocol with a dummy member D whom GA creates using an ability to issue a user. Then, GA can identify U by using the information of D and the ability of identifying members. Also, Strong Detector Resistance can never be satisfied against GA with all of the abilities. The reason is that if U belongs to G, Uoutputs accept certainly executing a handshake protocol with a dummy member D whom GA creates using an ability to issue a user.

Therefore, in order to discuss the anonymity against GA, we split the role of GA into the two; issue authority and trace authority. The issue authority (IA) issues the certificate to users. The trace authority (TA) has an ability to identify a member. Co-Traceability is defined against an adversary who has an ability of either TA or IA. Strong Detector Resistance is defined against an adversary who has an ability of TA.

*Co-Traceability:* If a secret handshake scheme adopt whistle-blowing, handshake players do not want to reveal own identification against even GA. However, tracing of handshake players is useful for the handshake player in case the evidence of handshaking is required.

So, we introduce new security requirement Co-Traceability. Intuitively, Co-Traceability means that TA alone cannot identify handshake players, but TA can identify players by cooperating with another. In our proposed secret handshake scheme, another is a handshake player. If Alice and Bob execute a secret handshake, TA alone cannot identify Alice and Bob, but if TA cooperates with Alice, TA could identify Bob.

At the same time, in this paper, we will define this algorithm "SHS.Co-Trace". This new algorithm is useful in the situation which a handshake player wants to know the other handshake partner when the output of the protocol is *accept*. In previous secret handshake schemes, identifications of handshake players always reveal against everyone. By using SHS.Co-Trace, a handshake player can execute a handshake protocol hiding own identification and can know the identification of the other handshake player by revealing own identification to TA.

Strong Detector Resistance: In previous works, in Secret Handshake system, information about the group of the player must not be leaked, if players does not belong to the same group. There are the cases that a player wants to hide the information of his own group from TA as well as members and users. If TA can know whether a handshake player belongs to his own group G or not, GA might examine the frequency that a member of group G executes a handshake protocol. This consideration leads to the new security property, Strong Detector Resistance.

Previous Security	Issuer Authority	Trace Authority
Impersonator Resistance	Uncorrupted Uncorrupt	
Detector Resistance	Uncorrupted	Uncorrupted
Unlinkability	Uncorrupted	Uncorrupted
Proposed Security	Issuer Authority	Trace Authority
Co-Traceability (against TA)	Uncorrupted	Corrupted
Co-Traceability (against IA)	Corrupted	Uncorrupted
Strong Detector Resistance	Uncorrupted	Corrupted

Table 1. Levels of trust in authorities for each of security requirements

In this paper, we discuss the anonymity and groups which members belong to in the case TA or IA is corrupted. Here, "TA (IA) is corrupted" means that an adversary  $\mathcal{A}$  can get a secret key of TA (IA). The levels of trust for each authority for each requirement are summarized in Table  $\square$ 

In order to realize *Strong Detector Resistance* and *Co-Traceability*, we introduce a new algorithm SHS.Co-Trace. SHS.Co-Trace, given a group public key, a group authority secret key, and "an internal information of a player", outputs ID of the other player. Concretely, we realize SHS.Co-Trace and *Co-Traceability* with a secret key to execute SHS.Co-Trace separately by GA and a handshake player.

# 2 Definition of Secret Handshake

#### 2.1 Entity

In SHS, there exist three entities in the group G as follows.

- **User:** the entity which does not belong to the group.  $A \notin G$  means that the user A does not belong to the group G.
- **Member:** the entity which is made belong to the group, by the Group Authority.  $A \in G$  means that the member A belongs to the group G.
- **GA (Group Authority):** the manager of a group. GA is responsible for adding users into the group he manages. GA maintains a list  $\mathcal{L}$ , which includes certificates and registration data of all members.

#### 2.2 The Model of Secret Handshake

A secret handshake scheme SHS consists of the following five algorithms:

- SHS.Setup: generates the public parameters *param* which is common to all groups.
- SHS.CreateGroup: generates a key pair *gpk* (group public key) and *gsk* (private key for GA), using *param*. SHS.CreateGroup is run by GA.

- SHS.AddMember: is executed between a player U and a GA of some group. Inputs of SHS.AddMember are gsk, param, and gpk, and outputs are a membership certificate  $(cert_U)$  and a secret key  $(sk_U)$
- SHS.Handshake: is the authentication protocol executed between U and V, based on the public input param. The group public keys  $(gpk_U, gpk_V)$  and certificates  $((cert_U, sk_U), (cert_V, sk_V))$  of U and V are input to handshake protocol. The result of the algorithm is either reject or accept.  $U \stackrel{Handshake}{\longleftrightarrow} V$ means the above situation.
- SHS.Trace: , given gpk, gsk and a transcript  $\mathcal{T}$  of the handshaking of U and V, outputs the member U (or V).

## 2.3 Security Properties of SHS Scheme

A secret handshake protocol must satisfy the following *basic* security properties: Correctness, Impersonator Resistance, Detector Resistance, Unlinkability 6.

- **Correctness:** If honest members U, V of the same group run handshake protocol, then both players always output "*accept*".
- **Impersonator Resistance (IR):** IR demands that the adversary  $\mathcal{A}$ , who does not belong to a group G, is not able to authenticate an honest who belongs to G.
- **Detector Resistance (DR):** DR demands that the adversary  $\mathcal{A}$ , who does not belong to a group G, is not able to distinguish whether some honest is a member of some group G.
- **Unlinkability** (Unlink): Unlink demands that the adversary  $\mathcal{A}$ , who does not belong to a group G, is not able to decried whether two executions of the handshake protocol were performed by the same party or not, even if both of them were successful.

# 3 Anonymity against Group Authority

In previous works of secret handshake schemes, anonymities against GA have not been discussed. In this section we will define two new anonymity requirements against GA, *Co-Traceability* and *Strong Detector Resistance* of secret handshake schemes.

## 3.1 Issue Authority and Trace Authority

In this subsection, let us classify the GA from the view point of abilities. GA has the two abilities. First ability is to issue the certificate to users. Second ability is to trace handshake players. We call the authority with the former ability *Issue authority* (IA). and the authority with the latter ability *Trace authority* (TA). IA has a public and secret key (ipk, isk) and TA has a public and secret key (tpk, tsk)

<sup>&</sup>lt;sup>1</sup> In previous definition, gpk = (ipk, tpk), gsk = (isk, tsk).

### 3.2 Co-traceability

In previous works of secret handshake schemes, two types of anonymities against the GA are discussed from the view point of revealing the identification of handshake players: 1)Even GA cannot identify players, namely nobody can identify them, 2)Only GA can identify players.

Tracing of handshake players is useful for the handshake player in case the evidence of handshaking is required. However when the anonymity is preferred as the case of prosecution from inside, a more strict anonymity is required ever for GA.

In the way of SHS.Trace, members except TA cannot join SHS.Trace process. From the view point of power of TA, this situation could be troublesome for members. Then, we introduce a new security requirement *Co-Traceability*. Intuitively, *Co-traceability* means that TA alone cannot identify handshake players.

Co-Traceability can never be satisfied against GA with all abilities, since if U belongs to G, U might be revealed identification executing a handshake protocol with a dummy member D who GA creates using an ability to issue a user. So, two different types of adversarys,  $\mathcal{A}_T$  and  $\mathcal{A}_I$ , should be concerned. The difference between  $\mathcal{A}_T$  and  $\mathcal{A}_I$  is the input. The input of  $\mathcal{A}_T$  is *param*, *gpk* and *tsk*. The input of  $\mathcal{A}_I$  is *param*, *gpk* and *isk*. The formal definition of Co-traceability is as follow:

**Definition 1 (Co-Traceability).** Co-Traceability (Co-Trace) means that an adversary  $\mathcal{A}$  who has all GA's secret key (isk, tsk) cannot identify the player which execute SHS.Handshake even though  $\mathcal{A}$  is not a member of G. Formally, we say that SHS guarantees Co-Trace if the following function  $\operatorname{Adv}_{\mathcal{A}}^{\operatorname{Co-Trace}}(k)$  is negligible. Let a member which executes according to the protocol be honest. We consider a PPT adversary  $\mathcal{A}$  that can access to the following oracles  $\mathcal{O}$ .

Formally, we say that SHS guarantees Co-Trace if the following function  $\operatorname{Adv}_{A}^{\operatorname{Co-Trace}}(k)$  is negligible for any PPT adversary  $\mathcal{A} = (\mathcal{A}_T, \mathcal{A}_I)$ .

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{Co-Trace}}(k) = \Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{Co-Trace}}(k) = 1]$$

Experiment  $\mathbf{Exp}_{\mathcal{A}}^{\mathsf{Co-Trace}}(k)$   $param \leftarrow \mathsf{SHS}.\mathsf{Setup}(k)$   $(ipk, tpk, isk, tsk) \leftarrow \mathsf{SHS}.\mathsf{CreateGroup}(param);$   $(cert_U, sk_U) \leftarrow \mathsf{SHS}.\mathsf{AddMember}(param, ipk, tpk, isk, U)$ honest $(param, gpk, sk_U, cert_U) \xrightarrow{Handshake} \mathcal{A}^{\mathcal{O}}$   $U' \leftarrow \mathcal{A}^{\mathcal{O}}$ If U = U', outputs 1. Otherwise outputs 0.

#### 3.3 Strong Detector Resistance

If a secret handshake scheme satisfies Co-Traceability, TA alone cannot identify handshake players. However, GA may be able to identify whether handshake

players belong to the own group. Then, GA can know the frequency that members of own group execute handshake. In order to achieve high anonymity against GA, even the frequency of handshake should be hidden for GA.

We then introduce the concept of *Strong Detector Resistance* in order to cope with this requirement. Intuitively, Strong Detector Resistance means that even if an adversary  $\mathcal{A} \notin G$  has all the ability of revealing the identity of members,  $\mathcal{A}$  cannot make judgment whether a handshaking player belongs to G or not.

Strong Detector Resistance never satisfies against IA, since if U belongs to G, U outputs *accept* executing a handshake protocol with a dummy member who GA creates using an ability to issue a user. The formal definition of Strong Detector Resistance is as follow:

**Definition 2 (Strong Detector Resistance).** Strong Detector Resistance (SDR) demand that an adversary  $\mathcal{A}$  who have "the all ability of revealing the identity of members" (e.g. trace key, member-list  $\mathcal{L}$ ) cannot distinguish whether some honest player, who is a member of group G, even though  $\mathcal{A}$  is not a member of G. SDR is easier security goal than SDR for an adversary  $\mathcal{A}$ .

Formally, we say that SHS guarantees SDR if the following function  $\operatorname{Adv}_{\mathcal{A}}^{\operatorname{SDR}}(k)$  is negligible for any polynomially-bounded adversary  $\mathcal{A}$ .

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{SDR}}(k) = \left| \Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{SDR}}(1,k) = 1] - \Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{SDR}}(0,k) = 1] \right|$$

SDR has a close relationship to the SHS.Trace algorithm. If GA can reveal an identify ID of a member [6,3], an adversary can break SDR by executing SHS.Trace. On the other hand, if SHS does not have algorithm revealing an identify [1], SDR is equivalent to DR.

Experiment  $\mathbf{Exp}_{\mathcal{A}}^{\mathsf{SDR}}(b, k)$   $param \leftarrow \mathsf{SHS}.\mathsf{Setup}(k)$   $(ipk, tpk, isk, tsk) \leftarrow \mathsf{SHS}.\mathsf{CreateGroup}(param);$   $(cert_U, sk_U) \leftarrow \mathsf{SHS}.\mathsf{AddMember}(param, ipk, tpk, isk, U)$   $\mathbf{player}_b := \mathsf{honest}(param, gpk, sk_U, cert_U);$   $\mathbf{player}_{1-b} := \mathsf{SIM}(param)$   $\mathbf{player}_0 \xrightarrow{Handshake} \mathcal{A}^{\mathcal{O}}(param, gpk, tsk)$   $\mathbf{player}_1 \xrightarrow{Handshake} \mathcal{A}^{\mathcal{O}}(param, gpk, tsk)$   $b' \leftarrow \mathcal{A}^{\mathcal{O}}(param, qpk, tsk)$ Return b'

## 4 Proposed Scheme

#### 4.1 Group Signature with Message Recovery (GSMR) 12

Our proposed SHS, including SHS.Co-Trace algorithm and satisfied Co-Traceability and Strong Detector Resistance, is based on the SHS using GSMR (Group Signature with Message Recovery) [12]. A group signature, first introduce by Chaum and van Heyst [7] and followed by [2]8], allows a member which belong to a group to sign messages on behalf of the group without a member reveal own identity. In group signature system, as the same secret handshake systems, there exists a manager (authority) of group. A manager, in the case of a dispute, can reveal an identity of any group signature make valid group signatures A standard group signature scheme involves consists of five algorithms GS.KeyGen, GS.Join, GS.Sign, GS.Verify, GS.Open. GS.KeyGen, is a key generation algorithm, is given security parameter and outputs a group public key gpk and a group manager secret key gmsk. GS.Join, given gmskand a member secret key sk, outputs membership certificate. GS.Sign, given gpk, a member secret key sk and a message m, outputs a group signature  $\sigma$ . GS.Verify, given gpk, m, and  $\sigma$ , outputs accept if  $\sigma$  is valid for m with respect to gpk. GS.Open, given  $m, \sigma$ , and gmsk, outputs signer's identity. The security requirements of a group signature scheme are traceability, anonymity, and nonframeability [4].

Intuitively, **12** achieve Unlink by using group signature (more precisely, anonymity of group signatures) and realized SHS.Trace by SHS.Open. In **112**, a group signature with message recovery (GSMR) is constructed from the standard group signature [2]. In order to apply GSMR to SHS, GSMR can be forged a signature corresponding to an arbitrary message using a valid signature and *gpk*. If GSMR does not this property, anyone can be convinced of handshake players' groups by carrying out GS.Verify.

### 4.2 Construction of Proposed Scheme

We show the construction of proposed SHS scheme. Our GSMR scheme is based on **S**. We assume that  $\mathbb{G}_1 = \langle G_1 \rangle, \mathbb{G}_2 = \langle G_2 \rangle$  of prime order p that have a bilinear map e and  $e(G_1, G_2)$  generates  $\mathbb{G}_T$ . Here,  $(\mathbb{G}_1, \mathbb{G}_2)$  is bilinear groups.

SHS.Setup: Given a security parameter k, generates  $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, G_1, G_2, e)$ and chooses hash functions  $\mathcal{H} : \{0, 1\}^* \to \mathbb{G}_1$  and  $\mathcal{G} : \{0, 1\}^* \to \mathbb{Z}_p$ .

SHS.CreateGroup: First, IA chooses  $w \leftarrow_R \mathbb{Z}_p$  and  $H \leftarrow_R \mathbb{G}_1$  and generates  $W = wG_2$ . IA outputs isk = w, ipk = (H, W). Next, TA chooses  $(t, s) \leftarrow_R (\mathbb{Z}_p)^2$  and generates T, S such that H = sS = tT. TA outputs tsk = (t, s), tpk = (T, S).

SHS.AddMember: First, user U, who wants to join to the group, chooses  $(x', z) \leftarrow_R (\mathbb{Z}_p)^2$  and generates  $H' = x'H + zG_1$ . U sends H' to IA. Next, IA chooses  $(x'', z') \leftarrow_R (\mathbb{Z}_p)^2$  and sends (x'', z') to U. U generates  $x_U = x'x'' + z'$  and  $H_U = x_U H$ . U sends  $H_U$  to IA. U proves in zeroknowlege to IA the knowledge of  $x_U$  and x''z satisfying  $H_U = x_U H$  and  $x''H' + z'H - H_U = x''zG_1$ . Finally, IA chooses  $y_U \leftarrow_R \mathbb{Z}_p$  and generates  $A_U = \frac{1}{w+y_U}(G_1 - H_U)$ . IA sends  $(A_U, y_U)$  to U. IA adds  $(U, A_U, y_U)$  to the group member-list  $\mathcal{L}$ . GSMR(param, gpk, sk, cert, m)  $\rightarrow \sigma$ :

1. U chooses s', t', S', T', H' s.t.  $s'S' = t'T' = H' ((s', t') \in (\mathbb{Z}_p)^2, (S', T', H') \in (\mathbb{G}_1)^3)$ . If U will want to execute SHS.Co-Trace, U has to memorize their parameters.

- 2. U chooses  $(\alpha, \beta, \alpha', \beta') \leftarrow_R \mathbb{Z}_p^*$  and generates  $R_1 = \alpha T, R_2 = \beta S, R_3 = \alpha' T', R_4 = \beta' S'$  and  $R_5 = (\alpha + \beta)H + (\alpha' + \beta')H' + A_U$
- 3. U chooses  $(r_x, r_y, r_\alpha, r_\beta, r_{y\alpha}, r_{y\beta}, r_{\alpha'}, r_{\beta'}, r_{y\alpha'}, r_{y\beta'}) \leftarrow_R (\mathbb{Z}_p^*)^{10}$  and generate  $R'_1 = r_\alpha T, R'_2 = r_\beta S, R'_3 = r_{\alpha'}T', R'_4 = r_{\beta'}S', R'_5 = e(R_5, G_2)^{r_y} e(H, W)^{-(r_\alpha + r_\beta)} e(H', W)^{-(r_{\alpha'} + r_{\beta'})} e(H, G_2)^{-(r_{y\alpha} + r_{y\beta}) + r_x} e(H', G_2)^{-(r_{y\alpha'} + r_{y\beta'})}, R'_6 = r_y R_1 r_{y\alpha} T, R'_7 = r_y R_2 r_{y\beta} S, R'_8 = r_y R_3 r_{y\alpha'}T'$  and  $R'_9 = r_y R_4 r_{y\beta'}S'$
- 4. U generates  $c' = \mathcal{H}(param, gpk, R_1, R_2, R_3, R_4, R_5, R'_1, R'_2, R'_3, R'_4, R'_5, R'_6, R'_7, R'_8, R'_9)$  and  $c = c' \oplus m$ .
- 5. U generates  $s_x := r_x + cx_U, s_y := r_y + cy_U, s_\alpha := r_\alpha + c\alpha, s_\beta := r_\beta + c\beta s_{y\alpha} := r_{y\alpha} + c(y\alpha), s_{y\beta} := r_{y\beta} + c(y\beta), s_{\alpha'} := r_{\alpha'} + c\alpha', s_{\beta'} := r_{\beta'} + c\beta', s_{y\alpha'} := r_{y\alpha'} + c(y\alpha') \text{ and } s_{y\beta'} := r_{y\beta'} + c(y\beta')$
- 6. U outputs  $\sigma = (R_1, R_2, R_3, R_4, R_5, s_x, s_y, s_\alpha, s_\beta, s_{y\alpha}, s_{y\beta}, s_{\alpha'}, s_{\beta'}, s_{y\alpha'}, s_{y\beta'}, c)$

 $\mathsf{MR}:(param, gpk, \sigma) \to m$ 

- 1. V is given param, gpk, and  $\sigma$ .
- 2. V generates  $R'_1 = s_{\alpha}T cR_1, R'_2 = s_{\beta}S cR_2, R'_3 = s_{\alpha}T' cR_3, R'_4 = s_{\beta}S' cR_4, R'_5 = e(R_5, G_2)^{s_y}e(H, W)^{-(s_{\alpha}+s_{\beta})}e(H, G_2)^{-(s_{y\alpha}+s_{y\beta})+s_x}$   $e(H', W)^{-(s_{\alpha'}+s_{\beta'})}e(H, G_2)^{-(s_{y\alpha'}+s_{y\beta'})} \left(\frac{e(R_5, G_2)}{e(G_1, G_2)}\right)^c, R'_6 = s_yR_1 - s_{y\alpha}T,$  $R'_7 = s_yR_2 - s_{y\beta}S, R'_8 = s_yR_3 - s_{y\alpha'}T', \text{ and } R'_9 = s_yR_4 - s_{y\beta'}S'$
- 3. V generates  $c' = \mathcal{H}(param, gpk, R_1, R_2, R_3, R_4, R_5, R'_1, R'_2, R'_3, R'_4, R'_5, R'_6, R'_7, R'_8, R'_9)$  and  $m = c \oplus c'$ . V outputs m.

SHS.Handshake: Suppose the member U with certificate  $cert_U = (A_U, y_U)$  and secret key  $sk_U = x_U$ , and the member V with certificate  $cert_V = (A_V, y_V)$  and secret key  $sk_V = x_V$ , engage in handshake protocol.

- 1. U and V generates  $(s'_U, t'_U, S'_U, T'_U, H'_U), (s'_V, t'_V, S'_V, T'_V, H'_V)$ s.t.  $s'_U S_U = t'_U T = H'_U, s'_V S_V = t'_V T = H'_V$  $(s'_U, t'_U, s'_V, t'_V \in (\mathbb{Z}_p)^4, S'_U, T'_U, H'_U, S'_V, T'_V, H'_V \in \mathbb{G}_1).$
- 2. U chooses  $r_U \leftarrow_R \mathbb{Z}_p^*$  and generates  $m_U := r_U G_1$  and  $\sigma_U \leftarrow \mathsf{GSMR}(param, gpk_U, sk_U, cert_U, m_U)$ . U send  $\sigma_U$  to VD
- 3. V chooses  $r_V \leftarrow_R \mathbb{Z}_p^*$  and generates  $m_V := r_V G_1$ ,  $\sigma_V \leftarrow \mathsf{GSMR}(param, gpk_V, sk_V cert_V, m_V)$  and  $m'_U \leftarrow \mathsf{MR}(param, gpk_V, \sigma_U)$ . V send  $\sigma_V$  to UD
- 4. U generates  $m'_V \leftarrow \operatorname{MR}(param, gpk_U, \sigma_V)$  and  $resp_U := \mathcal{G}(r_U m'_V, m_U)$  and send  $resp_U$  to V.
- 5. V generates  $resp_V := \mathcal{G}(r_V m'_U, m_V)$ . If  $resp_U = \mathcal{G}(r_V m'_U, m'_U)$ , V outputs *accept* and send  $resp_V$  to U. Otherwise V outputs *reject*.
- 6. If  $resp_V = \mathcal{G}(r_U m'_V, m'_V)$ , U outputs accept. Otherwise U outputs reject.

SHS.Co-Trace: Suppose the member U with  $cert_U = (A_U, y_U)$ , secret key  $sk_U = x_U$ , and the parameters  $(s'_U, t'_U, S'_U, T'_U, H'_U)$  used in SHS.Handshake.

1. TA is given  $cert_U = (A_U, y_U), sk_U = x_U$  from U. If  $e(A, W)e(H, G_2)^x e(A, G_2)^y = e(G_1, G_2)$ , TA executes the following. Otherwise TA outputs  $\perp$ .

Scheme	3	6	Ц	Proposed Scheme
Underlying Assumption	CBDH	CDH	SXDH and BDH	DL and DLDH
Number of rounds	3	4	2	4
Communication complexity (bits)	640	8512	684	5804
IR security	Yes	Yes	Yes	Yes
DR security	Yes	Yes	Yes	Yes
Unlink security	No	No	Yes	Yes
SDR security	No	No	Yes	Yes
Co-Trace security	No	No	No	Yes
SHS.Trace algorithm	Yes	Yes	No	No
SHS.Co-Trace algorithm	No	No	No	Yes

Table 2. Comparison among SHS

2. TA is given  $\sigma_V(=R_{V1}, R_{V2}, R_{V3}, R_{V4}, R_{V5}, s_{Vx}, s_{Vy}, s_{V\alpha}, s_{V\beta}, s_{Vy\alpha}, s_{Vy\beta}, s_{Vy\alpha'}, s_{Vy\alpha'}, s_{Vy\beta'}, c_V)$  (in the transcript of SHS.Handshake) and computes  $R_{V5} - (tR_{V1}sR_{V2} + t'R_{V3}s'R_{V4}) = A_V$  and outputs V such that  $(V, A_V, y_V) \in \mathcal{L}$ .

Among SHS, we show the comparisons with respect to important factors in Table [2] [3], [1] and proposed scheme use bilinear maps. We assume that  $\mathbb{G}_1 \neq \mathbb{G}_2$  such that the representation of  $G_1$  can be a 172-bit prime when |p| = 171. Also, we assume that [6] are instantiated on a 160-bits prime order subgroup of a prime finite field of 1024 bits. In [3], [6] and proposed scheme, a member has own ID. On the other hand, in [1], a member does not have ID, so [1] can not include both SHS.Trace and SHS.Co-Trace.

Our proposed scheme is secure under discrete logarithm assumption and Decisional Linear Diffie-Hellman (DLDH) assumption **5**. Proofs of the following theorems are given in **9**.

**Theorem 1.** The proposed scheme has Impersonator Resistance property, if the discrete logarithm problem is hard to solve.

**Theorem 2.** The proposed scheme has Strong Detector Resistance, Unlinkability and Co-Traceability property, if the Decisional Linear Diffie-Hellman (DLDH) problem is hard to solve.

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# An Extended Authentication and Key Agreement Protocol of UMTS

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Abstract. Identification, authentication and key agreement protocol of UMTS networks have some weaknesses to provide DoS-attack resistance, mutual freshness, and efficient bandwidth consumption. In this article we consider UMTS AKA and some other proposed schemes. Then we explain the known weaknesses in the previous frameworks suggested for UMTS AKA protocol. After that we propose a new UMTS AKA protocol (called EAKAP) for UMTS mobile network that combines identification stage and AKA stage of UMTS AKA protocol as well as eliminating disadvantages of related works and bringing some new features to improve the UMTS AKA mechanism such as reducing the interactive rounds of the UMTS AKA protocol.

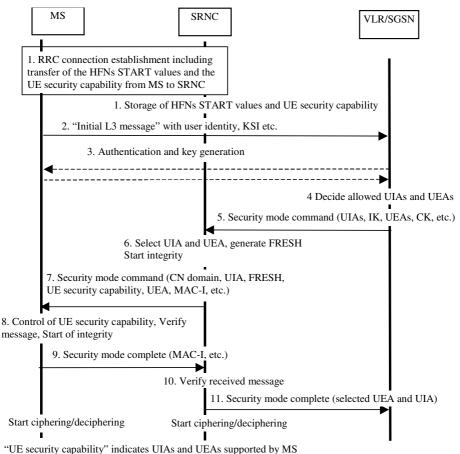
**Keywords:** Identification, Authentication and Key Agreement, UMTS, Mobile Network, Security Protocol.

# **1** Introduction

The wireless communications advances cause the ease of access to wireless services for individuals. Because of the air interface between the user and the network, the physical security of users' media is in serious danger with respect to the wired infrastructure. Physical layer security like spread spectrum methods for commercial usages is so expensive, hence wireless providers try to secure higher layers to obtain privacy and confidentiality features for their subscribers.

The most commonly used wireless communications are cellular communications. In the first-generation (1G) analog systems, security services were not addressed. Proper authentication of subscribers is an important feature for operators to charge them correctly. So in the second-generation (2G) digital cellular systems, the GSM communications, authentication and also confidentiality were taken into account and security measures were designed for these goals in 2G.

Some of the GSM security weaknesses are active attack by fake BTS, no secure communication between BTS and BSC and also between BSC and MSC, no data integrity check and weak stream cipher algorithm (A5/1,2) for confidentiality. These weaknesses were concentrated in the security design phase of UMTS, a major standard for the third-generation (3G). So, an enhanced authentication and key agreement protocol was considered for UMTS and integrity was added as well as using strong algorithms.



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Fig. 1. The UMTS Identification and AKA procedure [1]

Between features mentioned above, the authentication and key agreement protocol has vital importance which the most prominent security features are based on. In this protocol, other than authentication, user and network agree on the cipher and integrity keys CK and IK respectively. If existence of any vulnerability in this protocol, other than these keys, the subscriber secret key K may be compromised and so, we focus on this protocol for the purpose of improving it.

This paper is organized as follows; in section II we explain the UMTS authentication and key agreement (AKA) procedure. Section III outlines related challenges to improve the security and performance of the UMTS AKA protocol. In section IV the new proposed protocol is described that covers the previous known weaknesses. In section V the EAKAP is analyzed and evaluated from the security point of view. We conclude the paper in section VI. The descriptions of abbreviations are given in appendix.

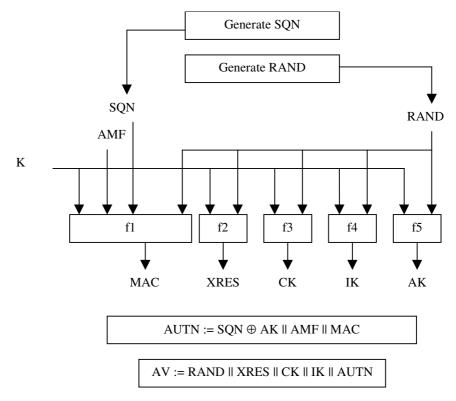


Fig. 2. Generation of Authentication Vectors [1]

# 2 UMTS AKA Description

The purpose of UMTS AKA is to authenticate the user and network to each other and also establish a new pair of cipher and integrity keys between the VLR/SGSN and the USIM [1]. MS, VLR/SGSN, and HLR/AuC (HE) are involved in UMTS AKA protocol. At identification stage, MS sends its identity to VLR/SGSN via SRNC.

The secret key (*K*) and cryptographic Algorithms including  $f_1$ ,  $f_1^*$ ,  $f_2$ ,  $f_3$ ,  $f_4$ ,  $f_5$ , and  $f_5^*$ , shared between MS and HE, are used for UMTS AKA process. Furthermore HE and MS track the value of a counter (*SQN<sub>HE</sub>*) and (*SQN<sub>MS</sub>*) respectively. These sequence numbers are used for the purpose of freshness checking of the received messages. As shown in figure 1, UMTS Identification, distribution of authentication data, and AKA procedure are performed as follows.

#### Identification

1- The MS sends the initial L3 message including its TIMSI and the KSI to VLR/SGSN via SRNC. By this message, the MS requests for services like Location Update, CM Service and Routing Area Update.

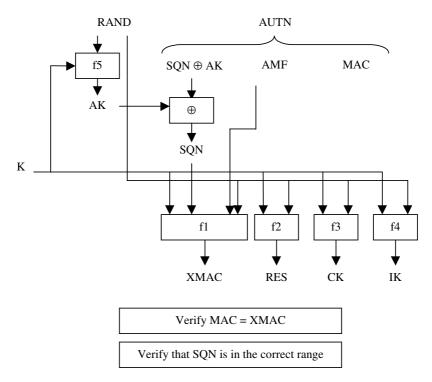


Fig. 3. User Authentication Function in the MS [1]

#### **Distribution of Authentication Data**

2- VLR/SGSN identifies MS by its TMSI and then sends the authentication data request including IMSI and requesting node type (PS or CS) to the HE.

3- Upon the receipt of the authentication data request, the HE sends an authentication data response back to the VLR/SGSN which contains an ordered array of n authentication vectors AV(1...n). Each AV includes parameters *RAND*, *XRES*, *CK*, *IK*, and *AUTN*. Generation of AV is shown in figure2.

#### Authentication and Key Management

4- The VLR/SGSN chooses the next unused AV from the ordered array of AVs in the VLR/SGSN database on the basis of first-in/first-out. Then the VLR/SGSN sends to the MS the random challenge (*RAND*) and an authentication token (*AUTN*) from the chosen AV.

5- When the MS receives RAND||AUTN, it proceeds as illustrated in figure3. The MS first computes the anonymity key  $AK=f5_K(RAND)$  and retrieves the sequence number. Then the MS computes  $XMAC=f1_K(SQN||RAND||AMF)$  and compares it with MAC included in *AUTN*. If they are the same, the MS verifies if the *SQN* is in the correct range. Then, the MS calculates  $RES=f2_K(RAND)$  and sends it to the VLR/SGSN. The VLR/SGSN compares the received *RES* with *XRES*. If they match, then the authentication process of the MS is successfully completed.

# **3 Related Works**

In this section we consider the security analysis of UMTS AKA Protocols. Many protocols have been suggested for UMTS AKA improvement, but we choose some protocols that have novelty in their design and use symmetric algorithms. Nevertheless they have some security or performance weaknesses in their structure that we try to explain.

The UMTS X-AKA protocol [7] applies a temporary key mechanism with timestamp instead of the sequence number. The function *f5* is used for generating temporary keys. The UMTS X-AKA protocol consists of two procedures. First, the user registers on HN and then HN distributes temporary key (TK) and authentication information to SN. Second, the authentication and key agreement procedure is executed between SN and MS. The SN uses TK and authentication information to carry out the mutual authentication between SN and MS and then an agreed key and a cipher key are provided. The UMTS X-AKA protocol uses timestamp to manage freshness of the messages. The timestamp usage needs a robust time synchronization infrastructure. Time synchronization structure of the network has no security feature, so the usage of an independent structure with no security to refresh the exchanged messages is hazardous. Also the HN could not recognize the shared session keys between MS and SN, because SN generates the pseudo-random number needed to construct the session keys.

In [8] an AKA protocol with robust user privacy protection has been proposed. In this scheme, temporary key mechanism to authenticate MS and prevent the location privacy attack is used. In addition, it has lower overhead on VLR. Since MS can easily compute the temporary key through the shared secret key, VLR can be authenticated by MS successfully. In this protocol, the VLR initiates the authentication process by sending a nonce to the MS without any MAC, so DoS attack is probable to be imposed on the MS. Also the protocol has seven steps without identification and security mode set-up stages.

J. Al-Saraireh and S. Yousef proposed an AKA protocol [9] in which, the MS generates the AVs sending to the network. They provided an efficient bandwidth consuming framework with minimal ways for the AKA procedure, but the proposed protocol doesn't support mutual authentication i.e. only the network authenticates the MS. The protocol has 3 steps. The man in the middle attack scenario on interworking of UMTS and GSM [10] could be applied on this protocol [9], because the MS doesn't recognize the validity of the network. Furthermore, the DoS attack on the MS is possible, because the MS could only verify the network until a MAC is received from the network. With some modification in this protocol, it will be mutual. If the VLR/SGSN sends the RES received from the HE to the MS, the mutual authentication would be satisfied by checking the XRES and RES in the MS side.

The security of the wireless network access has been enhanced by Harn and Hsin [11] that uses timestamp and hash chain to provide non-repudiation and freshness. As mentioned earlier, using the timestamp needs independent secure infrastructure. Also the hash chain construction consumes much computation load at the end user side. Furthermore, the number of the protocol ways is six and the protocol doesn't contain the identification and security mode set-up stages.

An extension of UMTS AKA protocol has been proposed by J. Al-Saraireh and S. Yousef in [12] that provides mutual freshness of the MS and the HE but it doesn't use sequence number mechanism and instead, both the MS and HE generates random numbers. So verifying the freshness of the messages could be done by searching in a large database that contains all of the previous random numbers generated by the parties. Applying such a huge database is more expensive and complex than using sequence number method. Besides, the power of DoS attack diminishes when the parties can check the integrity and freshness of the messages faster.

M. Zhang and Y. Fang enhanced the security of the 3GPP AKA by a protocol called AP-AKA. They have projected a special scenario of the redirection attack that UMTS AKA is weak towards it and AP-AKA is robust against it. But the MS traffic redirection by a virtual relay to the neighbor VLR could charge the MS more than usual because the location of the MS has been virtually changed. The first step of the AP-AKA has not integrity protection, so it could be forged. Also the attack [10] can be executed on AP-AKA while interworking with GSM because the VLR initiates the AKA procedure without integrity check. Furthermore, the identification and security mode set-up stages are not considered to provide a better performance for the six-way AP-AKA protocol.

## 4 UMTS Extended AKA Protocol

In this section, we propose a new authentication protocol of UMTS mobile networks. The stages of the proposed protocol are illustrated in figure4 and figure5. UMTS AKA has some weaknesses to provide simple DoS-attack resistance. Also previous proposed schemes (described in section III) do not provide strong mutual freshness. Furthermore, the steps of the pre-proposed protocols could be reduced to save bandwidth consumption.

The new proposed UMTS AKA protocol, named EAKAP, combines identification stage and AKA stage with security mode set-up of described UMTS protocol. EAKAP is done by a 5-way handshake protocol between the MS, the VLR/SGSN, and the HE. Most of the previous schemes were done by a 5-way handshake in the phase of AKA without security mode set-up, so we could enhance bandwidth efficiency. The EAKAP applies the secret key (K) and the cryptographic algorithms that are used in UMTS AKA protocol shared between the MS and the VLR/SGSN. But the usage of the f2 algorithm in EAKAP is not necessary because EAKAP does not generate the *RES* or the *XRES*.

Both the MS and the HE have sequence number and random number generator to provide complete freshness of their messages. On the basis of the f5 structure [2], we could improve the confidentiality of the SQN by changing its encryption method. To protect VLR/SGSN against DoS attack, we assume some security capabilities at VLR/SGSN side. These security features contain the shared cryptographic algorithms (fc and fi) between MS and VLR/SGSN. The fc algorithm is used for ciphering and the fi algorithm is used to generate integrity check. The EAKAP procedure works as follows (shown in figure4).

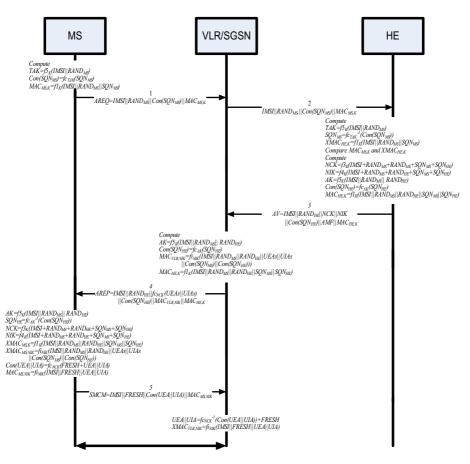


Fig. 4. EAKAP Structure when MS meets a new VLR/SGSN

#### **Authentication Request**

1- If no TMSI was set before, the MS generates the AREQ message as follows:

$$AREQ=IMSI ||RANDMS||Con(SQN_{MS})||MAC_{MS,K}$$
(1)

Where

$$TAK = f_{5K}(IMSI \| RAND_{MS})$$
(2)

$$Con(SQN_{MS}) = fc_{TAK}(SQN_{MS})$$
(3)

$$MAC_{MS,K} = f1_{K}(IMSI ||RAND_{MS}||SQN_{MS})$$
(4)

If the VLR/SGSN has allocated a TMSI encrypted by previous CK (OCK) paired with previous IK (OIK) to the MS, the MS would generate the AREQ message as follows:

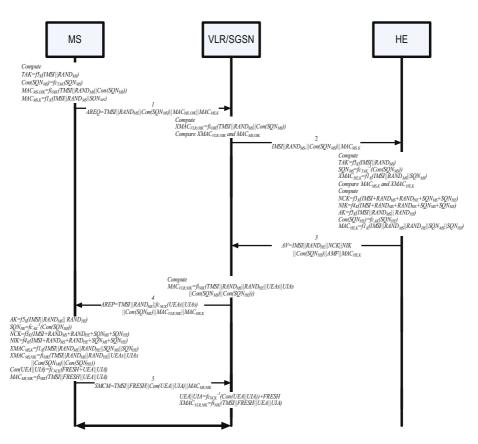


Fig. 5. EAKAP Structure when MS meets an old VLR/SGSN

$$AREQ=TMSI ||RAND_{MS}||Con(SQN_{MS})||MAC_{MS,OIK}||MAC_{MS,K}$$
(5)

Where

$$\Gamma AK = f_{5K}(IMSI \| RAND_{MS})$$
(6)

$$Con(SQN_{MS}) = fc_{TAK}(SQN_{MS})$$
(7)

$$MAC_{MS,OIK} = fi_{OIK}(TMSI ||RAND_{MS}||Con(SQN_{MS}))$$
(8)

$$MAC_{MS,K} = f1_{K}(IMSI ||RAND_{MS}||SQN_{MS})$$
(9)

Then the MS sends the calculated AREQ to the VLR/SGSN.

#### **Distribution of Authentication Data**

2- The VLR/SGSN identifies the MS by its sent IMSI via HE or TMSI via old VLR/SGSN. If TMSI related to current VLR/SGSN is sent, the VLR/SGSN computes the  $XMAC_{VLR,OIK}$  (as given below) and compares this with  $XMAC_{MS,OIK}$  which is included in AREQ.

$$XMAC_{VLR,OIK} = fi_{OIK}(TMSI ||RAND_{MS}||Con(SQN_{MS}))$$
(10)

If they are different, VLR/SGSN does not distribute the authentication data request for HE and AKA process fails. Otherwise, VLR/SGSN sends the authentication data request included IMSI,  $RAND_{MS}$ ,  $Con(SQN_{MS})$  and  $MAC_{MS,K}$  to HE.

3- When the HE receives the authentication data request from the VLR/SGSN, it first retrieves sequence number of the MS as follows.

$$TAK = f_{5}(IMSI || RAND_{MS})$$
(11)

$$SQN_{MS} = fc_{TAK}^{-1}(Con(SQNMS))$$
(12)

Then the HE computes the  $XMAC_{HE,K}$  (as given below) and compares it with  $MAC_{MS,K}$  which is included in the VLR/SGSN's authentication data request.

$$XMAC_{HE,K} = f1_{K}(IMSI || RAND_{MS} || SQN_{MS})$$
(13)

If the  $MAC_{MS,K}$  and the  $XMAC_{HE,K}$  are different, the HE detects the MS as a fraud user and does not generate any AV. If they are the same, the HE verifies the SQN is in the correct range. If the  $SQN_{MS}$  is considered to be in the correct range, the HE sends an authentication data response back to the VLR/SGSN that contains an authentication vector (AV). The generated AV included IMSI,  $RAND_{HE}$ , new CK (NCK), new IK(NIK),  $Con(SQN_{HE})$ , AMF, and  $MAC_{HE,K}$  as shown in figure 5.

$$AV=IMSI||RAND_{HE}||NCK||NIK||Con(SQN_{HE})||AMF||MAC_{HE,K}$$
(14)

Where

$$NCK = f_{3_{K}}(IMSI + RAND_{MS} + RAND_{HE} + SQN_{MS} + SQN_{HE})$$
(15)

$$NIK = f4_{K}(IMSI + RAND_{MS} + RAND_{HE} + SQN_{MS} + SQN_{HE})$$
(16)

$$AK = f_{5}(IMSI ||RAND_{MS}|| RAND_{HE})$$
(17)

$$Con(SQN_{HE}) = fc_{AK}(SQN_{HE})$$
(18)

$$MAC_{HE,K} = f1_{K}(IMSI ||RAND_{MS}||RAND_{HE}||SQN_{MS}||SQN_{HE})$$
(19)

#### **Authentication Reply**

4- Upon the receipt of the AV from the HE, the VLR/SGSN computes the *AREP* for the MS. The VLR/SGSN determines which UIAs and UEAs that are allowed to be used in order of preference. If no TMSI was set before, the VLR/SGSN generates the *AREP* message as follows:

$$\begin{aligned} AREP=IMSI ||RAND_{HE}||fc_{NCK}(UEAs||UIAs) \\ ||Con(SQN_{HE})||MAC_{VLR,NIK}||MAC_{HE,K} \end{aligned} \tag{20}$$

Where

$$AK = f_{K}(IMSI || RAND_{MS} || RAND_{HE})$$
(21)

$$Con(SQN_{HE}) = fc_{AK}(SQN_{HE})$$
(22)

$$MAC_{VLR,NIK} = fi_{NIK} (IMSI ||RAND_{MS} ||RAND_{HE} ||UEAs||UIAs||Con(SQN_{MS})||Con(SQN_{HE})))$$
(23)

$$MAC_{HE,K} = f1_{K}(IMSI \| RAND_{MS} \| RAND_{HE} \| SQN_{MS} \| SQN_{HE})$$
(24)

If the VLR/SGSN allocates a TMSI encrypted by previous *CK* (*OCK*) paired with old *IK* (*OIK*) to the MS, it would generate the *AREP* message as follows:

$$AREP=TIMSI ||RAND_{HE}||fc_{NCK}(UEAs||UIAs) ||Con(SQN_{HE})||MAC_{VLR,NIK}||MAC_{HE,K}$$
(25)

Where

$$\begin{aligned} &MAC_{VLR,NIK} = fi_{NIK}(TIMSI ||RAND_{MS} ||RAND_{HE} \\ &||UEAs||UIAs||Con(SQN_{MS})||Con(SQN_{HE})) \end{aligned} \tag{26}$$

5- Upon the receipt of the *AREP* message, the MS proceeds as illustrated in figure4 and figure5. If the MS has a TMSI, allocated by the VLR/SGSN, it first retrieves the HE's sequence number as follows.

$$AK = f_{5_{K}}(IMSI||RAND_{MS}||RAND_{HE})$$
(27)

$$SQN_{HE} = fc_{AK}^{-1}(Con(SQN_{HE}))$$
(28)

If the SQN is considered to be in the correct range, the MS calculates the new pair key, the new cipher key (NCK) and the new integrity key (NIK), as follows:

$$NCK = f_{3K}(IMSI + RAND_{MS} + RAND_{HE} + SQN_{MS} + SQN_{HE})$$
(29)

$$NIK = f4_{K}(IMSI + RAND_{MS} + RAND_{HE} + SQN_{MS} + SQN_{HE})$$
(30)

Then the MS calculates the  $XMAC_{MS,K}$  (as given below) and checks the correspondence of the  $MAC_{HE,K}$  and the  $XMAC_{MS,K}$ .

$$XMAC_{MS,K} = f1_{K}(IMSI ||RAND_{MS}||RAND_{HE}||SQN_{MS}||SQN_{HE})$$
(31)

If they are the same, the MS decrypts the  $fc_{NCK}(UEAs||UIAs)$  and gets the cryptography capabilities of the VLR/SGSN in order of preference. Then the MS computes  $XMAC_{MS,NIK}$  (as given below) with regard to the allocated TMSI.

$$\begin{aligned} XMAC_{MS,NIK} = f_{NIK}(TMSI ||RAND_{MS}||RAND_{HE} \\ ||UEAs||UIAs||Con(SQN_{MS})||Con(SQN_{HE})) \end{aligned} (32)$$

If no TMSI was allocated, the XMAC<sub>MS,NIK</sub> would be as follows:

$$\begin{aligned} XMAC_{MS,NIK} = f_{NIK}(IMSI ||RAND_{MS} ||RAND_{HE} \\ ||UEAs||UIAs||Con(SQN_{MS})||Con(SQN_{HE})) \end{aligned} (33)$$

The MS compares  $XMAC_{MS,NIK}$  with  $MAC_{VLR,NIK}$  included in the received AREP message. If they are different, the MS does not continue authentication process and terminate the connection. Otherwise, the MS generates the security mode complete message (SMCM) as follows:

$$SMCM=TIMSI||FRESH||Con(UEA||UIA)||MAC_{MS,NIK}$$
(34)

Where

$$Con(UEA||UIA) = fc_{NCK}(FRESH+UEA||UIA)$$
(35)

$$MAC_{MS,NIK} = fi_{NIK}(TMSI || FRESH || UEA || UIA)$$
(36)

The MS sends the SMCM to the VLR/SGSN and the VLR/SGSN decrypts the Con(UEA||UIA) by the new shared cipher key (*NCK*) to get preferred UEA and UIA. UEA||UIA=fc<sub>NCK</sub><sup>-1</sup>(Con(UEA||UIA))+FRESH

Then the VLR/SGSN verifies the integrity of the SMCM by generating the  $XMAC_{VLR,NIK}$  with regard to the chosen UEA and UIA by the MS and comparing it with the  $MAC_{MS,NIK}$ .

$$XMAC_{VLR,NIK} = fi_{NIK}(TMSI || FRESH || UEA || UIA)$$
(37)

Consequently the IAKA (Identification and AKA) process with security mode setup is complete i.e. the parties authenticate each other mutually and the preferred algorithms for ciphering and integrity checking are chosen by them.

## 5 Security and Performance Features of the Proposed Protocol

In this section, we analyze the security features of the new proposed protocol explained in previous section and then we evaluate the performance of the suggested protocol relatively. The EAKAP provides the main security services issued in literature like authentication, confidentiality and integrity [3]. Furthermore, the EAKAP is more robust than the UMTS AKA and the previously suggested protocols against the DoS attack so with using the EAKAP, availability service would be provided properly.

#### **Performance Evaluation towards Previous Works**

In the EAKAP, three sections of the UMTS protocol including identification, AKA and security mode set-up are combined to set-up connection. In EAKAP, security mode setup is performed after key establishment so the attacker has no information about the shared algorithms for ciphering and integrity check. The EAKAP is completed after five-way interactions. Previous works do not consider the identification and security mode set-up steps. Furthermore, if we consider the whole protocol, the number of interactions has been reduced largely. So the load of the network signaling is reduced and performance of the IAKA procedure grows efficiently.

#### Confidentiality, Integrity, and Authentication

As described in previous section, the only shared secret between the MS and the HE is a private key (K). Sometimes the old session pair keys (*OCK*, *OIK*) are shared between MS and VLR/SGSN if the MS has visited the area of the VLR/SGSN already. After the IAKA process, the MS and the VLR/SGSN share the new session pair key (*NCK*, *NIK*). These keys are hidden from the sight of adversary and we show that they could not be guessed or eavesdropped easily.

The TMSI/IMSI fields are sent public without any encryption because anonymity service is out of the scope of the article. Also RAND fields are sent as plain text because they are needed for the other side to generate necessary fields. The sequence numbers, UEA, and UIA fields are encrypted by the shared keys (K or *NCK*). So the confidentiality service is provided properly. Also *MAC* fields are protected by the shared keys (K, *OIK* or *NIK*) and hence the integrity service is afforded appropriately.

The HE could authenticate the identity (IMSI) of the MS by verifying the integrity of the  $MAC_{MS,K}$  generated by the MS. The MS could confirm the identity of its home network by checking the  $MAC_{HE,K}$  calculated by the HE. So two parties authenticate each other suitably referred to mutual authentication. Besides, the MS and the VLR/SGSN could authenticate each other by verifying the  $MAC_{MS,OIK}$ ,  $MAC_{MS,NIK}$  and  $MAC_{VLR,NIK}$ , as the session keys (*OIK*, *NIK*) could be computed or available in both sides (MS side and VLR/HE side). So the mutual authentication with security mode set-up is done by the EAKAP.

#### Sequence Number Protection vs. Private Key Derivation

MS increments the  $SQN_{MS}$  for every IAKA procedure and HE runs the  $SQN_{HE}$  for every generated AV. MS and HE use sequence number and random number generator to achieve strong freshness of their messages. However pseudo-random numbers could provide the freshness of the ways of the EAKAP, but both MS and HE need a large directory to save used random numbers in it. Also to avoid the usage of the repeated random numbers they have to search in their directory or sort and update the directory for every procedure that it consumes a large amount of energy and memory.

In the EAKAP, anonymity protection of the sequence number is higher than the UMTS AKA because the sequence number of the UMTS AKA is XOR-ed by anonymity key derived from nonce and the private key [2] like stream ciphers. As sequence number value is guessable because it starts from zero and changes incrementally slowly, the known plaintext attack is imaginable on the f5 algorithm to get private key (K). However the core of the f5 algorithm is based on Advanced Encryption Standard (AES) [4] that the recent proposed attacks are not practical on it [5]. In the EAKAP, the sequence number is encrypted by a block cipher algorithm called fc with derived key from f5 algorithm and so the privacy of the sequence number is strongly established.

#### Availability (Robustness against DoS Attack)

If the MS enters area of a new VLR/SGSN, it won't have a TMSI allocated by VLR/SGSN. So the MS must use its IMSI to initiate the procedure. In this situation, the MS actually aims its HE not VLR/SGSN. In fact SRNC and VLR/SGSN role as

relays to forward the MS messages to the HE. Here we don't struggle with the privacy establishment of the MS.

First, we consider the condition that no TMSI was set before. The MS and the VLR/SGSN have no shared key before and so, as mentioned in step1 of the previous section, the *AREQ* message has been sent to the HE via the intermediate SRNC and VLR/SGSN. As mentioned in step3, the HE could check the integrity of the  $MAC_{MS,K}$ , so the spam messages generated by spurious users are discarded by the HE. Furthermore, the DoS attack organized by unauthentic MS at the first step, could be detected at the HE side and no more traffic would be procreated in the network.

Second, we consider the situation where the VLR/SGSN has allocated the MS a TMSI so the MS and the VLR/SGSN have shared a pair key (*OCK* and *OIK*) with each other. Consequently, as explained in step2, the VLR/SGSN checks the integrity of the  $MAC_{MS,OIK}$  and rejects the forged messages. Furthermore, with the usage of the pre-shared information of the MS, the network could prevent DoS attack of first step at the VLR/SGSN side.

In the other steps of the EAKAP, by using MAC mechanism, the vulnerability towards DoS attack is reduced as mentioned above. Although in some cases, the *SQN* should be computed before the MAC so the computation load would be increased. But no further signaling load would be injected to the network and the elements of the network could decide about the genuineness of the message by verifying the MAC integrity.

#### Mutual Freshness and Unguessable Keys

The  $SQN_{MS}$  is encrypted by temporary anonymity key (*TAK*) derived from the output of the *f5* algorithm by a high entropy seed which is *IMSI*||*RAND<sub>MS</sub>*. The  $SQN_{HE}$  is protected by anonymity key (*AK*) derived from the *f5* algorithm by concatenation of the IMSI, *RAND<sub>MS</sub>* and *RAND<sub>HE</sub>* that is an entropic seed to generate a fresh key. The seed entropy of the *f5* algorithm depends on two random numbers *RAND<sub>MS</sub>* and *RAND<sub>HE</sub>* and hence is improbable to be guessed by adversary. So the sequence numbers could not be revealed. Also if an anonymity key is compromised at a session, no next-generated keys will be concealed i.e. our scheme supports forward security towards session key compromising. The MAC fields like the *SQN* fields are fresh. The entropic seed of the MAC fields is derived from the pseudo random numbers as well as IMSI and sequence number.

### 6 Conclusion

Most of the proposed protocols do not consider the previous and next stages of the UMTS AKA protocol and they try to improve the UMTS AKA protocol solely. Our proposed protocol, so called EAKAP, merges all of the stages in five ways so it improves performance by reducing the load of the network signaling. Also EAKAP support mutual freshness and is more robust against DoS attack by applying MAC mechanism between the MS and the network.

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# **APPENDIX** (Abbreviations)

- 3GPP Third-Generation Partnership Project
- AK Anonymity Key
- AKA Authentication and Key Agreement
- AMF Authentication Management Field
- AREP Authentication Reply
- AREQ Authentication Request
- AuC Authentication Center
- AUTN Authentication Token
- AV Authentication Vector

BSC	Base Station Controller
BTS	Base Transceiver Station
CK	Cipher Key
CM	Connection Management
CS	Circuit Switched
DoS	Denial of Service
EAKAP	Extended AKA Protocol
FRESH	Pseudo-Random Number Generated by MS
GSM	Global System for Mobile
HE	Home Environment included HLR and AuC
HLR	Home Location Register
HN	Home Network
IAKA	Identification and AKA
IK	Integrity Key
IMSI	International Mobile Subscriber Identity
KSI	Key Set Identifier
MAC	Message Authentication Code
MAC-I	Message Authentication Code for Integrity
MS	Mobile Station
MSC	Mobile Switching Center
PS	Packet Switched
RAND <sub>X</sub>	Pseudo-Random Number Generated by X
RES	Response
SGSN	Serving GPRS Support Node
SMCM	Security Mode Complete Message
SQN	Sequence Number
SRNC	Serving Radio Network Controller
TAK	Temporary Anonymity Key
TIMSI	TMSI or IMSI
SMCM	Security Mode Complete Message
SRNC	Serving Radio Network Controller
TAK	Temporary Anonymity Key

# Hash-Based Key Management Schemes for MPEG4-FGS

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Abstract. We propose two symmetric-key management schemes for the encryption of scalable compressed video content. The schemes are applicable to MPEG-4 Fine Grain Scalability video coding. Our constructions make only use of hash functions and achieve the optimal bound regarding the minimum number of keys and the time complexity in computing all the decryption keys. We also formalize new security notions about collusion-resistance. Unlike prior key management schemes, our second scheme resists to certain collusion attacks. The collusionresistance achieved is practical and hence sufficient for encryption of scalable video streams.

**Keywords:** symmetric-key encryption, key management, MPEG-4 Fine Grain Scalability.

### 1 Introduction

Scalability has become a fundamental feature in video coding. It allows one to encode a video once and to send it to many users with different usage profiles and capabilities (high-end PCs, high-definition displays, cell phones,...). Each user has access to a particular portion or quality of the content. Efficient access control methods are required in scalable video transmission in order to grant access only to authorized users for the video quality they purchased for.

In scalable video coding, the compressed bitstream is composed of two layers: a non scalable part called base layer and a scalable part containing many enhancement layers. If only the base layer is decoded, this will result in low quality version of the original content. If the enhancement layers are decoded, combined with the base layer, this produces an improved quality of the video, proportional to the Enhancement portion decoded. MPEG-4 Fine Grain Scalability (MPEG4-FGS) [4] provides flexibility in supporting:

- PSNR scalability by defining different quality bitplanes;
- Bitrate scalability by arbitrary bitplanes truncation.

Scalable compressed video is particularly adapted to flexible services provided by DRM (Digital Rights Management) technologies [15]. Scalable encryption

techniques if combined with a suitable key management scheme enables to improve access control processes as proposed for MPEG4-FGS [8].

To enforce a scalable access control, a typical approach is to encrypt the different portions in the enhancement layers with a different encryption key. The keys will be distributed selectively to legitimate users who should access only the keys that render the authorized video quality. In other words, the scalable access control problem for the video content is transformed into scalable access control on the encryption key.

In this paper, we focus on a layered access control scheme called Scalable Multi-Layer FGS Encryption (SMLFE) [8,10] that supports both PSNR and bit-rate scalability for the MPEG4-FGS coding. In SMLFE, the enhancement layers are encrypted into a single stream with multiple quality layers divided according to PSNR values and bit-rates. Enhancement layers for each frame are partitioned into n bit-rates and t PSNR layers independently.

**Background and Related Works.** In the trivial scheme [S], each segment  $S_{i,j}$   $(1 \le i \le n, 1 \le j \le t)$ , is at the intersection of one bit-rate and one PSNR layer and is encrypted with a different *segment encryption key*. Therefore,  $n \cdot t$  segment keys are randomly generated to encrypt all the segments. The protected content is next transferred to an appropriate server (distribution server) for the distribution while the  $n \cdot t$  segment keys are stored in a license server. When the user receives the protected MPEG4-FGS content he requests for a license. The license server generates it according to the quality and/or resolution requested. For instance, if a user acquires the rights for the full quality of the video, license server sends all the  $n \cdot t$  segment decryption keys. In this trivial scheme, the license server has to securely store all the  $n \cdot t$  segment keys.

Another method based on the Diffie-Hellman key agreement protocol is described in  $\square$ . In this scheme, only two secret keys are maintained by the license server (instead of  $n \cdot t$ ). for the full content quality, n public keys sent along

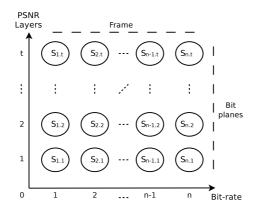


Fig. 1. Segments in the Enhancement Layers

Schemes	Number of Keys			Time-complexity		
Symmetric Key	server	user	length	hashes	exponentiations	
Trivial Basic Scheme Enhanced Scheme	1	1	128 bits 128 bits 128 bits		0 0 0	
Asymmetric Key			length 256 bits		$\frac{\text{exponentiations}}{n \cdot t}$	

 Table 1. Comparing Key Management Schemes Efficiency

with the protected content plus one secret key, sent in the license, are needed to compute all the segment keys. This system makes use of the hybrid encryption method where the asymmetric algorithm helps in computing a symmetric key used for segments encryption.

**Notation.** A 2*l*-bit integer *x* is an integer such that  $2^{2l-1} \leq x < 2^{2l}$ . By  $\lfloor x \rfloor$  we mean the highest integer less than or equal to *x*, i.e.  $\lfloor x \rfloor = \max \{n \in \mathbb{N} \mid n \leq x\}$ . The *l* most significant bits of *x* are noted  $\{x\}_L$  and *l* least significant bits  $\{x\}_R$ , i.e.  $x = \{x\}_L || \{x\}_R$  and could be written  $\{x\}_L 2^l + \{x\}_R$ . To ease the presentation, we do not distinguish between an integer and its representation. For a set S, we denote by |S| the number of elements in S.

**Our contribution.** In this work, we describe two symmetric-key management schemes for scalable multilayer fine grain scalability encryption. In the proposed schemes, we consider that a user having access to a given high resolution should access all lowest resolutions, and similarly if he has access to a given quality he should also have access to lowest qualities.

The "basic" scheme is the simplest construction that satisfies the SMLFE architecture. It is the most efficient in terms of number of keys sent to user and time complexity in recovering the segment decryption keys. The basic scheme is however vulnerable to some collusion attacks. We therefore propose the "enhanced" scheme to address the collusion problem.

Table Compares our schemes with the previous proposal in terms of number of keys maintained by the license server, number of keys and time complexity needed at user side to recover all the segment keys. We do not distinguish between secret keys and public keys sent along with the content. We consider the symmetric-key trivial scheme proposed by Yuan et al. S as well as the public-key scheme ZFL proposed by Zhu et al. S. Regarding the key lengths, asymmetric algorithm keys must be longer for equivalent resistance to attack than symmetric keys. To compare systems with roughly the same level of security, 128 bits for the symmetric-key schemes are compared with 256 bits for ZFL public key scheme using elliptic curves G.

#### 2 Basic Scheme

In this section, we propose a basic key management scheme. The scheme design is based on an iterative application of a one-way function. The one-way chain idea has been earlier proposed by Lamport in [3] to construct a one-time password scheme. [2] generalizes the idea behind Lamport's scheme through the use of several one-way functions.

**Definition 1.** A key scheme respecting the SMLFE architecture is said simply secure if the knowledge of any secret key  $k_{i,j}$  associated to  $S_{i,j}$  does not permit to get the secret key of any higher segment  $S_{k,l}$  such that k > i or l > j.

The definition here tells that the access to a given low quality of the video content should not lead to the access of a higher quality of this video.

Let us consider that there are  $n \cdot t$  different segments in the enhancement layers. Then each different segment will be encrypted with a different key. More precisely, the content provider generates at random a 2*l*-bit master encryption key K. This key encrypts the segment  $S_{n,t}$  that is in the higher quality level for both parameters PSNR and bit-rate. Segment keys  $k_{i,j}$ , which encrypt lower quality are computed recursively as follow:

$$k_{i,j} = \mathsf{h}^{(n-i)}(\{K\}_L) \parallel \mathsf{h}^{(t-j)}(\{K\}_R) , \qquad (1)$$

where  $h : \{0,1\}^l \to \{0,1\}^l$  is a one-way (hash) function.  $h^{(n)}(x)$  denotes the result after applying *n* times the one-way function h to *x*.

The master key K is sent to the license server and encrypted content to distribution server.

At the other end of the process, a user acquires a given quality of the video. Let us consider that the requested quality is obtained by accessing segments below PSNR level s and below bit-rate r. Then, the license server computes using formula (

$$K_{r,s} = \mathsf{h}^{(n-r)}(\{K\}_L) \parallel \mathsf{h}^{(t-s)}(\{K\}_R),$$

and sends it to user in a license. From this key, the user is able to compute the  $r \cdot s$  keys of lower quality segments.

**Proposition 1.** The basic scheme is simply secure.

*Proof.* This immediately follows from the one-wayness of h. If an attacker knows a key  $k_{i,j}$ , finding  $k_{x,y}$  for some x > i or some y > j requires the inversion of the one-way function h which is assumed to be computationally infeasible.

We now discuss about the complexity of the scheme. From equation (1) we can see that it requires for a user at most n + t - 2 calls to a one-way function to compute all the segment keys. It also requires for the license server to maintain one key and whatever the quality of the scalable content to transmit only one key to user.

Another advantage of the basic scheme is the possibility to derive from one secret value two orthogonal segment decryption keys. The license server have then the possibility to use both parameters PSNR and bit-rate to answer a requested quality. In the ZFL scheme, public keys are distributed with the encrypted content, then the license server can later only use one parameter (i.e. PSNR) for varying the quality of the content. A security concern in SMLFE is the *collusion-resistance*. A user could purchase different low quality versions of a video and combine the encryption keys in order to compromise the full quality of the video. Encryption keys should then be constructed to protect high quality keys from collusions of low quality encryption keys. The basic scheme is not resistant to such collusion attacks.

# 3 Collusion Resistance

We now formalize the model of a collusive attack against a key management scheme for SMLFE.

**Definition 2 (Basis).** A basis of a set  $\mathcal{U}$ , containing different segment keys of the enhancement layers, is the minimal set of segment keys that permits to compute all other segment keys in  $\mathcal{U}$ .

*Remark 1.* A basis may contain one single key. Any set  $\mathcal{U}$  can be then represented by its basis  $(k_1, k_2, \ldots, k_m)$ .

**Definition 3 (Collusion-Resistance).** A key management scheme is called collusion-resistant (CR) to any set C containing low quality segments (PSNR or Bit-rate), if for every high quality segment  $S_{i,j} \notin C$ , the knowledge of all secrets associated to segments in C does not permit to obtain the secret associated to  $S_{i,j}$ .

The definition states that if a coalition of users has the keys associated to segments in  $C = \{S_1, S_2, \ldots, S_k\}$  then the sharing of secrets by a coalition should not permit to get an advantage of knowing segment keys that are not in C.

**Definition 4 (Orthogonality).** Two different segment  $S_{i,j}$  and  $S_{x,y}$  are orthogonal when the right to access one of the two segments does not necessarily give the right access the other and vice-versa. A sequence of k different segments is said orthogonal if any two different segments in the k-sequence are orthogonal.

**Definition 5 (Orthogonal Collusion-Resistance).** The collusion-resistance is orthogonal (OCR) if the colluders set keys is generated by at least 2 keys and the knowledge of any couple  $(k_{i,j}, k_{u,v})$  in the basis does not permit to obtain the secrets associated to any segment in S, with

$$\mathcal{S} = \left\{ \{ \mathsf{S}_{x,y} \mid i < x \le n \text{ or } j < y \le t \} \} \cap \{ \mathsf{S}_{x,y} \mid u < x \le n \text{ or } v < y \le t \} \right\}.$$

A scheme is called k-OCR if it is collusion-resistant to any set C generated by at most k orthogonal segments. Out of these general definitions about collusion-resistance we will only consider in this paper collusion with 2 orthogonal segment decryption keys.

The scheme described in previous section is not 2-OCR since knowing any two orthogonal keys permits to compute a higher quality segment key. For instance, if an adversary query two orthogonal decryption keys of the form

$$k_{n,j} = \{K\}_L \parallel \mathsf{h}^{(t-j)}(\{K\}_R), \qquad k_{i,t} = \mathsf{h}^{(n-i)}(\{K\}_L) \parallel \{K\}_R$$

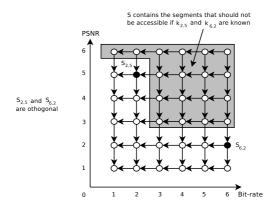


Fig. 2. Orthogonality and Orthogonal Collusion-Resistance

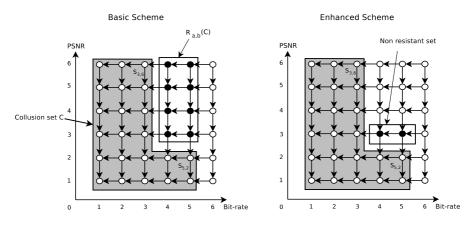


Fig. 3. Non 2-OCR Segments in Basic and Enhanced Scheme

he is able to reconstruct the key  $K = \{K\}_L \parallel \{K\}_R$  and to access all segments in the enhancement layers.

The segments that do not resist to a collusion set generated by  $(k_{i,j}, k_{u,v})$  is the rectangle  $\mathcal{R}_{a,b}(\mathcal{C})$  containing the segments  $\{S_{a,b}\}$  such that

$$\min(i, u) < a \le \max(i, u); \quad \min(j, v) < b \le \max(j, v).$$

There are then  $(\max(i, u) - \min(i, u)) \cdot (\max(j, v) - \min(j, v))$  segments that are vulnerable to the collusion. All other segments not in  $\mathcal{C}$  and outside the rectangle  $\mathcal{R}_{a,b}(\mathcal{C})$  remain resistant to the collusion. Figure 4 illustrates an example where  $\mathcal{C} = (k_{3,6}, k_{5,2}).$ 

The maximum number of the non collusion-resistant segments is reached for  $C = (k_{n,1}, k_{1,t})$  where there are  $(n-1) \cdot (t-1)$  non collusion-resistant segments keys. In the next section we proposes a scheme that better resists to a 2-orthogonal collusion by reducing the number of non collusion-resistant segments.

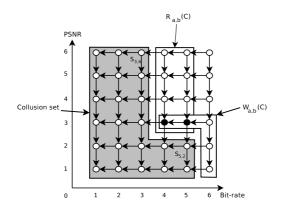


Fig. 4. Non Collusion-Resistant Set

### 4 Enhanced Scheme

In this section, we propose an enhanced key management scheme. We brought some modifications to the basic scheme to get better collusion-resistance properties.

The content provider generates randomly two 2l-bits master encryption keys  $K_1$ and  $K_2$ .  $K_1 \oplus K_2$  is the key that encrypts the segment  $S_{n,t}$  and the values of n and t are made public. Segment keys  $k_{i,j}$  are computed by the license server as follows:

- compute  $k_{i,j}^1$  from  $K_1$  using equation (II),
- compute next z(i, j):

$$z(i,j) = \begin{cases} 0 & \text{if } n-i=0 \text{ and } t-j=0, \\ 2^{n-i}-1 & \text{if } t-j=0 \text{ and } n-i\neq 0, \\ 2^{t-j}-1 & \text{if } n-i=0 \text{ and } t-j\neq 0, \\ 2^{n-i}+2^{t-j}-2 \text{ for } 1\leq i< n \text{ and } 1\leq j< t. \end{cases}$$
(2)

- compute  $k_{i,j}^2$  from  $K_2$ :

$$k_{i,j}^2 = \mathsf{h}_2^{(z(i,j))}(K_2)$$
 where  $\mathsf{h}_2 : \{0,1\}^{2l} \to \{0,1\}^{2l}$ ,

- segment key  $k_{i,j}$  is calculated by generating  $k_{i,j} = k_{i,j}^1 \oplus k_{i,j}^2$ .

For the quality corresponding to PSNR level s and bit-rate level r, license server delivers to user two keys  $K_{r,s}^1$  and  $K_{r,s}^2$  such that:

$$K_{r,s}^{1} = \mathsf{h}_{1}^{(n-r)}(\{K_{1}\}_{L}) \| \mathsf{h}_{1}^{(t-s)}(\{K_{1}\}_{R}), \qquad K_{r,s}^{2} = \mathsf{h}_{2}^{(z(r,s))}(K_{2}),$$

where z(r, s) is calculated using formula (2).

From these two keys, the user is able to compute all necessary  $r \cdot s$  segment decryption keys. Let us suppose that the user want to compute a lower-segment

key  $k_{\alpha,\beta}$ . From  $k_{\alpha,\beta}^1$ , it is easy for the user to guess the number of times that the one-way function  $h_1$  should be applied to the  $K_{rs}^1$  knowing  $\alpha, \beta, r$  and s. Indeed,

$$k_{\alpha,\beta}^{1} = \mathsf{h}_{1}^{(r-\alpha)}(\{K_{r,s}^{1}\}_{L}) \| \mathsf{h}_{1}^{(s-\beta)}(\{K_{r,s}^{1}\}_{R}) .$$

However, it is not as easy for the second part of the key  $k_{\alpha,\beta}^2$ . The formula (2) gives the method to compute the value of  $z(\alpha, \beta)$  in case the derivation is done using the master key  $K_2$ . The derivation process can also be performed in a relative way, i.e from a key  $K_{r,s}^2$  where  $K_{r,s}^2$  is not necessarily the second master key. Let  $\Delta_{(r,s)} z(\alpha,\beta)$  an integer that depends on the distance between  $S_{r,s}$  and  $\mathsf{S}_{\alpha,\beta}$ , the key  $k_{\alpha,\beta}^2$  is calculated as follow:

 $\begin{array}{l} - \mbox{ compute } z(r,s) \mbox{ for } K^2_{r,s} \mbox{ using formula (2), e.g. } z(r,s) = 2^{n-r} + 2^{t-s} - 2 \ , \\ - \mbox{ compute } z(\alpha,\beta) \mbox{ for } k^2_{\alpha,\beta} \mbox{ using formula (2), e.g. } z(\alpha,\beta) = 2^{n-\alpha} - 1 \ , \\ - \mbox{ compute } \Delta_{(r,s)} z(\alpha,\beta) = z(\alpha,\beta) - z(r,s) \ , \\ - \mbox{ generate } k^2_{\alpha,\beta} = {\sf h}_2^{(\Delta_{(r,s)} z(\alpha,\beta))}(K^2_{r,s}) \ . \end{array}$ 

The user needs to know n and t (public values) to compute each lower segment key  $k_{\alpha,\beta}^2$ . Finally,  $k_{\alpha,\beta}$  is calculated by generating  $k_{\alpha,\beta} = k_{\alpha,\beta}^1 \oplus k_{\alpha,\beta}^2$ . In this scheme, the user needs at most  $n + t + 2^{n-r} + 2^{t-s} - 4$  calls to one-way

functions for the computation of all the segment keys. In addition, all segment keys are derived recursively from two common master keys, thus the license server maintains two keys and whatever the quality requested by user only two keys are transmitted. A use case example is given in the Appendix A

We figure out that there are particular cases where the enhanced scheme is 2-OCR. These are cases where all the segment keys that should not be accessible remain resistant to a 2-orthogonal collusion attack. This is formalized by proposition 2

**Proposition 2.** Enhanced scheme is 2-OCR for any set  $\mathcal{C} = (k_{i,i}, k_{u,v})$  such that  $n - i \neq t - j$  and either n - i = t - v or t - j = n - u.

#### *Proof.* See Appendix A.

Proposition 2 states that the enhanced scheme is 2-orthogonal collusion-resistant in the case n-i=t-v or t-j=n-u. There are no other cases where this scheme is 2-orthogonal collusion-resistance. To compare it with the basic scheme, we studied the number of segments keys that are not 2-OCR in the general case, i.e when  $n-i \neq t-v$  and  $t-j \neq n-u$ . Proposition 3 gives the maximum number of the non 2-OCR segments keys and figure 3 illustrates an example where non 2-OCR segments are filled with black.

**Proposition 3.** For all (n > 1, t > 1), the number of segments keys that are not resistant to any 2-orthogonal collusion is bounded by  $\left\lfloor \frac{t}{2} \right\rfloor^2$  if  $n \leq t$  or  $\left\lfloor \frac{n}{2} \right\rfloor^2$ if n > t.

*Proof.* See Appendix A

In proposition 3 the number of non resistant segments keys is bounded by a smaller value. Enhanced scheme has therefore better collusion-resistance properties than basic scheme.

### 5 Conclusion

We have proposed two symmetric-key management schemes for protecting a multilayer scalable part of an MPEG4-FGS video content. Compared to previous work, our schemes reduces the number of keys managed by a license server and the number of keys needed by the user to access the protected scalable video. On the top of requiring less computational resources, symmetric algorithms permit to handle short keys as well, which makes the proposed schemes attractive and practical.

Besides, we focused on collusion-resistance and enhanced scheme has interesting collusion-resistance property. An open problem is the design of a symmetrickey system that is at least as efficient as the enhanced scheme and which remains fully secure no matter how many orthogonal segment keys are at the disposal of the attacker.

Acknowledgments. We thank Alain Durand, Marc Joye and Davide Alessio for valuable discussions and suggestions. We also thank Eric Diehl for helpful comments.

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### A Appendix

#### A.1 Enhanced Scheme Example

In this section, we give an example for the enhanced scheme where n = 5 and t = 4. The content provider generates two random 128-bit keys  $K_1$  and  $K_2$ , which are delivered to license server.  $K_1 \oplus K_2$  is the key that encrypts the segment  $S_{5,4}$ . Segment  $S_{i,j}$ , where  $1 \le i \le 5$  and  $1 \le j \le 4$ , is encrypted with a key that results from a XOR between  $K_{i,j}^1$  and  $K_{i,j}^2$ . Table 2 illustrates how these values are computed.

Computation of $K_{i,j}^1$								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$								
Computation of $K_{i,j}^2$								
		Co	mp	utation of	$K_{i,}^2$	j		
$h_2^{15}(K_2)$	<i>←</i>			$h_2^3(K_2)$	- ,	4	←	$K_2$
$\downarrow$		$\stackrel{h_2^7(K_2)}{\downarrow}$	~		~ ~	$h_2(K_2)$ $\downarrow$		Ļ
$\stackrel{\downarrow}{\stackrel{h_{2}^{16}(K_{2})}{\stackrel{\downarrow}{\downarrow}}}$	←	$\begin{array}{c} h_2^7(K_2) \\ \downarrow \\ h_2^8(K_2) \\ \downarrow \end{array}$	$\leftarrow$	$\stackrel{h_2^3(K_2)}{\downarrow}$	← ←	$\begin{array}{c} h_2(K_2) \\ \downarrow \\ h_2^2(K_2) \\ \downarrow \end{array}$	$\leftarrow$	$\stackrel{\downarrow}{\stackrel{h_2(K_2)}{\stackrel{\downarrow}{\stackrel{\downarrow}{\stackrel{\downarrow}{\stackrel{\downarrow}{\stackrel{\downarrow}{\stackrel{\downarrow}{\stackrel{\downarrow}{$

 Table 2. Segment Decryption Keys

One way functions  $h_1$  and  $h_2$  can use MD5 [7], which provides 128-bit ouput size. Computing all the segment encryption keys will then require 41 MD5 hashes and 20 XOR operations.

For the quality associated to PSNR level 3 and bit-rate level 2, license server delivers to user two keys:  $K_{2,3}^1 = \mathbf{h}_1^3(L) \parallel \mathbf{h}_1(R)$  and  $K_{2,3}^2 = \mathbf{h}_2^8(K_2)$ . The user is next able to compute  $K_{2,2}^1$ ,  $K_{2,1}^1$ ,  $K_{1,3}^1$ ,  $K_{1,2}^1$  and  $K_{1,1}^1$  by applying  $\mathbf{h}_1$  to  $K_{2,3}^1$  as illustrated in the table 2 Knowing the value of n and t and the coordinates of  $S_{2,3}$  i.e. (2,3), the user finds the number of times the one-way function  $\mathbf{h}_2$  is applied to  $K_{2,3}^2$  and obtains the other keys  $K_{2,2}^2$ ,  $K_{2,1}^2$ ,  $K_{1,3}^2$ ,  $K_{1,2}^2$  and  $K_{1,1}^2$ . Finally, segment decryption keys are calculated by generating  $K_{i,j} = K_{i,j}^1 \oplus K_{i,j}^2$  with  $1 \le i \le 2$  and  $1 \le j \le 3$ . In this example, the user performs 19 MD5 hashes and 6 XOR operations.

<sup>&</sup>lt;sup>1</sup> Given that a smaller input/output size is needed for  $h_1$ , we can apply the hash function to 64 bits part of the key to be hashed and the resulting output is truncated by selecting leftmost 64 bits and discarding the rightmost 64 bits output.

#### A.2 Proof of Proposition 2

Since  $k_{i,j}$  and  $k_{u,v}$  are orthogonal either i < n or j < t. Let suppose that i < n. If n - i = t - v and t - j = n - u then from (2) we have

$$k_{i,j}^{2} = k_{u,v}^{2} = \begin{cases} \mathsf{h}_{2}^{(2^{n-i}-1)}(K_{2}), \text{ if } t-j = 0\\ \mathsf{h}_{2}^{(2^{n-i}+2^{t-j}-2)}(K_{2}), \text{ otherwise }. \end{cases}$$
(3)

If  $n-i \neq t-v$  or  $t-j \neq n-u$  then  $\exists (m_1, m_2) \in \{1, \ldots, 2^{n-1}+2^{t-1}-2\}^2$  such that  $k_{i,j}^2 = \mathsf{h}_2^{(m_1)}(K_2)$  and  $k_{u,v}^2 = \mathsf{h}_2^{(m_2)}(K_2)$  with either  $m_1 < m_2$  or  $m_1 > m_2$ . The knowledge of one key enables to compute the other one. It is then sufficient to consider  $k_{i,j}^2$  by supposing  $m_1 < m_2$ .

 $k_{i,j}^1$  and  $k_{u,v}^1$  do not reveal the keys that are outside  $\mathcal{R}_{a,b}$ . It suffices then to find the keys in  $\mathcal{R}_{a,b}$  that can be computed when  $k_{i,j}^2$  is known. In the case n-i=t-v or t-j=n-u,  $\mathcal{R}_{a,b}$  contains the segment  $S_{a,b}$  such that

$$\min(i,j) < a, b \le \max(i,j)$$

\$

$$\min(n-i,t-j) \le n-a, t-b < \max(n-i,t-j) .$$

Consider that  $n - i = \max(n - i, t - j)$ . The segment key  $\{k_{a',b'}\}_2 \in \mathcal{R}_{a,b}$  that has the greatest hash exponent verifies (n - a', t - b') = (n - i - 1, n - i - 1)

$$\{k_{a',b'}\}_2 = \begin{cases} K_2, \text{ if } n-i-1=0\\ \mathsf{h}_2^{(2^{n-i}-2)}(K_2), \text{ otherwise }. \end{cases}$$
(4)

From (B) and (A) we have then  $\{k_{i,j}\}_2 = \mathsf{h}_2^{(x)}(K_2)$  and  $\{k_{a',b'}\}_2 = \mathsf{h}_2^{(y)}(K_2)$ with y < x. Since  $\mathsf{h}_2$  is non-invertible  $\Rightarrow NCRS_{\mathcal{C}} = \emptyset$ .

#### A.3 Proof of Proposition 3

Collusion set is  $\mathcal{C} = (k_{i,j}, k_{u,v})$ .  $k_{i,j}^1$  and  $k_{u,v}^1$  reveal first part of the keys in  $\mathcal{R}_{a,b}(\mathcal{C})$ . Let  $k_{a,b}^2$  s.t.  $(a,b) \in \{1,\ldots,n\} \times \{1,\ldots,t\}$  that can be computed from  $k_{i,j}^2 = \mathsf{h}_2^{(2^{n-i}+2^{t-j}-2)}(K_2)$ . We have then

$$2^{n-i} + 2^{t-j} \le 2^{n-a} + 2^{t-b}.$$
(5)

Let suppose that  $n - i \leq t - j$ , therefore three cases are possible:

- if  $\max(n-a, t-b) > t-j$ , (b) has always a solution (because  $\sum_{i=0}^{k} 2^i < 2^{k+1}$ ), - if  $\max(n-a, t-b) = t-j$ , (b) has a solution only if  $\min(n-a, t-b) \ge n-i$ ,
- if  $\max(n-a, t-b) < t-j$ , (5) has no solution.

The solution set verifies then

$$\max(n-a,t-b) > \max(n-i,t-j)$$
or

 $\left(\max(n-a,t-b) = \max(n-i,t-j) \text{ and } \min(n-a,t-b) \ge \min(n-i,t-j)\right).$ 

The keys that can be computed from  $k_{ij}^2$  and  $k_{uv}^2$  are associated to the segments in  $\mathcal{W}_{a,b}$  where

$$\mathcal{W}_{a,b} = \left\{ \mathsf{S}_{a,b} \notin \mathcal{C} \middle| \big( \max(n-a,t-b) > \max(n-i,t-j) \big) \text{ or} \right. \\ \left( \max(n-a,t-b) = \max(n-i,t-j) \text{ and } \min(n-a,t-b) \ge \min(n-i,t-j) \big) \right\},$$

and non collusion-resistant set  $NCRS_{\mathcal{C}} = \mathcal{R}_{a,b} \cap \mathcal{W}_{a,b}$ . We now give the bound  $NCRS_{\mathcal{C}}$  in the worst case.

- if  $n \ge t$ , ∃  $j \in \{1, t-1\}$  s.t.  $NCRS_{\mathcal{C}}$  has maximum size for  $\mathcal{C}' = (\mathsf{S}_{n,j}, \mathsf{S}_{1,t})$ , - if n < t, ∃  $i \in \{1, n-1\}$  s.t.  $NCRS_{\mathcal{C}}$  has maximum size for  $\mathcal{C}' = (\mathsf{S}_{n,1}, \mathsf{S}_{i,t})$ ,

$$\Rightarrow |NCRS_{\mathcal{C}'}| \le \begin{cases} \max_{\substack{j=1\\n-1\\i=1}}^{t-1} ((n-t+j-1)\cdot(t-j)), \text{ if } t = \min(n,t), \\ \max_{i=1}^{n-1} ((t-n+i-1)\cdot(n-i)), \text{ if } n = \min(n,t). \end{cases}$$

Let  $n = \max(n, t)$ , then  $\max_{\substack{j=1\\j=1}}^{t-1} \left( (n-t+j-1) \cdot (t-j) \right) = k \cdot (k-1) \le k^2$  if n is even (i.e. n = 2k); or  $\max_{\substack{j=1\\j=1}}^{t-1} \left( (n-t+j-1) \cdot (t-j) \right) = k^2$  if n is odd (i.e. n = 2k+1). This implies  $\max_{\substack{j=1\\j=1}}^{t-1} \left( (n-t+j-1) \cdot (t-j) \right) \le k^2 = \lfloor \frac{n}{2} \rfloor^2$ . The same result holds for t when  $t = \max(n, t) \Rightarrow \forall \mathcal{C}, |NCRS_{\mathcal{C}}| \le \max(\lfloor \frac{n}{2} \rfloor^2, \lfloor \frac{t}{2} \rfloor^2)$ .

# TWISTER – A Framework for Secure and Fast Hash Functions

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Abstract. In this paper we present TWISTER, a new framework for hash functions. TWISTER incorporates the ideas of wide pipe and sponge functions. The core of this framework is a – very easy to analyze – Mini-Round providing both extremely fast diffusion as well as collision-freeness for one Mini-Round. The total security level is claimed to be not below  $2^{n/2}$  for collision attacks and  $2^n$  for 2nd pre-image attacks. TWISTER instantiations are secure against all known generic attacks. We also propose three instances TWISTER-*n* for hash output sizes n = 224, 256, 384, 512. These instantiations are highly optimized for 64-bit architectures and run very fast in hardware and software, e.g TWISTER-256 is faster than SHA2-256 on 64-bit platforms and TWISTER-512 is faster than SHA2-512 on 32-bit platforms. Furthermore, TWISTER scales very well on low-end platforms.

**Keywords:** Hash Function, Sponge Function, AES, HAIFA, Wide pipe, Randomized Hashing.

### 1 Introduction

One of the most used primitives in modern cryptography are hash functions. A hash function  $H : \{0,1\}^* \to \{0,1\}^n$  takes an input of arbitrary size and computes an *n*-bit fingerprint out of it. Established security requirements for cryptographic hash functions are collision-, pre-image and 2nd pre-image resistance – but ideally, cryptographers expect a good hash function to behave like a random function. Nearly all iterative hash functions are designed using the MERKLE-DAMGÅRD construction **[16] 31**. A MERKLE-DAMGÅRD hash function is an iterated hash function that uses a fixed length compression function  $C : \{0,1\}^{n_c} \times \{0,1\}^m \to \{0,1\}^{n_c}$  where  $n_c$  is the size of the chaining value and m the size of a message block. We have  $n = n_c$  for hash functions using the MERKLE-DAMGÅRD construction. By assuming a padded message  $M = (M_1, \ldots, M_l), |M_i| = m, 1 \le i \le l$  and an internal chaining value  $h_i \in \{0,1\}^{n_c}$  ( $h_0$  is called the initial value) the computation of the hash value  $h_l$  for such a message M is as follows:  $h_i = C(h_{i-1}, M_i), i = 1, \ldots, l$ .

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The main benefit of the MERKLE-DAMGÅRD transformation is that it preserves collision resistance: if the compression function C is collision resistant, then so is the hash function. Unfortunately, this result does not extend to preand 2nd preimage resistance. Recent results highlight some intrinsic limitations of the MERKLE-DAMGÅRD approach. This includes being vulnerable to multicollision attacks [22], long 2nd preimage attacks [24], and herding [23]. Even though the practical relevance of these attacks is unclear, they highlight some security issues which designers are well advised to avoid or take care of.

**Related Work.** Most popular hash functions such as MD5 [38], SHA-0 [34] or SHA-1 [33] have weaknesses in their design, leading to a huge amount of attacks [6], [7], [13], [18], [37], [40], [41], [42]. But also most new hash functions [2], [21], [25], [26] which try to take care for weaknesses in the MERKLE-DAMGÅRD -construction itself were broken soon after their publications [29], [36], [35], [30], [20].

The concept of sponge functions [4] uses for example a big internal state that absorbs a message of arbitrary length and that later squeezes out a hash value of variable size. RADIOGATÚN [3] with XOR sponges and GRINDAHL [26] with truncate-overwrite sponges are the first hash function that use this framework. GRINDAHL was shown to be vulnerable to several attacks [35], [20].

**Our contribution.** The design of secure and practical hash functions is of great interest since most hash functions have been broken. Due to the SHA-3 [32] competition many new proposals for hash function primitives will be published in the next months. In this paper we present a new hash function framework called TWISTER. Our proposal is based on a sponge construction [5] as well as on an wide pipe approach [27]. It also includes randomized hashing as proposed in the HAIFA framework [8]. The main goal of our approach is to present a fast, secure hash function which is flexible and simple to analyze. We can show that one cannot find a collision after one so called Mini-Round and that we obtain full diffusion after two application of a Mini-Round.

More precise, it uses XOR-sponges with a big internal state as proposed in [27]. The randomized hashing is built in via a salt value - a method proposed in the HAIFA framework [8]. In some sense we learn from the GRINDAHL design [26], but our approach is different in many ways. We take advantage of the well studied basic operations of AES [15] and adopt several of them, including some optimization techniques.

Due to recent breakdowns of many proposed hash functions we analyze the resistance of TWISTER against all known generic attacks on hash functions. We find, that the TWISTER framework resists all of them if the size of the internal chaining value is at least double the size of the hash output.

**Outline.** In Section 2 we present an informal description of the TWISTER hash function family. Then, in Section 3, we give an detailed description of the general TWISTER framework and we propose three instances: TWISTER-256, 384 and 512 for 256, 384 and 512 bit hash sizes. In Section 4 we discuss some security issues

of TWISTER. Finally, we conclude our paper in Section **5** Some performance aspects of TWISTER are presented in Appendix **A**.

## 2 An Informal Description of the TWISTER Framework

In this section, we give a general description of the TWISTER hash function family and its building blocks. For a complete description of TWISTER, see the formal specifications in Section 3 TWISTER follows a very simple and clear design goal. It consists of an iterated compression function and of an output function.

### 2.1 The Compression Function

The compression function of TWISTER consists of building blocks called Mini-Rounds which are grouped into Maxi-Rounds. Each Mini-Round is a combination of well studied primitives, which are easy to analyze and fast to implement in software and hardware. The instances of the TWISTER hash function family differ only in their construction of a Maxi-Round. We will give an informal description of these principles below.

A Mini-Round consists of the following primitives:

- MessageInjection inserts a 64-bit message block into the last row of the state matrix,
- SubBytes applies a non-linear S-box table look up on each byte of the state matrix in parallel,
- AddTwistCounter XOR's a round dependent counter into the second column of the state matrix,
- ShiftRows rotates row i by i 1 positions to the left,
- $\mathsf{MixColumns}$  applies a linear diffusion on each column of the state matrix in parallel

A visualization of a mini round is shown in Appendix B

A Maxi-Round contains between three and four Mini-Rounds and zero or one blank rounds. A blank round is a Mini-Round with no message input, which is equivalent to the all zero message block. Each Maxi-Round uses also a feed forward operation, i.e., the state before a Maxi-Round is feed forwarded with the state after a Maxi-Round. Figure I gives a high level description of a Maxi-Round.

### 2.2 The Output Function

The Output-Round of TWISTER contains a global feed forwards as well as some Mini-Rounds depending on the size of the hash output. First a Mini-Round is applied on the state  $H_{i-1}$ , then the resulting state is XOR'ed with  $H_{i-1}$  another Mini-Rounds is applied which gives the state  $H_i$ . Let  $H_f$  be the final state after the last compression function call. A 64-bit output stream *out<sub>i</sub>* is then obtained

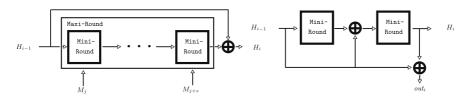


Fig. 1. Left: A Maxi-Round

Right: An Output-Round

by XORing the first column of  $H_i$  with the first column of  $H_f$ . This procedure takes place until the needed amount of output bits are obtained. The last output stream can be varied between 32 bits and 64 bits by taking only the first half of *out<sub>i</sub>*. This allows to vary the output size for a huge amount of applications. Figure  $\square$  gives a high level description of a Output-Round.

## 3 Specification: The TWISTER Hash Function Family

In this section we present our hash function. We start off with a description of the general design strategy. The design is based on a block cipher that is iterated using the Davies-Meyer (DM) mode of operation [10]. TWISTER is a byte-oriented framework that operates on a square state matrix. The building block of the block cipher is called Mini-Round. It takes a sub portion of the message and processes it into the state S whereas S is a  $N \times N$  byte-matrix,  $N \in \mathbb{N}$ . N should be at least of size 4 to obtain a valuable structure at all. After two Mini-Rounds, the state is guaranteed to have full diffusion. Also, two subsequent iterations of the Mini-Round is can be proved to be collision free. See Section 4 for a detailed discussion on our security issues.

After processing the padded message (i.e. the message is completely absorbed by the state S), the output follows. This technique follows the design ideas of the sponge function [4] by not presenting the complete internal state to the attacker at once but slice by slice.

The following notations are used in the following:

$\mathcal{S} = (S_{i,j})_{1 \le i,j \le N}$	internal state matrix
$\mathcal{C} = (C_{i,j})_{1 \le i,j \le N}$	internal checksum matrix
$N$ $\sim$	number of rows and columns of the internal state
	matrix
$msg_{size}$	size of unpadded message (in bits)
$R_{total}$	number of total Mini-Rounds per compression
	function
$R_{msg}$	number of N-byte blocks processed per compression
	function
m	size of the padded message, measured in
	$N\cdot R_{msg}$ -byte blocks, i.e. the number of compression

function calls needed to absorb the message into the
state $\mathcal{S}$
padded message to be handled by the TWISTER hash
function
unpadded message
size of the hash value measured in N-byte blocks, i.e.
$out = n/(8 \cdot N)$
size of the hash value in bits, i.e. $n = out \cdot 8 \cdot N$ ,
where $n \leq (8 \cdot N)^2$
number of $N$ -byte blocks of the hash output
TwistCounter

#### **3.1** TWISTER Components

This section describes in detail the TWISTER components.

The State S. TWISTER operates on a square state matrix  $S = (S_{i,j}), 1 \leq i, j \leq N$ , consisting out of N rows and columns, where each cell  $S_{i,j}$  represents one byte.

$S_{1,1}$	$S_{1,2}$		$S_{1,N}$
$S_{2,1}$	$S_{2,2}$		$S_{2,N}$
:	:	·	:
$S_{N,1}$	$S_{N,2}$		$S_{N,N}$

Notation:  $S_{(i \to)} := (S_{i,1}, \ldots, S_{i,N})$  denotes the *i*-th row vector and  $S_{(j \downarrow)} := (S_{1,j}, \ldots, S_{N,j})$  the *j*-th column vector.

**Checksum** C. The checksum enlarges the state of TWISTER-384 and TWISTER-512 to stick to our wide pipe design [27] decision. In other words: using the checksum we can double the internals state.

The checksum is as the state S a square checksum matrix  $C = (C_{i,j}), 1 \leq i, j \leq N$ , consisting out of N rows and columns, where each cell  $C_{i,j}$  represents one byte.

$C_{1,1}$ $C_{1,2}$		$C_{1,N}$
$C_{2,1}$ $C_{2,2}$		$C_{2,N}$
: :	•.	:
$C_{N,1} C_{N,2}$		$\frac{\cdot}{C_{N,N}}$

**TwistCounter**  $\phi$ . The TwistCounter  $\phi$  is a unsigned 64 bit integer that is added and decreased within a Mini-Round to prevent slide attacks.

#### 3.2 The Compression Function

The TWISTER-hash function calls the compression function using the DM mode of operation. After the message is absorbed in the internal state the output function is called for every N bytes of output.

Recall that the compression function of TWISTER consists of building blocks called Mini-Rounds which are grouped into Maxi-Rounds. The instances of the TWISTER hash function family differ only in their construction of a Maxi-Round.

The compression function takes a 512-bit block and processes them into the internal state matrix S. As a Maxi-Round only indicates the position of the local feed forward XOR-operation we will normally only discuss a compression function as a set of Mini-Rounds. The local feed-forward operation is an optional security feature and is discussed in Section 4. More general, the compression function works as follows. Let R be the number of Mini-Rounds in a compression function. (Note: In Figure 2 we have R = 9.)

TWISTER-224 and TWISTER-256. The TWISTER-224 and TWISTER-256 compression function consists of three Maxi-Rounds. Each Maxi-Rounds is followed by a feed-forward XOR-operation. The first and second Maxi-Round consist of three Mini-Rounds. The last Maxi-Round consists of two Mini-Rounds and one blank round. Figure 2 illustrates the compression function.

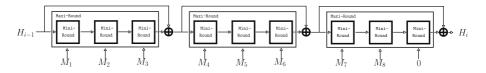


Fig. 2. The compression function of TWISTER-224 and TWISTER-256

TWISTER-384 and TWISTER-512. The TWISTER-384 and TWISTER-512 compression function consists of three Maxi-Rounds, too. The first Maxi-Rounds consists of three Mini-Rounds. The second Maxi-Rounds consists of a Mini-Round followed by a blank round and an other Mini-Round. The last Maxi-Rounds consists of three Mini-Rounds followed by a blank round. Figure 3 illustrates the compression function.

Mini-Round. The Mini-Round is the underlying building-block of any TWISTER hash function. It's main purpose is to inject the message (Message-Injection) and to take care for the diffusion of the state matrix S. It is visualized in Figure Twister can handle at most  $2^{64}$  Mini-Rounds. This limitation causes by the Add-TwistCounter operation where a 64-bit counter is added. Each Mini-Round can process 64 bit of message data. Therefore, with a native usage of a Mini-Round it is possible to process up to  $2^{64} \cdot 64$  message bits. If this limitation became in the future a real world issue it is possible to increase the size of the TwistCounter to 128 bit with almost no performance loss.

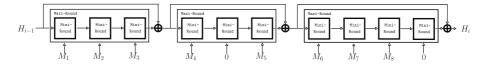


Fig. 3. The compression function of TWISTER-384 and TWISTER-512

Message injection. A 64 bit message block m is inserted (via XOR  $\oplus$ ) into the last row. By using the notation  $m = (m[1], \ldots, m[N])$  whereas the length of m[N] is one byte, and

$$S_{(\rightarrow j)} \oplus m := (S_{1,j} \oplus m[1], \dots, S_{N,j} \oplus m[N])$$

we define the message injection process by

$$S_{(\to 1)} = S_{(\to N)} \oplus m.$$

AddTwistCounter. The TwistCounter  $\phi$  is a unsigned 64 bit integer. The initial state is the maximum value (0xFFFFFFFFFFFFFFFF). By using the notation  $\phi = (\phi[1], \ldots, \phi[N])$  whereas the length of  $\phi[N]$  is one byte, and  $\phi[1]$  is the most significant byte of  $\phi$ . The counter is added byte by byte - via the XOR operation - into the second column of the state S.

$$S_{2,\downarrow} \oplus \phi := S_{2,1} \oplus \phi[1], \dots, S_{2,N} \oplus \phi[N])$$

We define the TwistCounter addition by

$$S = S_{2,\downarrow} \oplus \phi$$

After the addition  $\phi$  is decreased by one.

SubBytes. The function is defined as an bijection

$$SubBytes: \{0,1\}^8 \longrightarrow \{0,1\}^8$$

and is used as an S-box for each byte. It should, among other properties, be highly non-linear. A discussion on how to obtain such cryptographically strong S-boxes (for 8x8 S-boxes) can be found in [43]. TWISTER uses the well known and studied AES S-box. It can be found in [15].

We define the SubBytes operation by

$$S_{i,j} = SB(S_{i,j}) \quad \forall i, j.$$

ShiftRows. ShiftRows is a cyclic left shift similar to the ShiftRows operation of AES. It rotates row j by  $(j-1) \mod N$  bytes to the left.

We define the ShiftRows operation by

$$S_{(i,j-1)} := S_{(i,j)} \quad \forall i, j.$$

*MixColumns*. The MixColumn step is a permutation operation on the state. It applies a  $N \times N$ -MDS A (a maximum distance separable matrix as defined below) to each column, i.e. performs the operation  $A \cdot S_{(j\perp)}$ .

**Definition 1.** An [n,k,d] code with generator matrix

$$G = [I_{k \times k} \ A_{k \times (n-k)}]$$

is an MDS code if every square submatrix of A is non singular. The matrix A is called a MDS-matrix.

Our chosen MDS matrix is cyclic, i.e., its i-th row can be obtained by a cyclic right rotation of (02 01 01 05 07 08 06 01) by *i* entries. It has a branch number of 9 meaning that if two 8 byte input vectors differ in  $1 \le k \le 8$  bytes, the output of MixColumns differs in at least 9 - k bytes. The  $8 \times 8$ -MDS matrix used for all proposed instances of TWISTER is:

$$MDS = \begin{pmatrix} 02 & 01 & 01 & 05 & 07 & 08 & 06 & 01 \\ 01 & 02 & 01 & 01 & 05 & 07 & 08 & 06 \\ 06 & 01 & 02 & 01 & 01 & 05 & 07 & 08 \\ 08 & 06 & 01 & 02 & 01 & 01 & 05 & 07 \\ 07 & 08 & 06 & 01 & 02 & 01 & 01 & 05 \\ 05 & 07 & 08 & 06 & 01 & 02 & 01 & 01 \\ 01 & 05 & 07 & 08 & 06 & 01 & 02 & 01 \\ 01 & 01 & 05 & 07 & 08 & 06 & 01 & 02 \end{pmatrix}$$

All of the byte entries are considered to be elements of  $\mathbb{F}_{2^8}$ . An element  $\sum_{i=0}^{7} a_i x^i \in \mathbb{F}_{2^8}$  is represented by  $\sum_{i=0}^{7} a_i 2^i$ . The reduction polynomial m(x) of  $\mathbb{F}_{2^8}$  is defined as

$$m(x) = x^8 + x^6 + x^3 + x^2 + 1.$$
 (1)

Properties of MDS matrices/codes can be found e.g. in [28]. A discussion on how to obtain suitable MDS matrices can be found in the full version of the paper.

Maxi-Round. A Maxi-Round contains of several Mini-Rounds, blank round and a optional checksum updates. We defined a checksum update operation as

$$C_{(i,\downarrow)} = C_{(i,\downarrow)} \oplus C_{(i+1,\downarrow)} \boxplus S_{(i,\downarrow)}$$

Maxi-Rounds use also a feed forward operation where the state before a Maxi-Round  $S^k$  is feed forwarded with the state after a Maxi-Round  $S^{k+1}$ . We defined the feed forward operation as

$$S_{(i,j)} := S_{(i,j)}^k \oplus S_{(i,j)}^{k+1} \quad \forall i, j.$$

Figure 11 illustrates a Maxi-Round.

#### 3.3 Postprocessing

This section describes the TWISTER finalization process.

Postprocessing shall take place after the message processed by the compression function. This post processing consists of two steps:

- 1. Padding the message M
- 2. Message digest computation

**Padding.** The message, M, shall be padded before hash computation begins. For the padding, the well known 10-padding rule is applied (see e.g. 33)

Message digest computation. The Output-Round computes the message digest. It contains a global feed forwards as well as some Mini-Rounds depending on the size of the hash output. For every 64 message digest bits Mini-Round is applied on the state S, then the resulting state is XOR'ed with  $S^{k-1}$  another Mini-Rounds is applied which gives the state  $S^k$ . Let  $S^f$  be the final state after the last compression function call. A 64-bit output stream  $out_i$  is then obtained by XORing the first column of  $S^k$  with the first column of  $S^{k-1}$ . This procedure takes place until the needed amount of message digest bits are obtained. The last output stream can be varied between 32 bits and 64 bits by taking only the first half of  $out_i$ . This allows to vary the output size for a huge amount of applications. Figure 1 illustrates the Output-Round.

#### 4 Security

In this section we analyze the security of TWISTER and show that it is resistant to all known generic attacks.

#### 4.1 Generic Attacks

Length-Extension Attacks. Given an MERKLE-DAMGÅRD based hash function H. If one can find a collision for two messages M, M' with  $M \neq M'$ , such that H(M) and H(M') collide, then one can apply a length extension attack. For any message N one can easily produce a collision for M||N and M'||N as H(M||N) = H(M'||N). Our padding rule avoids such type of attacks since we concatenate the length of the message to the message itself. Another attack can be as follows. For a known hash value H(M) one can compute the hash value H(M||X||N) for any suffix N, if the length of an unknown message M is known as well as the padding X of M. We prevent TWISTER to this kind of attack by two countermeasures: (i) By knowing only the hash value an attacker can not easily determine the state S after the last compression function call as he has only access to the result of the final() function. This function squeezes out some bits of the state applying output transformation and squeezes out some bits again. The bits of a squeezing process do not leave enough information to

recover the internal state. (ii) The multiple feed-forward does also prevent any attacker to successfully gain any knowledge of prior state information. In each squeezing process the one feed forward takes place.

Multi-Collision Attacks. Joux 22 found that when iterative hash functions are used, finding a set of  $2^k$  messages all colliding on the same hash value (a  $2^k$ multi-collision) is as easy as finding k single collision for the hash function. Finding a collision in the compression function, i.e., a single block collision one can find k of such collisions each starting from the chaining value produced by the previous one-block collision. In other words, one have to find two messages blocks  $M_i$  and  $M'_i \neq M_i$  with  $C(h_{i-1}, M_i) = C(h_{i-1}, M'_i)$ , where  $C(\cdot)$  represents the compression function and  $h_i$  the chaining value. Then it is possible to construct  $2^k$  messages with the same hash value by choosing for block *i* either the message block  $M_i$  or  $M'_i$ . Joux also showed that the concatenation of two different hash functions is not more secure against collision attacks than the strongest one. This attack can find  $2^k$ -way internal multi-collisions with a complexity of  $k \cdot 2^{n_c/2}$ . An instance of TWISTER fully resists the multi-collision attack if  $8 \cdot N^2 \ge 2n_c$ , since the complexity is determined by  $k \cdot 2^{(8 \cdot N^2)/2}$ . All instances of TWISTER have this feature, although the state of TWISTER-384 and TWISTER-512 is not big enough to prevent this attack alone, including the check sum can be viewed as an enlarging of the state, which then guarantees to prevent for this attack.

Herding Attacks. The herding attack [23] works as follows. An attacker takes  $2^k$  chaining values which are fixed or randomly chosen. Then he chooses  $O(2^{n_c/2-k/2})$  message blocks. He computes the output of the compression function for each chaining value and each block. It is expected that for each chaining value there exists another chaining value, such that both collide to the same value. The attacker stores the message block that leads to such a collision in a table and repeats this process again with the newly found chaining values. Once the attacker has only one chaining value, it is used to compute the hash value to be published. To find a message whose chaining value is among the  $2^k$  original values, the attacker has to perform  $O(2^{n_c-k})$  operations. For such a message the attacker can retrieve from the stored messages the message blocks that would lead to the desired hash value. The time complexity of this attack is about  $O(2^{n_c/2+k/2})$  operations for the first step and  $O(2^{n_c-k})$  operations for the second step. The whole attack on an n-bit hash function requires approximately  $2^{(2n_c-5)/3}$  work. For TWISTER we have  $n_c = 8 \cdot N^2$  and with  $8 \cdot N^2 \ge (3n_c+5)/2$ the attack has the same complexity as for for a (second) pre-image attack on a random oracle. The complexity of this attack decreases with increased size of the message. If the message is of size about  $2^t$ , then the complexity of the attack is  $2^{(2n_c-5)/3-t}$ . One has to choose N such that the hash function is protected against this kind of attack for a given upper bound. All of our proposed instances of TWISTER resist this kind of attack.

Long 2nd pre-image Attacks. Dean  $\square$  found that fix-points in the compression function can be used for a second-pre-image attack against long messages in time  $O(n_c \cdot 2^{n_c/2})$  and memory  $O(n_c \cdot 2^{n_c/2})$ , where  $n_c = 8 \cdot N^2$  is the size of the internal state (which is equal to the size of the hash output for a plain MERKLE-DAMGÅRD constructions). Kelsey and Schneier  $\square$  extend this result and provide an attack to find a 2nd pre-image on a MERKLE-DAMGÅRD construction with MERKLE-DAMGÅRD strengthening much faster than the expected workload of  $2^{n_c}$ . The complexity of the attack is determined by the complexity of finding expandable messages. These are messages of varying sizes such that all these messages collide internally for a given IV. Expandable message can either be found using internal collisions or fixed points between a one-block message and an  $\alpha$ -block message for varying values of  $\alpha$ . The complexity of the generic attack to find a 2nd pre-image for a  $2^k$ -message block is about  $k \cdot 2^{n_c/2+1} + 2^{n_c-k+1}$  compression function calls.

Long 2nd preimage attacks in this form cannot be applied to the TWISTER framework for three reasons. First, we include the twist counter which does not allow to find expandable messages. Second, we make use of multiple feed-forwards and, third, the internal chaining value is in general much larger than n. This make it harder to find collisions and fix points since we essentially have a constructions similar to the wide pipe design [27].

Andreeva et al. [1] have shown that a combination of the attacks from [17, [24, [23]] can be mounted to dithered hash functions, which gives the attacker more control on the second-pre-image, since he can choose about the first half of the message in an arbitrary way. This attack can be done in time  $2^{n_c/2+k/2+2}+2^{n_c-k}$ . Although it is more expensive than the attack of Kelsey and Schneier [24], it works even when an additional input to the compression function (dithering) is given. One have to make the dithering as huge as possible, such that there are no small cycles. TWISTER includes the twist counter  $\phi$  which is very large, i.e., the twist counter is of size of the maximal message length. The larger this counter is the longer cycles we have, which increases the protection against this type of attack.

Slide Attacks. Slide attacks are common in block cipher cryptanalysis, but they also applicable to hash functions. Given a hash function H and two messages M and M' where M is a prefix of M', one can find a slid pair of messages (M, M') such that the the last message input block of the longer message M' performs only an additional blank round, e.g. for sponge constructions. These two messages are then slid by one blank round. This attack allows to recover the internal state of a slid pair of messages an even backward computation as shown in [20]. The twist counter  $\phi$  avoids the possibility of slide attacks, since XOR-ing a different value in each Mini-Round into the state matrix does not allow to find slid pairs of messages. Furthermore, the last inserted message block cannot be the all zero block due to the padding rules. Thus slide attacks are unavailable for TWISTER.

Differential Attacks. The essential idea of differential attack on hash functions [12], as used to break MD5 and SHA-0/1, is to exploit a high-probability input/output differential over some component of the hash function, e.g. under the form of a "perturb-and-correct" strategy for the latter functions, exploiting high probability linear/non-linear characteristics. In the design of the TWISTER framework, we applied the following countermeasures against differential attacks:

- High-Speed-Diffusion. The strong diffusion capabilities of the Mini-Round in combination with the non-linear S-box make the exploit of linear approximations highly implausible. As it is impossible to find a collision after one Mini-Round, any attacker has to trail presumably very long paths to be able to find a collision.
- *Nested feed-forward*. The internal feed-forward operations aim at strengthening the function against differential paths.
- Optional internal wide pipe. this makes internal collision unlikely, and the output-rounds make the differences much harder to predict in the hash value.
- Using different operators (e.g.  $\boxplus$  and  $\oplus$ ) highly complicates the computation of good differential paths.

### 4.2 Security Proofs for TWISTER

TWISTER was designed such that a single Mini-Round is proven to be collision free. This is expressed by the following lemma.

**Lemma 1.** From any state  $h_{i-1}$  one cannot find input message blocks  $M, M' \neq M$  such that

$$Mini-Round(h_{i-1}, M) = Mini-Round(h_{i-1}, M')$$

for all  $M, M' \neq M$ .

*Proof.* Assume that  $h_i^M$  is the state after inserting the message block M and  $h_i^{M'}$  is the state after inserting M'. Then, if M and M' are different in byte j the states  $h_i^M$  and  $h_i^{M'}$  are different in column j in at least 9 - k bytes. This is due to the MDS property of our diffusion layer, which has a branch number of 9.

We can also show that TWISTER offers full diffusion after two input blocks.

**Lemma 2.** Given an internal state  $h_{i-1}$  and two input blocks  $M_1$  and  $M'_1$  where  $M_1 \neq M'_1$ . Then, we have full diffusion of the state after two Mini-Rounds.

*Proof.* The message distribution process is visualized in Figure 4. Two messages  $M_1$  and  $M'_1 \neq M_1$  are different in at least one byte. Due to the diffusion of MixColumns at least one state column differs in 8 bytes. The ShiftRows of the following Mini-Round with no message input will distribute the all difference column into a one byte difference in each state column. After that MixColumns generates a difference in all state bytes. Which leads to full diffusion.

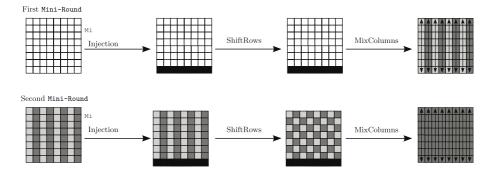


Fig. 4. Visualization of the diffusion of a message after two Mini-Rounds

### 5 Conclusion

In this paper we proposed a family of hash functions which overcome several identified weaknesses of the commonly used MDx family of hash functions (MD4, MD5, SHA-X). The weaknesses are addressed by several methods. By using some of the well analyzed building blocks and ideas of Rijndael we obtain a design for which we claim that no efficient differential collision structures exist. In addition, we limit access to the internal structure and take care that any possible difference quickly diffuses into the internal state. Furthermore it is highly scalable as there are – as proposed in e.g. in TWISTER-256 and TWISTER-512 – many possible ways to adopt our main building block, the Mini-Round.

We proposed three instantiations of the TWISTER framework, TWISTER-n, where n = 256, 384, 512. The claimed security level for TWISTER-256 with respect to collision resistance is  $2^{128}$  and with respect to (2nd) pre-image resistance  $2^{256}$ . For TWISTER-384, we claim a collision resistance of  $2^{192}$  and a (2nd) pre-image resistance of  $2^{384}$ . For TWISTER-512, the claimed security level for collision resistance is  $2^{256}$  and for (2nd) pre-image resistance  $2^{512}$ . The TWISTER family of hash functions exploits mathematical structures (i.e. MDS matrices) and, at the same time, having comparable speed as the SHA-2 family. Thus, instances of the TWISTER framework are suitable for a huge range of applications from low-end 8-bit platforms up to high-end 64-bit architectures.

TWISTER is submitted to the NIST SHA-3 competition. A new slightly modified version of TWISTER as well as the full submission document can be found on www.twister-hash.com.

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# A Performance

TWISTER was especially designed with 64-platforms in mind by making it possible to aggregate 8 times an 8-bit table lookup into one single 64-bit table lookup. The following performance measurements were conducted on:

Processor: Core2Duo T7300 Clock Speed: 2000 MHz Memory: 2048 MB Operating System: Linux, GNU Debian *Lenny*, Kernel 2.6.26-1 x64 Compiler: GCC 4.3, Optimization settings: -Os

For comparison, performance measurement results for SHA-2 on the this platform are given in Table  $\blacksquare$ 

	Output Size				
Algorithm	Platform	224/256	384/512		
SHA-2	64	20.1	13.1		
TWISTER	64	15.8	17.5		
SHA-2	32	29.3	55.2		
TWISTER	32	35.8	39.6		
TWISTER	8	200	220		

 Table 1. Performance comparison of TWISTER and SHA-2

All value are measured in cycles per byte.

# B Visualization of a Mini-Round

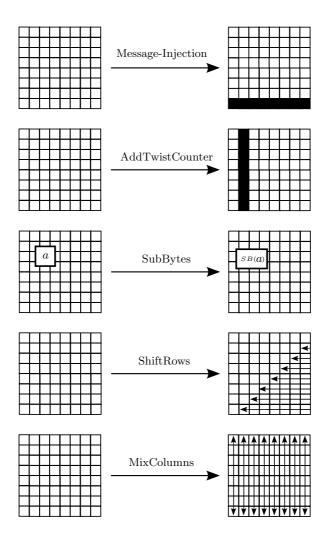


Fig. 5. A Mini-Round

# Preimage Attack on Hash Function RIPEMD

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Abstract. RIPEMD is a cryptographic hash function devised in the framework of the RIPE project (RACE Integrity Primitives Evaluation, 1988-1992). It consists of two parallel lines, and each line is identical to MD4 except for some internal constants. It has been broken by the collision attack, but no preimage attack was given. In this paper, we give a preimage attack on the compression function of the 26-step reduced RIPEMD with complexity  $2^{110}$  compression function to an attack on the 26-step reduced RIPEMD with complexity  $2^{115.2}$  instead of  $2^{128}$ . Then we extend the attack on 26 steps to the attack on 29 steps with the same complexity. Moreover, we can reduce the complexity of the preimage attack on the full RIPEMD without the padding rule by 1 bit compared with the brute-force attack.

Keywords: hash function, RIPEMD, cryptanalysis, preimage attack.

### 1 Introduction

Hash function is defined as a mapping  $h : \{0,1\}^* \longrightarrow \{0,1\}^n$ , where  $\{0,1\}^*$  denotes the set of bit strings of arbitrary length, and  $\{0,1\}^n$  denotes the set of strings of n-bit length.

From the security perspective, a hash function h with inputs x, x' and outputs y, y' should satisfy some or all of the properties. For the formal definition of them we refer to  $\blacksquare$ .

- Preimage Resistance: for any output y, it is computationally infeasible to get an input x such that h(x) = y.
- Second-preimage Resistance: for any input x, it is computationally infeasible to get another input  $x' \neq x$  such that h(x) = h(x').

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- Collision Resistance: it is computationally infeasible to get any two distinct inputs x, x' such that h(x) = h(x').

Now most attacks on hash functions focus on finding collisions, and the most well-known generic attack to find collisions is the birthday attack. By a reasoning similar to the birthday paradox, there is a good chance to find a collision if we compute hash values of about  $2^{\frac{n}{2}}$  messages, where the hash value is n-bit. While finding preimage is relatively harder, and the only generic attacks to find second preimage or preimage are the brute-force which both have the complexity of  $2^{n}$ .

A hash function with n-bit hash value is considered academically broken if it is possible to find collisions, second preimage or preimage in less than  $2^{\frac{n}{2}}$ ,  $2^{n}$  or  $2^{n}$  hash computations respectively.

There has been a great progress in finding the collision of hash functions in recent years. Wang et al. [3][4][5][6][7], Biham et al. [8] and Mendel et al. [9] found collisions of many hash functions based on MD4, such as HAVAL, MD5, RIPEMD, SHA-0 and SHA-1 etc.. These hash functions are considered not secure now. However, some of them are still widespread used when collision resistance is not required.

Preimage resistance is a weaker security notion than collision resistance. In some applications, collision resistance is unimportant, but preimage resistance is needed. The work on preimage attacks is much less than collision attacks in the cryptanalysis of hash functions. 10 showed that preimages of the first 2-round MD4 can be found, which was improved by 11. They found the preimage of 2 rounds and 7 steps of MD4. 12 presented a preimage attack on the full MD4. 13 gave an example of the preimage attack against MD2. This work was improved in 14. In 15, they presented preimage attacks on 3-pass HAVAL and 47-step MD5. 20 gave preimage attacks on 3, 4, and 5-pass HAVAL.

RIPEMD [2] was developed in the European RIPE project (RACE Integrity Primitives Evaluation, 1988-1992). Its compression function consists of two parallel lines of MD4 [17] compression function. In this paper, we present an inversion of the compression function of the 26-step reduced RIPEMD with the complexity of  $2^{110}$  compression function computations. Then the preimage attack on the compression function is extended to a preimage attack on the 26-step reduced RIPEMD with complexity  $2^{115.2}$  and memory requirement of  $2^{23}$  bytes. The attack on 26-step reduced RIPEMD can be extended to 29 steps with the same complexity. Moreover, we present a preimage attack on the full RIPEMD without the padding rule with complexity  $2^{127}$ , which optimizes the complexity order for brute-force attack.

The rest of the paper is organized as follows. Section 2 describes the RIPEMD algorithm. In Section 3, we show how to invert the compression function of the 26-step reduced RIPEMD and extend the preimage attack on the compression function to the preimage attack on hash function. Moreover, we extend the attack on 26-step reduced RIPEMD to 29-step reduced RIPEMD. Section 4 shows the preimage attack on the full RIPEMD without the padding rule by speeding up the brute force attack. Finally, we summarize the paper in Section 5.

### 2 Description of RIPEMD

The hash function RIPEMD compresses any arbitrary length message into a message with the length of 128 bits. Firstly, RIPEMD pads any given message into a message with the length of 512 bits multiple. For each 512-bit message block, RIPEMD compresses it into a 128-bit hash value by a compression function.

The initial value of RIPEMD is:

 $(a_0, b_0, c_0, d_0) = (0x67452301, 0xefcdab89, 0x98badcfe, 0x10325476)$ 

The compression function of RIPEMD consists of two parallel lines of MD4, which are denoted by Line1 operation and Line2 operation respectively. Each operation is identical to MD4 except for some internal constants. The nonlinear functions in each round are as follows:

$$F(X, Y, Z) = (X \land Y) \lor (\neg X \land Z)$$
  

$$G(X, Y, Z) = (X \land Y) \lor (X \land Z) \lor (Y \land Z)$$
  

$$H(X, Y, Z) = X \oplus Y \oplus Z$$

Here X, Y, Z are 32-bit words. The operations of the three functions are all bitwise.  $\land$ ,  $\oplus$  and  $\lor$  are bitwise AND, XOR and OR respectively.  $\neg$  represents the bitwise complement of X. Each round of the compression function is composed of 16-step operations.

In the following descriptions,  $\ll s_i (i = 1, \dots, 48)$  represents the circular shift  $s_i$ -bit positions to the left. + denotes addition modulo  $2^{32}$ , - denotes subtract modulo  $2^{32}$ .

Line 1 operation process. For a 512-bit block M,  $M = (m_0, m_1, \ldots, m_{15})$ , Line1 operation process is as follows:

- 1. Let  $(aa_0, bb_0, cc_0, dd_0)$  be the input of Line1 process for M. If M is the first block to be hashed,  $(aa_0, bb_0, cc_0, dd_0)$  is the initial value. Otherwise it is the output of the previous block compressing.
- 2. Perform the following 48 steps (three rounds):
  - (a) For  $i = 1, \dots, 16$ , do the following 16 operations:

$$aa_{i} = dd_{i-1}$$
  

$$bb_{i} = (aa_{i-1} + F(bb_{i-1}, cc_{i-1}, dd_{i-1}) + m_{\sigma(i)}) \ll s_{i}$$
  

$$cc_{i} = bb_{i-1}$$
  

$$dd_{i} = cc_{i-1}$$

(b) For  $i = 17, \dots, 32$ , do the following 16 operations:

$$aa_{i} = dd_{i-1}$$
  

$$bb_{i} = (aa_{i-1} + G(bb_{i-1}, cc_{i-1}, dd_{i-1}) + m_{\sigma(i)} + 0x5a827999) \ll s_{i}$$
  

$$cc_{i} = bb_{i-1}$$
  

$$dd_{i} = cc_{i-1}$$

(c) For  $i = 33, \dots, 48$ , do the following 16 operations:

$$\begin{aligned} aa_{i} &= dd_{i-1} \\ bb_{i} &= (aa_{i-1} + H(bb_{i-1}, cc_{i-1}, dd_{i-1}) + m_{\sigma(i)} + 0x6ed9eba1) \ll s_{i} \\ cc_{i} &= bb_{i-1} \\ dd_{i} &= cc_{i-1} \end{aligned}$$

The orders of message words can be seen in Table 11. The details of the shift positions can be seen in Table 22.

Line 2 operation process. For a 512-bit block M,  $M = (m_0, m_1, \ldots, m_{15})$ , Line2 operation process is as follows:

- 1. Let  $(aaa_0, bbb_0, ccc_0, ddd_0)$  be the input of Line2 process for M. If M is the first block to be hashed,  $(aaa_0, bbb_0, ccc_0, ddd_0)$  is the initial value. Otherwise it is the output of the previous block compressing.
- 2. Perform the following 48 steps (three rounds) :
  - (a) For  $i = 1, \dots, 16$ , do the following 16 operations:

 $\begin{aligned} aaa_{i} &= ddd_{i-1} \\ bbb_{i} &= (aaa_{i-1} + F(bbb_{i-1}, ccc_{i-1}, ddd_{i-1}) + m_{\sigma(i)} + 0x50a28be6) \ll s_{i} \\ ccc_{i} &= bbb_{i-1} \\ ddd_{i} &= ccc_{i-1} \end{aligned}$ 

(b) For  $i = 17, \dots, 32$ , do the following 16 operations:

$$aaa_{i} = ddd_{i-1}$$
  

$$bbb_{i} = (aaa_{i-1} + G(bbb_{i-1}, ccc_{i-1}, ddd_{i-1}) + m_{\sigma(i)}) \ll s_{i}$$
  

$$ccc_{i} = bbb_{i-1}$$
  

$$ddd_{i} = ccc_{i-1}$$

(c) For  $i = 33, \dots, 48$ , do the following 16 operations:

$$\begin{split} &aaa_{i} = ddd_{i-1} \\ &bbb_{i} = (aaa_{i-1} + H(bbb_{i-1}, ccc_{i-1}, ddd_{i-1}) + m_{\sigma(i)} + 0x5c4dd124) \ll s_{i} \\ &ccc_{i} = bbb_{i-1} \\ &ddd_{i} = ccc_{i-1} \end{split}$$

The orders of message words can be seen in Table []. The details of the shift positions can be seen in Table [2].

Add the output of Line1 to the output of Line2.

$$H_0 = b_0 + cc_{48} + ddd_{48}$$
$$H_1 = c_0 + dd_{48} + aaa_{48}$$

$$H_2 = d_0 + aa_{48} + bbb_{48}$$
$$H_3 = a_0 + bb_{48} + ccc_{48}$$

If M is the last message block of the message MM, then  $H(MM) = H_0 * H_1 * H_2 * H_3$  is the hash value for the message MM, where \* denotes the bit concatenation. Otherwise take  $(H_0, H_1, H_2, H_3)$  as inputs, and repeat the compression process for the next 512-bit message block.

Table 1. Word Processing Orders

$round_1$																
$round_2$	0	4	8	12	1	5	9	13	2	6	10	14	3	7	11	15
$round_3$	0	8	4	12	2	10	6	14	1	9	5	13	3	11	7	15

Table 2. Shift positions

$round_1$	3 7 11 19 3 7 11 19 3 7 11 19 3 7 11 19 3 7 11 19
	$3\ 5\ 9\ 13\ 3\ 5\ 9\ 13\ 3\ 5\ 9\ 13$
$round_3$	$3 \ 9 \ 11 \ 15 \ 3 \ 9 \ 11 \ 15 \ 3 \ 9 \ 11 \ 15$

# 3 Preimage Attack on the 26-Step Compression Function

In this section, first, we will recall some nice properties of the Boolean function F, then, we will describe our preimage attack on the compression function of the 26-step reduced RIPEMD, which is denoted by CRIPEMD. We can invert CRIPEMD by using the strategy of **[12]15** and exploiting the nice properties of Boolean function F and the order of the message words. A preimage attack against CRIPEMD is given with the complexity of  $2^{110}$  CRIPEMD computations. Then we extend the preimage attack on CRIPEMD to an preimage attack on the 26-step reduced RIPEMD with complexity  $2^{115.2}$ .

### 3.1 Some Basic Propositions

In this section we will recall some popular used properties of the nonlinear function F in our attack. These properties were presented in **10,16,3** etc.. We can use these key properties to absorb some difference.

**Proposition.** For the nonlinear function  $F(X, Y, Z) = (X \land Y) \lor (\neg X \land Z)$ , the following properties hold:

1.  $F(x, y, z) = F(x, \neg y, z)$  if and only if x = 0.

2.  $F(x, y, z) = F(x, y, \neg z)$  if and only if x = 0xfffffff.

#### 3.2 Preimage Attack on CRIPEMD

The key tricks used in our attack are the absorption of changes in  $c_0$  and the exploitation of the order of the message words. We will give the following observations.

**Observation 1.** From the algorithm of RIPEMD, it is easy to get the following conclusions about Line1 operation and Line2 operation:

- 1. At each step i  $(1 \le i \le 48)$ , only the chaining variable  $b_i$  is renewed, and  $b_i = c_{i+1} = d_{i+2} = a_{i+3}$   $(0 \le i \le 45)$ ,  $b_{46} = c_{47} = d_{48}$ ,  $b_{47} = c_{48}$ .
- 2. At each step i  $(1 \le i \le 48)$ , from the chaining variables  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$  and message word  $m_{\sigma(i)}$ , we can compute  $a_{i-1}$ ,  $b_{i-1}$ ,  $c_{i-1}$ ,  $d_{i-1}$ .

**Observation 2.** From the order of the message words in Table 1. we know that  $m_2$  is used at Step 3 (the very beginning of CRIPEMD) and Step 25 (the very end of CRIPEMD). Therefore, we can modify  $m_2$  to alter  $cc_{26}$  (and don't alter  $ddd_{26}$ ), so that  $H_0 = b_0 + cc_{26} + ddd_{26}$  is equal to  $H_0^*$ . Then according to the Proposition, we know that F can "absorb" the change in  $c_0$  at Step 1 and Step 2 if the conditions  $b_0 = 0$  and  $b_1 = 0$ xffffffff are added respectively.

The general procedure of the attack is as follows. Firstly, we choose chaining variables  $(a_0, b_0, c_0, d_0)$  with certain constraints. Secondly, we choose a 512-bit message block  $M = (m_0, \dots, m_{15})$  with certain constraints. Thirdly, we modify the message word  $m_2$  such that  $cc_{26}$  is changed (and  $ddd_{26}$  isn't changed) to ensure that  $H_0 = b_0 + cc_{26} + ddd_{26}$  is equal to  $H_0^*$ . Fourthly, we correct  $c_0$  such that the change in  $m_2$  doesn't change  $b_1, b_2, b_3, b_4$ , so doesn't change the subsequent chaining variables.

**Description of the Attack.** Suppose we want to get the output of CRIPEMD  $(H_0^*, H_1^*, H_2^*, H_3^*)$ , our goal is to find a 512-bit message block M and input chaining variables  $(a_0, b_0, c_0, d_0)$  such that the compressing output is equal to  $(H_0^*, H_1^*, H_2^*, H_3^*)$ , i. e. CRIPEMD $((a_0, b_0, c_0, d_0), M) = (H_0^*, H_1^*, H_2^*, H_3^*)$ .

We will describe the attack in Algorithm 1. The algorithm first sets  $b_0 = 0$ and  $bbb_1 = 0$ xffffffff to ensure that a change in  $c_0$  doesn't affect  $bbb_1$  and  $bbb_2$  of Line2 operation. If  $bbb_1 = 0$ xffffffff holds, then the Hamming weight (i. e. the number of "1" bits in the binary sequence) of  $bb_1$  in Line1 operation is 18, which will be proved below. Therefore, a change in  $c_0$  doesn't affect  $bb_2$ of Line1 operation with the probability of  $2^{-14}$ . Obviously, the condition  $b_0 = 0$ ensure that a change in  $c_0$  doesn't affect  $bb_1$ . Then the algorithm will modify  $m_2$  to ensure that  $H_0$  meets the requirement. Finally,  $c_0$  is corrected to ensure that the change of  $m_2$  doesn't affect  $bb_3$  and  $bbb_3$ . So the modification of  $c_0$ is absorbed with probability  $2^{-14}$ , i. e. the probability of the forward stage unchanged with the new  $m_2$  is  $2^{-14}$ . If the forward stage unchange, then the 32-bit of the 128-bit image is satisfied, and we can get the compressing values  $(H_0, H_1, H_2, H_3) = (H_0^*, H_1^*, H_2^*, H_3^*)$  by exhaustively search the remaining 96 bits values.

#### Algorithm 1 Preimage Attack on CRIPEMD

Input:  $H_0^*$ ,  $b_0^* = 0$ . Output:  $M = (m_0, \dots, m_{15})$  and  $(a_0, b_0, c_0, d_0)$  st.  $(H_0, H_1, H_2, H_3) = (H_0^*, H_1^*, H_2^*, H_3^*)$ . 1. repeat 2.Pick an initial variable with  $b_0 = 0$  and  $a_0, c_0, d_0$  being arbitrary values. 3. Choose  $m_0$  such that  $bbb_1 = 0$ xffffffff. 4. Choose arbitrary values for  $m_1, \dots, m_{15}$ . 5.Do the Line1 operation to get  $aa_{26}$ ,  $bb_{26}$ ,  $cc_{26}$ ,  $dd_{26}$ . 6. Do the Line2 operation to get  $aaa_{26}$ ,  $bbb_{26}$ ,  $ccc_{26}$ ,  $ddd_{26}$ . 7. Modify  $m_2$  to change  $cc_{26}$ , so that  $cc_{26} = H_0^* - b_0 - ddd_{26}$ . 8. Correct  $c_0$  to keep  $bb_3$  and  $bbb_3$  unchanged. 9. Compute the final compressing value  $(H_0, H_1, H_2, H_3)$ . 10.If  $(H_0, H_1, H_2, H_3) = (H_0^*, H_1^*, H_2^*, H_3^*)$ , then return  $(a_0, b_0, c_0, d_0)$  and  $M = (m_0, \dots, m_{15})$ .

#### **Correctness of the Attack**

1. In order to make Line 3 of Algorithm 1 feasible, we modify  $m_0$  as follows:

$$m_0 \leftarrow (\texttt{0xffffffff} >>> 3) - a_0 - F(b_0, c_0, d_0) - 0x50a28be6$$

2. We will show that if  $bbb_1 = 0$ xffffffff, then the Hamming weight of  $bb_1$  in Line1 operation is 18. From the algorithm of RIPEMD, we can get the equations (1) and (2), combined with the equation (3), we know that  $bb_1 = (0xffffffff - 0x50a28be6) \ll 3 = 0xaf5d7419 \ll 3 = 0x7aeba0cd$ . Therefore, the Hamming weight of  $bb_1$  is 18.

$$bb_1 = (a_0 + F(b_0, c_0, d_0) + m_0) \ll 3$$
<sup>(1)</sup>

$$bbb_1 = (a_0 + F(b_0, c_0, d_0) + m_0 + 0x50a28be6) \ll 3$$
<sup>(2)</sup>

$$bbb_1 = \mathsf{Oxfffffff} \tag{3}$$

- 3. For Line2 operation, according to Proposition 1, we know that the condition  $b_0 = 0$  ensures the change of  $c_0$  results in no change in  $bbb_1$ , and according to Proposition 2, the condition  $bbb_1 = 0$ xffffffff ensures that the change of  $c_0$  results in no change in  $bbb_2$ . Therefore, we can modify  $c_0$  to correct a change in  $m_2$ , and in the same time not alter the chaining variables  $b_0$ ,  $bbb_1$ ,  $bbb_2$ ,  $bbb_3$ , so not alter the subsequent chaining variables.
- 4. For Line1 operation, according to Proposition 1, we know that the condition  $b_0 = 0$  ensures the change of  $c_0$  results in no change in  $bb_1$ , and according to Proposition 2, the condition  $bb_1 = 0x7aeba0cd$  ensures that the change of  $c_0$  results in no change in  $bb_2$  with the probability of  $2^{-14}$ . Therefore, we can modify  $c_0$  to correct a change in  $m_2$  without altering the chaining variables  $b_0, bb_1, \dots, bb_{26}$  with probability  $2^{-14}$ .
- 5. In the second round of Line1 operation, changing  $m_2$  results in altering  $cc_{26}$ . In the second round of Line2 operation, changing  $m_2$  doesn't change  $ddd_{26}$ .

$$H_0 = b_0 + cc_{26} + ddd_{26} \tag{4}$$

$$cc_{26} = bb_{25} = (aa_{24} + G(bb_{24}, cc_{24}, dd_{24}) + m_2 + 0x5a827999) \ll 3$$
(5)

Combined with equation (4) and equation (5), we can modify  $m_2$  as follows in order to set  $H_0 = H_0^*$ .

$$m_2 \leftarrow (H_0^* - b_0 - ddd_{26}) >>> 3 - aa_{24} - G(bb_{24}, cc_{24}, dd_{24}) - 0x5a827999$$

6. We can change  $c_0$  in order to correct the change of  $m_2$  in Line1 operation, i. e. to keep  $bb_3$  unchanged.  $c_{0,1}^*$  denotes the  $c_0$  after changed in Line1 operation. Denote  $m'_2$  by the original message word before changed.  $m''_2$  is the message word after changed. From the algorithm of RIPEMD, we can get the following equation (6) and equation (7):

$$bb_{3} = (aa_{2} + F(bb_{2}, cc_{2}, dd_{2}) + m'_{2}) \ll 11$$
  
=  $(c_{0} + F(bb_{2}, bb_{1}, b_{0}) + m'_{2}) \ll 11$  (6)

$$bb_3 = (c_{0,1}^* + F(bb_2, bb_1, b_0) + m_2) \ll 11$$
(7)

From equation (6) and equation (7) we can get  $c_0 + m'_2 = c^*_{0,1} + m''_2$  (8)

7. Similarly, we can change  $c_0$  to correct the change of  $m_2$  in Line2 operation, i. e. to keep  $bbb_3$  unchanged.  $c_{0,2}^*$  denotes the  $c_0$  after changed in Line2 operation. From the algorithm of RIPEMD, we can get the following equation (9) and equation (10):

$$bbb_{3} = (c_{0} + F(bbb_{2}, bbb_{1}, b_{0}) + m_{2}^{'}) \ll 11$$
(9)

$$bbb_3 = (c_{0,2}^* + F(bbb_2, bbb_1, b_0) + m_2') \ll 11$$
(10)

From equation (9) and equation (10) we can get  $c_0 + m'_2 = c^*_{0,2} + m''_2$  (11) From equation (8) and equation (11), easily we get  $c^*_{0,1} = c^*_{0,2}$ .

Therefore, the change in 
$$m_2$$
 can be corrected by changing  $c_0$  as follows:  
 $c_0 \leftarrow (bb_3 >>> 11) - F(bb_2, bb_1, b_0) - m_2^{''} = (bbb_3 >>> 11) - F(bbb_2, bbb_1, b_0) - m_2^{''} - 0x50a28be6$ 

With the new value  $c_0$ , the chaining variables  $bb_3$  in Line1 operation and  $bbb_3$  in Line2 operation are not altered.

All in all, firstly, we modify  $m_2$  to get the output  $H_0^*$ , then we alter  $c_0$  to correct the change in  $m_2$  in Line2 operation, and to correct the change in  $m_2$  in Line1 operation with probability  $2^{-14}$ . We can get the outputs  $H_1^*$ ,  $H_2^*$  and  $H_3^*$  by exhaustively search with  $2^{96}$  trials. Thus the total cost is  $2^{110} (= 2^{96} \times 2^{14})$  trials.

#### 3.3 Preimage Attack on the 26-Step Reduced RIPEMD

In order to extend the preimage attack on CRIPEMD to the 26-step reduced RIPEMD, we take into consideration of two aspects: one is the padding rule of RIPEMD, and the other is the standard initial value. The padding rule only forces some constraints on the last message words, and there aren't any restrictions on the last message words in our attack. So the padding rule is not an obstacle. However, the initial value is a problem, the initial value in our attack

is different from the standard one in the RIPEMD algorithm. We use two approaches to extend our attack to the attack based on the standard initial value. These approaches are also used in 15.

**Basic Meet-in-the-Middle Approach.** We will use the basic meet-in-themiddle approach to turn the attack to the preimage attack on the 26-step reduced RIPEMD. The approach is similar to the unbalanced meet-in-the-middle attack in [18]. If the complexity of the attack on the compression function is  $2^x$ , then we hash  $2^{\frac{n+x}{2}}$  random message blocks with the standard initial value, and compute  $2^{\frac{n-x}{2}}$  preimages for the targeting value  $H^*$  by using the preimage attack on the compression function. By the birthday paradox, we expect one match. The complexity of the preimage attack on hash functions is  $2^{1+\frac{n+x}{2}}$ .

In our attack, we have x = 110. We hash  $2^{119}$  random message blocks with the standard initial value with complexity  $2^{119}$ , and compute  $2^9$  preimages for the targeting value  $H^*$  with complexity  $2^{119} = 2^9 \times 2^{110}$ . Therefore, the complexity of our attack on 26-step reduced RIPEMD is about  $2^{120}$  hash computations.

**Tree Approach.** This approach is the meet-in-the-middle approach combined with a tree-based approach. The details of the approach can refer to **12** which describes the approach exactly. **19** describes a similar approach. Firstly, we compute  $2^{16}$  multi-block preimages of the targeting value  $H^*$  by using the tree-based approach, which costs  $2^{115} = 16 \times 2^{111}$  computations. Then we hash  $2^{112}$  random message blocks with the standard initial value, and we expect to find one match according to the birthday paradox. Therefore, the preimage attack on 26-step reduced RIPEMD has the time complexity of about  $2^{115.2} (= 2^{115} + 2^{112})$ , and needs to store  $2^{17}$  message blocks  $(2^{23}$  bytes).

**Delayed-Start Attack.** We can extend the attack to 29-step reduced RIPEMD, which is from step 2 to 30 by using the message word  $m_3$  instead of  $m_2$ . The attack has the same complexity as the attack on 26 steps.

# 4 Preimage Attack on the Full RIPEMD

In this section, by the approach of speeding up the brute-force attack proposed in [20], we can reduce the complexity of the preimage attack on the full RIPEMD without the padding rule by 1 bit.

We will recall the speeding up approach as follows. Given the initial value IV and hash value H, we want to find a message block such that the hash value is equal to H under the initial value IV. Assume  $m_i$  and  $m_j$  form a local collision in the first round, and  $m_i$ ,  $m_j$  appear at steps  $i_1$ ,  $j_1$  ( $i_1 < j_1$ ) in the second round, then the values of the chaining variables from step 1 to  $i_1$  can be reused with all  $m_i$  and corresponding  $m_j$ . Let  $m_i$ ,  $m_j$  appear at steps  $i_2$ ,  $j_2$  ( $i_2 < j_2$ ) in the last round, then the results from step  $j_2$  to the last can be reused. Similarly, if  $m_i$  and  $m_j$  form a local collision in the last round, then this technique can also be achieved. The memory requirement of the attack is negligible.

We make a local collision from step 12 to step 16 in Line2 operation. Then the results from step 1 to 30 can be reused. Assume the initial value is IV and the hash value is  $H = (H_0H_1H_2H_3)$ . We use the speeding up approach to invert the full RIPEMD in the following steps.

- 1. Randomly choose the message words  $m_i$  (i=0,...,9), and compute the chaining variables  $aaa_i$ ,  $bbb_i$ ,  $ccc_i$  and  $ddd_i$  (i=1,...,10).
- 2. Choose  $m_{10}$  such that  $bbb_{11} = bbb_{10}$ , i. e.  $ccc_{12} = ddd_{12}$ .
- 3. Randomly choose  $m_{11}$ . Choose  $m_{12}$  such that  $bbb_{13} = 0$ . Choose  $m_{13}$  such that  $bbb_{14} = 1$ .
- 4. Randomly choose other message words. Compute  $aa_i$ ,  $bb_i$ ,  $cc_i$  and  $dd_i$  (i=1,...,11). Compute  $aaa_i$ ,  $bbb_i$ ,  $ccc_i$  and  $ddd_i$  (i=1,...,30). Store the values  $aaa_{30}$ ,  $bbb_{30}$ ,  $ccc_{30}$  and  $ddd_{30}$  in a table.
- 5. For all  $2^{32}$   $m_{11}$ , compute the corresponding  $m_{15}$  such that  $bbb_{16}$  doesn't change. Compute  $aa_i$ ,  $bb_i$ ,  $cc_i$  and  $dd_i$  for i = 12, ..., 48.
- 6. Let  $aaa_{48} = H_0 aa_{48}$ ,  $bbb_{48} = H_1 bb_{48}$ ,  $ccc_{48} = H_2 cc_{48}$  and  $ddd_{48} = H_3 dd_{48}$ , compute  $aaa_i$ ,  $bbb_i$ ,  $ccc_i$  and  $ddd_i$  in the reverse direction and get the value  $aaa_{33}$ . Check whether  $aaa_{33} = bbb_{30}$  is in the table or not. If it is in the table, compute  $bbb_{32}$ ,  $bbb_{31}$  and  $bbb_{30}$  and check all values are matched. Otherwise, choose  $m_{11}$  and repeat the process.

The complexity of the attack is  $2^{127} = 2^{96} \times 2^{31}$ . The reason is that the complexity of the above procedure is about  $2^{31} = 2^{32} \times \frac{(48-11)+(48-33)}{48+48}$ , and the success probability is  $2^{-96} = 2^{-128} \times 2^{32}$ .

### 5 Conclusion

In this paper, firstly, we present an inversion of the compression function of the 26-step reduced RIPEMD with complexity  $2^{110}$ , and extend the preimage attack on the compression function to a preimage attack on the 26-step reduced RIPEMD with complexity  $2^{115.2}$  and memory requirement of  $2^{23}$  bytes. Then we extend the attack on 26-step reduced RIPEMD to 29 steps with the same complexity. In the last, we give a preimage on the full RIPEMD without the padding rule and reduce the complexity by 1 bit compared with the brute-force attack. Though the attacks require very large number of compression function evaluations, they are successful attacks on reduced RIPEMD theoretically. Moreover, it shows that the hash functions composed of two parallel lines are not secure as we expected. As far as we know, this is the first preimage attack on RIPEMD.

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# Full Key-Recovery Attack on the HMAC/NMAC Based on 3 and 4-Pass HAVAL

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Abstract. In this paper, we give the full key-recovery attacks on the HMAC/NMAC instantiated with 3 and 4-Pass HAVAL using our new differential paths. The complexity to recover the inner key is about  $2^{103}$  MAC queries for the 3-Pass HAVAL and  $2^{123}$  MAC queries for the 4-Pass HAVAL. The complexity to recover the outer key is about  $2^{69}$  MAC queries and  $2^{198}$  offline computations for the 3-Pass HAVAL based HMAC/NMAC. For the 4-Pass HAVAL case, the number of MAC queries for outer key-recovery is about  $2^{103}$  and the offline work is about  $2^{180}$  4-Pass HAVAL computations.

**Keywords:** HMAC, NMAC, key-recovery, 3-Pass HAVAL, 4-Pass HAVAL.

### 1 Introduction

Recently the analysis of the MACs based on hash functions have attracted extensive attention in the field of cryptographic analysis. A message authentication code(MAC) is a function which takes a message and a secret key as inputs and produces an output called an *authentication tag.* HMAC [2] is a standardized hash-based MAC algorithm which is widely used as a MAC algorithm and a pseudorandom function generator. NMAC [2] is a generalized version of HMAC. They are proven secure under the assumption that the compression function of the underlying hash function is a pseudorandom function [1]. However, the attacks on hash functions [8],9],10,111,12,13] have shown that the prevailing hash functions such as MD4, HAVAL, MD5, SHA-0, SHA-1 are not collision resistant. Therefore research on the HMAC/NMAC based on those hash functions has become a hot topic. There are three kinds of attacks on HMAC/NMAC: *distinguishing attack, forgery attack* and *key-recovery attack*. We focus on the keyrecovery attack by trying to recover the inner and outer key of HMAC/NMAC instantiated by 3 and 4-Pass HAVAL.

At Asiacrypt'06, Contini and Yin 🗓 used collision techniques to obtain forgery and partial key-recovery attacks on HMAC/NMAC instantiated with

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MD4, MD5, SHA-0 and reduced SHA-1. At SCN 2006, Kim *et.al* **[5]** give distinguishing, forgery and partial key-recovery attacks on the HMAC/NMAC based on full or reduced HAVAL, MD4, MD5, SHA-0 and SHA-1. At FC 2007, Rechberger and Rijmen **[7]** proposed a full key-recovery attack in the related-key setting on NMAC with full MD5 and 34-step SHA-1. At Crypto 2007, Fouque *et.al* presented the first full key-recovery attacks on HMAC/NMAC based on MD4 using the IV-dependent collision differential path. Their attack can derive bits of outer key  $k_1$  by observing whether or not collisions occurred for the outer MD4. At Eurocrypt 2008, Wang *et.al* **[14]** gave a new outer key-recovery attack on HMAC/NMAC-MD4 by using the near-collision differential path. They also extended the attack of **[3]** into a full key-recovery attack on NMAC-MD5 in the related-key setting. In FSE 2008 **[6]**, Lee *et.al* found a 3-Pass HAVAL collision differential path with probability  $2^{-114}$ , and that allowed to inner key-recovery attack on HMAC/NMAC based on 3-Pass HAVAL with  $2^{122}$  MAC queries and  $2^{96}$  offline computations.

In this paper, we extend the technique of Contini-Yin [3] to recover the inner key for the HMAC/NMAC based on 3 and 4-Pass HAVAL using our new differential paths. For the outer key-recovery, we joint the IV-dependent technique in [4] and the near-collision technique in [14] together to recover more key bits by using our new IV-dependent near-collision differential path. Our inner key-recovery attack on HMAC/NMAC based on 3-Pass HAVAL is more efficient than that of Lee *et.al*'s [6]. According to our knowledge, this is the first full key-recovery attack for the HMAC/NMAC based on 3 and 4-Pass HAVAL. The main results of this paper are listed in Table [].

Hash	type	Probability of	Online queries	Offline computations
functions		differentials		
3-Pass HAVAL	Inner K.	$2^{-96}$	$2^{103}$	$2^{70}$
4-Pass HAVAL	Inner K.	$2^{-121}$	$2^{123}$	$2^{39}$
3-Pass HAVAL	Outer K.	$2^{-64}$	$2^{69}$	$2^{198}$
4-Pass HAVAL	Outer K.	$2^{-98}$	$2^{103}$	$2^{180}$

This paper is organized as follows. In section 2, we describe the algorithms of HAVAL and HMAC/NMAC briefly. In section 3, we introduce our inner key-recovery attack on the HMAC and NMAC based on 3 and 4-Pass HAVAL. In section 4, we give the outer key-recovery attack on the HMAC/NMAC based on 3 and 4-Pass HAVAL. Finally we conclude this paper in section 5.

# 2 Algorithm Description

### 2.1 Description of 3 and 4-Pass HAVAL

The original description of the HAVAL algorithm can be found in 15. In this section, we only describe the compression functions for the 3 and 4-Pass HAVAL

for our purpose of attacks. The compression function of HAVAL takes a 1024-bit message  $M = (m_0, m_1, ..., m_{31})$  and a 256-bit initial value  $h_{in} = (a_0, b_0, ..., h_0)$  as inputs, and produces a 256-bit hash value  $h_{i+1}$  as output. The compression function is defined in the case of 3 and 4-Pass as follows:

- Initialize chaining variables (a, b, ..., h) as  $(a_0, b_0, ..., h_0)$ .
- Perform the following  $32 \times 3$  (or 4):
  - For i=0 to 2 (or 3) For j = 0 to 31  $p := f_{i+1}(g, f, e, d, c, b, a)$   $r := (p \gg 7) + (h \gg 11) + m_{ord(i,j)} + k_{i,j}$ (a, b, c, d, e, f, g, h) := (r, a, b, c, d, e, f, g)

The constant  $k_{i,j}$  and the order of the messages words in each pass can be found in [15]. The Boolean functions (round functions) involved in the 3 and 4-Pass HAVAL are defined in Table [4].

- Output the 256-bit value  $h_{out} := (a + a_0, b + b_0, ..., h + h_0).$ 

Some main properties of the round function  $f_1$  of 3-Pass HAVAL that are used to find differential paths are listed in Tables of Appendix. It is easy to deduce the similar properties of the other round functions for the 3 and 4-Pass HAVAL.

#### 2.2 Description of NMAC and HMAC

HMAC and NMAC are both hash based MACs. Let H be the underlying hash function and f be the compression function. The basic design approach for NMAC is to replace the fixed IV in H with a secret key. Using the notations of the paper [2], let  $f_k(x) = f(k, x)$  denote the keyed compression function and  $H_k(x) = H(k, x)$  denote the keyed hash function. Let  $(k_1, k_2)$  be a pair of independent keys, the NMAC algorithm, on input message M and the secret key  $(k_1, k_2)$ , is defined as :

$$NMAC_{(k_1,k_2)}(M) = H_{k_1}(H_{k_2}(M)).$$

HMAC is a variant of NMAC that uses a fixed IV so that it can be implemented by simply calling the existing hash functions. The function HMAC works on an arbitrary length input message M and a fixed length random string k as its keys:

$$HMAC_k(M) = H(\overline{k} \oplus opad||H((\overline{k} \oplus ipad)||M)),$$

where  $\overline{k}$  is the key k padded to full b-bit block size by appending zero bits. *ipad* and *opad* are two b-bit constants. Let  $k_1 = f(\overline{k} \oplus opad)$  and  $k_2 = f(\overline{k} \oplus ipad)$ , then HMAC can be seen as a special case of NMAC. So in the following attacks, we only focus on the key-recovery attack on the NMAC and it is also applicable to HMAC.

### 3 Inner Key-Recovery on NMAC Based on 3 and 4-Pass HAVAL

In this section, we extend Contini-Yin's inner key-recovery technique [3] on NMAC to the 3 and 4-Pass HAVAL.

#### 3.1 Inner Key-Recovery on NMAC Based on 3-Pass HAVAL

In this section, we find five differential paths with almost the same probability  $2^{-98}$  for inner key-recovery attack on NMAC based on 3-Pass HAVAL. The message difference  $\Delta M = (\Delta m_i)_{(0 \le i \le 31)}$  selected for these paths is

$$\Delta m_i = \begin{cases} 2^{30}, \, i = 20\\ 0, \quad 0 \le i \le 31, \, i \ne 20. \end{cases}$$

The five differential paths and their precise probabilities are shown in Table **6**. The total probability of the differential paths in Table **6** is  $2^{-96}$ , i.e., for any random message M, the message pair  $(M, M + \Delta M)$  obeys one of the five paths with probability  $2^{-96}$ .

For a reasonable success rate, we need  $2^2 \times 2^{96} = 2^{98}$  chosen message pairs to get the collide messages  $(M, M + \Delta M)$ , and they obey one of the five paths in Table 6 No matter which path they obey, some variable conditions generated by M are determined. If we can recover the values of successive eight chaining variables  $a_i$  to  $a_{i+7}$  generated by M, the inner key  $k_2$  can be recovered by backward computation. Let  $P_r$  is the probability of the differential path from step r to the last step. In order to carry out our work, we list the sufficient and necessary conditions in the chaining variables  $a_{20}$  to  $a_{27}$  and the corresponding probability  $P_r$  in Table 2

Utilizing the conditions in  $a_{20}$ ,  $a_{21}$ ,  $a_{22}$ ,  $a_{23}$ ,  $a_{25}$  and  $a_{27}$ , we can recover the least significant 31 bits in these chaining variables by changing the corresponding bit in  $m_{19}$ . For a reasonable success rate, it needs about  $\frac{2^3}{Pr_{i+1}}$  message pairs  $(M^*, M^* + \Delta M)$  to recover one bit of  $a_i$ . The structure of the message  $M^*$  and the detail bit-recovery technique can be referred to  $\square$ . The recovered bits in  $a_{20}$  to  $a_{27}$  and the corresponding data complexity are also given in Table  $\square$ .

The total number of recovered bits in the eight successive chaining variables  $a_{20}$  to  $a_{27}$  is  $31 \times 6 = 186$ . We guess the unknown 256 - 186 = 70 bits and compute backward to get the input  $k_2$ . The right key  $k_2$  will result to  $H_{k_2}(M) = H_{k_2}(M + \Delta M)$ .

The data complexity of this attack includes two parts: searching the collision message pairs and recovering the chaining variables. Adding two parts, we get a complexity about  $2^{103}$ . The success rate of recovering a single bit in Table 2 is  $1 - \frac{1}{e^3}$ , which is very close to 1. In total, the success rate for recovering all 186 bits is about 0.94. The attack does not require any storage, its time complexity is only the time required to compute the MAC values for the chosen messages.

#### 3.2 Inner Key-Recovery on NMAC Based on 4-Pass HAVAL

For 4-Pass HAVAL, we select the message difference  $\Delta m_5 = 2^{19}$  and construct the 4-Pass differential paths. The paths are composed of two inner collisions, step 6 to 33 and step 95 to 123. The two inner collisions are shown in Tables 7 and 8 The first inner collision includes two paths and each of them has the probability  $2^{-59}$ . The second inner collision from step 95-123 includes six paths

step	Output	Condition on relevant bit	Pr	Recovered bit	Data complexity
20	$a_{20}$	$a_{20,30} = a_{19,30}$		least significant 31 bits	2
21	$a_{21}$	$a_{21,30} = 0$	$P_{22} = 2^{-92.26}$	least significant 31 bits	
22	$a_{22}$			least significant 31 bits	
23	$a_{23}$	$a_{23,19} = 0, a_{23,30} = 1$	$P_{24} = 2^{-89.26}$	least significant 31 bits	$2^{98.16}$
24	$a_{24}$	No conditions		0	
25	$a_{25}$		$P_{26} = 2^{-87.26}$	least significant 31 bits	$2^{96.16}$
26	$a_{26}$	No conditions		0	
27	$a_{27}$	$a_{27,19} = 0, a_{27,30} = 0$	$P_{28} = 2^{-85.26}$	least significant 31 bits	$2^{94.16}$

**Table 2.** Recovered bits from  $a_{20}$  to  $a_{27}$  for inner key-recovery attack on NMAC based on 3-Pass HAVAL

**Table 3.** Recovered bits on  $a_{22}$  to  $a_{29}$  for inner key-recovery attack on HMAC based on 4-Pass HAVAL

step	Output	Condition on relevant bit	Pr	Recovered bit	Data complexity
22	$a_{22}$				
23	$a_{23}$	$a_{23,30} = 0$		least significant 31 bits	
24	$a_{24}$	$a_{24,30} = 0$	$P_{25} = 2^{-67.27}$	least significant 31 bits	$2^{76.17}$
25	$a_{25}$	$a_{25,30} = 1$	$P_{26} = 2^{-66.27}$	least significant 31 bits	$2^{75.17}$
26	$a_{26}$	20,00		least significant 31 bits	
27	$a_{27}$	$a_{27,30} = 0$	$P_{28} = 2^{-64.27}$	least significant 31 bits	$2^{73.17}$
28	$a_{28}$	$a_{28,30} = 0$	$P_{29} = 2^{-63.27}$	least significant 31 bits	$2^{72.17}$
29	$a_{29}$	No conditions		0	

that contribute a total probability of  $2^{-63.23}$ . So the probability for the entire 4-Pass HAVAL differential paths is about  $2^{-121}$ .

The first step for the inner key-recovery attack is to find a pair of collided messages  $(M, M + \Delta M)$  and this step has a data complexity about  $O(2^{123})$  according to the differential paths in Table 7 and 8. Using the Boolean function properties of 4-Pass HAVAL, we can deduce a set of sufficient and necessary conditions on the eight chaining variables  $(a_{22}, a_{23}, ..., a_{29})$ . Then we can give the recovered bits in Table 3.

Now we have recovered  $31 \times 7 = 217$  bits of variables and the remaining 39 bits can be exhaustively searched. The data complexity is determined by searching the collision message pairs as for the case of the 4-Pass HAVAL. The success rate of this attack is about 0.93.

### 4 Outer Key-Recovery on NMAC Based on 3 and 4-Pass HAVAL

In this section, we find the new near-collision differential paths for 3 and 4-Pass HAVAL. Combining the IV-dependent technique in [14] and the near-collision technique in [14], we give the first outer key recovery attack on NMAC based on 3 and 4-Pass HAVAL using our new differentials.

#### 4.1 Outer Key-Recovery on NMAC Based on 3-Pass HAVAL

Suppose that the inner key  $k_2$  of NMAC based on 3-Pass HAVAL is known and we want to recover the outer key  $k_1$ , which will be decomposed as eight 32-bit variables  $k_a$ ,  $k_b$ ,  $k_c$ ,  $k_d$ ,  $k_e$ ,  $k_f$ ,  $k_g$  and  $k_h$ . Because HAVAL uses the Davies-Meyer mode, the 256-bit output of NMAC based on 3-Pass HAVAL is  $(k_a + a_{96}, k_b + a_{95}, k_c + a_{94}, k_d + a_{93}, k_e + a_{92}, k_f + a_{91}, k_g + a_{90}, k_h + a_{89})$ , denoted as  $(h_a, h_b, h_c, h_d, h_e, h_f, h_g, h_h)$ . Here,  $a_{96}$  down to  $a_{89}$  are the final values of the eight 32-bit chaining variables.

Select the message difference  $\Delta m_5 = 2^i$ , we can find eight IV-dependent nearcollision differential paths named  $DP_1$ ,  $DP_2$ , ...,  $DP_8$  orderly in Table  $\Omega$  and they are all based on a local collision from step 6 to 33. The message word  $m_5$  occurs at step 6, 33 and 95. The differential paths  $DP_1$ ,  $DP_2$ ,  $DP_3$  and  $DP_4$  work for the cases  $i = 0 \sim 2, 6 \sim 9, 11 \sim 14, 18 \sim 30$ . And the differential paths  $DP_5$ ,  $DP_6$ ,  $DP_7$  and  $DP_8$  work for the cases  $i = 0 \sim 9, 11 \sim 13, 17 \sim 24, 28 \sim 31$ . The other values of i fail because of bit expansion. Each of the local collision path holds with probability about  $2^{-67}$ . So the message pair  $(M, M + \Delta M)$  obey one of the eight paths with probability  $2^{-64}$ . These paths are all dependent with  $k_a$ of IV. The outer key-recovery attack on NMAC based on 3-Pass HAVAL can be divided into the following process.

**Recovering the key**  $k_a$ : According to the properties of the 3-Pass HAVAL round functions listed in Table 5, we know that any one-block message pair  $(M, M + \Delta M)$  with  $\Delta m_5 = 2^i$   $(i \neq 10, 15, 16)$  will obey one of the eight paths in Table 5 and cause the collision in  $(h_c, h_d, h_e, h_f, h_g, h_h)$  with probability about  $2^{-61}$  when  $k_{a,i+1} = 0$ , otherwise when  $k_{a,i+1} \neq 0$ , M and  $M + \Delta M$  consist of a collision on the six output values with probability lower than  $2^{-69}$ .

So for each *i* of the message difference  $\Delta m_5 = 2^i$  where *i* runs from 0 to 31 and  $i \neq 10, 15, 16$ , we can recover one bit on  $k_a$  of IV using the technique in  $\square$  as follows:

- 1. Generate pairs of message satisfying  $H_{k_2}(M') = H_{k_2}(M) + \Delta M$  where  $\Delta M$  is  $\Delta m_5 = 2^i$ .
- 2. Send M and M' to the NMAC based on 3-Pass HAVAL oracle. Once roughly  $2^{61}$  pairs of messages (M, M') are queried, if a collision of  $(h_c, h_d, h_e, h_f, h_g, h_h)$  is obtained, we judge the outer key bit  $k_{a,i} = 0$ , otherwise we tell  $k_{a,i} = 1$ . So with  $2^{62}$  queries, we will recover one bit of  $k_a$ .
- 3. Change i, and repeat step 1 and 2 until all values of i are used.

In this way, we can recover 29 bits of  $k_a$ .

**Detecting the bits in**  $a_{95}$  and  $a_{92}$ : For those *i* that result to the *i*-th bit of  $k_a$  equal to zero, we use the near-collision technique in [14] to recover  $k_e$  and  $k_b$  as follows:

1. Generate the message pairs which make  $H_{k_2}(M') = H_{k_2}(M) + \Delta M$  where  $\Delta M$  is  $\Delta m_5 = 2^i$ .

- 2. Send such messages M and M' to the NMAC based on 3-Pass HAVAL oracle to obtain each of the following two kinds of near-collisions:
  - Pairs  $(M_e^i, M_e^{i'})$ : such that  $\Delta h_a = 0$ ,  $\Delta h_b = 2^i$ ,  $\Delta h_c = 0, ..., \Delta h_h = 0$ . - Pairs  $(M_b^i, M_b^{i'})$ : such that  $\Delta h_a = \pm 2^{i+1-7}$ ,  $\Delta h_b = 2^i$ ,  $\Delta h_c = 0, ..., \Delta h_h = 0$ .
- 3. Change *i*, and repeat step 1 and 2 until all values of *i* that result to the *i*-th of  $k_a = 0$  are used.

Once we obtain the pairs  $(M_e^i, M_e^{i'})$ , according to Boolean function properties of 3-Pass HAVAL, we can tell  $a_{92,i} = 1$ . If we get the pairs  $(M_b^i, M_b^{i'})$ , we can judge  $a_{95,i} = 1$ .

In total, the number of the message pairs  $(M_e^i, M_e^{i'})$  and  $(M_b^i, M_b^{i'})$  that we have obtained such that  $a_{92,i} = 1$  and  $a_{95,i} = 1$  is about 29.

**Recovering the key**  $k_b$  and  $k_e$ : We can recover the key  $k_b$  and  $k_e$  by the offline work using the technique in [14]. The methods to recover  $k_b$  and  $k_e$  are the same. We only pick  $k_b$  as an example.

- 1. Guess the bits  $k_{b,10}$ ,  $k_{b,15}$ ,  $k_{b,16}$ , and the bits  $k_{b,i}$  corresponding to  $k_{a,i} = 1$  that we fail obtaining  $M_b^i$ . On average, the total number of possibilities is about  $2^{17}$ .
- 2. Calculate other bits of  $k_b$  from the least significant to the most significant bit using  $M_b^i$ . Suppose we have get the first t bits of  $k_b$ , the (t+1)-th bit of  $k_b$ will be calculated using the obtained  $M_b^{t+1}$ . Compare  $k_{b,t\sim0}$  with  $h_{b,t\sim0}$ . If  $k_{b,t\sim0} > h_{b,t\sim0}$ , there exists a carry from bit t to t+1 during the computation of  $k_b + a_{95}$ . Otherwise, there will be no carry from bit t to t+1 during the computation of  $k_b + a_{95}$ . Since  $a_{95,t+1} = 1$ , the carry influence is known, and the value  $h_{b,t+1}$  is known, so the value  $k_{b,t+1}$  can be calculated.

**Complexity computation:** As explained in section 4.1, the key number we can obtain on average is 29 + 29 = 58 by online work, and the remaining keys can be search exhaustively. So the complexity of the offline exhaustive search needs  $2^{198}$  3-Pass HAVAL computations.

Now we only need to analyze the complexity of online work. To recover  $k_a$ , we need  $29 \times 2^{61}$  message pairs. The message  $(M_b^i, M_b^{i'})$  can be obtained with the probability  $2^{-61} \times 2^{-2} = 2^{-63}$ . The message  $(M_e^i, M_e^{i'})$  can be obtained with the probability  $2^{-61} \times 2^{-3} = 2^{-64}$ . In this way, the total number of message pairs required to recover the 58 key bits is about  $29 \times 2^{64} < 2^{69}$ . The cost of the message generation is the offline work because we have known  $k_2$ . Using the birthday attack, it need about  $2^{163}$  3-Pass HAVAL computations to get the  $2^{69}$  message pairs needed in the outer functions.

Overall, the complexity to recover the outer key of NMAC based on 3-Pass HAVAL is about  $2^{69}$  online MAC queries and  $2^{198}$  offline operations.

#### 4.2 Outer Key-Recovery on NMAC Based on 4-Pass HAVAL

Let the 256-bit output value of HMAC/NMAC based on 4-Pass HAVAL be  $(h_a, h_b, h_c, h_d, h_e, h_f, h_g, h_h)$ , where  $h_a = k_a + a_{128}$ ,  $h_b = k_b + a_{127}$ ,  $h_c = k_c + a_{126}$ ,  $h_d = k_d + a_{125}$ ,  $h_e = k_e + a_{124}$ ,  $h_f = k_f + a_{124}$ ,  $h_g = k_g + a_{122}$ ,  $h_h = k_h + a_{121}$ .

In the case of 4-Pass HAVAL, we use the near-collision differential path in Table  $\square$  to recover the out keys  $k_1$  of the NMAC. The message difference is  $\Delta m_5 = 2^i$ . The near-collision differential path can be divided into two parts. The first local collision path is from step 6 to 33 which include two path and each have a probability  $2^{-59}$ . And the second near collision is from step 95 to 128.

According to the Boolean functions properties of 4-Pass HAVAL, the nearcollision differential paths in Table [10] is independent with the IV. We can recover the keys according to the shape of the near-collision.

Extracting the bit values  $a_{122}$ ,  $a_{125}$ ,  $a_{126}$  and  $a_{127}$ : For each  $0 \le i \le 31$ ,  $i \ne 10, 15, 16$ , the key recovery processor as follows:

- 1. Generate pairs of message satisfying  $H_{k_2}(M') = H_{k_2}(M) + \Delta M$  where  $\Delta M$  is  $\Delta m_5 = 2^i$ .
- 2. Send such messages M and M' to the NMAC based on 4-Pass HAVAL oracle to obtain the following near-collisions shape:
  - $(M_i, M'_i): \Delta h_a = \pm 2^{i-18}, \Delta h_b = 2^{i-12}, \Delta h_c = 0, \Delta h_d = 0, \Delta h_e = 0, \Delta h_f = 2^i, \Delta h_g = 0, \Delta h_h = 0.$
- 3. Change i, and repeat step 1 and 2 until all values of i are used.

Once we obtain the pair  $(M_i, M'_i)$ , we can judge that  $a_{122,i} = 1, a_{125,i} = a_{123,i}, a_{126,i+1} = 1, a_{127,i-12} = 1$ . So it easy to recover the 29 key bits of  $k_b, k_c, k_d$  and  $k_q$  utilizing the technique described in section 4.1.

Thus, we can recover  $29 \times 4 = 116$  key bits by the online work. The left 140 bits can be obtained by exhaustive search. The probability to obtain a pair  $(M_i, M'_i)$  is about  $2^{-58} \times 2^{-40} = 2^{-98}$ , where  $2^{-58}$  is the probability of local collision from 6 to 33 and  $2^{-40}$  is the near-collision probability from step 95 to 128. So the total online complexity to recover the 116 key bits is about  $29 \times 2^{98} \approx 2^{103}$ . In order to obtain the  $2^{103}$  online message pairs, it needs about  $2^{180}$  offline 4-Pass HAVAL computations.

# 5 Conclusion

This paper give the full key recovery attack on the HMAC/NMAC based on the 3 and 4-Pass HAVAL. Our main contribution is to find series of collision and near-collision differential paths which is appropriate to attack the HMAC/ NMAC based on 3 and 4-Pass HAVAL utilizing the recent technique [3]4[14].

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# Appendix

Table 4.	Round	functions	of 3	and	4-Pass	HAVAL	

Pass	Round functions
3-Pass	$f_1(g, f, e, d, c, b, a) = cd \oplus ag \oplus bf \oplus ce \oplus e$
	$f_{2}(g, f, e, d, c, b, a) = adf \oplus bcf \oplus ef \oplus ef \oplus ac \oplus df \oplus bd \oplus bc \oplus fg \oplus g$
	$f_3(g, f, e, d, c, b, a) = def \oplus cf \oplus be \oplus dg \oplus ad \oplus a$
4-Pass	$f_1(g, f, e, d, c, b, a) = bd \oplus fg \oplus ce \oplus ad \oplus a$
	$f_2(g, f, e, d, c, b, a) = abg \oplus bcf \oplus bg \oplus cg \oplus bd \oplus af \oplus cf \oplus be \oplus e$
	$f_3(g, f, e, d, c, b, a) = acg \oplus cd \oplus ae \oplus bg \oplus fg \oplus f$
	$f_4(g,f,e,d,c,b,a) = bcf \oplus ace \oplus afg \oplus ab \oplus cg \oplus af \oplus ef \oplus fg \oplus ae \oplus ag \oplus ad \oplus d$

$\Delta a$ $\angle$	$\Delta b$ .	$\Delta c$	$\Delta d$	$\Delta e$	$\Delta f$	$\Delta g$	$\Delta f_1 = 0$	$\Delta f_1 = 1$	$\Delta f_1 = -1$
0	0	0	0	0	0	0	1	_	_
1	0	0	0	0	0	0	g = 0	$g = 1,  cd \oplus bf \oplus ce \oplus e = 0$	$g = 1,  cd \oplus bf \oplus ce \oplus e = 1$
-1	-	0	0	0	0	0	g = 0	$g = 1, cd \oplus bf \oplus ce \oplus e = 1$	$g=1,cd\oplus bf\oplus ce\oplus e=0$
0	1	0	0	0	0		f = 0	$f = 1, cd \oplus ag \oplus ce \oplus e = 0$	$f = 1, cd \oplus ag \oplus ce \oplus e = 1$
0	-1	0	0	0	0	0	f = 0	$f=1, cd \oplus ag \oplus ce \oplus e=1$	$f = 1,  cd \oplus ag \oplus ce \oplus e = 0$
0	0	1	0	0	0	0	e = d	$e \neq d,  ag \oplus bf \oplus e = 0$	$e \neq d, ag \oplus bf \oplus e = 1$
0	0	-1	0	0	0	0	e = d	$e \neq d, ag \oplus bf \oplus e = 1$	$e \neq d, ag \oplus bf \oplus e = 0$
0	0	0	1	0	0	0	c = 0	$c = 1, ag \oplus bf \oplus ce \oplus e = 0$	$c = 1, ag \oplus bf \oplus ce \oplus e = 1$
0	0	0	-1	0	0	0	c = 0	$c = 1,  ag \oplus bf \oplus ce \oplus e = 1$	$c = 1, ag \oplus bf \oplus ce \oplus e = 0$
0	0	0	0	1	0	0	c = 1	$c = 0, cd \oplus ag \oplus bf = 0$	$c = 0, cd \oplus ag \oplus bf = 1$
0	0	0	0	-1	0	0	c = 1	$c = 0, cd \oplus ag \oplus bf = 1$	$c = 0,  cd \oplus ag \oplus bf = 0$
0	0	0	0	0	1	0	b = 0	$b = 1, cd \oplus ag \oplus ce \oplus e = 0$	$b = 1, cd \oplus ag \oplus ce \oplus e = 1$
0	0	0	0	0	-1	0	b = 0	$b=1,cd\oplus ag\oplus ce\oplus e=1$	$b=1,cd\oplus ag\oplus ce\oplus e=0$
0	0	0	0	0	0	1	a = 0	$a = 1, cd \oplus bf \oplus ce \oplus e = 0$	$a = 1, cd \oplus bf \oplus ce \oplus e = 1$
0	0	0	0	0	0	-1	a = 0	$a=1,cd\oplus bf\oplus ce\oplus e=1$	$a = 1, cd \oplus bf \oplus ce \oplus e = 0$

**Table 5.** Some properties for the first round function  $f_1$  of the 3-Pass HAVAL

Table 6. Collision differential paths for HMAC based on 3-Pass HAVAL with probability  $2^{-96.26}$ 

Step	$m_{i-1}$	$\Delta m_i$	$DP_1$	$DP_2$	$DP_3$	$DP_4$	$DP_5$
21	$m_{20}$	$2^{30}$	$a_{21}[30]$	$a_{21}[30]$	$a_{21}[30]$	a <sub>21</sub> [30]	$a_{21}[30]$
22	$m_{21}^{-0}$		a22	a22	a22	a22	a22
23	$m_{22}$		a23	a23	a23	a23	a23
24	$m_{23}$		a24	a24	a24	a24	a24
25	$m_{24}$		a25	a25	a25	a25	a25
26	$m_{25}$		a26	a26	a26	a26	a26
27	$m_{26}$		a27	a27	a27	a27	a27
28	$m_{27}^{$		a28	a28	a28	a28	a28
29	$m_{28}^{-1}$		$a_{29}[19]$	$a_{29}[19]$	$a_{29}[19]$	$a_{29}[19]$	$a_{29}^{-}[19]$
30	$m_{29}^{-0}$		a30	a30	a30	a30	a30
31	$m_{30}$		a31	a31	a31	a31	a31
32	$m_{31}$		a32	a32	a32	a32	a32
33	$m_5$		a33	a33	a33	a33	a33
34	$m_{14}^{\circ}$		a34	a34	a34	a34	a34
35	$m_{26}$		a35	a35	a35	a35	a35
36	m18		a36	a36	a <sub>36</sub>	a <sub>36</sub>	a <sub>36</sub>
37	$m_{11}^{10}$		$a_{37}[-8, -9, 10]$	$a_{37}[-8, -9, 10]$	$a_{37}[-8, -9, 10]$	$a_{37}[-8, -9, 10]$	$a_{37}[-8, -9, 10]$
38	$m_{28}$		$a_{38}[3, 4, 5, -6]$	a38	a38	a38	a <sub>38</sub>
39	$m_7^2$		a39	$a_{39}[3, 4, 5, -6]$	a39	a39	a39
40	$m_{16}$		a40	a40	$a_{40}[3, 4, 5, 6]*$	a40	$a_{40}^{a_{40}}$
41	$m_0$		a41	a41	a <sub>41</sub>	$a_{41}[3, 4, 5, 6]*$	a41
42	$m_{23}$		$a_{42}$	$a_{42}$	$a_{42}$	$a_{42}$	$a_{42}[3, 4, 5, 6]*$
43	$m_{20}$	$2^{30}$	$a_{43}$	$a_{43}$	$a_{43}$	$a_{43}$	$a_{43}$
44	$m_{22}^{20}$		$a_{44}^{40}[-31]$	$a_{44}^{40}[-31]$	$a_{44}^{40}[-31]$	$a_{44}^{40}[-31]$	$a_{44}^{40}[-31]$
45	$m_1^{22}$		a45	a45	a45	a45	a45
46	$m_{10}^{1}$		a46	a46	a46	a46	a46
47	$m_4$		a47	a47	a47	a47	a47
48	$m_8$		a48	a48	a48	a48	a48
49	$m_{30}$		a49	a49	a49	a49	a49
50	$m_3$		a 50	a50	a50	a50	a50
51	$m_{21}$		$a_{51}^{\circ}$	$a_{51}^{\circ}$	$a_{51}^{\circ \circ}$	a51	a51
52	$m_9$		$a_{52}[-20]$	$a_{52}[-20]$	$a_{52}[-20]$	$a_{52}[-20]$	$a_{52}^{-}[-20]$
53	$m_{17}^{\circ}$		a53	a53	a53	a53	a53
54	$m_{24}$		a54	$a_{54}$	$a_{54}$	a54	a54
55	$m_{29}$		a55	a55	a55	a55	a55
56	m6		a56	a56	a56	a56	a56
57	$m_{19}$		a57	a57	a57	a57	a57
58	$m_{12}$		a58	a58	a 58	a58	a58
59	$m_{15}$		a59	$a_{59}$	$a_{59}$	a59	$a_{59}$
60	$m_{13}$		$a_{60}[-9]$	$a_{60}[-9]$	$a_{60}[-9]$	$a_{60}[-9]$	$a_{60}[-9]$
61	$m_2$		a61	a61	a61	a61	a61
62	$m_{25}$		a62	a62	a62	a62	a62
63	$m_{31}$		a63	a63	a63	a63	a63
64	$m_{27}$		a64	$a_{64}$	a64	a64	a64
65	$m_{19}$		$a_{65}$	$a_{65}$	a65	a65	a65
66	$m_9$		a66	a66	<sup>a</sup> 66	<sup>a</sup> 66	a66
67	$m_4$		a67	a <sub>67</sub>	a67	a67	a67
68	$m_{20}$	$2^{30}$	a68	a <sub>68</sub>	a68	a68	a <sub>68</sub>
			$pr=2^{-98.96}$	$pr=2^{-98.96}$	$pr=2^{-97.21}$	$pr=2^{-99.21}$	$pr=2^{-100.52}$

ľ	step	$m_i$		$DP_1$	$DP_2$
	6	$m_5$	$2^{19}$	$a_6[-19, 20]$	a <sub>6</sub> [19]
	7	$m_6$		a7	a7
	8	$m_7$		a.8	a8
	9	$m_8$		a9	a9
	10	$m_{9}$		$a_{10}[13, 14, 15, -16]$	a9
	11	$m_{10}$		a11	a <sub>11</sub>
	12	$m_{11}$		a <sub>12</sub>	a <sub>12</sub>
	13	$m_{12}$		a13	a <sub>13</sub>
	14	$m_{13}$		a <sub>14</sub>	$a_{14}[-8, 9]$
	15	$m_{14}$		a <sub>15</sub>	a <sub>15</sub>
	16	$m_{15}$		a16	a <sub>16</sub>
	17	$m_{16}$		$a_{17}[-9]$	a <sub>17</sub>
	18	$m_{17}$		a <sub>18</sub>	$a_{18}^{-1}[2, 3, 4, -5]$
	19	$m_{18}$		a19	a <sub>19</sub>
	20	$m_{19}$		a20	a20
	21	$m_{20}$		a21	a <sub>21</sub>
	22	$m_{21}$		a22	a22
	23	$m_{22}$		a23	a23
	24	$m_{23}$		a24 [ 20]	a24 [ 0.0]
	25	$m_{24}$		$a_{25}[-30]$	$a_{25}[-30]$
	26 27	$m_{25}$		<sup>a</sup> 26	a <sub>26</sub>
	27	$m_{26}$		<sup>a</sup> 27	a <sub>27</sub>
	28 29	$m_{27}$		a28	a28
	29 30	$m_{28}$		a29	a29
	30 31	$m_{29}$		a30	a <sub>30</sub>
	32	m <sub>30</sub>		a31	a <sub>31</sub>
	-	$m_{31}$	2 <sup>19</sup>	a <sub>32</sub>	a <sub>32</sub>
ŀ	33	$m_5$	2-5	$a_{33}$ $pr = 2^{-59}$	$a_{33}$ $pr = 2^{-59}$

Table 7. First inner collision differential path (step 6-33) for NMAC based on 4-Pass HAVAL with probability  $2^{-58}$ 

Table 8. Second inner collision differential path (step 95-123) for NMAC based on 4-Pass HAVAL with probability  $2^{-63.27}$ 

step	$m_i$	$\Delta m_i$	$DP_1$	$DP_2$	$DP_3$	$DP_4$	$DP_5$	$DP_6$
95	$m_5$	2 <sup>19</sup>	$a_{95}[-19, 20]$	$a_{95}[-19, 20]$	$a_{95}[-19, 20]$	a <sub>95</sub> [19]	a <sub>95</sub> [19]	a <sub>95</sub> [19]
	$m_2$		a 96	a96	a96	a96	a96	a96
97	$m_{24}$		a97	a97	a97	a97	a97	a97
98	$m_4$		a98	a 98	a98	a98	a98	a98
	$m_0$		a99	a99	a99	a99	a99	a99
100	$m_{14}$		$a_{100}[13, 14, 15, 16]^*$	a100	a100	a100	a100	a100
101	$m_2$		a101	$a_{101}[13, 14, 15, 16]^*$	a101	a101	a101	a101
102			a102	a102	$a_{102}[13, 14, 15, 16]^*$	a102	a102	a102
103	$m_{28}$		a103	a103	a103	$a_{103}[-8, 9]$	$a_{103}[-8, 9]$	$a_{103}[-8, 9]$
104	$m_{23}^{-5}$		a104	a <sub>104</sub>	a <sub>104</sub>	a104	a104	a <sub>104</sub>
105	$m_{26}^{-0}$		a105	a105	a105	a105	a105	a105
106	$m_6$		a106	a106	a106	a106	a106	a106
107	$m_{30}^{\circ}$		$a_{107}[-9]$	$a_{107}[-9]$	$a_{107}[-9]$	a107	a107	a107
108	$m_{20}$		a108	a108	a108	$a_{108}[2, 3, 4, 5]^*$	a108	a108
109	$m_{18}^{-5}$		a109	a109	a109	a109	$a_{109}^{103}[2, 3, 4, 5]^*$	a109
	$m_{25}$		a110	a110	a110	a110	a110	$a_{110}[2, 3, 4, 5]^*$
111	$m_{19}$		a111	a <sub>111</sub>	a <sub>111</sub>	a111	a <sub>111</sub>	a111
112	$m_3$		a112		a112	a112	a112	a112
113	$m_{22}^{0}$		a113	a113	a113	a113	a113	a113
114	$m_{11}^{22}$		a114	a114	a114	a114	a114	a114
115	$m_{31}$		$a_{115}[-30]$	$a_{115}[-30]$	$a_{115}[-30]$	$a_{115}[-30]$	$a_{115}[-30]$	$a_{115}[-30]$
116	$m_{21}$		a116	a116	a116	a116	a116	a116
117	$m_8$		a117	a <sub>117</sub>	a <sub>117</sub>	a117	a117	a117
118	$m_{27}$		a <sub>118</sub>	a118	a118	a118	a118	a118
119	$m_{12}$		a119	a119	a119	a <sub>119</sub>	a119	a119
$120 \\ 121$			a120	a120	a120	a120	a120	a120
121	<sup>m</sup> 1		a <sub>121</sub>	a121	a121	a121	a121	a121
	$m_{29}^{1}$	2 <sup>19</sup>	a122	a122	a122	a122	a122	a122
123	$m_5$		a123 64.00	a123 67.47	a123 65 10	a123	a123	a123
			$pr = 2^{-64.99}$	$pr = 2^{-67.47}$	$pr = 2^{-65.19}$	$pr = 2^{-65.77}$	$pr = 2^{-68.05}$	$pr = 2^{-65.77}$

Step	$m_i$	$\Delta m_i$	$DP_1$	$DP_2$	$DP_3$	$DP_4$
6		$2^i$		-		
	$m_5$	21	$a_6[-i, i+1]$	$a_6[-i, i + 1]$	$a_6[-i, i + 1]$	$a_6[-i, i+1]$
7	$m_6$		a7	a7	a7	a7
8	$m_7$		a8	a8	a8	a8
9	$m_8$		a9	a9	a9	a9
10	$m_9$		$a_{10}[i - 6, i - 5,$	a <sub>10</sub>	a10	a10
	5		i - 4, -(i - 3)]	10	10	10
11	$m_{10}$		a <sub>11</sub> (* 3)]	$a_{11}[i-6, i-5, i-4, -(i-3)]$	a <sub>11</sub>	a <sub>11</sub>
12	$m_{11}$		a <sub>12</sub>	a <sub>12</sub>	$a_{12}[i-6, i-5, i-4, -(i-3)]$	a <sub>12</sub>
13	$m_{12}$		a <sub>13</sub>	a <sub>13</sub>	a <sub>13</sub>	$a_{13}[i-6, i-5, i-4, -(i-3)]$
14	$m_{13}$		a <sub>14</sub>	a <sub>14</sub>	a <sub>14</sub>	a <sub>14</sub>
15	$m_{14}$		a15	a15	a15	a15
	$m_{15}^{14}$		a <sub>16</sub>	a <sub>16</sub>	a16	a16
	$m_{16}^{13}$		$a_{17}^{10}[-(i-10)]$	$a_{17}^{10}[-(i-10)]$	$a_{17}^{10}[-(i-10)]$	$a_{17}^{10}[-(i-10)]$
18	$m_{17}^{10}$				a <sub>18</sub>	
			a18	a18		a18
19	$m_{18}$		a <sub>19</sub>	a <sub>19</sub>	a <sub>19</sub>	a <sub>19</sub>
20	$m_{19}$		a20	a20	a20	a20
	$m_{20}$		a <sub>21</sub>	a21	a21	a21
	$m_{21}$		a22	a22	a22	a22
	$m_{22}$		a23	a23	a23	a23
	$m_{23}^{$		a24	a24	a24	a <sub>24</sub>
	$m_{24}^{20}$		$a_{25}^{24}[-(i-21)]$	$a_{25}^{24}[-(i-21)]$	$a_{25}^{24}[-(i-21)]$	$a_{25}^{24}[-(i-21)]$
	$m_{25}^{24}$		a26	a26	a26	a26
	$m_{26}^{m_{25}}$		a26 a27	a26 a27	a26 a27	a26 a27
	$m_{27}$		a28	a28	a28	a28
20	$m_{28}$		a29	a29	a29	a29
	$m_{29}$		a <sub>30</sub>	a <sub>30</sub>	a <sub>30</sub>	a <sub>30</sub>
	$m_{30}$		a31	a <sub>31</sub>	a31	a <sub>31</sub>
32	$m_{31}$		a <sub>32</sub>	a <sub>32</sub>	a32	a <sub>32</sub>
33	$m_5$	$2^{i}$	a33	a <sub>33</sub>	a33	a <sub>33</sub>
		-	- 33	-33	-33	-33
		$2^{i}$		r - 1	r - 1	r - 1
95	$m_5$	2	$a_{95}[i]$	$a_{95}[i]$	$a_{95}[i]$	$a_{95}[i]$
						9011
96	$m_2$		a96	a96	a96	a96
96 Step	$m_2$ $m_i$	$\Delta m_i$				a96 DP8
Step	$m_2$ $m_i$		a <sub>96</sub> DP <sub>5</sub>	<sup>a</sup> 96 DP <sub>6</sub>	a <sub>96</sub> DP <sub>7</sub>	a <sub>96</sub> DP <sub>8</sub>
Step 6	$m_2$ $m_i$ $m_5$	$\frac{\Delta m_i}{2^i}$	$a_{96}$ $DP_5$ $a_6[i]$	$a_{96}$ $DP_{6}$ $a_{6}[i]$	$\frac{a_{96}}{DP_7}$ $a_6[i]$	$\frac{a_{96}}{DP_8}$ $a_6[i]$
Step 6 7	$m_2 \\ m_i \\ m_5 \\ m_6$		$a_{96}$ $DP_5$ $a_6[i]$ $a_7$	$a_{96}$ $DP_{6}$ $a_{6}[i]$ $a_{7}$	$a_{96}$ $DP_7$ $a_6[i]$ $a_7$	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$
Step 6 7 8	$m_2 \\ m_i \\ m_5 \\ m_6 \\ m_7 \\ m_7 \\ m_7 \\ m_7 \\ m_7 \\ m_1 \\ m_2 \\ m_1 \\ m_2 $		$a_{96}$ $DP_5$ $a_6[i]$ $a_7$ $a_8$	$a_{96}$ $DP_{6}$ $a_{6}[i]$ $a_{7}$ $a_{8}$	$a_{96}$ $DP_7$ $a_6[i]$ $a_7$ $a_8$	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$ $a_8$
Step 6 7 8 9	$m_2 \ m_1 \ m_5 \ m_6 \ m_7 \ m_8$		$a_{96}$ $DP_5$ $a_6[i]$ $a_7$ $a_8$ $a_9$	$a_{96}$ $DP_6$ $a_6[i]$ $a_7$ $a_8$ $a_9$	a <sub>96</sub> DP <sub>7</sub> a <sub>6</sub> [i] a <sub>7</sub> a <sub>8</sub> a <sub>9</sub>	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$ $a_8$ $a_9$
Step 6 7 8 9 10	$m_2 \ m_1 \ m_5 \ m_6 \ m_7 \ m_8 \ m_9$		$a_{96}$ $DP_5$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$	$a_{96}$ $DP_{6}$ $a_{6}[i]$ $a_{7}$ $a_{8}$ $a_{9}$ $a_{10}$	$a_{96}$ $DP_7$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$
Step 6 7 8 9 10 11	$m_2 \ m_i \ m_5 \ m_6 \ m_7 \ m_8 \ m_9 \ m_{10}$		$a_{96}$ $DP_5$ $a_6 [i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$	$a_{96}$ $\overline{DP_6}$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$	$a_{96}$ $\overline{DP_7}$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$	$a_{96}$ $\overline{DP_8}$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$
Step 6 7 8 9 10 11 12	$m_2 \ m_1 \ m_5 \ m_6 \ m_7 \ m_8 \ m_9 \ m_{10} \ m_{11}$		$a_{96}$ $DP_5$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$	$a_{96}$ $DP_6$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$	$a_{96}$ $DP_{7}$ $a_{6}[i]$ $a_{7}$ $a_{8}$ $a_{9}$ $a_{10}$ $a_{11}$ $a_{12}$	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$
Step 6 7 8 9 10 11 12 13	$m_2$ $m_i$ $m_5$ $m_6$ $m_7$ $m_8$ $m_9$ $m_{10}$ $m_{11}$ $m_{12}$		$a_{96}$ $D_{75}$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$	$a_{96}$ $DP_6$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$	$a_{96}$ $DP_7$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$
Step 6 7 8 9 10 11 12 13 14	$m_2$ $m_i$ $m_5$ $m_6$ $m_7$ $m_8$ $m_9$ $m_{10}$ $m_{11}$ $m_{12}$ $m_{13}$		$a_{96}$ $D_{75}$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \end{array}$	$a_{96}$ $DP_7$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$	$a_{96}$ $DP_8$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$
Step 6 7 8 9 10 11 12 13 14 15	$m_2$ $m_i$ $m_5$ $m_6$ $m_7$ $m_8$ $m_9$ $m_{10}$ $m_{11}$ $m_{12}$ $m_{13}$ $m_{14}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \end{array}$	$a_{96}$ $DP_7$ $a_6[i]$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $a_{13}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \end{array}$
Step 6 7 8 9 10 11 12 13 14 15	$m_2$ $m_i$ $m_5$ $m_6$ $m_7$ $m_8$ $m_9$ $m_{10}$ $m_{11}$ $m_{12}$ $m_{13}$ $m_{14}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \end{array}$
Step 6 7 8 9 10 11 12 13 14 15 16	$m_2$ $m_i$ $m_5$ $m_6$ $m_7$ $m_8$ $m_9$ $m_{10}$ $m_{11}$ $m_{12}$ $m_{13}$ $m_{14}$ $m_{15}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{16} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \end{array}$
Step 6 7 8 9 10 11 12 13 14 15 16 17	$\begin{array}{c} m_2 \\ m_1 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$
Step 6 7 8 9 10 11 12 13 14 15 16 17	$m_2$ $m_i$ $m_5$ $m_6$ $m_7$ $m_8$ $m_9$ $m_{10}$ $m_{11}$ $m_{12}$ $m_{13}$ $m_{14}$ $m_{15}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18}[i-17, i-16, \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \end{array}$
Step 6 7 8 9 10 11 12 13 14 15 16 17 18	$\begin{array}{c} m_2 \\ m_1 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         19	$\begin{array}{c} m_2 \\ m_1 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \\ m_{17} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \end{array}$
Step         6           7         8           9         10           11         12           13         14           15         16           17         18           19         20           21         21	$\begin{array}{c} m_2 \\ m_1 \\ m_5 \\ m_6 \\ m_7 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ i_{-15}, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \end{array}$
Step         6           7         8           9         10           11         12           13         14           15         16           17         18           19         20           21         22	$\begin{array}{c} m_2 \\ m_1 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \\ m_{21} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{16} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \end{array}$
Step         6           7         8           9         10           11         12           13         14           15         16           17         18           19         20           21         22	$\begin{array}{c} m_2 \\ m_1 \\ m_5 \\ m_6 \\ m_7 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ i_{-15}, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{23} \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \end{array}$
Step         6           7         8           9         10           11         12           13         14           15         16           17         18           19         20           21         22           23         23	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ \underline{m_5} \\ \underline{m_6} \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \\ m_{21} \\ m_{22} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \end{array}$	$\begin{array}{c} a_{96} \\ \hline pP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{23} \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_8 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20         21           22         23         24         24	m2           mi           m5           m6           m7           m8           m9           m11           m12           m13           m14           m16           m17           m18           m19           m20           m21           m22           m23		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{22} \\ a_{23} \\ a_{24} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15           16         17         18           19         20         21           22         23         24           25         5         5	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ \underline{m_5} \\ \underline{m_5} \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_8 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \end{array}$
$\begin{array}{ c c c c c }\hline Step \\\hline 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ \end{array}$	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ \underline{m_5} \\ \underline{m_5} \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{11} \\ m_{15} \\ m_{16} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \end{array}$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ a_{21} \\ [i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} \\ [-(i-21)] \\ a_{26} \\ \end{array}$
Step         6           6         7         8           9         10         11           12         13         14           15         16         17           18         19         20         21           22         23         24         25           26         27         27	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ \underline{m_5} \\ \underline{m_6} \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{12} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{26$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{16} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ \end{array}$	$\begin{array}{c} \underline{a_{96}} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ \end{array}$
$\begin{array}{ c c c c c c c }\hline Step \\\hline 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ 27 \\ 28 \\ \end{array}$	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{12} \\ m_{11} \\ m_{12} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{26} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{14} \\ a_{16} \\ a_{16} \\ a_{16} \\ a_{17} \\ a_{18} \\ i_{16} \\ a_{17} \\ i_{17} \\ a_{18} \\ i_{17} \\ a_{18} \\ i_{17} \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} \\ [-(i-21)] \\ a_{28} \\ a_{28} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ \end{array}$	$\begin{array}{c} \frac{a_{96}}{DP_7} \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{28} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{27} \\ a_{28} \\ a_{28} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20         21           22         23         24         25         26           27         28         29         29	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{11} \\ m_{14} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{29} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{29} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ a_{21} \\ [i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{27} \\ a_{28} \\ a_{29} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         9         20         21           21         22         23         24         25         26           22         23         24         25         26         27         28         29         30	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{12} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{29} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$	$\begin{array}{r} \underline{a_{96}} \\ \hline DP_{7} \\ a_{6}[i] \\ a_{7} \\ a_{8} \\ a_{9} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$	$\begin{array}{r} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20           21         22         23         24           25         26         27         28           29         30         31	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{11} \\ m_{14} \\ m_{17} \\ m_{18} \\ m_{19} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{29} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{29} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{29} \\ a_{30} \\ a_{31} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ a_{21} \\ [i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{27} \\ a_{28} \\ a_{29} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20           21         22         23         24           25         26         27         28           29         30         31         1	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{11} \\ m_{12} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{29} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{17} \\ a_{18} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{29} \\ a_{30} \\ a_{31} \\ \end{array}$	$\begin{array}{r} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20           21         22         23         24           25         26         27         28           29         30         31         32	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{29} \\ m_{20} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{29} \\ m_{20} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\$		$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$	$\begin{array}{r} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$	$\begin{array}{r} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20           21         22         23         24           22         23         24         25         26           26         27         28         29         30         31           32         33         33         33         33	$\begin{array}{c} \underline{m_2} \\ \underline{m_1} \\ \underline{m_5} \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{11} \\ m_{12} \\ m_{11} \\ m_{12} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{23} \\ m_{23} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\$	2 <sup><i>i</i></sup>	$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{15} \\ a_{16} \\ a_{16} \\ a_{16} \\ a_{17} \\ a_{18}[i-17,i-16, \\ a_{18}[i-17,i-16, \\ a_{18}[i-17,i-16, \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_6 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \\ \end{array}$	$\begin{array}{r} \underline{a_{96}} \\ \hline DP_8 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19} \\ a_{20} \\ a_{21} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ a_{33} \\ \vdots \end{array}$
Step         6           7         8         9           10         11         12           13         14         15         16           17         18         19         20           21         22         23         24           25         26         27         28           29         30         31         32	$\begin{array}{c} \underline{m_2} \\ \underline{m_i} \\ m_5 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{29} \\ m_{20} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{25} \\ m_{26} \\ m_{27} \\ m_{28} \\ m_{29} \\ m_{20} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\ m_{22} \\ m_{22} \\ m_{21} \\ m_{22} \\$	2'	$\begin{array}{c} a_{96} \\ \hline DP_5 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ i-15, -(i-14)] \\ a_{19} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$	$\begin{array}{r} a_{96} \\ \hline DP_6 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{19}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$	$\begin{array}{c} a_{96} \\ \hline DP_7 \\ \hline a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} [-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} [i-17, i-16, \\ i-15, -(i-14)] \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} [-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$	$\begin{array}{r} a_{96} \\ \hline DP_8 \\ a_6[i] \\ a_7 \\ a_8 \\ a_9 \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14}[-(i-11), i-10] \\ a_{15} \\ a_{16} \\ a_{17} \\ a_{18} \\ a_{20} \\ a_{21}[i-17, i-16, \\ i-15, -(i-14)] \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25}[-(i-21)] \\ a_{26} \\ a_{27} \\ a_{28} \\ a_{29} \\ a_{30} \\ a_{31} \\ a_{32} \\ \end{array}$

Table 9. The near-collision differential path of NMAC based on 3-Pass HAVAL with probability  $2^{-64}$ 

step	$m_i$	$\Delta m_i$	$DP_1$	$DP_2$
6	$m_5$	$2^i$	$a_6[-i, i + 1]$	$a_6[i]$
7	$m_6$		a7	a7
8	$m_7^0$		a8	a8
9	$m_8'$		a9	a9
10	$m_9$		$a_{10}^{\circ}[i-6, i-5, i-4, -(i-3)]$	a <sub>10</sub>
11	$m_{10}$		a <sub>11</sub>	a <sub>11</sub>
12	$m_{11}^{10}$		a <sub>12</sub>	a <sub>12</sub>
13	$m_{12}^{11}$		a <sub>13</sub>	a13
14	$m_{13}^{12}$		a <sub>14</sub>	$a_{14}^{i}[-(i-11), i-10]$
15	$m_{14}^{10}$		a15	a <sub>15</sub>
16	$m_{15}^{11}$		a <sub>16</sub>	a16
17	$m_{16}^{10}$		$a_{17}^{10}[-(i-10)]$	a17
18	$m_{17}^{10}$		a18	$a_{18}[i-17, i-16, i-15, -(i-14)]$
19	$m_{18}^{11}$		a19	a19
20	$m_{19}^{10}$		a20	a20
21	$m_{20}^{10}$		a21	a21
22	$m_{21}^{20}$		a22	a22
23	$m_{22}^{-1}$		a23	a23
24	$m_{23}^{22}$		a24	a24
25	$m_{24}^{20}$		$a_{25}^{24}[-(i-21)]$	$a_{25}^{24}[-(i-21)]$
26	$m_{25}^{24}$		a26	a26
27	$m_{26}^{20}$		a27	a27
28	$m_{27}^{20}$		a28	a28
29	$m_{28}^{-1}$		a29	a29
30	$m_{29}^{20}$		a30	a30
31	$m_{30}^{20}$		a <sub>31</sub>	a <sub>31</sub>
32	$m_{31}^{30}$		a <sub>32</sub>	a <sub>32</sub>
33	$m_5$	$2^{i}$	a <sub>33</sub>	a <sub>33</sub>
		-	-33	-33
95		$2^{i}$	a [i]	a [i]
96	$m_{5}$ $m_{2}$	4	a95[i]	a95[i]
90 97			a96	<sup>a</sup> 96
98	$m_{24}$		a97	a97
99	$m_4$		a98	a98
100	<i>m</i> <sub>0</sub>		a99	a99
101	m <sub>14</sub>		a100	a100
101	$m_{2} \\ m_{7}$		a101	<sup>a</sup> 101
102			$a_{102}^{a_{102}}$ $a_{103}[i - 11]$	$a_{102} a_{103}[i-11]$
103	$m_{28}$ $m_{23}$			
104			a104	a104
106	$m_{26} m_6$		$a_{105} a_{106}$	$a_{105} a_{106}$
107	$m_{30}$			
101	$m_{20}^{m_{30}}$		a <sub>107</sub> a <sub>108</sub>	$a_{107}^{a_{107}}$
109	$m_{18}^{m_{20}}$		a 108 a 109	a108 a109
110	$m_{18}^{m_{18}}$		a <sub>109</sub> a <sub>110</sub>	a <sub>109</sub> a <sub>110</sub>
111	$m_{19}^{m_{25}}$		$a_{111}^{i_{110}}[i-22]$	$a_{1110}^{a_{1110}} a_{1111}^{a_{1110}} [i - 22]$
112	$m_{19}$ $m_{3}$		a <sub>111</sub> [ <i>i</i> = 22] a <sub>112</sub>	$a_{111} [i = 22]$ $a_{112}$
113	$m_{22}^{m_3}$	[	a112 a113	a112 a113
114	$m_{11}^{22}$		a <sub>113</sub> a <sub>114</sub>	a113 a114
115	$m_{31}^{m_{11}}$		a <sub>115</sub>	a <sub>115</sub>
116	$m_{21}^{m_{31}}$	[	a115 a116	a115 a116
117	m <sub>8</sub>		a <sub>117</sub>	a <sub>117</sub>
118	$m_{27}^{m_8}$		a117 a118	a117 a118
119	$m_{12}^{m_{27}}$		$a_{119}^{a_{118}}[i-1]$	$a_{119}^{a_{118}}[i-1]$
120	m9		a119[i = 1] a120	$a_{119[i} = 1$ $a_{120}$
120	$m_1$		a120 a121	a120 a121
121	$m_{29}^{m_1}$		a121 a122	a121 a122
122	$m_{29}$ $m_{5}$	$2^{i}$		
123		2	a <sub>123</sub> [i]	a <sub>123</sub> [ <i>i</i> ]
124	m <sub>15</sub>		a124	a124
125	$m_{17}$		a125	a125
	$m_{10} m_{16}$	[	$a_{126}^{a_{126}}$ $a_{127}[i - 12]$	$a_{126} a_{127}[i-12]$
128	$m_{13}^{m_{16}}$		@127[0 - 12] @100	
120	13		a128	a128

Table 10. The near-collision differential path of NMAC based on 4-Pass HAVAL with probability  $2^{-98}$ 

# Memoryless Related-Key Boomerang Attack on the Full Tiger Block Cipher

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Abstract. In this paper we present the first attack on the full 24 round internal block cipher of Tiger [1]. Tiger is a hash function proposed by Biham and Anderson at FSE'96. It takes about ten years until the first cryptanalytic result was presented by Kelsey and Lucks [10] at FSE'06. Up to now, the best known attack on the internal block cipher of Tiger is able to break 22 rounds. Our attack on the full 24 rounds of the Tiger block cipher has a data complexity of  $2^{3.5}$  chosen plaintexts and ciphertexts, which can be called memoryless. This is since we do not have to store all the data generated in our attack. The time complexity is about  $2^{259.5}$  24-round Tiger encryptions. Moreover, we have further reduced the time complexity using a bit fixing technique to  $2^{195.5}$  24round encryptions.

**Keywords:** differential cryptanalysis, related-key boomerang attack, Tiger block cipher.

### 1 Introduction

Tiger II is a 160-bit hash function based on MERKLE-DAMGÅRD I3, 6, which operates on 512-bit message blocks.

The first cryptanalytic result for Tiger was a collision attack on 17 of 24 rounds of Tiger proposed by Kelsey and Lucks [10] using  $2^{49}$  compression function calls. They also presented a near-collision attack on 20-round Tiger with complexity  $2^{49}$ . Mendel et al. [11] found a collision attack on 19-round Tiger with complexity  $2^{62}$  and also a pseudo-near-collision for 22-round Tiger which needs about  $2^{44}$ compression function calls. At AsiaCrypt'07 Mendel and Rijmen [12] improved the previous results and presented a pseudo-near-collision attack on the full 24-round Tiger hash function with complexity  $2^{47}$  and a pseudo collision with the same complexity. Indesteege and Preneel [9] proposed a pre-image attack on 12 and 13 round Tiger and also a (2nd) pre-image attack on Tiger reduced to 12 rounds. The best cryptanalytic result on the Tiger block cipher is a 22-round related-key boomerang and rectangle attack [7].

In this paper we present the first attack that can break the full 24-round Tiger encryption mode. We use a related-key boomerang attack which has a data complexity of  $2^{3.5}$  chosen plaintexts and ciphertexts. The time complexity is about  $2^{259.5}$  24-round Tiger encryptions. Moreover we have reduced the time complexity using a bit fixing technique to  $2^{195.5}$  24-round Tiger encryptions.

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The paper is organized as follows: In Section 2 we give a brief description of the Tiger encryption mode. In Section 3 we describe the related-key boomerang attack. In Section 4 we present our related-key boomerang attack on the full Tiger encryption mode. Section 5 concludes the paper.

# 2 Description of the Tiger Block Cipher

The following notations are used in this paper:

 $\begin{array}{l} \boxplus : \text{addition modulo } 2^{64} \text{ operation} \\ \blacksquare : \text{subtraction modulo } 2^{64} \text{ operation} \\ \boxtimes : \text{multiplication modulo } 2^{64} \text{ operation} \end{array}$ 

 $X \ll i$ : shift of word X by i bits to the left

 $X \gg i$ : shift of word X by i bits to the right

Tiger's compression function is based on applying an internal "block cipher like" function, which takes a 192-bit "plaintext" and a 512 bit key to compute a 192-bit "ciphertext". The "block cipher like" function is applied according to the Davies-Meyer construction: a 512-bit message block is basically used as a key to encrypt the 192-bit chaining value, and then the input chaining value is fed forward to make the whole thing non-invertible. In the remainder of this section, we will describe Tiger in sufficient detail to follow the course of our attack; for a more detailed description of the hash function and its design rationale, the reader is referred to [I]. If, for a given input chaining value, we generate two different keys with the same output chaining value, then we have found a collision for Tiger.

Tiger has been designed with 64-bit architectures in mind. Accordingly, we will denote a 64-bit unsigned integer as a "word". We will represent a word as a hexadecimal number. Tiger uses arithmetic operations (addition, subtraction and multiplication by small constants), bit-wise XOR, NOT, logical shift operations and S-Box applications. The arithmetic operations over words are modulo  $2^{64}$ . The chaining value is represented internally as three 64-bit words, the message block as eight 64-bit words.

#### 2.1 The Tiger Round Function

In the terminology of [14], Tiger's block cipher like function is a "target-heavy unbalanced Feistel cipher". The block is broken into three words, labeled A, B, and C. Each round, a message word X is XORed into C:

$$C := C \oplus X.$$

Then A and B are modified:

$$A := A \boxminus \operatorname{even}(C),$$
  
$$B := B \boxplus \operatorname{odd}(C),$$
  
$$B := B \boxtimes (\operatorname{const}),$$

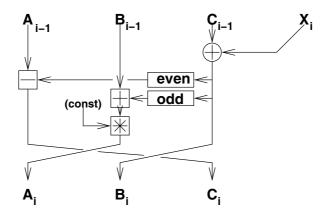


Fig. 1. The round function of Tiger

with a round-dependent constant (const)  $\in \{5, 7, 9\}$ . The results are then shifted around, so that A, B, C becomes B, C, A. See Figure  $\blacksquare$ . For the definition of **even** and **odd**, consider the word C being split into eight bytes  $C_0, \ldots, C_7$ , with  $C_0$ as the most significant byte. The functions **even** and **odd** employ four S-Boxes  $T_1, \ldots, T_4 : \{0, 1\}^8 \to \{0, 1\}^{64}$  as follows:

$$\mathbf{even}(C) := T_1[C_0] \oplus T_2[C_2] \oplus T_3[C_4] \oplus T_4[C_6],$$
$$\mathbf{odd}(C) := T_1[C_7] \oplus T_2[C_5] \oplus T_3[C_3] \oplus T_4[C_1].$$

We will refer to  $C_0, C_2, C_4$ , and  $C_6$  as the "even bytes of C".

The round function spreads changes around very quickly. A one-bit difference introduced into C in the first round will change about half the bits of the block by the end of the third round.

#### 2.2 The Key Schedule

Tiger consists of 24 rounds. Each round uses one message word  $X_i$  as its round key. The first eight round keys  $X_0, \ldots, X_7$  are identical to the 512-bit cipher key (or rather, to the 512-bit message block). The remaining 16 round keys are generated by applying the key schedule function:

$$(X_8, \dots, X_{15}) := \text{KeySchedule}(X_0, \dots, X_7)$$
$$(X_{16}, \dots, X_{23}) := \text{KeySchedule}(X_8, \dots, X_{15})$$

The key schedule function uses logical shifts on words, denoted by  $\ll$  and  $\gg.$  E.g.,

 $1111\,5555\,9999\,FFFF \ll 5 = 222A\,AAB3\,333F\,FFE0 \quad \text{and} \\ 222A\,AAB3\,333F\,FFE0 \gg 9 = 0011\,1555\,5999\,9FFF.$ 

Further, it uses the bit-wise NOT function, e.g. for  $X = \text{EEEE} \text{AAAA} 6666\,0000$ , the negation  $\overline{X}$  of X is  $\overline{X} = 1111\,5555\,9999\,\text{FFFF}$ . The key schedule modifies its input  $(x_0, \ldots, x_7)$  in two passes:

first pass second pass 1.  $x_0 := x_0 \boxminus (x_7 \oplus \text{Const}_1)$ 9.  $x_0 := x_0 \boxplus x_7$ 2.  $x_1 := x_1 \oplus X_0$ 10.  $x_1 := x_1 \boxminus (x_0 \oplus (\overline{x_7} \ll 19))$ 3.  $x_2 := x_2 \boxplus X_1$ 11.  $x_2 := x_2 \oplus x_1$ 4.  $x_3 := x_3 \boxminus (x_2 \oplus (\overline{x_1} \ll 19))$ 12.  $x_3 := x_3 \boxplus x_2$ 5.  $x_4 := x_4 \oplus x_3$ 13.  $x_4 := x_4 \boxminus (x_3 \oplus \overline{x_2} \gg 23))$ 6.  $x_5 := x_5 \boxplus x_4$ 14.  $x_5 := x_5 \oplus x_4$ 7.  $x_6 := x_6 \boxminus (x_5 \oplus (\overline{x_4} \gg 23))$ 15.  $x_6 := x_6 \boxplus x_5$ 8.  $x_7 := x_7 \oplus x_6$ 16.  $x_7 := x_7 \boxminus (x_6 \oplus \text{Const}_2)$ 

The final values  $(x_0, \ldots, x_7)$  are used as the key schedule output. The constants are Const<sub>1</sub> = A5A5 ... A5A5 and Const<sub>2</sub> = 0123 ... CDEF.

### 3 The Related-Key Boomerang Attack

The related-key boomerang attack was first published in 3. It is a combination of the related-key attack 2 and the boomerang attack 15.

Related-key attacks consider the information that can be extracted from encryptions using related but unknown keys. Such ciphers with a weak key schedule are vulnerable to this kind of attack. The key scheduling algorithms of many block ciphers inherit obvious relationships between keys, which are called related-keys.

The boomerang attack is an extension to differential cryptanalysis [4] using adaptive chosen plaintexts and ciphertexts to attack block ciphers. An attacker can only apply a chosen plain- and ciphertext attack while choosing the relation between related, but unknown, keys.

We now describe the related-key boomerang attack in more detail. But first, we have to give some definitions.

**Definition 1.** Let P, P' be two bit strings of the same length. The bit-wise XOR of P and  $P', P \oplus P'$ , is called the difference of P, P'.

**Definition 2.**  $\alpha \to \beta$  is called a differential if  $\alpha$  is the plaintext difference  $P \oplus P'$ before some non-linear operation  $f(\cdot)$  and  $\beta$  is the difference after applying these operation, i.e,  $f(P) \oplus f(P')$ . The probability p is linked on a differential saying that an  $\alpha$  difference turns into a  $\beta$  difference with probability p.

**Theorem 1.** ([15, Sec. 6].) Let  $\alpha \to \beta$  be a differential with probability p as in Definition 2. Then the backward direction of this differential  $\beta \to \alpha$  has also probability p.

Two texts (P, P') are called a *pair*, while two pairs (P, P', O, O') are called a *quartet*. Differential cryptanalysis exploits high probability differentials  $\alpha \to \beta$  between the first and a later round of a cipher. In general, an attacker tries to find high probability differentials over the first N-1 out of N rounds to exploit

some subkey bits of the last (i.e. N'th) round. An attacker first computes the expected output of the N-1-th round differential and then guesses some subkey bits of the last round. He decrypts some ciphertexts under the guessed key bits one round and compares the resulting difference with the output difference of the N-1 round differential. For every matching he increases a counter to the used key bits and takes the key bits with the highest counter as the 'correct' one. If the differential is well chosen, i.e., its probability is high enough, these subkey bits can be computed much faster than applying exhaustive search.

The related-key boomerang attack extends differential cryptanalysis. The cipher is split into two sub-ciphers and the attacker tries to exploit a differential for each sub-cipher. These two differentials alone only cover a few rounds but together the whole cipher. Regularly, the differential probability decreases the more rounds are included. Therefore two short differential covering only a few rounds each will be used instead of a long one covering the whole cipher. In this way key bits can be computed with much fewer plaintexts than compared to classic differential cryptanalysis, since the differential probabilities are normally higher. Related-keys are used to exploit some weaknesses of the key schedule to enhance the probability of the differentials being used. We call such differentials related-key differentials, since different but related keys are used for encryption and decryption. We split the related-key boomerang attack into two steps. The related-key boomerang distinguisher step and the key recovery step. The relatedkey boomerang distinguisher is used to find all plaintexts sharing a desired difference that depends on the choice of the differential. These plaintexts are used in the key recovery step afterwards to recover subkey bits for the initial round kev.

**Distinguisher Step.** During the distinguisher step we assume that the cipher can be treated as a cascade of two sub-ciphers  $E_K(P) = E1_K(E0_K(P))$ , where K is the key used for encryption and decryption. We assume that the related-key differential  $\alpha \to \beta$  for E0 occurs with the probability p, while the related-key differential  $\gamma \to \delta$  for E1 occurs with probability q, where  $\alpha, \beta, \gamma$  and  $\delta$  are differences. The backward direction  $E0^{-1}$  and  $E1^{-1}$  of the related-key differential for E0 and E1 are denoted by  $\alpha \leftarrow \beta$  and  $\gamma \leftarrow \delta$  and occur, because of Theorem  $\square$ with probability p and q respectively. The related-key boomerang distinguisher involves four or more unknown but related keys

$$\Delta K' = K^a \oplus K^b = K^c \oplus K^d,$$
  
$$\Delta K^* = K^a \oplus K^c = K^b \oplus K^d,$$

where  $\Delta K'$  and  $\Delta K^*$  are a known key differences. The attack works as follows:

- For i = 1, 2, ..., s do
  - 1. Choose plaintext  $P_{0,i}^a$  and plaintext  $P_{0,i}^b = P_{0,i}^a \oplus \alpha$ , where the subindex 0 defines the plaintext.
  - 2. Ask for the encryption of  $P_{0,i}^{a}$  under  $K^{a}$ , i.e.,  $P_{n,i}^{a} = E_{K^{a}}(P_{0,i}^{a})$  and  $P_{0,i}^{b}$  under  $K^{b}$ , i.e.,  $P_{n,i}^{b} = E_{K^{b}}(P_{0,i}^{b})$ .

- 3. Compute the new ciphertexts  $P_{n,i}^c = P_{n,i}^a \oplus \delta$  and  $P_{n,i}^d = P_{n,i}^b \oplus \delta$ .
- 4. Ask for decryption of  $P_{n,i}^c$  under  $K^c$ , i.e.,  $P_{0,i}^c = E_{K^c}^{-1}(P_{n,i}^c)$  and  $P_{n,i}^d$  under  $K^d$ , i.e.,  $P_{0,i}^d = E_{K^d}^{-1}(P_{n,i}^d)$ .
- 5. If  $P_{0,i}^c \oplus P_{0,i}^d = \alpha$  store the quartet  $(P_{0,i}^a, P_{0,i}^b, P_{0,i}^c, P_{0,i}^d)$  in the set  $\Theta$ .

Assume that a pair  $(P_{0,i}^a, P_{0,i}^b)$ ,  $i \in \{1, \ldots, s\}$  with the difference  $\alpha$  satisfies the differential  $\alpha \to \beta$  with the probability p. Let  $0 < b < n \leq N$  be an intermediate round, where N denotes the final round. Subcipher E0 covers rounds 1 to b and subcipher E1 rounds b + 1 to N. The output of E0 is  $P_{b,i}^a$  and  $P_{b,i}^b$ , i.e.,  $P_{b,i}^a =$  $E0_{K^a}(P_{0,i}^a)$  and  $P_{b,i}^b = E0_{K^b}(P_{0,i}^b)$ .  $P_{n,i}^a$  and  $P_{n,i}^b$  have a difference  $\beta = P_{b,i}^a \oplus P_{b,i}^b$ with probability p. Using the ciphertexts  $P_{n,i}^a$  and  $P_{n,i}^b$  (encrypted via  $E_{K^a}$  and  $E_{K^b}$  we can compute the new ciphertexts  $P_{n,i}^c = P_{n,i}^a \oplus \delta$  and  $P_{n,i}^d = P_{n,i}^b \oplus \delta$ . Let  $P_{b,i}^c = E \mathbb{1}_{K^c}^{-1}(P_{n,i}^c)$  and  $P_{b,i}^d = E \mathbb{1}_{K^d}^{-1}(P_{n,i}^d)$  be the decryption of  $P_{n,i}^c$  and  $P_{n,i}^d$  under E1. A difference  $\delta$  turns into a difference  $\gamma$  after passing  $E1^{-1}$  with probability q. Since  $\delta = P_{n,i}^a \oplus P_{n,i}^c = P_{n,i}^b \oplus P_{n,i}^d$  we know that  $P_{b,i}^a \oplus P_{b,i}^c = \gamma$ and  $P_{b,i}^b \oplus P_{b,i}^d = \gamma$  with probability  $q^2$ . As we know that  $P_{b,i}^a \oplus P_{b,i}^b = \beta$  with probability p, it follows that  $(P_{b,i}^c \oplus P_{b,i}^d) = (P_{b,i}^c \oplus P_{b,i}^a) \oplus (P_{b,i}^a \oplus P_{b,i}^b) \oplus (P_{b,i}^b \oplus P_{b,i}^b)$  $P_{b,i}^d$  =  $\gamma \oplus \beta \oplus \gamma = \beta$  holds with probability  $pq^2$ . A  $\beta$  difference turns into an  $\alpha$  difference after passing the differential  $E0^{-1}$  with probability p. Thus, a pair of ciphertexts  $(P_{0,i}^a, P_{0,i}^b)$  with  $P_{0,i}^a \oplus P_{0,i}^b = \alpha$  generates a new pair of plaintexts  $(P_{0,i}^c, P_{0,i}^d)$  where  $P_{0,i}^c \oplus P_{0,i}^d = \alpha$  with probability  $(pq)^2$ . A quartet containing these two pairs is defined as:

**Definition 3.** A quartet  $(P_{0,i}^a, P_{0,i}^b, P_{0,i}^c, P_{0,i}^d)$  which satisfies

$$\begin{split} P^{a}_{0,i} \oplus P^{b}_{0,i} &= P^{c}_{0,i} \oplus P^{d}_{0,i} = \alpha, \\ P^{a}_{b,i} \oplus P^{b}_{b,i} &= P^{c}_{b,i} \oplus P^{d}_{b,i} = \beta, \\ P^{a}_{b,i} \oplus P^{c}_{b,i} &= P^{b}_{b,i} \oplus P^{d}_{b,i} = \gamma, \\ P^{a}_{n,i} \oplus P^{c}_{n,i} &= P^{b}_{n,i} \oplus P^{d}_{n,i} = \delta, \end{split}$$

is called a correct related-key boomerang quartet which occurs with probability  $Pr_c = (pq)^2$ . A quartet  $(P_{0,i}^a, P_{0,i}^b, P_{0,i}^c, P_{0,i}^d)$  which only satisfies the condition  $P_{0,i}^a \oplus P_{0,i}^b = \alpha = P_{0,i}^c \oplus P_{0,i}^d$  is called a false related-key boomerang quartet.

Figure 2 displays the structure of the related-key boomerang distinguisher step.

Any attacker that applies a related-key boomerang distinguisher does not know the internal states  $P_{b,i}^{a}$ ,  $P_{b,i}^{b}$ ,  $P_{b,i}^{c}$ ,  $P_{b,i}^{d}$ , and since he can only apply a chosen plaintext and ciphertext attack on the cipher. The set  $\Theta$ , which is the output of the related-key boomerang distinguisher, therefore contains correct and false related-key boomerang quartets. It is impossible to form another distinguisher which separates the correct and the false related-key boomerang quartets, since the interior differences  $\beta$  and  $\gamma$  cannot be computed.

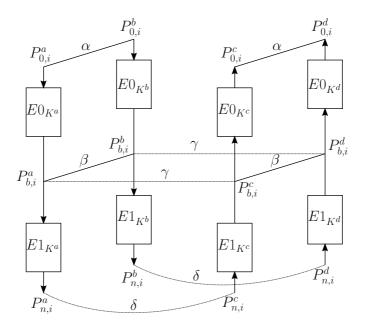


Fig. 2. The related-key boomerang distinguisher

**Key Recovery Step.** The second step of the related-key boomerang attack is the key recovery step. From now on, an attacker operates on the set  $\Theta$  that was stored by the related-key boomerang distinguisher. Let  $K_0^a, K_0^b, K_0^c, K_0^d$  be the 0-th round keys derived from the master keys  $K^a, K^b, K^c, K^d$ . Let  $enc_{K_0^i}(P)$  be the one round partial encryption of P under the round key  $K_0^i, i \in \{a, b, c, d\}$ . The round keys are related as

$$\Delta K_0' = K_0^a \oplus K_0^b = K_0^c \oplus K_0^d,$$
  
$$\Delta K_0^a = K_0^a \oplus K_0^c = K_0^b \oplus K_0^d,$$

where  $\Delta K_0^i$ ,  $i \in \{a, b, c, d\}$  is the key difference of the 0-th round keys. The key recovery step works as follows:

- For each key-bit combination of  $K_0^a, K_0^b, K_0^c$  and  $K_0^d$ 
  - 1. Initialize a counter with zero.
  - For all quartets  $(P^a_{0,i},P^b_{0,i},P^c_{0,i},P^d_{0,i})$  stored in  $\varTheta$ 
    - 2. Encrypt the plaintext quartet  $(P_{0,i}^{a}, P_{0,i}^{b}, P_{0,i}^{c}, P_{0,i}^{d})$  one round under the guessed round keys  $K_{0}^{a}, K_{0}^{b}, K_{0}^{c}$  and  $K_{0}^{d}$  respectively, i.e.,  $P_{1,i}^{a} = enc_{K_{0}^{a}}(P_{0,i}^{a}), P_{1,i}^{b} = enc_{K_{0}^{b}}(P_{0,i}^{b}), P_{1,i}^{c} = enc_{K_{0}^{c}}(P_{0,i}^{c})$  and  $P_{1,i}^{d} = enc_{K_{0}^{d}}(P_{0,i}^{d}).$
    - 3. Test whether the differences  $P_{1,i}^a \oplus P_{1,i}^b$  and  $P_{1,i}^c \oplus P_{1,i}^d$  have a desired difference an attacker would expect depending on the related-key

differential being used. Increase a counter for the used key-bits if the difference is fulfilled in both pairs.

4. Output the round keys  $K_0^a, K_0^b, K_0^c$  and  $K_0^d$  with the highest counter as the correct one.

In Step 3, four cases can be distinguished, since  $\Theta$  contains correct and false related-key boomerang quartets and the round keys  $K_0^a, K_0^b, K_0^c$  and  $K_0^d$  can either be correct or false. A correct related-key boomerang quartet encrypted using the correct round key will have the desired difference needed to pass the test in Step 3. Hence, the counter for the correct round key is increased.

The other three cases are: a correct related-key boomerang quartet is used with a false round key  $(Pr_{cK_f})$ , a false related-key boomerang quartet is used with a correct round key  $(Pr_{fK_c})$  or a false related-key boomerang quartet is used with a false round key  $(Pr_{fK_f})$ . We assume that the cipher acts like a random permutation. In these case we can assume that

$$Pr_{cK_f} = Pr_{fK_c} = Pr_{fK_f} =: Pr_{filter}.$$

The probability that a quartet in one of the three undesirable cases is counted for a certain round key is  $Pr_{filter}$ . The related-key differentials have to be chosen such that the counter for the correct round key is significantly higher than the counter for each false round key. If the differentials have a high probability the key recovery step outputs the correct round key in Step 4 with a high probability much faster than exhaustive key search.

### 4 A Memoryless Related-Key Boomerang Attack on the Full Tiger Block Cipher

In this section, we propose a 22-round related-key boomerang distinguisher, which is used for our memoryless related-key boomerang attack on the full 24-round Tiger encryption mode. We make extensive use of the following property in our attack.

Property 1. Switching between an additive and an XOR difference holds with some probability. e.g., if  $X - Y = 2^i \mod 2^{64}$ , then  $\Pr[X \oplus Y = 2^i] = 2^{-1}$ . If i = 63, however, we have  $\Pr[X \oplus Y = 2^i] = 1$ . I.e., switching between the additive difference  $I = 2^{63}$  and the XOR-difference I is for free.

#### 4.1 A 22-Round Related-Key Boomerang Distinguisher

Let K be a master key which can be written as  $K = x_0, x_1, \ldots, x_7$ , where  $x_i$  is a 64-bit word. We use four different but related master keys  $K^a, K^b, K^c$  and  $K^d$ to mount our related-key rectangle attack on the full Tiger encryption mode. The master key differences are as follows:

$$\Delta K' = K^{a} \oplus K^{b} = K^{c} \oplus K^{d} = (I, I, 0, 0, 0, I, 0, 0),$$
  
$$\Delta K^{*} = K^{a} \oplus K^{c} = K^{b} \oplus K^{d} = (0, 0, 0, I, 0, 0, 0, I)$$

Since the key schedule of Tiger offers a high degree of linearity we can determine most of the round key differences derived from the master key differences  $\Delta K'$ and  $\Delta K^*$  respectively. Using the above key schedule we can derive the round key differences from the master key differences  $\Delta K'$  and  $\Delta K^*$ . The round key differences for E0 propagate as

$$(I, I, 0, 0, 0, I, 0, 0) \longrightarrow (0, 0, 0, 0, 0, I, 0, I) \longrightarrow (I, 0, I, 0, ?, ?, ?, ?)$$

The ? indicates an unknown value of a key difference. We obtain the round key differences for E1 as  $\frac{2}{3}$ 

$$(0, 0, 0, I, 0, 0, 0, I) \longrightarrow (0, I, 0, 0, 0, 0, 0, I) \longrightarrow (0, 0, 0, 0, 0, 0, 0, I)$$

For our attack we use an 11-round related-key differential from round 1 to 12 for  $E0 \ (\alpha \to \beta)$  using the master key difference  $\Delta K'$ . The related-key differential is:

 $(0,0 \boxminus odd(I), I) \rightarrow (0,0,0)$ 

The differential for  $E0^{-1}$  ( $\beta \to \alpha$ ) is the differential for E0 in reverse direction. The related-key differential E0 and  $E0^{-1}$  are shown in Table  $\square$  We exploit

$\operatorname{Round}(i)$	$\Delta A_i$	$\Delta B_i$	$\Delta C_i$	$\Delta k_i$	Prob.
1	0	$0 \boxminus odd(I)$	Ι	—	$2^{-32}$
2	0	Ι	0	0	1
3	Ι	0	0	0	1
4	0	0	Ι	0	1
5	0	0	0	Ι	1
6	0	0	0	0	1
	•		•	•	
:	:	:	:	:	:
12	0	0	0	0	_

another 11 rounds related-key differential for  $E1^{-1}$  ( $\delta \rightarrow \gamma$ ) that covers round 23 to 12 using the master key difference  $\Delta K^*$ . The related-key differential is:

$$(odd(I), I, 0) \rightarrow (0, I, 0)$$

The related-key differential  $E1^{-1}$  is shown in Table 2 Our 22-round related-key boomerang distinguisher holds with probability  $2^{-128}$ , since we have  $p = 2^{-32}$  and  $q = 2^{-32}$  which leads to  $(p \cdot q)^2 = 2^{-128}$ .

<sup>&</sup>lt;sup>1</sup> This related keys were also used in the attack on the Tiger encryption mode from **7**.

<sup>&</sup>lt;sup>2</sup> The same master key differential was used by pseudo-collision attack of Mendel and Rijmen 12.

$\operatorname{Round}(i)$	$\Delta A_i$	$\Delta B_i$	$\Delta C_i$	$\Delta k_i$	Prob.
23	odd(I)	Ι	0	-	$2^{-32}$
22	0	0	0	Ι	1
21	0	0	0	0	1
:	•••	••••	••••	••••	:
16	0	0	0	0	1
15	0	0	0	0	1
14	0	0	1	1	1
13	Ι	0	0	0	1
12	0	1	0	0	—

**Table 2.** The Related-Key Differential  $E1^{-1}$ 

#### 4.2 The Attack

The attack works as follows:

- 1. Guess two 64-bit round keys  $k_0^a, k_1^a$  and compute  $k_0^i, k_1^i$ ,  $i \in \{b, c, d\}$  using the known round key differences  $\Delta K'$  and  $\Delta K^*$ .
  - For  $i = 1, 2, \dots, 2^{129.5}$  do
    - 2. Set A = odd(I). Choose a plaintext  $P_{1,i}^a$  uniformly at random and compute  $P_{1,i}^b = P_{1,i}^a \oplus \alpha$ , where  $\alpha = (0, 0 \boxminus A, I)$ . Decrypt  $P_{1,i}^a$  and  $P_{1,i}^b$  under  $k_1^a, k_0^a$  and  $k_1^b, k_0^b$  respectively and obtain the plaintexts  $P_{-1,i}^a$  and  $P_{-1,i}^b$ . With a chosen plaintext attack scenario, encrypt the plaintexts  $P_{-1,i}^a$  and  $P_{-1,i}^b$  under  $K^a$  and  $K^b$  respectively to obtain the ciphertexts  $P_{23,i}^a$  and  $P_{23,i}^b$ .
    - 3. Compute the ciphertexts  $P_{23,i}^c = P_{23,i}^a \oplus \delta$  and  $P_{23,i}^d = P_{23,i}^b \oplus \delta$ ,  $\delta = (A, I, 0)$ . With a chosen ciphertext scenario, decrypt the ciphertexts  $P_{23,i}^c, P_{23,i}^d$  under  $K^c$  and  $K^d$  respectively and obtain the plaintexts  $P_{-1,i}^c$  and  $P_{-1,i}^d$ .
    - 4. Partially encrypt  $P_{-1,i}^c, P_{-1,i}^d$  under  $k_0^c, k_1^c$  and  $k_0^d, k_1^d$  respectively and obtain  $P_{1,i}^c$  and  $P_{1,i}^d$ . Check if  $P_{1,i}^c \oplus P_{1,i}^d = \alpha$ . If true, store the quartet  $(P_{1,i}^a, P_{1,i}^b, P_{1,i}^c, P_{1,i}^d)$  in  $\Theta$ .
- 5. If the number of quartets in  $\Theta$  is at least two, record the round keys  $k_0^i, k_1^i$   $(i \in \{a, b, c, d\})$  and go to Step 6. Otherwise, go to Step 1 with another key candidates.
- 6. For a suggested (k<sup>a</sup><sub>0</sub>, k<sup>a</sup><sub>1</sub>), do an exhaustive search for the remaining 512−3⋅64 = 384 key bits by trial encryption. If a 512-bit key is suggested, output it as the master key K<sup>a</sup> of the full Tiger encryption mode (as well as K<sup>b</sup> = K<sup>a</sup> ⊕ K', K<sup>c</sup> = K<sup>a</sup> ⊕ K<sup>\*</sup> and K<sup>d</sup> = K<sup>a</sup> ⊕ K' ⊕ K<sup>\*</sup>). Otherwise restart the algorithm.

#### 4.3 Analysis of the Attack

From  $\#Q = 2^{129.5}$  boomerang quartets we expect about  $\#C = 2^{129.5} \cdot 2^{-128} \approx 3$  correct boomerang quartets in  $\Theta$ , since the probability of the related-key boomerang distinguisher is  $2^{-128}$ . A random permutation of a difference  $P_{1,i}^c \oplus$ 

 $P_{1,i}^d$  has difference  $\alpha$  with probability  $Pr_f = 2^{-192}$ . We expect that  $\#F = \#Q \cdot Pr_f = 2^{129.5} \cdot 2^{-192} = 2^{-62.5}$  false related-key boomerang quartets will pass the test in Step 4.

The data complexity of Step 1 to 4 is about  $2^2 \cdot 2^{1.5} = 2^{3.5}$  chosen plain- and ciphertexts, since we expect to have only  $2^{1.5}$  quartets stored in  $\Theta$  that satisfy the condition in Step 4, not  $2^{129.5}$  as one might respect since we do not have to store all the quartets. Step 1 will be applied  $2^{128}$  times in the worst case. The time complexity of Step 2 is determined by two 24-round Tiger encryptions. The time complexity of Step 3 is negligible, while the time complexity of Step 4 is the same as for Step 3. Step 1 to 4 will be computed  $2^{129.5}$  times. And thus the overall time complexity is bounded by  $2^{128} \cdot 2^{129.5} \cdot 2^2 = 2^{259.5}$ .

A false combination of quartets is counted in Step 6 with probability  $Pr_{filter} = 2^{-2 \cdot 192}$ . At least  $\#cK_c \approx 3$  correct related-key boomerang quartets and additionally  $\#fK_c = \#F \cdot Pr_{filter} = 2^{-62.5} \cdot 2^{-384} = 2^{-446.5}$  false related-key boomerang quartets are counted with the correct key bits. Note that a correct boomerang quartet is always counted with probability one for a correct round key. We double the filtering since we can filter on both pairs of a quartet. About  $\#cK_c + \#fK_c \approx 3 + 2^{-446.5} \approx 3$  quartets are counted for the correct key bits.

About  $\#cK_f = \#C \cdot Pr_{filter} \approx 3 \cdot 2^{-384} = 2^{-382.5}$  correct related-key boomerang quartets and  $\#FK_f = \#F \cdot Pr_{filter} = 2^{-62.5} \cdot 2^{-384} = 2^{-446.5}$  false relatedkey boomerang quartets are counted with a false round key, which are approximately  $\#cK_f + \#fK_f \approx 2^{-382.5} + 2^{-446.5} \approx 2^{-382.5}$  counts for each false round key.

Using the Poisson distribution we can compute the success rate of our attack. The probability that the number of remaining quartets for each false round key combination is larger than 1 is  $Y \sim Poisson(\mu = 2^{-446.5})$ ,  $\Pr(Y \ge 2) \approx 0$ , since the expected number of quartets counted with a false round key is  $2^{-446.5}$ . We expect to have a count of at least 3 quartets for the correct key bits. The probability that the number of quartets counted for the correct round keys is at least 2 is  $Z \sim Poisson(\mu = 3)$ ,  $\Pr(Z \ge 2) \approx 0.8$ . The data complexity of our attack is  $2^{3.5}$  chosen plaintexts and ciphertexts, while the time complexity is about  $2^{259.5}$  24-round Tiger encryptions. The attack has a success rate of 0.8.

#### 4.4 Further Improvements

The time complexity of our attack can be reduced using the following techniques. One can fix some of the plaintext bits that form an  $\alpha$  difference. This will lead to a probability one differential for E0. The complexity can be reduced by a factor of about  $2^{64}$  in this way. The time complexity of the improved attack is therefore bounded by  $2^{195.5}$  24-round Tiger encryptions.

### 5 Conclusion

In this paper we present the first cryptographic attack on the full 24-round Tiger encryption mode. Our related-key boomerang attack uses less memory, which is of about  $2^{3.5}$  chosen plaintexts and ciphertexts. The time complexity of our attack is  $2^{259.5}$  24-round Tiger encryptions. Moreover we shown how time complexity of the attack can be reduced to  $2^{195.5}$  encryptions by fixing some of the plaintext bits.

Our result shows that Tiger used as a block cipher can be distinguished from an ideal cipher. Nevertheless, we point out that Tiger used as a hash function offers some weaknesses, which should be considered.

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# Memoryless Related-Key Boomerang Attack on 39-Round SHACAL-2

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Abstract. SHACAL-2 is a 64-round block cipher based on the compression function of the hash function standard SHA-256. It has a 256-bit block size and a variable length key of up to 512 bits. Up to now, all attacks on more than 37 rounds require at least  $2^{235}$  bytes of memory. Obviously such attacks will never become of practical interest due to this high amount of space. In this paper we adopt the relate-key boomerang attack and present the first memoryless attack on 39-round SHACAL-2. Our attack only employs  $2^{8.5}$  bytes of memory and thus improves the data complexity of comparable attacks up to a factor of at least  $2^{230}$ , which is a substantial improvement. We do not need to store all the data which gives this low data complexity. The related-key boomerang attack presented in this paper can also be seen as a starting point for more advanced attacks on SHACAL-2. The main advantage of our new attack is that we can proceed the data sequentially instead of parallel as needed for other attacks, which reduces the memory requirements dramatically.

**Keywords:** SHACAL-2, block cipher, differential cryptanalysis, related-key boomerang attack, memoryless attacks.

### 1 Introduction

SHACAL-2 [4] is a 256-bit block cipher with 512-bit keys which is based on the compression function of the hash function standard SHA-256 [15]. SHACAL-2 as well as SHACAL-1, which is based one SHA-1 [14], were submitted to the NESSIE (New European Schemes for Signatures, Integrity and Encryption) project [13]. SHACAL-2 was recommended to be one of the NESSIE projection selections, while SHACAL-1 was not selected, because of some concerns of the key schedule of SHACAL-1.

SHACAL-2 has 64-rounds and supports key lengths from 128 up to 512 bits. For key lengths below 512 bit, the key gets simply padded with zeros. The first cryptanalytic result on SHACAL-2 is an impossible differential attack **5** on a 30-round reduced version of SHACAL-2. A differential-nonlinear attack **16** and a square-nonlinear attack **16** were introduced in the following which are able to break up to 32 and 28 rounds. Including related keys leads to an improved attack of up to 37 rounds which was called the related-key differential-nonlinear attack **9**. The best cryptanalytic results one SHACAL-2 are the related-key rectangle

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attacks on 42 rounds by Lu et al. [12] and a 43-round attack by Wang [13]. The disadvantage of this attacks is the huge memory requirements which we will address in this paper. We present the first memoryless attack on SHACAL-2 which can break up to 39 rounds using two related keys. For our results we use the related-key boomerang attack, which improves the data complexity of comparable attacks by a factor of at least  $2^{226.5}$ . This new technique allows to proceed the data sequentially and thus reduces the memory requirements. This is also the main advantage of our related-key boomerang attack, since all other attacks have to store a huge amount of data so that the attack will work. Table [1] summarizes the results known from literature and our new results on SHACAL-2.

Attack	# Rounds	Data	Time	Memory	Source
Square-Nonlinear	28	$463 \cdot 2^{32}$ CP	$2^{494.1}$	$2^{45.9}$	16
Impossible Differential	30	744 CP	$2^{495.1}$	$2^{14.5}$	5
Differential-Nonlinear	32	$2^{43.4}$ CP	$2^{504.2}$	$2^{48.4}$	16
<b>RK</b> Differential-Nonlinear	35	$2^{42.32}$ RK-CP	$2^{452.10}$	$2^{47.32}$	9
RK Rectangle	37	$2^{235.16}$ RK-CP	$2^{486.95}$	$2^{240.16}$	9
RK Boomerang	39	$2^{3.5}$ RK-CPCC	$2^{483.5}$	$2^{8.5}$	Sec. 4
RK Rectangle	40	$2^{243.38}$ RK-CP	$2^{448.43}$	$2^{247.38}$	12
RK Rectangle	42	$2^{243.38}$ RK-CP	$2^{488.37}$	$2^{247.38}$	12
RK Rectangle <sup>†</sup>	43	$2^{240.38}$ RK-CP	$2^{480.4}$	$2^{245.38}$	18
RK Rectangle	44	$2^{233}$ RK-CP	$2^{497.2}$	$2^{238}$	11

Table 1. Comparison of attacks on SHACAL-2

RK: Related-Key, CP: Chosen Plaintexts, CC: Chosen Ciphertexts.

<sup> $\dagger$ </sup> : The attack has a flaw pointed out in **11**.

Boomerang Attack. The boomerang attack **[17]** is a strong extension to differential cryptanalysis for breaking more rounds than differential attacks can do, since the cipher is treated as a cascade of two sub-ciphers, using short differentials in each sub-cipher. These differentials are combined in an adaptive chosen plaintext and ciphertext attack to exploit properties of the cipher that have a high probability.

Related-Key Attack. The related-key attack [1,10] applies differential cryptanalysis to the cipher with different, but related, keys and considers the information that can be extracted from encryptions using these keys. Ciphers with a weak key schedule are vulnerable to this kind of attack. The idea of related-key differentials was presented in [7], while two encryptions under two related-keys are used. Several combinations of related-key and differential attacks were introduced in the following. Related-Key Boomerang and Rectangle Attack. The related-key boomerang and rectangle attack was published first in [2, 6, 8]. The related-key boomerang attack utilizes the fact, that the boomerang attack uses short differentials and so the lower diffusion in the differences of the round keys can be better exploited than in the original related-key differential attack. The attack combines features of the boomerang and the related-key attack.

**Outline.** In Section 2 we briefly describe the SHACAL-2 block cipher. In Section 3 we introduce the related-key boomerang attack, which is used in Section 4 to break 39 rounds of SHACAL-2 without using any noteworthy amount of memory. Finally, we conclude our paper in Section 5

# 2 Description of SHACAL-2

SHACAL-2 is composed out of 64 rounds using a variable key length of at least 128 bits and up to 512 bits. For keys smaller than 512 bits zeros are padded until the key length reaches 512 bits. A 256-bit plaintext

$$P_0 = A_0 ||B_0||C_0||D_0||E_0||F_0||G_0||H_0$$

is divided into eight 32-bit words  $A_0, B_0, C_0, D_0, E_0, F_0, G_0, H_0$ . The corresponding ciphertext  $P_{64}$  is denoted by

$$P_{64} = A_{64} ||B_{64}||C_{64}||D_{64}||E_{64}||F_{64}||G_{64}||H_{64}.$$

The following notations are used in this paper:

⊕ : bitwise XOR operation ∧ : bitwise AND operation + : addition modulo  $2^{32}$  operation ¬ : bitwise complement operation  $e_i$  : a 32-bit word with zeros in all positions except for bit i,  $(0 \le i \le 31)$   $e_{i_1,...,i_l}$  :  $e_{i_1} \oplus \cdots \oplus e_{i_l}$ The round function of round i can be described as follows:

$$\begin{split} T_{i+1}^{1} &= K_{i} + \Sigma_{1}(E_{i}) + Ch(E_{i},F_{i},G_{i}) + H_{i} + W_{i}, \\ T_{i+1}^{2} &= \Sigma_{0}(A_{i}) + Maj(A_{i},B_{i},C_{i}), \\ H_{i+1} &= G_{i}, \\ G_{i+1} &= F_{i}, \\ F_{i+1} &= E_{i}, \\ E_{i+1} &= D_{i} + T_{i+1}^{1}, \\ D_{i+1} &= C_{i}, \\ C_{i+1} &= B_{i}, \\ B_{i+1} &= A_{i}, \\ A_{i+1} &= T_{i+1}^{1} + T_{i+1}^{2}, \end{split}$$

where  $K_i$  is the *i*-th round key and  $W_i$  is the *i*-th round constant. The four functions in the encryption algorithm are defined as follows:

$$Ch(X, Y, Z) = (X \land Y) \oplus (\neg X \land Z),$$
  

$$Maj(X, Y, Z) = (X \land Y) \oplus (X \land Z) \oplus (Y \land Z),$$
  

$$\Sigma_0(X) = S_2(X) \oplus S_{13}(X) \oplus S_{22}(X),$$
  

$$\Sigma_1(X) = S_6(X) \oplus S_{11}(X) \oplus S_{25}(X),$$

where  $S_j(X)$  represents the right rotation of X by j bits. The bit positions of a 32-bit word are labeled as  $31, 30, \ldots, 1, 0$ , where bit 31 is the most significant bit and bit 0 is the least significant bit.

The key schedule algorithm of SHACAL-2 takes as input a 512-bit master key. As stated above, as many zeros as necessary will be padded to get a full 512-bit master key. The 512-bit master key K is divided into sixteen 32-bit words  $K_0, K_1, \ldots, K_{15}$ . These are the round keys for the first 16 rounds. The *i*-th round key ( $16 \le i \le 63$ ) is computed as

$$K_{i} = \sigma_{1}(K_{i-2}) + K_{i-7} + \sigma_{0}(K_{i-15}) + K_{i-16},$$
  
$$\sigma_{0}(X) = S_{7}(X) \oplus S_{18}(X) \oplus R_{3}(X),$$
  
$$\sigma_{1}(X) = S_{17}(X) \oplus S_{19}(X) \oplus R_{10}(X),$$

where  $R_i(X)$  represents right shift of X by j bits.

In the following we present some basic properties of the  $Ch(\cdot)$  and  $Maj(\cdot)$  functions which will be needed in our attack.

**Proposition 1.** (from [18]) For the nonlinear function  $Ch(X, Y, Z) = (X \land Y) \oplus (\neg X \land Z)$ , there are the following properties:

- 1.  $Ch(x, y, z) = Ch(\neg x, y, z)$  if and only if y = z. Ch(0, y, z) = 0 and Ch(1, y, z) = 1 if and only if y = 1 and z = 0. Ch(0, y, z) = 1 and CH(1, y, z) = 0 if and only if y = 0 and z = 1.
- 2.  $Ch(x, y, z) = Ch(x, \neg y, z)$  if and only if x = 0. Ch(x, 0, z) = 0 and Ch(x, 1, z) = 1 if and only if x = 1.
- 3.  $Ch(x, y, z) = Ch(x, y, \neg z)$  if and only if x = 1. Ch(x, y, 0) = 0 and Ch(x, y, 1) = 1 if and only if x = 0.

**Proposition 2.** (from [18]) For the nonlinear function  $Maj(X, Y, Z) = (X \land Y) \oplus (X \land Z) \oplus (Y \land Z)$ , there are the following properties:

- Maj(x, y, z) = Maj(¬x, y, z) if and only if y = z. Maj(0, y, z) = 0 and Maj(1, y, z) = 1 if and only if y = ¬z.
   Maj(x, y, z) = Maj(x, ¬y, z) if and only if x = z. Maj(x, 0, z) = 0 and Maj(x, 1, z) = 1 if and only if x = ¬z.
- 3.  $Maj(x, y, z) = Maj(x, y, \neg z)$  if and only if x = y. Maj(x, y, 0) = 0 and Maj(x, y, 1) = 1 if and only if  $x = \neg y$ .

## 3 The Related-Key Boomerang Attack

Recall that the related-key boomerang attack was first published in [2]. It is a combination of the related-key attack [1] and the boomerang attack [17].

Related-key attacks consider the information that can be extracted from encryptions using related but unknown keys. Such ciphers with a weak key schedule are vulnerable to this kind of attack. The key scheduling algorithms of many block ciphers inherit obvious relationships between keys, which are called related-keys.

The boomerang attack is an extension to differential cryptanalysis [3] using adaptive chosen plaintexts and ciphertexts to attack block ciphers. An attacker can only apply a chosen plain- and ciphertext attack while choosing the relation between related, but unknown, keys.

We now describe the related-key boomerang attack in more detail. But first, we have to give some definitions.

**Definition 1.** Let P, P' be two bit strings of the same length. The bit-wise XOR of P and  $P', P \oplus P'$ , is called the difference of P, P'.

**Definition 2.**  $\alpha \to \beta$  is called a differential if  $\alpha$  is the plaintext difference  $P \oplus P'$ before some non-linear operation  $f(\cdot)$  and  $\beta$  is the difference after applying these operation, i.e,  $f(P) \oplus f(P')$ . The probability p is linked on a differential saying that an  $\alpha$  difference turns into a  $\beta$  difference with probability p.

**Theorem 1.** ([17], Sec. 6].) Let  $\alpha \to \beta$  be a differential with probability p as in Definition 2. Then the backward direction of this differential  $\beta \to \alpha$  has also probability p.

Two texts (P, P') are called a *pair*, while two pairs (P, P', O, O') are called a *quartet*. Differential cryptanalysis exploits high probability differentials  $\alpha \to \beta$  between the first and a later round of a cipher. In general, an attacker tries to find high probability differentials over the first N-1 out of N rounds to exploit some subkey bits of the last (i.e. N'th) round. An attacker first computes the expected output of the N-1-th round differential and then guesses some subkey bits of the last round. He decrypts some ciphertexts under the guessed key bits one round and compares the resulting difference with the output difference of the N-1 round differential. For every matching he increases a counter to the used key bits and takes the key bits with the highest counter as the 'correct' one. If the differential is well chosen, i.e., its probability is high enough, these subkey bits can be computed much faster than applying exhaustive search.

The related-key boomerang attack extends differential cryptanalysis. The cipher is split into two sub-ciphers and the attacker tries to exploit a differential for each sub-cipher. These two differentials alone only cover a few rounds but together the whole cipher. Regularly, the differential probability decreases the more rounds are included. Therefore two short differential covering only a few rounds each will be used instead of a long one covering the whole cipher. In this way key bits can be computed with much fewer plaintexts than compared to classic differential cryptanalysis, since the differential probabilities are normally higher. Related-keys are used to exploit some weaknesses of the key schedule to enhance the probability of the differentials being used. We call such differentials related-key differentials, since different but related keys are used for encryption and decryption. We split the related-key boomerang attack into two steps. The *related-key boomerang distinguisher step* and the *key recovery step*. The relatedkey boomerang distinguisher is used to find all plaintexts sharing a desired difference that depends on the choice of the differential. These plaintexts are used in the key recovery step afterwards to recover subkey bits for the initial round key.

**Distinguisher Step.** During the distinguisher step we assume that the cipher can be treated as a cascade of two sub-ciphers  $E_K(P) = E1_K(E0_K(P))$ , where K is the key used for encryption and decryption. We assume that the related-key differential  $\alpha \to \beta$  for E0 occurs with the probability p, while the related-key differential  $\gamma \to \delta$  for E1 occurs with probability q, where  $\alpha, \beta, \gamma$  and  $\delta$  are differences. The backward direction  $E0^{-1}$  and  $E1^{-1}$  of the related-key differential for E0 and E1 are denoted by  $\alpha \leftarrow \beta$  and  $\gamma \leftarrow \delta$  and occur, because of Theorem  $\square$ with probability p and q respectively. The related-key boomerang distinguisher involves two different unknown – but related – keys  $K, K^* = K \oplus \Delta K$ , where  $\Delta K$  is a known key differences. The attack works as follows:

- For i = 1, 2, ..., s do
  - 1. Choose plaintext  $P_{0,i}^1$  and plaintext  $P_{0,i}^2 = P_{0,i}^1 \oplus \alpha$ , where the subindex 0 defines the plaintext.
  - 2. Ask for the encryption of  $P_{0,i}^1$  under K, i.e.,  $P_{n,i}^1 = E_K(P_{0,i}^1)$  and  $P_{0,i}^2$  under  $K^*$ , i.e.,  $P_{n,i}^2 = E_{K^*}(P_{0,i}^2)$ .
  - 3. Compute the new ciphertexts  $P_{n,i}^3 = P_{n,i}^1 \oplus \delta$  and  $P_{n,i}^4 = P_{n,i}^2 \oplus \delta$ .
  - 4. Ask for decryption of  $P_{n,i}^3$  under K, i.e.,  $P_{0,i}^3 = E_K^{-1}(P_{n,i}^3)$  and  $P_{n,i}^4$  under  $K^*$ , i.e.,  $P_{0,i}^4 = E_{K^*}^{-1}(P_{n,i}^4)$ .
  - 5. If  $P_{0,i}^3 \oplus P_{0,i}^4 = \alpha$  store the quartet  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  in the set  $\Theta$ .

Assume that a pair  $(P_{0,i}^1, P_{0,i}^2)$ ,  $i \in \{1, \ldots, s\}$  with the difference  $\alpha$  satisfies the differential  $\alpha \to \beta$  with the probability p. Let  $0 < b < n \leq N$  be an intermediate round, where N denotes the final round. Subcipher E0 covers rounds 1 to b and subcipher E1 rounds b + 1 to N. The output of E0 is  $P_{b,i}^1$  and  $P_{b,i}^2$ , i.e.,  $P_{b,i}^1 = E0_K(P_{0,i}^1)$  and  $P_{b,i}^2 = E0_{K^*}(P_{0,i}^2)$ .  $P_{n,i}^1$  and  $P_{n,i}^2$  have a difference  $\beta = P_{b,i}^1 \oplus P_{b,i}^1$  with probability p. Using the ciphertexts  $P_{n,i}^1$  and  $P_{n,i}^2$  (encrypted via  $E_K$  and  $E_{K^*}$ ) we can compute the new ciphertexts  $P_{n,i}^3 = P_{n,i}^1 \oplus \delta$  and  $P_{n,i}^4 = P_{n,i}^2 \oplus \delta$ . Let  $P_{b,i}^3 = E1_K^{-1}(P_{n,i}^3)$  and  $P_{b,i}^4 = E1_{K^*}^{-1}(P_{n,i}^4)$  be the decryption of  $P_{n,i}^3$  and  $P_{n,i}^4$  under E1. A difference  $\delta$  turns into a difference  $\gamma$  after passing  $E1^{-1}$  with probability q. Since  $\delta = P_{n,i}^1 \oplus P_{n,i}^3 = P_{n,i}^2 \oplus P_{n,i}^4$  we know that  $P_{b,i}^1 \oplus P_{b,i}^3 = \gamma$  and  $P_{b,i}^2 \oplus P_{b,i}^4 = \gamma$  with probability  $q^2$ . As we know that  $P_{b,i}^1 \oplus P_{b,i}^2 = \beta$  with probability p, it follows that  $(P_{b,i}^3 \oplus P_{b,i}^4) = (P_{b,i}^3 \oplus P_{b,i}^1) \oplus (P_{b,i}^1 \oplus P_{b,i}^2) \oplus (P_{b,i}^2 \oplus P_{b,i}^4) \oplus P_{b,i}^2 = \beta$  holds with probability  $pq^2$ . A  $\beta$  difference turns into an  $\alpha$  difference after passing the differential  $E0^{-1}$  with probability p. Thus, a pair

of ciphertexts  $(P_{0,i}^1, P_{0,i}^2)$  with  $P_{0,i}^1 \oplus P_{0,i}^2 = \alpha$  generates a new pair of plaintexts  $(P_{0,i}^3, P_{0,i}^4)$  where  $P_{0,i}^3 \oplus P_{0,i}^4 = \alpha$  with probability  $(pq)^2$ . A quartet containing these two pairs is defined as:

**Definition 3.** A quartet  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  which satisfies

$$\begin{split} P^{1}_{0,i} \oplus P^{2}_{0,i} &= P^{3}_{0,i} \oplus P^{4}_{0,i} = \alpha, \\ P^{1}_{b,i} \oplus P^{2}_{b,i} &= P^{3}_{b,i} \oplus P^{4}_{b,i} = \beta, \\ P^{1}_{b,i} \oplus P^{3}_{b,i} &= P^{2}_{b,i} \oplus P^{4}_{b,i} = \gamma, \\ P^{1}_{n,i} \oplus P^{3}_{n,i} &= P^{2}_{n,i} \oplus P^{4}_{n,i} = \delta, \end{split}$$

is called a correct related-key boomerang quartet which occurs with probability  $Pr_c = (pq)^2$ . A quartet  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  which only satisfies the condition  $P_{0,i}^1 \oplus_{0,i}^2 = \alpha = P_{0,i}^3 \oplus P_{0,i}^4$  is called a false related-key boomerang quartet.

Figure II displays the structure of the related-key boomerang distinguisher step.

Any attacker that applies a related-key boomerang distinguisher does not know the internal states  $P_{b,i}^1, P_{b,i}^2, P_{b,i}^3, P_{b,i}^4$ , and since he can only apply a chosen plaintext and ciphertext attack on the cipher. The set  $\Theta$ , which is the output of the related-key boomerang distinguisher, therefore contains correct and false related-key boomerang quartets. It is impossible to form another distinguisher which separates the correct and the false related-key boomerang quartets, since the interior differences  $\beta$  and  $\gamma$  cannot be computed.

**Key Recovery Step.** The second step of the related-key boomerang attack is the key recovery step. From now on, an attacker operates on the set  $\Theta$  that was stored by the related-key boomerang distinguisher. Let  $K_0, K_0^*$  be the 0-th round keys derived from the master keys  $K, K^*$ . Let  $enc_{K_0}(P)$  be the one round partial encryption of P under the round key  $K_0$ . The round keys are related as  $K_0^* = K_0 \oplus \Delta K_0$ , where  $\Delta K_0$  is the key difference of the 0-th round keys. The key recovery step works as follows:

- For each key-bit combination of  $K_0$  and  $K_0^*$ 
  - 1. Initialize a counter with zero.
  - For all quartets  $(P_{0,i}^1,P_{0,i}^2,P_{0,i}^3,P_{0,i}^4)$  stored in  $\varTheta$ 
    - 2. Encrypt the plaintext quartet  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  one round under the guessed round keys  $K_0$  and  $K_0^*$  respectively, i.e.,  $P_{1,i}^1 = enc_{K_0}(P_{0,i}^1), P_{1,i}^2 = enc_{K_0^*}(P_{0,i}^2), P_{1,i}^3 = enc_{K_0}(P_{0,i}^3)$  and  $P_{1,i}^4 = enc_{K_0^*}(P_{0,i}^4).$
    - 3. Test whether the differences  $P_{1,i}^1 \oplus P_{1,i}^2$  and  $P_{1,i}^3 \oplus P_{1,i}^4$  have a desired difference an attacker would expect depending on the related-key differential being used. Increase a counter for the used key-bits if the difference is fulfilled in both pairs.

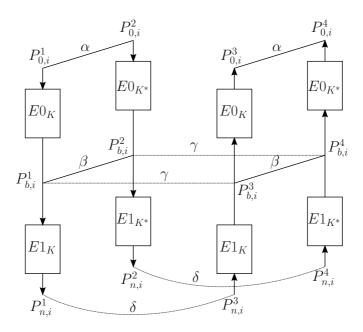


Fig. 1. The related-key boomerang distinguisher

4. Output the round keys  $K_0$  and  $K_0^*$  with the highest counter as the correct one.

In Step 3, four cases can be distinguished, since  $\Theta$  contains correct and false related-key boomerang quartets and the round keys  $K_0, K_0^*$  can either be correct or false. A correct related-key boomerang quartet encrypted using the correct round key will have the desired difference needed to pass the test in Step 3. Hence, the counter for the correct round key is increased.

The other three cases are: a correct related-key boomerang quartet is used with a false round key  $(Pr_{cK_f})$ , a false related-key boomerang quartet is used with a correct round key  $(Pr_{fK_c})$  or a false related-key boomerang quartet is used with a false round key  $(Pr_{fK_f})$ . We assume that the cipher acts like a random permutation. In these case we can assume that

$$Pr_{cK_f} = Pr_{fK_c} = Pr_{fK_f} =: Pr_{filter}.$$

The probability that a quartet in one of the three undesirable cases is counted for a certain round key is  $Pr_{filter}$ . The related-key differentials have to be chosen such that the counter for the correct round key is significantly higher than the counter for each false round key. If the differentials have a high probability the key recovery step outputs the correct round key in Step 4 with a high probability much faster than exhaustive key search.

## 4 Memoryless Related-Key Boomerang Attack on 39-Round SHACAL-2

In this section we propose a 39-round memoryless related-key boomerang attack on SHACAL-2 using two related keys. The cipher E can be treated as a cascade of two sub-ciphers  $E = E1 \circ E0$ , E0 is a sub-cipher containing rounds 1 to 25 while the sub-cipher E1 covers rounds 25 to 35. Our related-key differentials are based on the differentials used in **[18]**. We use these differentials to build a 35-round related-key boomerang distinguisher, which can be used in a memoryless attack to break 39 rounds of SHACAL-2. The main advantage of our attack is that we do not need to store all the quartets as in the related-key rectangle attack of **[9, [12, [18, [11]**. Thus, we only require a very small amount of memory to successfully mount our attack.

The notations used in the attack will be as follows:

- $K, K^*$  master keys (512 bit).
- $-K_i, K_i^*$  round keys of round *i*, where  $i \in \{0, 1, 2, ..., 38\}$  (32 bit).
- $-\Delta K_i$  is the *i*-th round key difference derived from  $\Delta K$

In the following we will describe the related-key differentials used during our attack.

### 4.1 The Related-Key Differential E0 and $E0^{-1}$

The related-key differential for E0 covers rounds 1 to 25, which is

 $(e_{6,9,18,20,25,29}, e_{31}, 0, 0, e_{6,20,25}, e_{31}, 0, 0) \rightarrow (0, 0, e_M, e_{31}, 0, e_{9,13,19}, e_{18,29}, e_{31}),$ 

$A^{1}_{0,i}$	$B^{1}_{0,i}$	$E_{0,i}^{1}$	$F_{0,i}^{1}$
$a_{31}^1 = b_{31}^1,  a_k^1 = c_k^1$	$b_k^1 = \neg f_k^1 \ (k = 19, 30)$	$e_{31}^1 = 0, \ e_k^1 = 0$	$f_k^1 = g_k^1$
(k = 6, 9, 18, 20, 25, 29)	$b_{9}^{1} = e_{9}^{1}$	(k = 18, 29)	(k = 9, 13, 19)
$a_k^1,  b_k^1,  c_k^1,  e_k^1,  f_k^1,  g_k^1$ a	re the k-th bits of $A_{0,s}^1$	$_{i}, B^{1}_{0,i}, C^{1}_{0,i}, E^{1}_{0,i}$	$F_{0,i}^1, G_{0,i}^1.$

Table 2. The fixed plaintext bits for SHACAL-2

**Table 3.** The Related-Key Differential E0 and  $E0^{-1}$  in reverse order

i	$\Delta A_i$	$\Delta B_i$	$\Delta C_i$	$\Delta D_i$	$\Delta E_i$	$\Delta F_i$	$\Delta G_i$	$\Delta H_i$	$\Delta K_i$	Prob.
0	0	$e_M$	$e_{31}$	0	$e_{9,13,19}$	$e_{18,29}$	$e_{31}$	$\Delta_H$	$e_{31}$	1
1	0	0	$e_M$	$e_{31}$	0	$e_{9,13,19}$	$e_{18,29}$	0	0	$2^{-11}$
2	$e_{31}$	0	0	$e_M$	0	0	$e_{9,13,19}$	$e_{18,29}$	0	$2^{-10}$
3	0	$e_{31}$	0	0	$e_{6,20,29}$	0	0	$e_{9,13,19}$	0	$2^{-7}$
4	0	0	$e_{31}$	0	0	$e_{6,20,25}$	0	0	0	$2^{-4}$
5	0	0	0	$e_{31}$	0	0	$e_{6,20,25}$	0	0	$2^{-3}$
6	0	0	0	0	$e_{31}$	0	0	$e_{6,20,25}$	0	$2^{-4}$
7	0	0	0	0	0	$e_{31}$	0	0	0	$2^{-1}$
8	0	0	0	0	0	0	$e_{31}$	0	0	$2^{-1}$
9	0	0	0	0	0	0	0	$e_{31}$	$e_{31}$	1
10	0	0	0	0	0	0	0	0	0	1
• • •	• • •	• • •	• • •	•••	• • •	•••	•••	•••	• • •	•••
23	0	0	0	0	0	0	0	0	0	1
24	0	0	0	0	0	0	0	0	•	$2^{-6}$
25	$e_{13,24,28}$	0	0	0	$e_{13,24,28}$	0	0	0	•	—

 $M = \{6, 9, 18, 20, 25, 29\}, \ \Sigma_1(E_0 \oplus e_{9,13,19}) - \Sigma_1(E_0) + \Delta_H = 0.$ 

we can assume that the probability for a differential remains the same if it runs in backward direction (Theorem 1). Thus,  $\Pr(\alpha \to \beta) = \Pr(\alpha \leftarrow \beta)$  always holds for the related-key differential E0 and  $E^{0^{-1}}$ . Thus, Table 3 also represents the related-key differential  $E0^{-1}$ , which is starts from the bottom of the table (round 25) and goes up to the top (round 1). The related-key differential E0 occurs with probability  $\Pr(\alpha \to \beta) = 2^{-47}$  and therefore the related-key differential  $E0^{-1}$ occurs also with probability  $\Pr(\alpha \leftarrow \beta) = 2^{-47}$  too.

#### 4.2 The Related-Key Differential $E1^{-1}$

Our related key differential  $E1^{-1}$  covers rounds 35 to 25, which can be written as

 $(e_{6,9,18,20,25,29}, e_{31}, 0, 0, e_{6,20,25}, e_{31}, 0, 0) \to (e_{31}, e_{31}, e_{M'}, 0, 0, e_{9,13,19}, e_{18,29,31}, 0).$ 

The related-key differential  $E1^{-1}$  occurs with probability  $\Pr(\gamma \leftarrow \delta) = 2^{-65}$  and can be found in Table 4.

i	$\Delta A_i$	$\Delta B_i$	$\Delta C_i$	$\Delta D_i$	$\Delta E_i$	$\Delta F_i$	$\Delta G_i$	$\Delta H_i$	Prob.
35	$e_{6,9,18,20,25,29}$	$e_{31}$	0	0	$e_{6,20,25}$	$e_{31}$	0	0	$2^{-11}$
34	$e_{31}$	0	0	0	$e_{31}$	0	0	0	1
33	0	0	0	0	0	0	0	$e_{31}$	$2^{-1}$
32	0	0	0	0	0	0	$e_{31}$	$e_{31}$	1
31	0	0	0	0	0	$e_{31}$	$e_{31}$	$e_{31}$	$2^{-4}$
30	0	0	0	0	$e_{31}$	$e_{31}$	$e_{31}$	$e_{6,20,25}$	$2^{-7}$
29	0	0	0	$e_{31}$	$e_{31}$	$e_{31}$	$e_{6,20,25}$	0	$2^{-8}$
28	0	0	$e_{31}$	$e_{31}$	$e_{31}$	$e_{6,20,25}$	0	0	$2^{-7}$
27	0	$e_{31}$	$e_{31}$	$e_{31}$	$e_{6,20,25}$	0	0	$e_{9,13,19}$	$2^{-12}$
26		$e_{31}$	$e_{31}$	$e_{M'}$	0	0	$e_{9,13,19}$	$e_{18,29,31}$	$2^{-15}$
25	$e_{31}$	$e_{31}$	$e_{M'}$	0	0	$e_{9,13,19}$	$e_{18,29,31}$	0	_

**Table 4.** The Related-Key Differential  $E1^{-1}$ 

 $M' = \{6, 9, 18, 20, 25, 29, 31\}.$ 

#### 4.3 The Attack

The attack works as follows:

- For  $i = 1, 2, \dots, 2^{225.5}$  do
  - Guess two 128-bit round keys  $(K_{38}, K_{37}, K_{36}, K_{35})$  and  $(K_{38}^*, K_{37}^*, K_{36}^*, K_{35}^*)$  and do the following:
    - 1. Choose plaintext  $P_{0,i}^1$  and  $\Delta H_i$ , such that the 16 bit conditions presented in Table 2 and the condition  $\Sigma_1(E_0 \oplus e_{9,13,19}) \Sigma_1(E_0) + \Delta H_i = 0$  are satisfied. Ask for encryption of  $P_{0,i}^1$  under K to obtain the plaintext  $P_{39,i}^1$ .
    - 2. Compute the intermediate value  $P_{35,i}^1$  by decrypting  $P_{39,i}^1$  under  $(K_{38}, K_{37}, K_{36}, K_{35})$ .
    - 3. Compute the intermediate value  $P^3_{35,i} = P^1_{35,i} \oplus \delta$ , where  $\delta = (e_{6,9,18,20,25,29}, e_{31}, 0, 0, e_{6,20,25}, e_{31}, 0, 0)$ . Compute  $P^3_{39,i}$  by encrypting  $P^3_{35,i}$  using  $(K_{35}, K_{36}, K_{37}, K_{38})$ .
    - 4. Ask for decryption of  $P_{39,i}^3$  under K and obtain  $P_{0,i}^3$ .
    - 5. Now, use the previously chosen  $P_{0,i}^1$  and the difference  $\alpha_i = (0, e_M, e_{31}, 0, e_{9,13,19}, e_{18,29}, e_{31}, \Delta H_i), \quad M = \{6, 9, 18, 20, 25, 29\}$ , to compute  $P_{0,i}^2 = P_{0,i}^1 \oplus \alpha_i$ . Ask for encryption of  $P_{0,i}^{2}$  under  $K^*$  to obtain  $P_{39,i}^{2}$ .
    - 6. Compute the intermediate value  $P^2_{35,i}$  by decrypting  $P^2_{39,i}$  under  $(K^*_{38}, K^*_{37}, K^*_{36}, K^*_{35})$  respectively.
    - 7. Compute the intermediate value  $P_{35,i}^4 = P_{35,i}^2 \oplus \delta$ . Compute  $P_{39,i}^4$  by encrypting  $P_{35,i}^4$  with keys  $(K_{35}^*, K_{36}^*, K_{37}^*, K_{38}^*)$ .
    - 8. Ask for decryption of  $P^4_{39,i}$  under  $K^*$  to obtain  $P^4_{0,i}$ .
    - 9. Check if  $P_{0,i}^3 \oplus P_{0,i}^4 = \alpha_i$ . If true, store the quartet  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  in  $\Theta$  as well as the used partial keys  $K_0, K_{35}, K_{36}, K_{37}, K_{38}$ .
- For all 64-bit round keys  $(K_0, K_0^*)$  do

- 10. Set counter = 0.
  - For all quartets  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  in  $\Theta$ 
    - 11. Encrypt  $(P_{0,i}^1, P_{0,i}^2, P_{0,i}^3, P_{0,i}^4)$  one round with  $K_0$  and  $K_0^*$  respectively.
    - 12. Check if  $P_{1,i}^1 \oplus P_{1,i}^2 = P_{1,i}^3 \oplus P_{1,i}^4 = \tau$ , where  $\tau = (0, 0, e_M, e_{31}, 0, e_{9,13,19}, e_{18,29}, 0)$ . If true, increase the *counter* by one.
    - 13. Take the guess of  $K_0$  and  $K_0^*$  with  $counter \ge 2$  as the correct round key pair.
- 14. For a suggested  $(K_0, K_{35}, K_{36}, K_{37}, K_{38})$  (as also saved in  $\Theta$  for every quartet), do an exhaustive search for the remaining  $512 5 \cdot 32 = 352$  key bits by trial encryption. If a 512-bit key is suggested, output it as the master key of the 39-round SHACAL-2. Otherwise restart the algorithm.

#### 4.4 Analysis of the Attack

A correct related-key boomerang quartet occurs with probability

$$Pr_c = \Pr(\alpha \to \beta) \cdot (\Pr(\gamma \leftarrow \delta))^2 \cdot \Pr(\alpha \leftarrow \beta)$$
$$= 2^{-47} \cdot (2^{-65})^2 \cdot 2^{-47} = 2^{-224},$$

since all related-key differential conditions are fulfilled. A random permutation of a difference  $P_{0,i}^3 \oplus P_{0,i}^4$  has difference  $\alpha$  with probability  $Pr_f = 2^{-256}$ . Let  $\#Q = 2^{225.5}$  be the number of initially chosen quartets and #C be the number of correct quartets as tested in Step 9. We expect that  $\#C = \#Q \cdot Pr_c = 2^{225.5} \cdot 2^{-224} = 2^{1.5} \approx 3$  correct quartets are found and that  $\#F = \#Q \cdot Pr_f = 2^{225.5} \cdot 2^{-226} = 2^{-30.5}$  false quartets are found. The data complexity of Step 1 to 9 is about  $2^2 \cdot 2^{1.5} = 2^{3.5}$  chosen plain– and ciphertexts, since we expect to have about  $\#C = 2^{1.5}$  correct quartets and  $\#F = 2^{-30.5}$  false quartets stored in  $\Theta$ . This amount occurs since we have a 256-bit filtering condition on one side of the boomerang. The time complexity is determined by  $2^2 \cdot 2^{225.5} \cdot 2^{8\cdot32} = 2^{483.5}$  39-round SHACAL-2 encryptions.

A false combination of quartets and key bits is counted in Step 12 with probability  $Pr_{filter} = 2^{-2\cdot32}$ . At least  $\#cK_c \approx 3$  correct related-key boomerang quartets and additionally  $\#fK_c = \#F \cdot Pr_{filter} = 2^{-30.5} \cdot 2^{-128} = 2^{-158.5}$ false related-key boomerang quartets are counted with the correct key bits. We double the filtering since we can filter on both pairs of a quartet. About  $\#cK_c + \#fK_c \approx 3 + 2^{-158.5} \approx 3$  quartets are counted for the correct key bits. Note that we need only  $\Theta \approx 2^{1.5}$  quartets for our key recovery step, since  $\Theta$ contains only correct quartets. This means that the correct key bits alway get a count of  $2^{1.5}$  which will distinguish them from all false key bit combinations.

About  $\#cK_f = \#C \cdot Pr_{filter} \approx 3 \cdot 2^{-64} = 2^{-62.5}$  correct related-key boomerang quartets and  $\#FK_f = \#F \cdot Pr_{filter} = 2^{-30.5} \cdot 2^{-64} = 2^{-94.5}$  false related-key boomerang quartets are counted with the false key bits, which are approximately  $\#cK_f + \#fK_f \approx 2^{-62.5} + 2^{-94.5} \approx 2^{-62.5}$  counts for each false key bit combination. Step 13 takes the correct key bits since we expect about  $2^{-62.5}$  counts for

each false key bit combination and about 3 counts for the correct key bits. The time complexity of Steps 10 to 13 is negligible small under these circumstances. ( $\approx 2^{64}$ ).

Using the Poisson distribution we can compute the success rate of our attack. The probability that the number of remaining quartets for each false key bit combination is larger than 1 is  $Y \sim Poisson(\mu = 2^{-62.5})$ ,  $\Pr(Y \ge 2) \approx 0$ , since the expected number of quartets counted with a false key bit combination is  $2^{-62.5}$ . We expect to have a count of at least 3 quartets for the correct key bits. The probability that the number of quartets counted for the correct key bits is at least 2 is  $Z \sim Poisson(\mu = 3)$ ,  $\Pr(Z \ge 2) \approx 0.8$ . The data complexity of our attack is  $2^{3.5}$  chosen plaintexts and ciphertexts, while the time complexity time complexity is about  $2^{483.5}$  39-round SHACAL-2 encryptions. Our attack has a success rate of 0.8.

## 5 Conclusion

In this paper we present the first memoryless attack on SHACAL-2. Using a related-key boomerang distinguisher we can mount an attack on 39-round SHACAL-2. This improves the memory requirements of existing attacks by a factor of about  $2^{235}$ . Furthermore our attack can be seen as a starting point/building block to form new attacks on SHACAL-2 using the related-key boomerang technique. This is the first application of the related-key boomerang attack used as a memoryless variant on a block cipher. The main advantage of our attack is that the data can be proceeded sequentially instead of parallel as needed for all the other attacks.

Further research. An improvement of our attack would be the following scenario: Instead of using one differential for E0 and E1 one could try to use all possible differentials which have the same input and output difference. This will decrease the time complexity but does not lead to attack more rounds. Another improvement might be to use more than two related keys which may give somewhat better differentials.

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# Some New Observations on the SMS4 Block Cipher in the Chinese WAPI Standard

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Abstract. SMS4 is a 128-bit block cipher used in the WAPI standard in wireless networks in China. The cipher has attracted much attention in the past two years. This paper consists of two parts. The first part is on the design of the linear diffusion layer L of SMS4. Some new observations on L are present, which open out the design rationales of L and such class functions to a great extent. The second part is on the differential attack against SMS4. A class of 18-round differential characteristics with a higher probability is given. Then a simple differential attack on 22-round SMS4 is present, which is an improvement of the previous work, thus our attack becomes the best known one on SMS4. Furthermore, we make a remark on the construction of differential characteristics of SMS4.

**Keywords:** WAPI, Block Cipher, SMS4, Diffusion Transformation, Differential Cryptanalysis.

## 1 Introduction

WAPI (WLAN Authentication and Privacy Infrastructure) is the Chinese national standard for WLANs (Wireless Local Area Networks) products. SMS4 is the underlying block cipher used in the WAPI standard. WAPI has been submitted to ISO for recognition as an international standard, but rejected in March 2006, partially because of uncertainties regarding the security of the unrevealed SMS4 cipher. However, the WAPI standard continues to be widely used in Chinese industries.

Chinese government released the SMS4  $\square$  cipher in January 2006 in a Chinese version, and [3] presented an English translation of [1]. SMS4 has a 128-bit block size, a 128-bit user key, and a total of 32 rounds. It employs a kind of generalized Feistel network structure, and only 32 bits are modified in each round. The main part of its round function (i.e., T function) is composed of 3 steps, firstly the subkey XOR-addition, next 4 S-boxes in parallel, lastly a linear transformation L.

Since its introduction, the SMS4 cipher has attracted much attention from cryptanalytic researchers, due to its simplicity and Chinese standard prominence. There are several public papers on SMS4 in the past two years. The first open

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analysis [10] uncovers the origin of its S-box, and presents an integral attack on 13-round SMS4. Next, some properties on a class of linear transformations based on cyclic-shift and Xor operations are presented [12], the results can be directly applied to the diffusion layer of SMS4. Then, a rectangle attack on 14 rounds and an impossible differential attack on 16 rounds are presented [11]. Later, a differential attack on 21 rounds [14] and a linear attack on 22 rounds [5] are respectively revealed. Recently, a linear attack and a differential attack both on 22-round SMS4 |9| are presented based on the work of |14|. Other papers on SMS4 include [4,7,15]. The previous best known attack are due to 5 and [9].

This paper consists of two parts. The first part is on the design of the linear transformation L of SMS4. L is very simple, composed of only 5 left rotations and 4 XORs. For all we know, the analogous modules only appeared in the hash standard SHA-256, namely  $\sigma_i$  and  $\Sigma_i$  (i = 0, 1) functions of SHA-256, which are all composed of 3 right rotations (or right shifts) and 2 XORs. In the following, we will present 3 observations on the linear transformation L of SMS4, which provides some insight into the design rationales of L and such class functions.

The second part is on the differential cryptanalysis of SMS4. We will show that there are 12 18-round differential characteristics for SMS4 each with a higher probability of  $2^{-114}$ , which are the best differential characteristics for SMS4 so far. Then we present a simple differential attack on 22-round SMS4 based on any one of the 12 differential characteristics, which improves the previous best known attacks due to J. Etrog and M.Robshaw<sup>[5]</sup> and T.Kim et.al.<sup>[9]</sup> with regard to attack complexity, thus our attack becomes the best one so far. Finally, we make a remark on constructing differential characteristics of SMS4.

#### 2 Description of SMS4

SMS4 is a 32-round block cipher with a 128-bit block size and a 128-bit key size. Its overall structure is a kind of generalized Feistel network, thus the encryption and decryption have the same procedure except that the round subkeys for decryption are used in the reverse order.

#### 2.1Notation

The following notations are used throughout this paper.

- $-Z_2^{32}$ : the set of 32-bit words;  $Z_2^8$ : the set of 8-bit bytes;  $Z_2 = \{0, 1\}$ .  $\ll i$ : left rotation by *i* bits;  $\gg i$ : right rotation by *i* bits.
- For any  $U \in \mathbb{Z}_2^{32}$ , let  $U = (U_0, U_1, U_2, U_3) \in (\mathbb{Z}_2^8)^4$ .
- $-K_i$ : the 32-bit subkey in Round  $i, K_i = (K_{i,0}, \tilde{K}_{i,1}, K_{i,2}, K_{i,3}) \in (\mathbb{Z}_2^8)^4$ ,  $(i = 0, 1, \ldots, 31).$
- $-\operatorname{Prob}_F(\alpha \to \beta)$ : the probability that output difference of function F is  $\beta$  given input difference is  $\alpha$ , F can be omitted when the context is clear.

<sup>1</sup> Actually, Theorem 3.1 in this paper can be directly obtained from Theorem 1 of [12]. We know this after we have obtained Theorem 3.1 and Theorem 3.2. Whereas the proof of Theorem 1 is not presented in [12], we remain Theorem 3.1 and its proof in this paper.

#### 2.2 Encryption Procedure of SMS4

Let  $(X_0, X_1, X_2, X_3) \in (Z_2^{32})^4$  and  $(Y_0, Y_1, Y_2, Y_3) \in (Z_2^{32})^4$  denote the 128-bit plaintext and ciphertext respectively. Let  $K_i \in Z_2^{32}$  (i = 0, 1, 2, ..., 31) denote the round subkeys.

The encryption procedure of SMS4 is as follows:

$$X_{i+4} = X_i \oplus T(X_{i+1} \oplus X_{i+2} \oplus X_{i+3} \oplus K_i), \quad \text{for} \quad i = 0, 1, \dots, 31$$
$$(Y_0, Y_1, Y_2, Y_3) = (X_{35}, X_{34}, X_{33}, X_{32}).$$

where the function  $T: Z_2^{32} \mapsto Z_2^{32}$  is composed of a non-linear transformation  $\tau$  and a linear transformation L, i.e.,  $T(\cdot) = L(\tau(\cdot))$ .

Fig.1 depicts one round of the encryption procedure of SMS4.

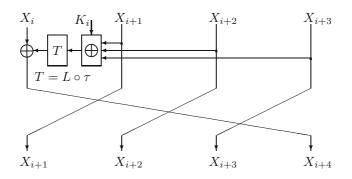


Fig. 1. The *i*-th round of SMS4

 $\tau$  applies 4 S-boxes in parallel. Let  $A = (a_0, a_1, a_2, a_3) \in (Z_2^8)^4$  be the input, where each  $a_i$  is a byte. Then the output  $B = \tau(A) = (b_0, b_1, b_2, b_3) \in (Z_2^8)^4$  is given by

$$(b_0, b_1, b_2, b_3) = (Sbox(a_0), Sbox(a_1), Sbox(a_2), Sbox(a_3))$$

where Sbox is a  $8 \times 8$  S-box.

The 32-bit output B of  $\tau$  will be the input of the linear transformation L. The output  $C = L(B) \in (\mathbb{Z}_2)^{32}$  is given by

$$C = L(B) = B \oplus (B \lll 2) \oplus (B \lll 10) \oplus (B \lll 18) \oplus (B \lll 24).$$

## 3 Observations on the Linear Transformation L

In this section, we will present 3 observations on the linear transformation L.

#### 3.1 Optimality of L

Branch number introduced by J.Daemen [2] has now become a widely-accepted concept for measuring the diffusion effect of a transformation (usually a linear transformation).

The following gives the definition of the branch number of L.

**Definition 3.1.** (Branch number) Let  $W(\cdot)$  denote the byte weight function, i.e., the number of nonzero bytes. The branch number of a function  $F: \mathbb{Z}_2^{32} \to \mathbb{Z}_2^{32}$  is defined by

$$\beta(F) = \min_{X \neq 0, X \in \mathbb{Z}_2^{32}} (W(X) + W(F(X))).$$

It is easy to know that  $\beta(F) \leq 5$ . As a rule, the larger the branch number, the better its diffusion effect. When  $\beta(F)$  reaches the maximum 5, we call F maximal.

For the linear transformation L of SMS4, the branch number reaches the maximum 5, which can be easily verified by a computer program.

For a diffusion transformation of a block cipher or a hash function, the implementation is another important factor besides its branch number. L is composed of 5 left-rotations and 4 XORs. The question posed here is : can we use less leftrotations and XORs to get a transformation whose branch number is also the best possible 5? In the following, we will show that the answer is "NO".

Let  $F(X, k, r_1, \ldots, r_k) = \bigoplus_{i=1}^k (X \ll r_i)$ , where  $X \in \mathbb{Z}_2^{32}$ , k is a natural number and  $0 \leq r_1 < r_2 < \ldots < r_{k-1} < r_k \leq 31$ . That is,  $F(X, k, r_1, \ldots, r_k)$  is composed of k left rotations and (k-1) XORs, and the rotations are specified by  $(r_1, \ldots, r_k)$ .

**Theorem 3.1.** If  $F(X, k, r_1, \ldots, r_k)$  is maximal for some value of  $(k, r_1, \ldots, r_k)$ , then  $k \ge 5$ .

**Proof.** Let  $X = (x_{31}, x_{30}, \ldots, x_1, x_0) \in \mathbb{Z}_2^{32}$ , its bit index  $31, 30, \ldots, 1, 0$  can be divided into 4 subsections of one byte each:  $31 \ldots 24$ ,  $23 \ldots 16$ ,  $15 \ldots 8$  and  $7 \ldots 0$ , we call them byte-index subsections.

Let  $X \in \mathbb{Z}_2^{32}$  and  $Y = X \lll r$ , then the *i*th bit of Y is  $y_i = x_{(i-r)mod 32}$   $(i = 0, 1, \ldots, 31)$ . Let  $e_i$  be the 32-bit string whose *i*-th bit equals to 1 and the other 31 bits all equal to 0, then we can get the value of  $F(e_i, k, r_1, \ldots, r_k)$ : the  $(i + r_1)$ th,  $(i + r_2)$ th,  $\ldots$ ,  $(i + r_k)$ th bits equal to 1 and the other (32 - k) bits equal to 0. Therefore, if  $F(X, k, r_1, \ldots, r_k)$  is maximal, then it must require  $W(F(e_i, k, r_1, \ldots, r_k)) = 4$  as  $W(e_i) = 1$ . That is to say, for each of the 4 byte-index subsections, at least one element of  $\{r_1, \ldots, r_k\}$  is falling into it (when considering  $e_0$ ), which also means  $k \ge 4$ .

On the other hand, it's easy to see that

$$F(0xffffffff, k, r_1, \dots, r_k) = \begin{cases} 0xffffffff & \text{if } k \text{ is odd} \\ 0 & \text{if } k \text{ is even} \end{cases}$$

Thus, the branch number of  $F(X, k, r_1, \ldots, r_k)$  is not more than 4 if k is even.

Therefore, it must require  $k \ge 5$  for a maximal  $F(X, k, r_1, \ldots, r_k)$ .

According to Theorem 3.1, we can know that the selection of the linear transformation L of SMS4 is optimal between the branch number and the number of rotations. Furthermore, we will show that there are few maximal  $F(X, k, r_1, \ldots, r_k)$  when k = 5.

Let  $F(X, k, r_1, \ldots, r_k) \ll h$  denote the function  $\left(\bigoplus_{i=1}^k (X \ll r_i)\right) \ll h$ , it is easy to see that:

$$F(X,k,r_1,\ldots,r_k) \lll h = \bigoplus_{i=1}^k (X \lll (r_i + h))$$

where "+" denotes addition modulo 32.

Firstly, we give the following property:

**Property 3.1.** For each value of k and each value of  $(r_1, \ldots, r_k)$ , the branch number of  $F(X, k, r_1, \ldots, r_k) \ll 8$ ,  $F(X, k, r_1, \ldots, r_k) \ll 16$  and  $F(X, k, r_1, \ldots, r_k) \ldots r_k \gg 24$  are all equal to the branch number of  $F(X, k, r_1, \ldots, r_k)$ .

The reason of Property 3.1 is that the output of each of the three functions is just a byte - permutation of the output of  $F(X, k, r_1, \ldots, r_k)$  given the same input X, so the branch number is unchanged.

When k = 5, there are 5 parameters  $r_1, r_2, r_3, r_4, r_5$ . If  $F(X, 5, r_1, \ldots, r_5)$  is maximal, then it must be that 2 parameters lie in a same byte-index subsection, and the other 3 parameters lie in the other three byte-index subsections each.

According to Property 3.1, we will assume:

 $0\leqslant r_1< r_2\leqslant 7, \quad 8\leqslant r_3\leqslant 15, \quad 16\leqslant r_4\leqslant 23, \quad 24\leqslant r_5\leqslant 31$ 

Thus, there are  $\frac{8\times7}{2} \times 8 \times 8 \times 8 = 14336$  different transformations  $F(X, 5, r_1, \ldots, r_5)$  in all in the sense of Property 3.1. An experiment is made to test which are maximal among all the 14336 transformations, which spent several days on two computers. The result is as follows:

**Theorem 3.2.** Among all of the 14336 transformations  $F(X, 5, r_1, \ldots, r_5)$ , only two are maximal, one is L, the other is :

 $L^{*}(X) = X \oplus (X \lll 6) \oplus (X \lll 14) \oplus (X \lll 22) \oplus (X \lll 24)$ 

 $L^*(X)$  and L(X) are related in the following way: substitute " $\gg$ " for " $\ll$ " in  $L^*(X)$ , we have

$$R^*(X) = X \oplus (X \implies 6) \oplus (X \implies 14) \oplus (X \implies 22) \oplus (X \implies 24)$$
$$= L(X) \lll 24$$

It's easy to know  $R^*(X)$  and  $L^*(X)$  have the same branch number. Also L(X) and  $L(X) \ll 24$  have the same branch number. Hence, our experimental result shows that the linear transformation L of SMS4 is the unique maximal one in essence among all the transformations  $F(X, 5, r_1, \ldots, r_5)$ , in view of Property 3.1 and diffusion effect.

#### 3.2 Bijectivity of $F(X, k, r_1, \ldots, r_k)$ Function

In [6], the authors proved that  $\Sigma_i$  (i = 0, 1) functions of SHA-256 are all one to one, their proof need the specific parameters of the two functions. In the following, we will present a more general result, which makes the above result in [6] a simple corollary.

**Theorem 3.3** Let X be a 32-bit variable. For any  $(r_1, \ldots, r_k)$ , satisfying  $0 \leq$  $r_1 < r_2 < \ldots < r_{k-1} < r_k \leq 31, F(X, k, r_1, \ldots, r_k)$  is bijective if k is odd, and non-bijective if k is even.

**Proof.** Let  $f^{n}(\cdot)$  denote the *n*-th iteration of  $f(\cdot)$ , i.e.,  $f^{n+1} = f \circ f^{n}$ , where  $f \circ g$  denotes function composition, that is,  $(f \circ g)(x) = f(g(x))$ .

We have

$$F^{2}(X, k, r_{1}, \dots, r_{k}) = \bigoplus_{j=1}^{k} ((\bigoplus_{i=1}^{k} (X \ll r_{i})) \ll r_{j}) = \bigoplus_{i,j=1}^{k} (X \ll (r_{i} + r_{j}))$$

$$= \bigoplus_{i=1}^{k} (X \ll (2 \times r_{i}))$$
(1)

where "+" denotes addition modulo 32 and " $\times$ " multiplication modulo 32.

Because X is a 32-bit variable, it is easy to see that  $X \ll 32 = X$  holds. Then, we can get the following equation by repeatedly using equation (1) 5 times:

$$F^{32}(X,k,r_1,\ldots,r_k) = \bigoplus_{i=1}^k (X \lll (32 \times r_i)) = \bigoplus_{i=1}^k X = \begin{cases} X & \text{if } k \text{ is odd} \\ 0 & \text{if } k \text{ is even} \end{cases}$$

Hence, if k is odd, then the inverse function of  $F(X, k, r_1, \ldots, r_k)$  is  $F^{31}(X, k, r_k)$  $r_1, \ldots, r_k$ ), thus F function must be a bijection. If k is even, then F must be non-bijective. 

Theorem 3.3 suggests that k must be odd when  $F(X, k, r_1, \ldots, r_k)$  is used as a function of cryptographic ciphers, such as block ciphers and hash functions.

#### 3.3Distribution of Input-Output Patterns of L

Let x be a byte, define the function  $\delta(x)$  as:

$$\delta(x) = \begin{cases} 1 \text{ if } x \neq 0\\ 0 \text{ if } x = 0 \end{cases}$$

Let  $X = (x_0, x_1, x_2, x_3) \in (\mathbb{Z}_2^8)^4$ , the pattern of X is defined by Pattern(X) = $(\delta(x_0), \delta(x_1), \delta(x_2), \delta(x_3))$ , i.e., Pattern(X) specifies the active and passive byte positions of X. It's easy to see that there are 15 possible patterns for a non-zero X.

Table 1 gives the distribution of input pattern and output pattern of Y =L(X), where the first column denotes the 15 possible input patterns of X, the first row denotes the 15 output patterns of Y, the element at position  $(i,j)(i,j \in \{0001,0010,\ldots,1111\})$  denotes the probability of Pattern(Y) = jgiven Pattern(X) = i, where

$$\begin{split} \varepsilon_1 &= \frac{1}{255}, \varepsilon_2 = \frac{251}{255}, \\ \varepsilon_3 &= \frac{1}{65025}, \varepsilon_4 = \frac{251}{65025}, \varepsilon_5 = \frac{64015}{65025}, \\ \varepsilon_6 &= \frac{1}{16581375}, \varepsilon_7 = \frac{251}{16581375}, \varepsilon_8 = \frac{64015}{16581375}, \varepsilon_9 = \frac{16323805}{16581375}. \end{split}$$

	0001	0010	0100	1000	0011	0101	0110	1001	1010	1100	0111	1011	1101	1110	1111
0001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0010	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0011	0	0	0	0	0	0	0	0	0	0	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_2$
0101	0	0	0	0	0	0	0	0	0	0	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_2$
0110	0	0	0	0	0	0	0	0	0	0	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_2$
1001	0	0	0	0	0	0	0	0	0	0	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_2$
1010	0	0	0	0	0	0	0	0	0	0	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_2$
1100	0	0	0	0	0	0	0	0	0	0	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_1$	$\varepsilon_2$
0111	0	0	0	0	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_5$
1011	0	0	0	0	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_5$
1101	0	0	0	0	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_5$
1110	0	0	0	0	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_4$	$\varepsilon_5$
1111	$\varepsilon_6$	$\varepsilon_6$	$\varepsilon_6$	$\varepsilon_6$	$\varepsilon_7$	$\varepsilon_7$	$\varepsilon_7$	$\varepsilon_7$	$\varepsilon_7$	$\varepsilon_7$	$\varepsilon_8$	$\varepsilon_8$	$\varepsilon_8$	$\varepsilon_8$	$\varepsilon_9$

**Table 1.** Distribution of Input-Output Patterns of L

The concept of branch number only roughly reflect the diffusion effect of a transformation. Whereas a distribution table as Table 1 reflect the diffusion effect more comprehensively. Such a distribution table may be very helpful in analyzing a cipher against differential, truncated differential or linear cryptanalysis, etc. For example, Zhang etc. [14] have used the pattern of Pattern(X) = Pattern(Y) = (0, 1, 1, 1) in their differential attack on 21-round SMS4.

Phan [13] has presented the distribution of input-output patterns of the Mix-Column transformation of AES, we have also experimentally verified this distribution. It's interesting to find that the distribution of input-output patterns of L of SMS4 is the same as that of the MixColumn transformation of AES. Hence, we can say that the diffusion effect of L is the same with that of the MixColumn transformation of AES.

## 4 A Simple Differential Attack on 22-Round SMS4

In [14], a class of 5-round iterative differential characteristics of SMS4 ( the amount is 7905) is presented, with an average probability of  $2^{-42}$  each. Then, a class of 18-round differential characteristics are easily derived by simply iterating the 5-round iterative differentials, with an average probability of  $2^{-126}$  each. Eventually, the authors present a differential attack on 21-round SMS4, which uses all of the 7905 18-round differential characteristics. It's worth noting that there are a few 5-round iterative differential characteristics with a higher probability of  $2^{-38}$  each, as the authors of [14] have also pointed out and used one to construct a 14-round rectangle distinguisher.

In this section, we will show that there are 3 out of 7905 5-round differential characteristics, which have a probability of  $2^{-38}$  each. Then 18-round differential characteristics are easily derived, with a probability of  $2^{-114}$  each. Using any of

these 3 18-round differential characteristics, we can present a simple differential attack on 22-round SMS4.

# 4.1 18-Round Differential Characteristics with a Probability of $2^{-114}$ Each

The 5-round iterative differential characteristics are as follows [14]:

$$(\alpha, \alpha, \alpha, 0) \xrightarrow{5 Rounds} (\alpha, \alpha, \alpha, 0)$$

with a probability of  $(Prob_T (\alpha \rightarrow \alpha))^2$ .

Because the branch number of L is 5, so it require that  $H(\alpha) \ge 3$  in order for  $Prob_T(\alpha \to \alpha) \ne 0$ . It's obvious that  $Prob_T(\alpha \to \alpha)$  is larger when  $H(\alpha) = 3$ . In [14], the authors show that there are 7905 candidates for  $\alpha$  when  $Pattern(\alpha) = (0, 1, 1, 1)$ . One can refer to [14] for more details.

Through an experiment, we find that there are only 3 out of the 7905 candidates for  $\alpha$ , which make  $Prob_T (\alpha \to \alpha)$  reach the maximal value  $2^{-19}$ . The three values are: 0x002cf5cd, 0x00d2c822 and 00c30290. Furthermore, let Y be one of the three values, i.e.,  $Y \in \{0x002cf5cd, 0x00d2c822, 00c30290\}$ . Then, when  $\alpha = (Y \ll 8)$  or  $\alpha = (Y \ll 16)$  or  $\alpha = (Y \ll 24)$ , the corresponding 5-round iterative differential characteristic also holds with a probability of  $2^{-38}$ , which is also verified by a computer program.

Iterating a 5-round differential characteristic three and a half times, then a 18-round differential characteristic is derived. Hence, there are 12 18-round differentials in total, with a probability of  $2^{-38*3} = 2^{-114}$  each, which are the best distinguishers for SMS4 so far.

#### 4.2 A Differential Attack on 22-Round SMS4

In this section, we fix  $\alpha$  as any of the three values: 0x002cf5cd, 0x00d2c822 or 00c30290. Then, we have the following 18-round differential characteristic:

$$(\alpha, \alpha, \alpha, 0) \xrightarrow{18 Rounds} (0, \alpha, \alpha, \alpha)$$

with a probability of  $2^{-114}$ .

Attack Procedure. We use Round 0, Round1, ..., Round 21 to number the rounds of the 22-round SMS4. Applying the above 18-round differential at Rounds 0 ~ 17. If the output difference of Round 17 is  $(0, \alpha, \alpha, \alpha)$ , then the input difference of T function in Round 18 equals to  $\alpha$ . For the S-box of SMS4, there are 127 possible output differences for any nonzero input difference, thus the output difference of T in Round 18 has only about  $2^{7*3} = 2^{21}$  possible values. Let  $\Lambda$  denote the set of these  $2^{21}$  possible values, then the output difference of Round 18 must belong to  $(\alpha, \alpha, \alpha, \Lambda)$ . Similarly, the output difference of Round 19 must belong to  $(\alpha, \alpha, \Lambda, ?)$ , the output difference of Round 20 must belong to  $(\alpha, \Lambda, ?, ?)$ , and the output difference of Round 21 must belong to  $(\Lambda, ?, ?, ?)$ , where ? denotes an unknown word. The attack procedure is as follows.

- 1. Let  $(P, P^*)$  denote a plaintext pair, and  $(C, C^*)$  the corresponding ciphertext pair. Select  $2^m$  plaintext pairs, satisfying  $P \bigoplus P^* = (\alpha, \alpha, \alpha, 0)$  for each pair.
- 2. For each ciphertext pair  $(C, C^*)$ , check if the first word of the ciphertext difference belongs to the set  $\Lambda$ . If not, discard the pair. Store the filtered ciphertext pairs in a table, about  $2^{m-11}$  ciphertext pairs are expected to be stored after this test.
- 3. For every guess of the 0th byte of  $K_{21}$ , i.e.,  $K_{21,0}$ , do as follows:
  - (a) For each remaining ciphertext pair  $(C, C^*)$ , let  $C = (C_0, C_1, C_2, C_3) \in (Z_2^{32})^4$ ,  $C^* = (C_0^*, C_1^*, C_2^*, C_3^*) \in (Z_2^{32})^4$ , partially decrypt to get the output difference of the 0th Sbox in Round 21:  $\gamma = Sbox((C_0 \oplus C_1 \oplus C_2)_0 \oplus K_{21,0}) \oplus Sbox((C_0^* \oplus C_1^* \oplus C_2^*)_0 \oplus K_{21,0})$ , and compute  $\delta = (L^{-1}(C_3 \oplus C_3^* \oplus \alpha))_0$ . If the guess of  $K_{21,0}$  is correct, then it must be  $\gamma = \delta$  for right pairs. If this is not the case, discard the pair. About  $2^{m-19}$  ciphertext pairs are expected to remain after this test.
  - (b) For each of the other 3 bytes of K<sub>21</sub>, the 4 bytes of K<sub>20</sub> and the 4 bytes of K<sub>19</sub>, continue making a guess, then compute γ and δ with the guessed subkey byte. Similar to step 3.a, discard the disqualified ciphertext pairs. About 2<sup>m-107</sup> ciphertext pairs are expected to remain after these tests.
  - (c) Guess  $K_{18,0}$ , do similarly to step 3.a. But a probability of about  $\frac{1}{2}$  pairs are already discarded because of the selection of the set  $\Lambda$ , hence about  $2^{m-107-7} = 2^{m-114}$  ciphertext pairs are expected to remain after this test.
  - (d) For each of the other three bytes of  $K_{18}$ , make a guess and do similarly to step 3.a. About  $2^{m-135}$  ciphertext pairs are expected to remain after these tests. If m = 116, thus the expectation of the remaining ciphertext pairs for a wrong key guess is about  $2^{116-135} = 2^{-19}$ , and the expectation of the remaining ciphertext pairs for the right key guess is about  $2^{116-114} = 2^2$ . Hence, if the number of the remaining ciphertext pairs is larger than 2, then keep the guess of  $K_{21}, K_{20}, K_{19}, K_{18}$  as the candidates of the right subkeys.
  - (e) For the survived candidates of the right subkeys in step 3.c, compute the seed key K using the key schedule of SMS4. Then, do an exhaustive search to find the unique right seed key.

The data complexity of the above attack is about  $2^{117}$  chosen plaintexts, and the memory complexity is about  $2^{110}$  bytes.

By the Poisson distribution,  $X \sim Poi(\lambda = 2^{-19})$ ,  $Prob[X > 2] \approx 2^{-59.5}$ , thus the expectation of subkey candidates suggested in step 3.e is about  $2^{128-59.5} = 2^{68.5}$ . By the Poisson distribution,  $X \sim Poi(\lambda = 2^2)$ ,  $Prob[X > 2] \approx 76.2\%$ , so the success rate of the above attack is about 76.2%.

The time complexity is dominated by step 3.a - 3.d. In step 3.a,  $2^{105}$  ciphertext pairs are treated with  $2^8$  subkey candidates for  $K_{21,0}$ , so the time complexity is about  $2^{105+8+1} \times \frac{1}{22} \times \frac{1}{4} \approx 2^{107.54}$  22-round SMS4 encryptions. Similarly, the time complexity of step 3.b is about  $2^{107.54} \times 11$ , the time complexity of step 3.c is about  $2^{107.54}$ , and the time complexity of step 3.d is about

Source	Number of Rounds	Data Complexity	Time Complexity	Attack Type
Ref.[10]	13	$2^{16}$ CP	$2^{114}$	Integral
$Ref.[11]^{*}$	14	$2^{121.82}$ CP	$2^{116.66}$	Rectangle
	16	$2^{105}$ CP	$2^{107}$	Imp.Diff
Ref.[4]	14	$2^{106.89}$ CP	$2^{87.97}$	Rectangle
	16	$2^{117.06}\mathrm{CP}$	$2^{95.09}$	Imp.Diff
Ref.[14]	16	$2^{125}$ CP	$2^{116}$	Rectangle
	21	$2^{118}$ CP	$2^{126.6}$	Differential
$\operatorname{Ref.}[5]$	22	$2^{118.4}$ KP	$2^{117}$	Linear
Ref.[9]	18	$2^{120}$ ACPS	$2^{116.83}$	Boomerang
	18	$2^{124}$ CP	$2^{128}$	Rectangle
	22	$2^{117}$ KP	$2^{109.86}Enc. + 2^{120.39}A.O$	. Linear
	22	$2^{118}$ CP	$2^{123}$	Differential
This paper	22	$2^{117}$ KP	$2^{112.3}$	Differential

Table 2. Comparison of Some Previous Attacks with Our New Attack

CP – Chosen plaintext, KP – Known plaintext.

ACPS - Adaptive Chosen Plaintext and Ciphertext.

A.O. – Arithmetic operation.

Time complexity is measured in encryption units (Enc.).

 $(2^{2+112+1} + 2^{-5+120+1} + 2^{-12+128+1}) \times \frac{1}{22} \times \frac{1}{4}$ . Hence, the total time complexity is about  $2^{112.3}$  22-round SMS4 encryptions.

We summarize our attack along with the previously known ones against SMS4 in Table 2. Notice that our attack is based on the work of [14] and [9]. The key point is that we exploit a single differential characteristic with a much higher probability, which also makes the attack more effective and more clear. Whereas a class of 7905 differential characteristics are exploited in [14] and [9], with a much lower average probability.

## 5 A Remark on Constructing Differential Characteristics of SMS4

Zhang etc. [14] present the following 5-round iterative differential characteristics

$$(\alpha, \alpha, \alpha, 0) \xrightarrow{5 Rounds} (\alpha, \alpha, \alpha, 0)$$

In fact, let  $\Delta = \{(\chi_0, \chi_1, \chi_2, \chi_3) | \chi_i = 0 \text{ or } \chi_i = \alpha \text{ for each } 0 \leq i \leq 3\}$ . For each of the non-zero elements  $(\chi_0, \chi_1, \chi_2, \chi_3) \in \Delta$ , we can get the following 5round iterative differential characteristic by restricting the output difference of T function is  $\alpha$  when its input difference is  $\alpha$ :

$$(\chi_0,\chi_1,\chi_2,\chi_3) \xrightarrow{5 Rounds} (\chi_0,\chi_1,\chi_2,\chi_3)$$

There are 15 non-zero elements in  $\Delta$ , thus 15 different 5-round iterative differential characteristics. Through a calculation, we find there are 10 differentials with a probability of  $(Prob_T (\alpha \rightarrow \alpha))^2$  each, the other 5 with a lower probability.

For the 10 differentials, there are two best ones when we consider iterating them for more rounds, corresponding respectively to  $(\chi_0, \chi_1, \chi_2, \chi_3) = (\alpha, \alpha, \alpha, 0)$ and  $(\chi_0, \chi_1, \chi_2, \chi_3) = (0, \alpha, \alpha, \alpha)$ . This is because the first 3-round differential holds with probability 1 when  $(\chi_0, \chi_1, \chi_2, \chi_3) = (\alpha, \alpha, \alpha, 0)$ , and the last 3-round differential holds with probability 1 when  $(\chi_0, \chi_1, \chi_2, \chi_3) = (0, \alpha, \alpha, \alpha)$ , thus we can add 3 rounds for free (one in the end, the other in the beginning).

Moreover, it is easy to show that there exists no 4-round differential characteristic with probability 1.

We have tried to construct other differential characteristics of SMS4, but no more than 18 rounds, and the 18-round differentials presented in section 4.1 are the best.

In [8], the authors present the upper bounds of the maximum differential and linear characteristic probabilities of Camellia, which would inspire a similar study on SMS4. But the overall structure of SMS4 makes the study so complex, the workload is so huge, we can't achieve this goal at present.

## 6 Summary

In the first part, we present 3 observations on the design of the linear transformation L of SMS4. Firstly, we prove that L is optimal between the branch number and the number of rotations, furthermore we show there are essentially only two transformations whose branch number reaches the maximum 5 if using only 5 rotations and 4 XORs. Secondly, we present a sufficient and necessary condition for such class functions to be bijective. Thirdly, we show that L has the same diffusion effect as the MixColumn transformation of AES, by comparing their distributions of input-output patterns. These observations reveal the design rationales of L and such class functions to a great extent, and also helpful in analyzing the security of ciphers using such functions.

In the second part, firstly we show that there are 12 18-round differential characteristics for SMS4, each with a higher probability of  $2^{-114}$ , which are the best differential characteristics for SMS4 so far. Then, we present a simple differential attack on 22-round SMS4. Compared with the attack complexity of the 22-round attacks in [5,9], ours is better, hence our attack is the best known one on SMS4 in the literature so far. Furthermore, we make a remark on the construction of differential characteristics presented in section 4.1 ( the amount is 12) with a probability of  $2^{-114}$  each maybe the best effective ones. Finally, we stress that the full 32-round SMS4 provides a sufficient safety margin against differential cryptanalysis, since the best attack can only reach 22 rounds so far.

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# On the Correctness of an Approach against Side-Channel Attacks

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Abstract. Side-channel attacks are a very powerful cryptanalytic technique. Li and Gu [ProvSec'07] proposed an approach against side-channel attacks, which states that a symmetric encryption scheme is IND-secure in side-channel model, if it is IND-secure in black-box model and there is no adversary who can recover the whole key of the scheme computationally in side-channel model, i.e. WKR-SCA  $\land$  IND  $\rightarrow$  IND-SCA. Our researches show that it is not the case. We analyze notions of security against key recovery attacks and security against distinguishing attacks, and then construct a scheme which is WKR-SCA-secure and IND-secure, but not IND-SCA-secure in the same side-channel environment. Furthermore, even if the scheme is secure again partial key recovery attacks in side-channel model, this approach still does not hold true.

**Keywords:** Provable security, Side-channel attack, Symmetric encryption.

## 1 Introduction

In traditional cryptanalysis, an adversary has only black-box access to cryptographic algorithms, i.e. the adversary can query the keyed cryptographic algorithm with input of its choice and get the corresponding output, but it can not get any other information of what's going on during the computation of the output. Unfortunately, in physical implementations, this kind of information sometimes can be easily obtained, such as timing information **S**, power consumption **T**, electromagnetic leakage **4**, etc. We call the attacks based on this leaked information side-channel attacks. The researches in the last decade show that side-channel attacks are a very powerful cryptanalytic technique. The security of some cryptographic algorithms may collapse suddenly given some tiny side-channel information, even though they are very secure in traditional cryptanalysis.

The black-box model illustrates a theoretical world in which we focus on design in the level of algorithm, on the other hand the side-channel model illustrates a practical world in which we have to face the leaked information in

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implementations. We have millions of experiences on designing secure cryptographic algorithms in black-box model. But how to guarantee the security of the algorithms in side-channel model?

Let's first look at some notions of security. The minimal security requirement is the privacy of the secret key. Key recovery attacks are often used to analyze the security of cryptographic primitives such as block cipher both in blackbox model [2]3] and side-channel model [6]. Recently the researches on how to establish a unified framework to evaluate the implementation security also focus on key recovery attacks [10][11][12]. If no adversary can recover the whole key computationally, we say it is WKR-secure (in black-box mode) or WKR-SCAsecure (in side-channel model)[]. If no adversary can recover any part of key computationally, we say it is PKR-secure (in black-box mode) or PKR-SCAsecure (in side-channel model)].

But as to a concrete cryptographic scheme, we need a corresponding security notion. For example, an encryption scheme requires the privacy of the plaintext, i.e. any adversary can not learn any information of the plaintext (except the length) computationally given a challenge ciphertext. This notion was firstly defined by Goldwasser and Micali as semantic security **5**, which is equivalent to indistinguishability of the ciphertexts **5**[1]. If no information about the plaintext (except the length) is revealed computationally by the ciphertext, we say the scheme is IND-secure (in black-box mode) or IND-SCA-secure (in side-channel model).

Li and Gu proposed an approach against side-channel attacks in ProvSec 2007 [9], which states that if a symmetric encryption scheme is both WKR-SCA-secure and IND-secure, then it is IND-SCA-secure. In other words, in order to guarantee the practically security of the scheme, we only need to guarantee the theoretically security of the scheme and there is no adversary who can recover the whole key of the scheme computationally in the practical world.

Unfortunately, it is not the case.

Our results are based on the following two basic observations:

- The notion of WKR (WKR-SCA) is much weaker than the notion of IND (IND-SCA).
- The security of the scheme in side-channel model is closely related to the leaked information.

**Our Contributions.** We first analyze the relations among PKR, WKR and IND both in black-box model and side-channel model. Please see Figure 11 and Figure 22, which show that WKR is the weakest notion, and PKR and IND are incomparable. We give proofs for all the implications of the notions and construct concrete examples for all the separations of the notions.

We then explore the security of notion combination. Our results show that WKR-SCA  $\wedge$  IND does not imply IND-SCA. Given a symmetric encryption scheme which is IND-secure, we can construct a scheme which is both

<sup>&</sup>lt;sup>1</sup> The notion of WKR is the same as the notion of UB in [9], which means "unbreakability of the key".

IND-secure and WKR-SCA-secure, but not IND-SCA-secure in the same sidechannel environment.

Furthermore, we show that even the scheme is PKR-SCA-secure and INDsecure, this approach still does not hold. Based on a symmetric encryption scheme which is both PKR-secure and IND-secure, we construct a new scheme which is both PKR-SCA-secure and IND-secure, but not IND-SCA-secure in the same side-channel environment.

## 2 Preliminaries

**Notations.** We write  $s \stackrel{\$}{\leftarrow} S$  to denote choosing a random element *s* from a set *S* by uniform distribution. An *adversary* is an (randomized) algorithm with access to one or more oracles which are written as superscripts. We write the adversary **A** with oracle  $\mathcal{O}$  outputing a bit *b* as  $\mathbf{A}^{\mathcal{O}} \Rightarrow b$ .  $\mathbf{Adv}_{SSS}^{GGG}(\mathbf{A})$  denotes the advantage of **A** attacking a scheme "SSS" with a goal of "GGG".

 $A \rightarrow B$  means any scheme meeting notion A also meets notion B and B  $\not \rightarrow A$  means there exists a scheme meeting notion B but do not meets notion A.

Symmetric Encryption Scheme. A symmetric encryption scheme  $\mathcal{SE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  consists of three algorithms. The randomized key generation algorithm  $\mathcal{K}$  generates a key K, denoted as  $K \leftarrow \mathcal{K}$ . The randomized or stateful encryption algorithm  $\mathcal{E}$  takes the key K and a plaintext M to return a ciphertext C, denoted as  $C \leftarrow \mathcal{E}_K(M)$ . The deterministic and sateless decryption algorithm  $\mathcal{D}$  takes the key K and a string C to return the corresponding plaintext M or the symbol  $\bot$ , denoted as  $x \leftarrow \mathcal{D}$ , where  $x \in \{0, 1\}^* \cup \{\bot\}$ . We require that  $\mathcal{D}_K(\mathcal{E}_K(M)) = M$  for any plaintext M.

In this paper, we focus on the security of symmetric encryption scheme under chosen plaintext attacks.

Side-Channel Leakage Function. In side-channel attacks, the adversary not only can query the encryption oracle  $\mathcal{E}_K$  and get the corresponding ciphertext, but also can get some side-channel information during the computation of the ciphertext. We notice that the side-channel information is relevant to the key Kand the queried plaintext M, so we treat it as a function L(K, M) and call it a leakage function. We define a new oracle  $\mathcal{E}_K^+(M) = (\mathcal{E}_K(M), L(K, M))$  which returns both the ciphertext and the leaked information. Therefore in side-channel attacks, the adversary has actually oracle access to  $\mathcal{E}_K^+(\cdot)$ .

Security against Key Recovery Attacks. Key recovery attacks aim to recover the whole or partial key of the cryptographic algorithm. We define two kinds of security of symmetric encryption scheme against key recovery attacks

<sup>&</sup>lt;sup>2</sup> In  $\square$ , the adversary has oracle access to  $\mathcal{E}_K(\cdot)$  and  $S_K^*(\cdot)$  in side-channel model, where the input to  $S_K^*(\cdot)$  is the side-channel information and the output of  $S_K^*(\cdot)$ is a key  $K' \in \{0,1\}^* \cup \{\bot\}$ . This formalization conceals where the side-channel information comes from, and brings about confusions in subsequent discussions.

both in black-box model and in side-channel model. In whole key recovery attacks, the adversary tries to recovery the full key.

**Definition 1 (WKR and WKR-SCA).** Let  $S\mathcal{E} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  be a symmetric encryption scheme. Consider following two advantages:

$$\mathbf{Adv}_{\mathcal{S}\mathcal{E}}^{\mathrm{WKR}}(\mathbf{A}) = \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}(\cdot)} \Rightarrow K' : K = K'],$$
$$\mathbf{Adv}_{\mathcal{S}\mathcal{E}}^{\mathrm{WKR-SCA}}(\mathbf{A}) = \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}^{+}(\cdot)} \Rightarrow K' : K = K'].$$

We say that  $S\mathcal{E}$  is secure against whole key recovery attacks in black-box model (in side-channel model), or WKR-secure (WKR-SCA-secure), if the advantage  $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{WKR}}(\mathbf{A})$  ( $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{WKR}}(\mathbf{A})$ ) is negligible for any adversary  $\mathbf{A}$  with feasible resources.

In partial key recovery attacks, the adversary tries to recover any information of the key. We adopt a simulator-based definition, in which we use a function f(K)to represent the targeted information of the key and define the security against partial key recovery attacks as whatever an adversary **A** with oracle  $\mathcal{E}_K$  ( $\mathcal{E}_K^+$ ) can do, a simulator **S** without oracle also can do.

**Definition 2 (PKR and PKR-SCA).** Let  $S\mathcal{E} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  be a symmetric encryption scheme. Consider following two advantages:

$$\begin{aligned} \mathbf{Adv}_{\mathcal{S}\mathcal{E}}^{\mathrm{PKR}}(\mathbf{A}, \mathbf{S}) \\ =& \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}(\cdot)} \Rightarrow b : f(K) = b] - \Pr[K \leftarrow \mathcal{K}, \mathbf{S} \Rightarrow b : f(K) = b], \\ & \mathbf{Adv}_{\mathcal{S}\mathcal{E}}^{\mathrm{PKR-SCA}}(\mathbf{A}, \mathbf{S}) \\ =& \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}^{+}(\cdot)} \Rightarrow b : f(K) = b] - \Pr[K \leftarrow \mathcal{K}, \mathbf{S} \Rightarrow b : f(K) = b], \end{aligned}$$

where  $f : \{0,1\}^* \to \{0,1\}^*$  is a function. We say that  $S\mathcal{E}$  is secure against partial key recovery attacks in black-box model (in side-channel model), or PKR-secure (PKR-SCA-secure), if for any function f and any adversary  $\mathbf{A}$  with feasible resources, there exists an algorithm  $\mathbf{S}$  (we often call it a simulator) with feasible resources, such that the advantage  $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{PKR}}(\mathbf{A}, \mathbf{S})$  ( $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{PKR-SCA}}(\mathbf{A}, \mathbf{S})$ ) is negligible.

**Security against Distinguishing Attacks.** We adopt the security definition of Real-Or-Random in **1** for symmetric encryptions, which define the security as indistinguishability of ciphertexts of required plaintexts and ciphertexts of random strings.

**Definition 3 (IND and IND-SCA).** Let  $S\mathcal{E} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  be a symmetric encryption scheme. Consider following two advantages:

$$\mathbf{Adv}_{\mathcal{S}\mathcal{E}}^{\mathrm{IND}}(\mathbf{A}) = \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}(\cdot)} \Rightarrow 1] - \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}(\$(\cdot))} \Rightarrow 1],$$
$$\mathbf{Adv}_{\mathcal{S}\mathcal{E}}^{\mathrm{IND}}(\mathbf{A}) = \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}^{+}(\cdot)} \Rightarrow 1] - \Pr[K \leftarrow \mathcal{K}, \mathbf{A}^{\mathcal{E}_{K}^{+}(\$(\cdot))} \Rightarrow 1],$$

where  $(\cdot)$  returns a random string with the same length of the input. We say that  $S\mathcal{E}$  is secure against distinguishing attacks in black-box model (in sidechannel model), or IND-secure (IND-SCA-secure), if the advantage  $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{IND}}(\mathbf{A})$ ( $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{IND-SCA}}(\mathbf{A})$ ) is negligible for any adversary  $\mathbf{A}$  with feasible resources.

## 3 KR vs. IND in Block-Box Model

In this section, we elaborate the implications or separations of the notions summarized in Figure  $\blacksquare$ 

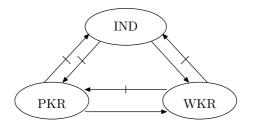


Fig. 1. Relations among PKR, WKR and IND

**Theorem 1 (PKR**  $\rightarrow$  **WKR).** Let SE be an encryption scheme. If SE is PKR-secure, then it is WKR-secure as well.

*Proof.* If **A** is an adversary against WKR-security, we construct an adversary **B** against PKR-security: run **A** and get K', then return K'. We set f = ID where ID is the identical transformation, then for any simulator **S**, we have  $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{PKR}}(\mathbf{B}, \mathbf{S}) \geq \mathbf{Adv}_{S\mathcal{E}}^{\mathrm{PKR}}(\mathbf{A}) - 1/2^k$ , where k is the length of the key.  $\Box$ 

**Theorem 2 (IND**  $\rightarrow$  **WKR).** Let SE be a symmetric encryption scheme. If SE is IND-secure, then it is WKR-secure as well.

*Proof.* If **A** is an adversary against WKR-security, we construct an adversary **B** against IND-security: run **A** and get K', then query  $M \in \{0,1\}^n$  and get C, if  $\mathcal{E}_{K'}(M) = C$ , then return 1, else return 0. We have that  $\mathbf{Adv}_{\mathcal{SE}}^{\mathrm{IND}}(\mathbf{B}) \geq \mathbf{Adv}_{\mathcal{SE}}^{\mathrm{WKR}}(\mathbf{A}) - 1/2^n$ .

**Proposition 1 (PKR**  $\rightarrow$  **IND).** There exists a symmetric encryption scheme which is PKR-secure, but not PKR-secure.

*Proof.* We construct a symmetric encryption scheme  $\mathcal{SE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ , where  $\mathcal{E}_K(M) = M$ , i.e. the encryption algorithm is an identical transformation and has nothing to do with the key K. Hence no matter how the adversary queries the encryption oracle, no information about K is obtained. More specifically, for any function f and adversary  $\mathbf{A}$  against PKR-security, the simulator  $\mathbf{S}$  just runs

 $\mathbf{A}^{ID(\cdot)}$  and returns whatever  $\mathbf{A}$  returns, where  $ID(\cdot)$  is the identical transformation. Then  $\mathbf{Adv}_{S\mathcal{E}}^{\mathrm{PKR}}(\mathbf{A}, \mathbf{S}) = 0.$ 

Furthermore, the identical transformation reveals all the information about the plaintext. More specifically, the adversary **B** just queries  $M \in \{0, 1\}^n$ , if the answer is M then return 1, else return 0. We have  $\mathbf{Adv}_{\mathcal{SE}}^{\text{IND}}(\mathbf{B}) = 1 - 1/2^n$ .  $\Box$ 

**Proposition 2 (IND**  $\rightarrow$  **PKR).** Given a symmetric encryption scheme SE which is IND-secure, we can construct a symmetric encryption scheme SE' which is also IND-secure, but not PKR-secure.

*Proof.* Suppose  $S\mathcal{E} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is IND-secure. We construct  $S\mathcal{E}' = (\mathcal{K}', \mathcal{E}', \mathcal{D}')$  as follows:

	SE	SE'
Key generation	$K \leftarrow \mathcal{K}$	$K_1 \leftarrow \mathcal{K}, K_2 \xleftarrow{\$} \{0, 1\}^n$
Encryption	$\mathcal{E}_K(M)$	$\mathcal{E'}_{K_1K_2}(M) = \mathcal{E}_{K_1}(M)    K_2$
Decryption	$\mathcal{D}_K(C)$	$\mathcal{D}'_{K_1K_2}(C) = \mathcal{D}_{K_1}(C)$

The new encryption scheme  $\mathcal{SE}'$  generates an extra key  $K_2 \in \{0,1\}^n$ , but reveals it in the ciphertext. If  $\mathcal{SE}$  is IND-secure, then  $\mathcal{SE}'$  is also IND-secure. More specifically, for any adversary **A** attacking  $\mathcal{SE}'$ , we construct adversary  $\mathbf{B}^{\mathcal{O}}$  attacking  $\mathcal{SE}: K_2 \stackrel{\$}{\leftarrow} \{0,1\}^n$ , run **A**, when **A** queries M, answer it with  $\mathcal{O}(M)||K_2$ , and return whatever **A** returns. We have  $\mathbf{Adv}_{\mathcal{SE}}^{\mathrm{IND}}(\mathbf{B}) = \mathbf{Adv}_{\mathcal{SE}'}^{\mathrm{IND}}(\mathbf{A})$ .

Furthermore, the ciphertext reveals the partial key of the  $\mathcal{SE}'$ , so it is not PKR-secure. More specifically, given the function  $f(K_1K_2) = K_2$  and the adversary **A** which returns  $K_2$  after arbitrary one query, any simulator **S** has no information about  $K_2$ , therefore  $\mathbf{Adv}_{\mathcal{SE}'}^{\mathrm{PKR}}(\mathbf{A}, \mathbf{S}) \geq 1 - 1/2^n$ .

Corollary 1 (WKR  $\rightarrow$  IND). There exists a symmetric encryption scheme  $S\mathcal{E}$  which is WKR-secure, but not IND-secure.

*Proof.* We have  $PKR \rightarrow WKR$  by Theorem **1** If  $WKR \rightarrow IND$ , then  $PKR \rightarrow IND$ . That contradicts Proposition **1**.

**Corollary 2 (WKR**  $\rightarrow$  **PKR).** There exists a symmetric encryption scheme  $S\mathcal{E}$  which is WKR-secure, but not PKR-secure.

*Proof.* We have IND  $\rightarrow$  WKR by Theorem 2 If WKR  $\rightarrow$  PKR, then IND  $\rightarrow$  PKR. That contradicts Proposition 2  $\Box$ 

### 4 KR vs. IND in Side-Channel Model

Attacks in black-box model can be regarded as special attacks in side-channel model when the leakage function returns nothing. Therefore the separations of the notions still hold in side-channel model. It is easy to verify the implications of the notions also hold in side-channel model. We summarize these results in Figure 2

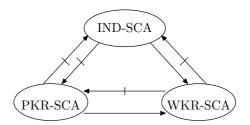


Fig. 2. Relations among PKR-SCA, WKR-SCA and IND-SCA

## 5 Failing Combination of Notions

From the above section, we know that the notion of WKR-SCA is much weaker than the notion of IND-SCA. Even if we combine the notion of WKR-SCA with that of IND, we can not get the notion of IND-SCA. Therefore we actually overturn the main result in **9**.

**Proposition 3 (WKR-SCA**  $\land$  **IND**  $\rightarrow$  **IND-SCA).** Given a symmetric encryption scheme  $S\mathcal{E}$  which is IND-secure, we can construct a symmetric encryption scheme  $S\mathcal{E}'$  which is both IND-secure and WKR-SCA-secure for some leakage function, but not IND-SCA-secure for the same leakage function.

*Proof.* Suppose  $S\mathcal{E} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  which is IND-secure. We construct  $S\mathcal{E}' = (\mathcal{K}', \mathcal{E}', \mathcal{D}')$  as follows:

	SE	SE'
Key generation	$K \leftarrow \mathcal{K}$	$K_1 \leftarrow \mathcal{K}, K_2 \xleftarrow{\$} \{0, 1\}^n$
Encryption	$\mathcal{E}_K(M)$	$\mathcal{E}'_{K_1K_2}(M) = \mathcal{E}_{K_1}(M)$
Decryption	$\mathcal{D}_K(C)$	$\mathcal{D}'_{K_1K_2}(C) = \mathcal{D}_{K_1}(C)$

The new encryption scheme  $\mathcal{SE}'$  generates an extra  $K_2 \stackrel{\$}{\leftarrow} \{0,1\}^n$ , but does not used in the encryption. The encryption algorithms of  $\mathcal{SE}$  and  $\mathcal{SE}'$  are the same, so  $\mathcal{SE}'$  is also IND-secure.

Now we consider the security of  $S\mathcal{E}'$  in side-channel model, given that the leakage function is  $L(K_1K_2, M) = K_1$ .

The encryption algorithm of  $\mathcal{SE}'$  does not use the key  $K_2$ , which is also not revealed by the leakage function, so it is WKR-SCA-secure. More specifically, for any adversary  $\mathbf{A}$ ,  $\mathbf{Adv}_{\mathcal{SE}'}^{\mathrm{WKR-SCA}}(\mathbf{A}) \leq 1/2^n$ .

Furthermore, the key  $K_1$  used in the encryption algorithm is revealed by the leakage function, so  $\mathcal{SE}'$  is not IND-SCA-secure. More specifically, the adversary **B** makes arbitrary query and gets  $K_1$  through the leakage function, and then queries  $M \in \{0,1\}^n$ , gets C. If  $C = \mathcal{E}'_{K_1}(M)$ , then return 1, else return 0. We have  $\mathbf{Adv}_{\mathcal{SE}}^{\mathrm{IND-SCA}}(\mathbf{A}) = 1 - 1/2^n$ .

Moreover, we show that even the scheme is PKR-SCA-secure and IND-secure, the approach in [9] still does not hold.

**Proposition 4 (PKR-SCA**  $\land$  **IND**  $\rightarrow$  **IND-SCA).** Given a symmetric encryption scheme  $S\mathcal{E}$  which is both PKR-secure and IND-secure, we can construct a symmetric encryption scheme  $S\mathcal{E}'$  which is both PKR-SCA-secure and IND-secure for some leakage function, but not IND-SCA-secure for the same leakage function.

*Proof.* Suppose  $\mathcal{SE} = (\mathcal{K}, \mathcal{E}, \mathcal{D})$  is both PKR-secure and IND-secure.

Now we consider the security of  $S\mathcal{E}$  in side-channel model, given that the leakage function is L(K, M) = M.

The leakage function does not reveal any information about the key K, so  $\mathcal{SE}$  is PKR-SCA-secure.

The leakage function reveals the queried plaintext, so  $\mathcal{SE}$  is not IND-SCAsecure. More specifically, the adversary **A** just queries  $M \in \{0,1\}^n$  and gets (C, M'). If M = M' then return 1, else return 0. We have  $\mathbf{Adv}_{\mathcal{SE}}^{\mathrm{IND-SCA}}(\mathbf{A}) = 1 - 1/2^n$ .

## 6 Conclusion

This paper gives implications or separations among the notions of IND, PKR and WKR both in black-box model and side-channel model, which show that the notion of WKR is much weaker than the notion of IND. Then we construct a concrete scheme to show that the approach against side-channel attacks proposed in [9] is flawed. The security against key recovery attacks does not help much for a practically cryptographic requirement. We note that the results are not limited to the security of symmetric encryption scheme, as to the security of the other cryptographic algorithms, such as block ciphers or authenticated encryption schemes, the corresponding results still hold true.

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## **Ranking Attack Graphs with Graph Neural Networks**

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**Abstract.** Network security analysis based on attack graphs has been applied extensively in recent years. The ranking of nodes in an attack graph is an important step towards analyzing network security. This paper proposes an alternative attack graph ranking scheme based on a recent approach to machine learning in a structured graph domain, namely, Graph Neural Networks (GNNs). Evidence is presented in this paper that the GNN is suitable for the task of ranking attack graphs by learning a ranking function from examples and generalizes the function to unseen possibly noisy data, thus showing that the GNN provides an effective alternative ranking method for attack graphs.

## 1 Introduction

A large computer system is built upon multiple platforms, runs different software packages and has complex connections to other systems. Despite the best efforts by system designers and architects, there will exist vulnerabilities resulting from bugs or design flaws, allowing an adversary (attacker) to gain a level of access to systems or information not desired by the system owners. The act of taking advantages of an individual vulnerability is referred to as an "atomic attack" or an "exploit". A vulnerability is often exploited by feeding a specially crafted chunk of data or sequence of commands to a piece of defective software resulting in unintended or unanticipated behaviour of the system such as gaining control of a computer system or allowing privilege escalation. Although an exploit may have only insignificant impact on the system by itself, an adversary may be able to construct a system intrusion that combines several atomic attacks, each taking the adversary from one system state to another, until attaining some state of interest, such as access to customer credit card data. To evaluate the security level of a large computer system, an administrator must not only take into account the effects of exploiting each individual vulnerability, but also considers global intrusion scenario where an adversary may combine several exploits possibly in a multi-stage attack to compromise the system.

In order to provide a global view on multi-stage attacks by combining several individual vulnerabilities, Sheyner *et al.* proposed the use of attack graphs [11]. An attack graph is a graph that consists of a set of nodes and edges, where each node represents a reachable system *state* and each edge represents an *atomic attack* that takes the system from one state to another. However, as the size and complexity of an attack graph usually greatly exceeds the human ability to visualize, understand and analyze, a scheme is required to identify important portions of attack graphs (which may be vulnerable to external attacks). Recently, Mehta *et al.* [15] proposed to rank states of an attack graph by their importance using PageRank [10], an algorithm used by the Google web search engine. This is possible since the class of attack graphs is a subset of a web graph with nodes representing web pages, and edges representing hyperlinks among the web pages. The PageRank algorithm would rank the importance of web pages based on its hyperlink structure only [10]. In other words, Mehta *et al.* demonstrated that the PageRank algorithm when applied to attack graphs allow the color coding of nodes in the graphs so as to highlight its important portions.

Questions such as "what happens if a particular vulnerability is patched" or "what happens if a firewall rule is altered, added or removed" are often raised in an organization before any planned network structural change takes place. Being able to answer such questions (without actually carrying the reconfiguration) is necessary for the network administrator to evaluate the results of the planned network changes. Clues to answering such questions can be obtained from analyzing the attack graph resulted from situation changes in the modeled system. However, the PageRank algorithm [10] is not of linear time complexity and thus it may be difficult to rank many attack graphs, each resulting from one of many possible changes in the modeled system.

In this paper, we consider an alternative scheme to estimate ranks of attack graphs based on the Graph Neural Network (GNN) [3], a new neural network model capable of processing graph structures. The applications of GNN have been successful in a number of areas where objects are naturally represented by a graph structure, the most notable example being the ranking of web pages [13]. In [13] it is shown that the GNN can be trained to simulate the PageRank function by using a relatively small set of training samples. Once trained, the GNN can rank large sets of web pages efficiently due to its ability to generalize over unseen data. In other words, the GNN learns the ranking function (whether explicit, as in the case of PageRank, or implicit) of web pages from a small number of training samples, and can then generalize over unseen examples, possibly contaminated by noise.

Because of the above stated properties, GNN may be considered suitable for the task of ranking attack graphs. Moreover, [12] provides a universal approximation theorem that the GNN is capable of learning to approximate any "reasonable" problem, reasonable in the sense that the problem is not a pathological problem, to any arbitrary degree of desired precision. The universal approximation theorem further justifies the suitability of ranking attach graphs using GNN. However on the other hand, some properties of attack graphs differ fundamentally from those of the web graph. The web graph is a single very large graph with a recursive link structure (web pages referring to themselves) whereas attack graphs can be numerous, are relatively small, and may be of strictly tree structured. This together with the observation that PageRank is a link based algorithm whereas the GNN is a node based approach in that it processes nodes more effectively than links, whether the GNN is suitable for the task of ranking attack graphs is unknown. One of the key questions that this paper wishes to answer is the suitability of

GNN for the purpose of ranking attack graphs. There were numerous prior approaches to the processing of attack graphs and similar poli-instantiation problems. Research in this area was particularly active in the 1980s and early 1990s (see for example [8]). Most of these work were of an automated proof nature, desiring to show if an attack graph is vulnerable or not. Such automated proof concept can be expressed in terms of graph structured data [5]. However, any attempts to use a machine learning approaches required the pre-processing of the graph structured data to "squash" the graph structure into a vectorial form as most popular machine learning approaches, e.g. multilayer perceptrons, self organizing maps 6, take vectorial inputs rather than graph structured inputs. Such pre-processing step can result in the loss of contextual information between atomic components of the data. The GNN is a relatively recent development of a supervised machine learning approach which allows the processing of generic graphs without first requiring the "squashing" of graph structured data into vectorial form. It is shown in [12] that the GNN algorithm is guaranteed to converge, and that it can model any set of graphs to any arbitrary degree of precision. Hence, to the best of our knowledge, this paper is the first reported work on ranking attack graphs (without pre-processing) using a machine learning approach.

Training a GNN can take a considerable period of time. It is shown in [3] that the computational burden of the training procedure can be quadratic. However, once trained, a GNN is able to produce output in linear time. This is an improvement over  $O(N\log \frac{1}{\epsilon})$ , the computational complexity of PageRank, where N is the number of links and  $\epsilon$  is the expected precision [10]. Moreover, GNN is able to learn the ranking scheme from a small number of training examples and then generalize to other unseen attack graphs. For reasons stated above, using GNN may be more suitable than the PageRank algorithm in the case where it requires ranking many attack graphs, each resulting from one of many possible changes in the modeled system.

The contributions of this paper include: (a) an alternative attack graph ranking method based on a supervised machine learning approach, viz., GNN is proposed, and (b) both the PageRank-based and the GNN-based ranking approaches are implemented and their results are compared. This provides an insight into the suitability of applying GNN to rank attack graphs.

The paper is organized as follows. Section 2 provides a brief review of relevant background. The proposed alternative ranking scheme is presented in Section 3 Section 4 provides the experimental results to show the effectiveness of the proposed scheme, and Section 5 concludes the paper.

### 2 Background

To evaluate the security condition of a computer system built upon multiple platforms, runs different software packages and has complex connections to other systems, a network administrator needs to consider not only the damages that can be done by exploiting individual vulnerabilities on the system, but also investigates the global effect on what an intruder can achieve by combining several vulnerabilities possibly in multi-stages which by themselves only have insignificant impact on the system. This requires an appropriate modeling of the system that takes into account information such as vulnerabilities and connectivity of components in the system, and is able to model a global intrusion scenario where an intruder combines several vulnerabilities to attack the system.

Sheyner et al. first formally defined the notion of attack graph to provide such a modeling of the system [11]. An *attack graph* describes security related attributes of the attacker and the modeled system using graph representations. States of a system such as the attacker's privilege level on individual system components are encapsulated in graph nodes, and actions taken by the attacker which lead to a change in the state of the system are represented by transition between nodes, *i.e.* edges of the graph. The root node of an attack graph denotes the starting state of the system where the attacker has not been able to compromise the system but is only looking for an entry point to the system. Consider an attack on a computer network as an example: a state may encapsulate attributes such as the privilege level of the attacker, the services being provided, access privileges of users, network connectivity and trust relations. The transitions correspond to actions taken by the attacker such as exploiting vulnerabilities to attain elevated privileges. The negation of an attacker's goal against the modeled system can be used as *security properties* that the system must satisfy in a secure state. An example of a security property in computer networks would be "the intruder cannot log in onto the central server". Nodes in an attack graph where the security properties are not satisfied are referred to as error states. A path from the root node to an error state indicates how an intruder exploits several vulnerabilities, each corresponding to one of the edges on the path, to finally reach a state where the system is considered compromised. An attack graph as a whole represents all the intrusion scenarios where an intruder can compromise the system by combining multiple individual vulnerabilities. Following these descriptions, an attack graph is formally defined as follows

Let AP be a set of atomic propositions. An Attack Graph is a finite automaton  $G = (S, \tau, s_0, S_s, l)$ , where S is a set of states in relation to a computer system,  $\tau \subseteq S \times S$  is the transition relation,  $s_0 \in S$  is the initial state,  $S_s \subseteq S$  is the set of error states in relation to the security properties specified for the system, and  $l : S \to 2^{AP}$  is a labeling of states with the set of propositions true in that state.

In other words, an attack graph is a tree structured graph where the root is the starting state, leaf nodes are final states denoting an intruder having reached a targeted system, and intermediate nodes are the intermediate states that a system can have during an attack. Typically, for mid- and large-sized organizations, an attack graph consists of several hundred or thousand nodes. Several attack graphs are formed to reflect the numerous various subnets or security configurations of a network.

Given a system and the related information such as vulnerabilities in the system, connectivity of components and security properties, model checking techniques can be used to generate attack graphs automatically [11].

#### 2.1 Multi-layer Perceptron Neural Networks

The application of neural networks [6] for pattern recognition and data classification has gained acceptance in the past. In this paper, *supervised* neural networks based on *layered* structures are considered. Multilayer perceptrons (MLPs) are perhaps the most well known form of supervised neural networks [6]. MLPs gained considerable acceptance among practitioners from the fact that a single-hidden-layer MLP has a universal

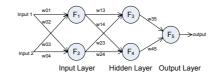


Fig. 1. An example of a multi-layered perceptron neural network, where  $F_1$ , and  $F_2$  form the input layer,  $F_3$  and  $F_4$  form the hidden layer, while  $F_5$  forms the output layer

approximation property [7], in that it can approximate a non-pathological nonlinear function to any arbitrary degree of precision.

The basic computation unit in a neural network is referred to as a *neuron* [6]. It generates the output value through a parameterized function called a *transfer function* f. An MLP can consist of several layers of neurons. The neuron layer that accepts external input is called the input layer. The dimension of the input layer is identical to the dimension of the data on which an MLP is trained. The layer that generates the output of the network is called the output layer. Its dimension is identical to the dimension of the target values. A layer between the input layer and the output layer is known as a hidden layer. There may be more than one hidden layer but here for simplicity we will only consider the case of a single hidden layer. Each layer is fully connected to the layer above or below. The input to each hidden layer neuron and the output layer neuron is the weighted sum from the previous layer, parameterized by the connecting weights, known as synaptic weights. The multiple layered structure described here has given MLP its name.

Figure II illustrates an example of a single-hidden-layered MLP. The transfer function associated with each of the neurons is  $F_j = f(\sum_{i=0}^n w_{ji}x_i)$ , where *n* is the total number of neurons in the previous layer,  $x_0 = 1$ ,  $x_i$  is the *i*-th input (either a sensory input or the output  $F_j$  from a previous layer), and  $w_{ji}$  is a real valued weight connecting neuron (or input) *i* with neuron *j*. The transfer function  $f(\cdot)$  often takes the shape of a hyperbolic tangent function. The MLP processes data in a *forward* fashion starting from the input layer towards the output layer.

Training an MLP [6] is performed through a *backward* phase known as error back propagation, starting from the output layer. It is known that an MLP can approximate a given unknown continuously differentiable function  $f(x_1, \ldots, x_n)$  by learning from a *training data set*  $\{x_{i1}, \ldots, x_{in}, t_i\}, 1 \le i \le r$  where  $t_i = g(x_{i1}, \ldots, x_{in})$ . It computes the output for each input  $X_i = \{x_{i1}, x_{i2} \ldots x_{in}\}$  and compares the output with the target  $t_i$ . Weights are then adjusted using the gradient descent algorithm based on the error towards the best approximation of the target  $t_i$ . The forward and the backward phases are repeated a number of times until a given prescribed accuracy (stopping criterion) is met *e.g.* the output is "sufficiently" close to the target.

Once the stopping criterion is met, the learning stage is finished. The MLP is then said to emulate the unknown nonlinear function  $g(\cdot, \ldots, \cdot)$ . Given any input  $X_i$  with

<sup>&</sup>lt;sup>1</sup> Fully connected layers are the most commonly studied and used architectures. There are some neural network architectures which are connected differently.

<sup>&</sup>lt;sup>2</sup> The function does not need to be continuously differentiable. However for our purpose in this paper, we will assume this simplified case.

unknown target  $t_i$ , it will be able to produce an output from g or it is said to be able to generalize from the training set to the unseen input  $X_i$ .

The application of MLP networks has been successful in many areas [3.13.6]. The inputs to the MLP would need to be in vectorial form, and as such, it cannot be applied to graph structured inputs or inputs which relate to one another.

### 2.2 Graph Neural Network

In several applications including attack graphs and web page ranking, objects and their relations can be represented by a graph structure such as a tree or a directed acyclic graph. Each node in the graph structure represents an object in the problem domain, and each edge represents relations between objects. For example, an attack graph node represents a particular system state, and an edge represents that the attacker can reach one state from the other. Graph Neural Network (GNN) is a recently proposed neural network model for graphical-based learning environments [3]. It is capable of learning topological dependence of information representing an object such as its rank relative to its neighbors, *i.e.* information representing neighboring objects and the relation between the object and its neighboring objects. The use of GNN for determining information on objects represented as nodes in a graph structure has been successfully shown in several applications [13].

The GNN is a complex model. A detailed description of the model and its associated training method cannot be described in this paper due to page limits, and hence, this section only summarizes the key ideas behind the approach as far as it is necessary to understand the method. The interested reader is referred to [3].

In order to encapsulate information on objects, a vector  $s_n \in \mathbb{R}^s$  referred to as *state*<sup>2</sup>, which represents information on the object denoted by the node, is attached to each node n. This information is dependent on the information of neighboring nodes and the edges connecting to its neighbors. For example, in the case of page rank determinations, the *state* of a node (page) is the rank of the page. Similarly, a vector  $e_{ij}$  may also be attached to the edge between node  $n_i$  and  $n_j$  representing attributes of the edge <sup>4</sup>. The state of a node n is determined by states of its neighbors and the labels of edges that connect it to one of its neighbors. Consider the GNN shown in Figure <sup>2</sup> the state of node  $n_1$  can be specified by Equation (1).

$$s_1 = f_w(s_2, s_3, s_5, e_{21}, e_{13}, e_{51}) \tag{1}$$

More generally, let S(n) denote the set of nodes connected to node n and  $f_{\mathbf{w}_1}$  be a function parameterized by  $\mathbf{w}_1$  that expresses the dependence of information representing a node on its neighborhood, then the state  $s_n$  is defined by Equation (2) where

<sup>&</sup>lt;sup>3</sup> For historical reasons this is called a "state", which carries slightly different meaning to the meaning of "state" in the attack graph literature. However, there should not be any risk of confusion as this concept of "state" in GNN is used in the training of the GNN model.

<sup>&</sup>lt;sup>4</sup> GNN can also accept labels on nodes; however this will not be used in this paper and is omitted here.

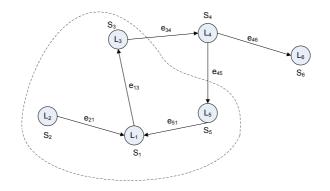


Fig. 2. The dependence of state  $s_1$  on neighborhood information

 $\mathbf{w}_1 \in \mathbb{R}$  is the parameter set of function f which is usually implemented as an MLP described in Section 2.1.

$$s_n = f_{\mathbf{w}_1}(s_u, e_{xy}), u \in \mathbb{S}(n), x = n \lor y = n$$

$$\tag{2}$$

The state  $s_n$  is then put through a parameterized *output network*  $g_{w_2}$ , which is usually also implemented as an MLP parameterized by  $w_2$ , to produce an output (*e.g.* a rank)  $o_n$ 

$$o_n = g_{\mathbf{w}_2}(s_n) \tag{3}$$

Equations (2) and (3) define a function  $\varphi_w(G, n) = o_n$  parameterized by weights  $\mathbf{w}_1$ , and  $\mathbf{w}_2$  that produces an output for the given graph G. The parameter sets  $\mathbf{w}_1$  and  $\mathbf{w}_2$  are adapted during the training stage such that the output  $\varphi$  is the best approximation of the data in the learning data set  $\mathcal{L} = \{(n_i, o_i) | 1 \le i \le q\}$ , where  $n_i$  is a node and  $t_i$  is the desired target for  $n_i$ .

#### 2.3 Existing Attack Graph Ranking Schemes

The PageRank algorithm [10] is used by Google to determine the relative importance of web pages in the World Wide Web. The PageRank is based on the behavioural model of a random surfer in a web graph. It assumes that a random surfer starts at a random page and keeps navigating by following hyperlinks, but eventually gets bored and starts on another random page. To capture the notion that a random surfer might get bored and restart from another random page, a *damping factor d* is introduced, where 0 < d < 1. The transition probability from a state is divided into two parts: d and 1 - d. The d mass is divided equally among the state's successors. Random transitions are added from that state to all other states with the residual probability 1 - d equally divided amongst them, modeling that if a random surfer arrives at a dangling page where no links are available, he is assumed to pick another page at random and continues surfing from that page. The computed rank of a page is the probability of a random surfer reaching that page, *i.e.*, consider a web graph with N pages linked to each other by hyperlinks, the PageRank  $x_p$  of page (node) p can be represented by Equation (4) where pa[p] denotes the set of nodes (pages) pointing to node (page) p

$$x_p = d \sum_{q \in pa[p]} \frac{x_q}{h_q} + \frac{1-d}{N}$$

$$\tag{4}$$

When stacking all the  $x_p$  into a vector **x**, and using iterative expressions, this can be represented as

$$\mathbf{x}(t) = dW\mathbf{x}(t-1) + \frac{1}{N}(1-d)\Pi_N$$
(5)

The computation of the PageRank can be considered a Markov process, as can be observed from Eq. (5). It has been shown hat after multiple iterations, Eq. (5) will converge to a stationary state where each  $x_p$  represents the probability of the random surfer reaching page p [15]. Time complexity required by PageRank is  $N\log \frac{1}{\epsilon}$  where N is the number of hyperlinks and  $\epsilon$  is the expected precision [10].

Based on the PageRank algorithm, Mehta et al. proposed a ranking scheme for attack graphs as follows [15]: given an attack graph  $G = (S, \tau, s_0, S_s, l)$ , the transition probability from each state is divided into d and 1 - d, modeling respectively an attacker is discovered and isolated from the system, or that the attacker remains undetected and proceeds to the next state with the intrusion. Similar to PageRank, the rank of a state in an attack graph is defined as the probability of the system being taken to that state by a sequence of exploits. The ranks of all states are computed using Equation (5). Breadth first search starting from the initial system state  $s_0$  is then performed for each atomic attack in  $\tau$  to construct the transition matrix W. The adjustment from PageRank, where a transition from each state pointing to all other states with probability 1 - d equally divided amongst all other states, is that a transition from each state pointing back to the initial state with probability 1 - d is added to model the situation where an attacker is discovered and has to restart the intrusion from the initial state. The rationale behind the use of the PageRank algorithm to rank attack graphs is based on the possibility of a hacker pursuing the hacking. In other words, a hacker being successful in obtaining root status may not be fully satisfied until further (or all possible) routes to obtaining root status are explored, and hence, may restart at any system state for further attempts to find vulnerabilities leading to a root status. Note that PageRank helps to find nodes that can more easily be reached by chance from one of the starting states. This describes the blind probing behavior of attackers. In practice, a human attacker is more likely to have some intuition that, say, getting root access on the mail server is more preferable than getting root access on one user's machine inside the network. In other words, PageRank is most likely to lead to states which are easiest to reach, and least likely to reach states which are less likely to be reached. There is work which modifies the PageRank algorithm slightly to allow control of the importance of some states. In the literature, this is refered to as "personalization" achieved through replacing the 1-vector  $\Pi_N$  in Eq. (5) by a parameterized vector (e.g. [14]). It is shown in [13] that the GNN can successfully encode the personalized PageRank algorithm, and hence, this paper does not attempt to explicitly show this aspect in the context of ranking attack graphs.

# 3 Ranking Attack Graph Using GNNs

The use of GNN for determining information on objects represented as nodes in a graph structure has been shown in several applications such as ranking web pages. In this paper, we develop an implementation of the GNN model as described in Section 2.2 applied to ranking attack graphs. For attack graphs, nodes represent computer system states and edges represent the transition relations between system states. Hence, the *state* of a node is determined by states and outgoing links of its parent nodes. Symbolic values 0 or 1 are also assigned to edge labels to distinguish between incoming and outgoing links. The function  $f_{w_1}$  that expresses the dependence of the state  $s_n$  of node n on its neighbors can be represented by Equation (6) where  $\mathbb{S}(n)$  denotes the set of nodes connected to node n,

$$s_n = f_{\mathbf{w}_1}(s_u, e_{uv}), u \in \mathbb{S}(n) \tag{6}$$

The output function  $g_{w_2}$  with input from the states produces the rank of node n.

$$o_n = g_{\mathbf{w}_2}(s_n) \tag{7}$$

The two computing units  $f_{\mathbf{w}_1}$  and  $g_{\mathbf{w}_2}$  are implemented by MLPs [1]. Inputs to the MLPs are the variables in Eqs. (6) and (7), *i.e.* states of neighboring nodes and labels of connecting edges. We use the *sigmoid* function [1] as the transfer function of  $f_{\mathbf{w}_1}$ . The *sigmoid* function is commonly used in back-propagation networks. The transfer function of the output network  $g_{\mathbf{w}_2}$  is a linear transfer function that stretches the output from  $f_{\mathbf{w}_1}$  to the desired target.  $f_{\mathbf{w}_1}$  and  $g_{\mathbf{w}_2}$  are thus parameterized by weight sets  $\mathbf{w}_1$  and  $\mathbf{w}_2$  of the underlying MLPs. The adaption of weights sets for the best approximation of training data is through a back-propagation process [1].

It is known that the computational complexity of the forward phase of MLP is O(n), where *n* is the number of inputs [2]. When considering that the functions  $f_{w_1}$  and  $g_{w_2}$ can be implemented as MLPs, it becomes clear that the computational complexity of the forward phase of GNN is also O(n). Note that a direct comparison of the computational complexity of PageRank is difficult since the computational complexity of PageRank depends on the number of links rather than on the number of nodes.

The GNN implemented is initialized by assigning random weights to the underlying MLPs. The training data is generated using Mehta *et al.*'s ranking scheme on a set of attack graphs. The trained GNN can take as input an attack graph and output through the trained network ranks of each node.

### 4 Experiments and Results

The objectives of the experiments are to verify the effectiveness of the proposed ranking approach: (1) if GNN can learn a ranking scheme applied to attack graphs, and (2) if it can generalize the results to unseen data. In particular we consider the PageRank scheme used by Mehta *et al.* to rank attack graphs.

It has been shown that GNN can learn PageRank on very large graphs [413]. Our objective is to ascertain if it can learn the ranking scheme when applied to attack graphs. We note that the WWW is a single very large graph and the aim of learning has been to generalize the results to this large graph by learning from selected small sub-graphs of WWW. However, in the case of ranking attack graphs, many small graphs of various

sizes exist. For a small network with few vulnerabilities, it can be a small graph with few nodes and edges, while for more complex networks, it can have hundreds of nodes and edges. One question is: what type/size of graph should be used for training? It would be ideal to train GNN with a set of attack graphs generated from networks of various complexity. However, while attack graphs can be generated quite easily from real networks, generation of realistic attack graphs of artificial examples requires significant manual labor as well as computational effort [11]. It is, in general, difficult to generate a large number of real attack graphs on artificial examples as training samples. To solve this problem, we experiment with training GNN with a set of automatically generated pseudo attack graphs that have similar shapes and node connection patterns to those of manually generated real attack graphs. Details for pseudo attack graph generation are provided in Section [4.3].

Note that it is time consuming to generate *artificial* attack graphs which resemble the type of attack graphs one would encounter in the real world. Real world attack graphs can be easily generated (the process may even be automated). However, due to a lack of access to large scale distributed system we were unable to obtain real world attack graphs. Instead we use the time consuming task of generating artificial data. We did not attempt to simulate attack graphs for small systems since the resulting graphs would be rather small. Existing approaches to ranking attack graphs are sufficient for small scale systems whereas the advantage of the proposed approach is most pronounced on larger graphs.

A related question is how we determine the effectiveness of the proposed ranking approach. GNN output produces a set of ranks for nodes of an attack graph. There are many ways to define the "accuracy" of a set of ranks compared to the set of ranks that is calculated by another scheme. In particular one may use the average distances of the sets, or the maximum distance over all elements of the set. For our particular application we use *Relative Position Diagram (RPD)* and *Position Pair Coupling Error (PPCE)* as described in Section [4.1]. The PPCE measure the agreement between the proposed approach and the ordering imposed on the nodes in an attack graph by PageRank. Hence, this ordering compares the proposed approach with an existing approach. No attempt is made to assess whether the PageRank order by itself makes sense.

### 4.1 Performance Measure

A ranking scheme can be denoted as a function that takes as input an attack graph (AG)and outputs the ranks, i.e. f(AG) = R where R is a real vector of the same size as the number of nodes in AG. A naive performance measure is to compare the output ranks by  $GNN f_G(AG) = R_G$  and that produced by Mehta's ranking scheme  $f_M(AG) = R_M$ . However, as ranks are used to identify important portions of an attack graph, ordering of ranks are considered more important than their actual numerical values. We therefore devised two performance measures to evaluate how well rank orderings are preserved. Relative Position Diagram (RPD): This visually illustrates how well rank orders are preserved, and is obtained by sorting  $R_M$  and  $R_G$  and plotting the order of each node in  $R_M$  against that in  $R_G$ . The X-axis and Y-axis represent the order of ranks in  $R_M$  and  $R_G$  respectively. Therefore, nodes that have the same rank orders by both schemes are plotted along the diagonal. Position Pair Coupling Error: An RPD only intuitively shows how well rank orders are preserved. We also provide a quantitative measure on rank order preservation as follows. Let  $r_i^M$  and  $r_i^G$  denote the i-th element in  $R_M$  and  $R_G$  respectively. For each pair of nodes, if  $r_i^M$  and  $r_j^M$  are not in the same order as  $r_i^G$  and  $r_j^G$ , then a position-pair-coupling-error (PPCE) is found. Performance of the GNN ranking scheme can therefore be quantitatively measured by the PPCE rate.

#### 4.2 Experimental Results

We found that the computational demand of the attack graph generation method in [11] is high. The generation of a single real world attack graph took several hours, and hence, we were not able to produce more than 14 graphs within a reasonable time frame for the experiments. The 14 attack graphs were generated using the same computer network example as in a previous work [9], with variations created by adding, removing, or varying vulnerabilities or network configurations. Eight of the attack graphs were randomly selected to form the training data set of the GNN. The remaining six attack graphs are used as the testing data set so as to examine the GNN's generalization ability in different situations. The GNN used for our experiments consists of two hidden layers, and each hidden layer has 5 neurons. The number of external inputs is also set to 5. Such a GNN has been shown to be successful when applied to ranking web pages [4.13], and it produces a sufficiently small network that allows for a fast processing of the data. However, this GNN was used successfully on a problem involving much larger number of nodes, and hence, it may be possible to use a somewhat smaller network when dealing with attack graphs.

We varied the number of training epochs from 500 to 20,000 and repeated the experiment 6 times with randomized initial weights of the underlying MLPs. The PPCE rate results are plotted in Fig. [3] It can be observed that when trained for a sufficiently large number of epochs (around 10,000) the PPCE rate reduces asymptotically to an optimal value. Improvement by further training can be observed but is not significant. Hence the number of training epochs is fixed to be 10,000 in the rest of the experiments for an appropriate effort/time tradeoff. However when applied to ranking attack graphs in practice the training epoch can be set to 20,000 or larger for more accurate results.

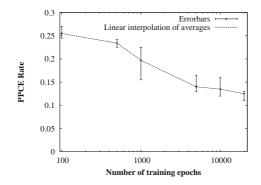


Fig. 3. Effect of number of training epochs

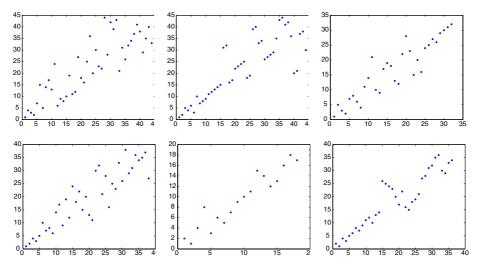


Fig. 4. Relative Position Diagram when trained on real-world attack graphs

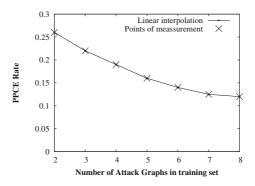


Fig. 5. Effect of number of attack graphs used in the training data set

From the above experimental results, it can be observed that the average PPCE rate can be reduced to around 10% when GNN is trained with sufficient number of training examples and training epochs. That being the case on an average the proposed GNN-based ranking scheme sacrificed around 10% accuracy in terms of relative position preservation. However, once trained, a GNN can produce output at an O(N) time complexity where N is the size of input. This may be an improvement compared with the  $O(N \log \frac{1}{\epsilon})$  computational complexity required by PageRank. Moreover, the generalization ability of GNN allows us to train GNN on only a small number of attack graphs and apply the results to unseen attack graphs.

Figure plots the RPDs resulting from ranking graphs from the testing data set with GNN to visualize the effectiveness of relative position preservation. Each subgraph represents the resulting RPD for one of the 6 attack graphs in the testing data set, and hence it is of different scale to other subgraphs. It can be observed that the order of ranks are preserved to a reasonable degree. Although only a relatively small portion of nodes

remains at exactly the same position as the PageRank scheme, most nodes remain centered around the diagonal and as a result important nodes remain important and unimportant nodes remain unimportant. The GNN-based ranking scheme therefore allows a system administrator to focus on most of the important nodes in an attack graph.

Finally, we experimented with reducing the number of training samples. The number of training attack graphs in the training data set is reduced from 8 to 2. The effect on reducing the number of attack graphs used in the training data set is plotted in Figure **5**. It can be observed that the number of attack graphs in the training data set has a significant impact on the accuracy in terms of relative positions on the training results. Sufficient attack graphs must be provided in the training data set to simulate the ranking scheme effectively.

#### 4.3 Training with Pseudo Attack Graphs

Generating a large number of attack graphs for the training data set is difficult due to significant manual labor and computational effort that it requires [11]. In the following, we provided an alternative scheme for automated training set generation, and train GNN with automatically generated pseudo attack graphs that have similar shapes and node connection patterns to those of manually generated realistic attack graphs.

That attack graphs are non-cyclic, tree shaped graphs resulting from a procedure as follows: at the initial system state, the intruder looks for an entry point and reaches the first layer of nodes in the attack graph by exploiting one of the vulnerabilities applicable to the initial state. With the escalated privilege by the initial exploit, the intruder obtains more intrusion options for the next intrusion step. This procedure repeats and keeps expanding the attack graph until the intruder can reach the intrusion goal. Leaf nodes in the attack graph are thus produced, and the attack graph begins to contract until all attack paths reach the intrusion goal. We generate pseudo attack graphs by

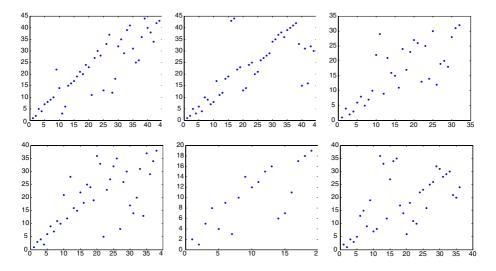


Fig. 6. Relative Position Diagram when training on pseudo attack graphs

	AG-1	AG-2	AG-3	AG-4	AG-5	AG-6	Avg
Min PPCE	0.16	0.11	0.14	0.12	0.12	0.11	0.12
Max PPCE	0.21	0.15	0.19	0.17	0.15	0.15	0.16

Table 1. Position Pair Coupling Error

simulating this procedure. A pseudo attack graph begins with the root node representing the initial state. It then expands with increasing number of nodes at each layer to simulate the expanding process of realistic attack graphs. This repeats until presumably the intruder begins to reach his goal. Then the pseudo attack graph contracts till all paths are terminated at a leaf node.

We repeat the experiments on the same 6 testing attack graphs but using pseudo attack graphs generated by the above procedure as the training data set. In total, 40 pseudo attack graphs are included in the training data set, and GNN is trained for 10,000 epochs for the asymptotic optimal result. The resulting Relative Position Diagrams are as shown in Figure []. It can be observed that orders of ranks are preserved also to a reasonable degree, but these results are not as accurate as those obtained when using real attack graphs in the training data set. Table [] lists maximum and minimum PPCE rate by repeating each experiment 3 times. The average PPCE is between 12% to 16%, indicating that on an average the proposed GNN-based ranking scheme sacrificed around 12% to 16% accuracy in terms of relative position preservation. Therefore, using pseudo attack graphs for training, but on the other hand it significantly reduces the effort required to generate real attack graphs.

# 5 Conclusions

This paper produced evidence that the GNN model is suitable for the task of ranking attack graphs. GNN learns a ranking function by training on examples and generalizes the function to unseen data. It thus provides an alternative and time-efficient ranking scheme for attack graphs. The training stage for GNN, which is required only once, may take a relatively long period of time compared with *e.g.* PageRank scheme. However, once trained the GNN may compute ranks faster than that required by the PageRank scheme.

Another advantage of using a machine learning approach is their insensitivity to noise in a dataset. This is a known property of GNN which is based on the MLP architecture. Existing numeric approaches such as PageRank or rule based approaches are known to be sensitive to noise in the data.

The GNN provides much more flexible means for ranking attack graphs. A GNN can encode labels that may be attached to nodes or links, and hence, is able to consider additional features such as the cost of an attack or time required for an exploit. This is not possible with the PageRank scheme which is strictly limited to considerations of the topology features of a graph. We plan to investigate how the ability to encode additional information can help to rank attack graphs in the future.

Despite the proven ability of the GNN to optimally encode any useful graph structure [12], the result in this paper showed that the performance of the GNN has a potential

for improvements. We suspect that this may be due to the choice of training parameters, or the chosen configuration of the GNN. A more careful analysis into this issue is left as a future task.

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# Implementing IDS Management on Lock-Keeper

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Abstract. Intrusion Detection System (IDS) management is an important component for most distributed IDS solutions. One of the main requirements is extensibility, which enables the integration of different types of IDS sensors as well as the deployment in different kinds of environments. Lock-Keeper is a simple implementation of the high level security idea, "Physical Separation". It works as a sluice to exchange data between two networks without having to establish a direct and physical connection. To enhance the security of the Lock-Keeper system itself, it is necessary to deploy IDS sensors on Lock-Keeper components. This paper proposes an extensible IDS management architecture, which can be easily integrated on the special hardware platform of Lock-Keeper. Unified interface and communication between different integrated IDS sensors are designed using the known IDS standard, IDMEF, and realized as several kinds of plugins, such as handlers, receivers, and senders. A prototype of implementation is presented and some practical experiments are carried out to show the extensibility and applicability of the proposed architecture.

### 1 Introduction

Intrusion detection systems (IDS) have been commonly used in practice for identifying malicious behaviors against protected hosts or network environments. An effective IDS should be capable of detecting various types of attacks and all the possible variants of a certain type of attack. Nowadays, a large number of commercial and open source IDS implementations have emerged, such as Snort [I], Samhain [2], Bro [3], F-Secure Linux Security [4] and Prelude [5]. Besides, many research works have been done or are being carried out around various topics, such as proposing new detection models for specific applications [6], accelerating efficiency of alert analysis [7], and improving precision of detection methods [8].

Based on the protected objective, IDS can be classified into host-based intrusion detection system (HIDS) and network-based intrusion detection system (NIDS) [9]. Another widely accepted classification is according to the used detection model, e.g., misuse detection [10] or anomaly detection [11]. Different detection models result in different design and implementation of IDS sensors. Therefore, it makes sense to provide integrated IDS solutions, which make it possible for users to simultaneously benefit from advantages of various kinds of IDS sensors. For this purpose, many distributed IDS (DIDS) solutions, which

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consists of multiple IDS sensors located in a loosely coupled environment, as well as hybrid IDS solutions, which contain both HIDS and NIDS, have been developed. The Intrusion Detection Message Exchange Format (IDMEF) [12], has been proposed as a standard to provide interoperability among different IDS approaches, including commercial, open source, and research systems. However, due to the highly heterogeneous architecture of these IDS implementations and poor acceptance of the IDMEF standard, integrating different types of IDS into a unified architecture remains to be a big challenge.

As an important implementation of the security concept, Physical Separation, Lock-Keeper has been offered as an efficient approach to separate private networks or sensitive hosts at any levels and permit secure data exchange simultaneously (**13**, **14**, and **15**). A Lock-Keeper system consists of three independent Single Board Computers (SBC): INNER, OUTER, and GATE, which are connected using a patented switch unit. The connection inside Lock-Keeper are restricted on the hardware layer by the switch unit that GATE can just be connected with only one partner, either INNER or OUTER. Since the OUTER component has to expose itself to the external world for providing Lock-Keeper supported services, the security of OUTER needs to be enhanced. Furthermore, normal operating system and application modules running on each Lock-Keeper component are another vulnerable points for being misused or even intentionally attacked. Deploying feasible IDS solutions on the main Lock-Keeper components is expected.

Therefore, we are motivated to combine the software based IDS solution with the hardware based Lock-Keeper system. An extensible IDS management architecture, which can easily aligned on the Lock-Keeper, is proposed in this paper. It includes several IDS sensors and a central management unit. A new design of the *Event Gatherer* for the management unit is provided through several *plugins*, i.e., *Sender*, *Receiver*, and *Handler*, which provide high flexibility in IDS deployment and support multiple types of communications, e.g., the file-based communication, network-based communication, etc. The IDMEF standard is implemented to represent and exchange the alarms. A standardized interface is designed to provide a unified view of alert reports. A prototype is implemented and practically deployed on the Lock-Keeper system.

The rest of the paper is organized as follows. Section 2 introduces some related works. Section 3 describes the proposed Lock-Keeper based IDS management architecture. Section 4 presents some experiments. Section 5 gives a short summary of our contributions and a brief outlook for the future work.

### 2 Related Works

In this section, we provide a short overview of existing works on IDS management. A practical IDS management system, called Prelude **5**, is described as an example. The IDMEF standard is briefly introduced. Furthermore, a review of the high level security concept, Physical Separation, and its Lock-Keeper implementation is provided as well.

#### 2.1 Overview of IDS Management

There are many previous works focusing on IDS management. In [16], a multiagent based approach is described, which uses SOAP-based Web Services for communications. The system is based on a layered architecture and aims to increase the readiness of information as well as decrease the time of information access. The system described in [17] is proposed to integrate wireless sensors. It is capable of analyzing data with data mining techniques, such as rule-based classifiers, spatial data mining and clustering methods, as well as decision tree models. The decentralized approach described in [18] consists of a so-called distributed hash table architecture and focuses on identifying large scale distributed attacks, such as scanning, worms, or Denial-of-Service (DoS) attacks. TRINETR [19] is an intrusion detection alert management system, which provides alert aggregation, knowledge-based alert evaluation and correlation. The correlation procedure is supported by several knowledge bases (KB), such as vulnerability KB, network and host asset KB, etc.

*Prelude* is a distributed IDS product, which is capable of connecting other separate IDS sensors. It can be used to collect, normalize, sort, aggregate, correlate and report most of security-related events. Although *Prelude* is relatively robust and can ensure high performance, it reveals several problems concerning extensibility. The interface to the *Prelude* management system is built based on the *Prelude* library. The change of source code of the target IDS sensor is always required for any kinds of extensions. Additionally, the connection between the management system and the sensors requires a stable TCP connection. That means it is not suitable for deploying in loosely coupled environment, where connections are not so reliable, e.g., in wireless network.

As a new standard, Intrusion Detection Message Exchange Format (*IDMEF*) provides a unified format for communications between IDS sensors, response systems and management units. It specifies a data model to represent the exchange data in XML files. There are two basic message types defined in *IDMEF*: Alert and Heartbeat. The alert message is used to represent and exchange security related events, such as a detected attack. The heartbeat message is used to inform the management system that a sensor is still active, i.e., it is used for maintenance of the sensors. By offering such a convenient communication channel, *IDMEF* has been accepted by the IDS community. However, there are still several existing IDS implementations, which do not support this standard.

#### 2.2 Physical Separation and Lock-Keeper

Based on the intuitive principle that "the ultimate method to secure a network is to disconnect it", the Lock-Keeper technology has been accepted as an efficient approach to guarantee the high-level security and prevent online network attacks by physically separating the protected hosts or networks (13, 15). Because of its simple idea and extensible architecture, the Lock-Keeper system can be easily and seamlessly integrated with any other security methods or solutions to provide thorough protection for most actual network-based applications. Currently,

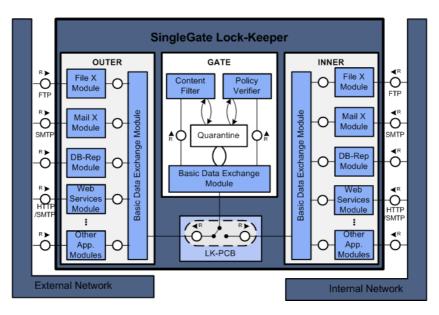


Fig. 1. Conceptual Architecture of the SingleGate Lock-Keeper

there are a lot of different "Physical Separation" implementations, such as Microsoft e-GAP-based Intelligent Application Gateway (IAG) [20], NRL Pump technology offered by U.S. Naval Research Laboratory [21], and DualDiode from Owl Computing Technologies [22], etc.

Lock-Keeper works as a sluice on the border of the protected network [15]. Because of such physical network separation, it can be guaranteed that hackers and malign data have no opportunities to break into the internal network by any means of online attacks. Currently, the commercial version of Lock-Keeper has already been developed and is now under the marketing extension by Siemens [14]. Here, we use a SingleGate Lock-Keeper system as an example to briefly explain what the Lock-Keeper is and how it works.

As shown in Fig. 1 a SingleGate Lock-Keeper system consists of three independent SBCs: INNER, OUTER and GATE. A patented switch unit, realized on a Printed Circuit Board (PCB), is used to autonomously control connections so that GATE can just be connected with only one partner, either INNER or OUTER. There are no ways to directly establish the connection between INNER and OUTER at any time. Besides these hardware components, there are also Lock-Keeper Secure Data Exchange (LK-SDE) software running in the Lock-Keeper system. LK-SDE software includes several application modules located on INNER and OUTER, which work as interfaces and provide popular network services to outside users, e.g., File eXchange (File-X) Module, Mail eXchange (Mail-X) Module, Database Replication (DB-Rep) Module and Web Services (WS) Module. Normal communication protocols, such as SMTP, HTTP, FTP, etc., are stopped and analyzed respectively by these application modules. Then, standard file-based Lock-Keeper Message Containers (LKMC) can be created to carry the data for respective services. These LKMCs will be transferred by "Basic Data Exchange Module" [15], which works based on the "Pull-and-Push" principle to avoid running any kinds of servers or server-similar programs on GATE so that there are no possibilities for outside hosts (even INNER and OUTER) to actively establish connection to GATE. In particular, because GATE is also a normal PC, it is possible to integrate Third-Party security software, e.g., virus scanning software, mail analysis tools, or content filtering methods, etc., into LK-SDE architecture, which help to check data traffic and prevent offline attacks, e.g., virus, malicious codes, etc.

# 3 Lock-Keeper IDS Management Architecture

To improve the accuracy of the intrusion detection effort, users need to use alerts from different audit sources. Therefore, it is necessary to propose an efficient management architecture, which can flexibly integrate different types of IDS sensors and automatically synthesize alerts. On the other hand, to enhance security of Lock-Keeper system, especially its main components, INNER and OUTER, a feasible distributed IDS solution is required. In this section, we present a new IDS Management architecture using the Lock-Keeper as the hardware base.

### 3.1 IDS Management: Architecture

As shown in Fig. 2, the proposed IDS Management Architecture consists of several *IDS Sensors* and an *IDS Management Unit*. The *IDS Management Unit* consists of four active components: *Event Database*, *Analysis Component*, *Remote Controller* and *Event Gatherer*.

*IDS Sensors* are responsible for detecting and reporting malicious behaviors. They are configured through the *Remote Controller*, which is responsible for

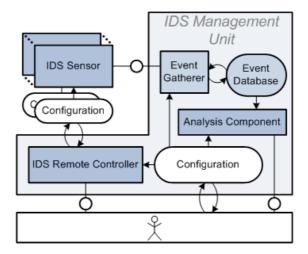


Fig. 2. IDS Management: Architecture

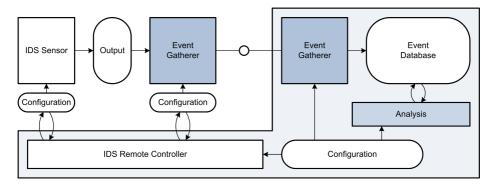


Fig. 3. IDS Management: Implementation

remotely configure and control all connected *IDS Sensors*. The *Remote Con*troller can be a remote controlling and monitoring interface, where *Users* have possibilities to directly access the *IDS Management Unit*. The control can also be realized by changing configuration files of the core components. *Event Database* is a passive storage that holds information on all received events. All the events are made persistent in the *Event Database* storage and can be accessed through the *Analysis Component*, which is responsible for representing the gathered events as well as analyzing these events, e.g., correlating multiple events in complex attack scenarios. Such an analysis component might be the front end to the database within the IDS management unit. The *Event Gatherer* is an important component for collecting and unifying all events generated from connected *IDS Sensors*. Similarly, the *Event Gatherer* can be configured by *Users* through its configuration file.

Based on the proposed theoretical model of IDS management, we make a detailed design of the IDS management as shown in Fig. 3 A new *Event Gatherer* component is introduced on the *IDS Sensor* side. This gatherer is used to standardize the outputs from different sensors, which solves the problem of non-standardized sensor output. Furthermore, it realizes the logical communication between the sensor and the management component.

### 3.2 IDS Management: Implementation

Fig. 4 shows a detailed view on the architecture of the *Event Gatherer*. The gatherer is the core component of the event flow. It is a plugin engine capable of loading several *Event Receivers* and one *Event Sender*. A receiver processes a certain output, creates standardized events in IDMEF format and writes all events into a queue for further processing. Possible receivers are *Snort CSV Receiver*, *Samhain Syslog Receiver* or *IDMEF Tcp Receiver*. A sender reads standardized events from the queue and forwards them to a specific channel or storage, e.g., to a defined host and port via TCP connection or to a database with a certain schema. Possible senders are the *MySQL Database Sender* or the *ID-MEF Tcp Sender*. To connect *Event Gatherers*, there are pairs of senders and

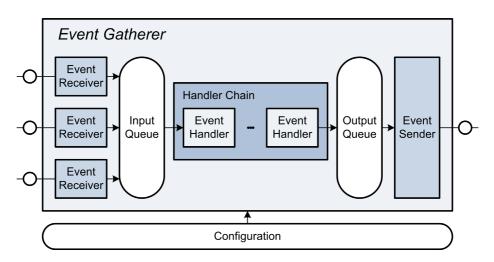


Fig. 4. Event Gatherer

receivers required, which can communicate with each other. This means that the sender creates output the receiver can process and understand. Possible pairs are the *IDMEF Tcp Sender and Receiver* or the *IDMEF File Sender and Receiver*. The senders and receivers are independent threads loaded at start up into the gatherer process. The gatherer can be configured to load several different receivers. This allows high flexibility in deployment and building intrusion detection scenarios.

### 3.3 A Lock-Keeper Based IDS Management Architecture

As shown in Figure 1, the proposed IDS management architecture is practically implemented on the Lock-Keeper system. We integrate Samhain and Snort on the OUTER and INNER component. On GATE, the Samhain and F-Secure Linux Security are deployed. The HIDS implementation Sambain is used to oversee the working state of each host. The NIDS snort is used to monitor network connections, which INNER/OUTER exposes to both sides. F-Secure Linux Security is used to do the application layer checking on the passing traffic, i.e., LKMC. Such known offline attacks as virus, worms, and other malicious codes, should be detected here.

There is an *Event Gatherer* running on each Lock-Keeper component to collect alerts from the connected sensors. The gatherer on OUTER includes the plugins: Samhain receiver, snort receiver, and IDMEF sender. The gatherer on GATE consists of the plugins: Samhain receiver, F-Secure receiver and IDMEF sender. The gatherer on INNER is running through the plugins: Samhain receiver, Snort receiver, IDMEF receiver, and SQL sender. The IDMEF sender on OUTER and GATE writes all alerts to a specified directory which is the interface to the Lock-Keeper SDE software. On INNER, the IDMEF receiver is listening on a specified directory where the alerts from OUTER and GATE will be put by the

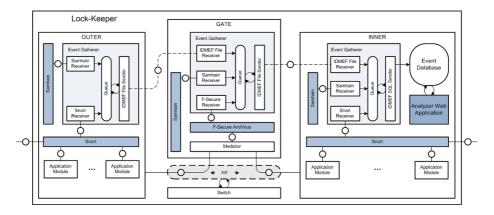


Fig. 5. Lock-Keeper based IDS Management Architecture

Lock-Keeper SDE software. The SQL sender is configured to write all the incoming alerts to a database for further processing.

As the host of the central IDS management unit, INNER has two additional important components: *Analyzer* and *Event Database*. The *analyzer* works on the event storage to provide simple event analysis and a unified view to end users. The database provides an easy way to store, query and process huge amounts of incoming alerts. Nevertheless, alerts can also be stored in other ways according to the practical requirements or environments, e.g., in a single XML file or in a specified directory with multiple alert files. A web application is built to connect *Analyzer* and *Event Database* as well as provide a user interface for displaying the unified view of events.

### 4 Experiment Results

As mentioned previously, we have two overall objectives: 1) improving the extensibility of IDS deployment and management, 2) enhancing the security of Lock-Keeper system. To evaluate the proposed Lock-Keeper based IDS management architecture and its implementation, we construct an experiment environment, as shown in Fig. <sup>6</sup> To test the functionality of each single integrated IDS sensor, e.g., Samhain, Snort, and F-Secure Linux Security, several simple attacks are executed within this environment.

A port scan towards the Lock-Keeper OUTER is executed from the *Attacker* by connecting to a port range of 1-1000. This port scan was performed using *Nmap* [23]. Similarly, this attack could also be carried out from inside towards INNER. The NIDS sensor, Snort, is supposed to detect this kind of attack.

Since Lock-Keeper also provides normal Email service, there are SMTP ports opened on OUTER. Therefore, attackers have the possibility to perform the SYN Flood DoS attack [24] towards OUTER. Snort on OUTER is responsible for recognizing this attack based on the preset threshold of the number of received SYN packets.

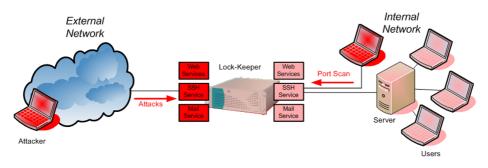
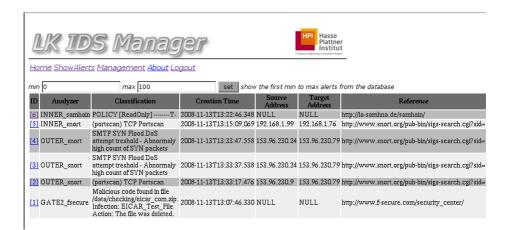


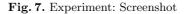
Fig. 6. Experiment: Testing Setup

To test the HIDS, we attempt to access and modify files in the sensitive directory on INNER, which is configured as "Read-Only". The deployed HIDS, Samhain, is used to identify such attacks.

To test the functionality of the F-Secure Linux Server Security running on GATE, which is used as an IDS sensor to find and prevent application layer attacks, we create a scenario by sending an Email from the *Attacker* side to Lock-Keeper. Within this test Email, we attached the *eicar Anti-Virus Test File* [25]. The Lock-Keeper Mail-X module handles the Email and passes it to GATE. On GATE, the integrated F-Secure virus scanner is triggered by the malicious Email message. An alert is created in the format of IDMEF and later sent to the management system on INNER.

As shown on Fig. 7. F-Secure Linux Security on GATE detects the Email with the eicar file (see alert [1]), Snort on OUTER identifies the port scan and SYN Flooding (see alerts [2], [3] and [4]). The port scan is identified as well on INNER (see alert [5]). Besides, the Samhain on INNER also successfully detects the illegal access to the protected file "/etc/passwd" (see alert [6]). The





- <idmef-message> - <alert messageid="alert 1226578966347_1731137144"> <analyzer analyzerid="lkids_samhain_INNER_samhain" class="hids" name="INNER_samhain" version="0.8.2.4.4"></analyzer> <createtimes 2008-11-13t13:22:45.40348<="" createtime=""> <detecttimes 2008-11-13t13:22:45-101:00<="" detecttime=""></detecttimes></createtimes></alert></idmef-message>
- <target></target>
- <file></file>
<pre><pre>cpath&gt;/etc/passwd</pre></pre>
- <classification text="POLICY [ReadOnly] T-"></classification>
- <reference></reference>
<url>http://la-samhna.de/samhain/</url>
- <additionaldata type="string"></additionaldata>
- <string></string>
CRIT : [2008-11-13T13:22:45+0100] msg= <policy [readonly]t-="">, path=, ctime_old=&lt;[2008-09-17T15:03:45]&gt;,</policy>
ctime new=<[2008-11-13T12:11:47]>, mtime_old=<[2008-09-17T15:03:45]>, mtime_new=<[2008-11-13T12:11:47]>,

Fig. 8. Experiment: IDMEF Message of Alert Nr. 6

proposed *Event Gatherer* transforms the output of different IDS sensors into the standard IDMEF messages, e.g., the alert [6] shown in the Fig. 8. With the help of respective sender plugins and receiver plugins as well as the Lock-Keeper SDE software, alerts generated on OUTER and GATE are transferred to the central management unit on INNER and displayed on the end user interface.

The experiments have shown that our system works well, which exactly supports the claimed benefits on both improving extensibility of IDS management and enhancing Security of Lock-Keeper. Different types of IDS implementations, e.g., Snort, Samhain and F-Secure Linux Security, are seamlessly integrated into a loosely coupled environment with several distributed hosts. Standardized events are correctly created, transferred, analyzed and reported using the central IDS management unit. On the other hand, the deployed IDS sensors strongly increase the security level of Lock-Keeper, especially on detecting and preventing the network attacks on INNER/OUTER and application layer attacks towards the internal users.

# 5 Conclusion

The paper proposes an extensible architecture for deploying different types of IDS on the Lock-Keeper system. A central management unit for adding, configuring, operating and removing the different IDS sensors, is designed and deployed on the Lock-Keeper components. A high amount of security related events produced by all the integrated sensors could be synchronized, unified, analyzed and reported in the distributed platform. The known IDS standard, IDMEF, is realized in several kinds of plugins, e.g., *Handler, Sender* and *Receiver*, for storage and communication between the involved IDS sensors and the central management unit. The possibility to connect *Event Gatherers* in the architecture provides a high flexibility in deployment. A prototype implementation is presented and testified by experiments. The architecture is developed with focus on flexibility, extensibility and portability. However, there are still many open issues. More assistant parts of the architecture need to be realized and integrated, e.g., an extended analysis of events. The analysis component should enable different modules and approaches for event correlation as well as extended management of events, such as sorting, filtering, tagging, etc. The number of available receivers and senders should be enriched. Performance is another important issue, which needs to be considered due to the complex integration and communication of heterogenous components. It is necessary to handle a certain amount of events within a tolerant time.

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# Ensuring Dual Security Modes in RFID-Enabled Supply Chain Systems

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Abstract. While RFID technology has greatly facilitated the supply chain management, designing a secure, visible, and efficient RFIDenabled supply chain system is still a challenge since the three equally important requirements (i.e., security, visibility, and efficiency) may conflict to each other. Few research works have been conducted to address these issues simultaneously. In this paper, we observe the different security requirements in RFID-enabled supply chain environments and differentiate the simplified model into two security levels. Accordingly, dual security modes are properly defined in our RFID setting. In the relatively secure environment, our system is set to the *weak security mode*, the tagged products can be processed in a highly efficient way. When in the *strong* security mode, our system guarantees a high level of security, while its efficiency is lower than that in the *weak security mode*. A set of RFID tag/reader protocols to facilitate the duel security modes are presented. Their security, visibility and efficiency are analyzed and compared with the relevant works.

# 1 Introduction

RFID systems consist of two main components: tags and readers. Tags are radio transponders attached to physical objects, while radio transceivers, or readers, query these tags for identifying information about the objects to which tags are attached. RFID technology, when combined with internet and networking technology, enables product information to be collected, integrated, shared, and queried at various levels (e.g., item, pallet, case, and container) in real time in a supply chain. RFID technology has been widely envisioned to have significant impact on the economy world-wide as an inevitable replacement of barcodes in the near future, which may facilitate the creation of secure, visible, and efficient supply chains. As a result, EPCglobal Network [2] is being formed to provide an open and standard interface to process RFID information in supply chain management.

The current EPCglobal Network standards depend entirely on honest supply chain partners to realize supply chain visibility. Few security mechanisms have been developed to ensure that the tracking services are confidential, verifiable,

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and accountable in the presence of realistic and malicious attacks, especially those coming from corrupted supply chain partners. Most of RFID technologies have focused on protecting a single RFID channel [5,6,7,8,9,10,11] without considering the relationship among supply chain parties. Such techniques cannot be directly used to protect the sharing of information among supply chain parties.

We develop a new security solution for RFID-enabled supply chain systems. In our solution, even a valid supply chain party, if not authorized, cannot track RFID tags after the ownership handover of the tags to other supply chain parties. On the other hand, an authorized party can have full supply chain visibility to track a tagged item in a supply chain. A root secret, shared between a trusted authority and each tag, is used to guarantee the anti-track and track/visibility properties. To achieve better efficiency, we classify the supply chain environments into two security levels. In an environment where insiders or outsiders can actively interact with a tag for the purpose of tracking, we set the system to strong security mode so as to maintain high security. In an environment where active attacks are not possible (e.g., within the territory of a supply chain party), we can set the system to *weak security mode* to achieve high processing speed. We use a binary switch on the tag to control the security modes. A set of RFID tag/reader protocols like tag reading protocol, security mode switching protocol and secret updating protocol, to facilitate the duel security modes are presented. We analyze the schemes in terms of security, visibility and efficiency. At last, our solution is compared with the relevant works.

The rest of this paper is organized as follows. In Section 2, we introduce a simplified system model and architecture for RFID-enabled supply chains. In Section 3, we present our protocols to protect RFID-enabled supply chain systems. In Section 4, we analyze our protocols in terms of security, visibility, and efficiency. In Section 5, we discuss the related works. Finally, Section 6 concludes this paper.

### 2 Model

Assumptions. In this paper, we focus on the attacks conducted on the wireless communications between RFID readers and tags. The adversaries can be either supply chain outsiders or insiders (i.e., dishonest supply chain parties). The adversaries are assumed to have the power to listen in communication channels, counterfeit as a valid supply chain party, to initiate, delete, modify, or transfer messages between RFID tags and readers. We do not consider the physical attacks, denial of service attacks, and side-channel attacks. We further assume that the communications between a supply chain partner and its RFID readers, and the communications between supply chain parties are secure, which can be protected by standard security techniques without limitation of resources.

*Requirements.* As discussed in existing RFID literatures, common security requirements of RFID tags include unlinkability, confidentiality, etc, nonetheless, applying RFID tags in supply chain environments introduces some unique needs such as supply chain visibility and extra efficiency. In the following, we list the details of the requirements for RFID system when deployed in the supply chain.

The security requirements of RFID-enabled supply chain are summarized in **12**. We increment the requirements to include forward and backward secrecy and de-synchronization resilience. The list of security requirements is given below: (i) Authoritative access: Only legitimate readers of an authorized party are allowed to identify and update a tag. (ii) Authenticity of tags: In a supply chain link, only legitimate RFID tags delivered by previous party will be accepted by the next party. (iii) Unlinkability: Weak unlinkability and strong unlinkability can be used to describe the security level of anti-tracking. Weak unlinkability requires that an unauthorized reader cannot link the responses of a tag interrogated before and after it is processed by an authorized party. Strong unlinkability requires that an unauthorized reader cannot link any two replies to the same tag. (iv) Forward and backward secrecy: If the communication between a tag and a party is compromised, it will not affect the security of the communication between the tag and any other party in the supply chain. (v) De-synchronization resilience: RFID communication protocol is resilient to the attacks that are targeted towards de-synchronizing a tag and a reader.

Besides security, supply chain visibility must be maintained in supply chain management. It means that the manager of the supply chain or any authorized party should be able to track the movement of RFID tags. The enhancement on supply chain visibility is the most attractive feature that RFID technology brings to the traditional supply chain management. It allows companies to track and monitor the progress of material flow without inefficient bar code scanning.

Due to mass product exchanging, the efficiency of RFID technology is crucial in supply chain management. Without incorporating security and visibility features, hundreds of read operations can be performed per second between a reader and tags. The processing speed should not be delayed by adding security and visibility features.

We stress that the three requirements (i.e., security, visibility, and efficiency) are equally important in supply chain environments. Our goal is to propose a practical solution for RFID-based supply chain systems under the three requirements with the lowest possible cost.

Architecture. Our solution involves four types of entities as shown in Figure  $\blacksquare$  (i) a supply chain manager which is a trusted authority, denoted by TA; (ii) independent supply chain parties denoted by  $P_i$ ; (iii) RFID readers inside a partner, which are collectively referred to as  $R_i$  controlled by its corresponding party  $P_i$ , and a back-end database referred to as  $D_i$ ; (iv) RFID tags denoted by  $T_j$ .

The architecture we proposed is suitable for various types of supply chain structures [3] and compatible with contemporary EPCglobal network architecture [2]. In a third-party logistics (3PL) supply chain, TA can be the shipping company, which is specialized in handling the shipping issues in the supply chain. In the vendor managed inventory (VMI) supply chains, the vendor manages all the delivering of products; thus, it is straightforward for the vendor to take the role of TA. For the collaborative planning, forecasting, and replenishment

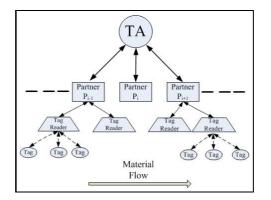


Fig. 1. A simplified RFID system architecture

(CPFR) supply chains, a supply chain hub is TA, which coordinates real-time sharing of supply chain information among supply chain parties. Finally, in supply network (SN), the situation is similar but more complex than in CPFR supply chains, where TA can be an existing supply chain hub or a dominant supply chain party.

### 3 Protocols

Since RFID tags will be used in vast numbers in supply chains, it is desired that the security design of the tags should be as cheap as possible. To achieve this goal, our protocols are designed to use passive tags that are equipped with pseudo-random number generators, standard XOR  $\oplus$  and hash  $H(\cdot)$  calculations.

A database is initialized by TA and sent to each supply chain party before the party can identify a batch of tags. With the help of the database, a supply chain party can switch security modes of RFID tags multiple times. Before a supply chain party sends a batch of tags to the next party in ownership handover, the current party needs to update the secret information in each tag. After ownership handover, the party can no longer identify the tags or track their movement.

#### 3.1 Initialization

Tag initialization. Before the first supply chain party  $P_1$  starts processing tagged products, TA will initiate three values  $(\alpha_j, \beta_{1\leftrightarrow j}, switch)$  and embed them in each tag  $T_j$  in a secure manner.

- $-\alpha_j$  is the tag root secret of length  $\ell$ , which is fixed and shared between TA and the tag. The root secret is used to identify the tag uniquely.
- $-\beta_{i\leftrightarrow j}$  is a temporary secret of length  $\ell$ , which is shared between supply chain party  $P_i$  and tag  $T_j$ . The two parameters i and j of  $\beta_{i\leftrightarrow j}$  denote the identity number of the supply chain party and the tag separately. This secret is initiated by TA to be  $\beta_{1\leftrightarrow j}$ . The temporal secret  $\beta_{i\leftrightarrow j}$  will be updated by  $P_i$  to  $\beta_{i+1\leftrightarrow j}$  before ownership handover to  $P_{i+1}$ .

switch is a binary value used to indicate the security mode of a tag. This
value is initiated to be 'on' for a strong security mode and it can be subsequently switched to 'off' for a weak security mode.

Database Initialization. Each party  $P_i$  maintains a database  $D_i$  in its local storage where each tuple in the database corresponds to a tag.  $D_i$  contains all RFID information with respect to a batch of tags except for tag root secrets  $\alpha$ .  $D_i$ consists of five attributes ( $\beta, x, p, s, switch$ ), where ( $\beta, switch$ ) are defined the same as in tag initialization, (x, p, s) are defined below.

- Tag response x: Tag response to be received from the corresponding RFID tag (in weak security mode). When the tag is on strong security mode, the value of this attribute is set to NULL.
- Pointer p: An octet string containing an address where the business information relevant to the tag is stored. An alternative approach is to store information in this field directly. Obviously, it trades the storage cost for communication efficiency.
- Status s: Binary bit; s = 1 means that the corresponding RFID tag has been processed; otherwise not. We call an entry is unmarked if its value is zero.

Reader initialization. When the  $P_i$  is to handover a batch of tagged products to  $P_{i+1}$ , it updates the tag temporal secrets and informs TA that  $P_{i+1}$  is the next party. Then TA will distribute the database  $D_{i+1}$  to  $P_{i+1}$  through a secure channel (e.g., SSL).

# 3.2 Tag Reading

Upon receiving  $D_i$  from TA and the tagged products from  $P_{i-1}$ , supply chain partner  $P_i$  can read any tag  $T_j$  in either the strong security mode if switch is on or in the weak security mode if switch is off.

- 1.  $R_i \to T_j {:}~r_1,$  where  $r_1$  is a random number of length  $\ell$  generated by the reader.
- 2.  $T_j \to R_i$ : On receiving  $r_1$ , the tag sends the reply  $(r_2, x)$  to the reader, where  $x = H(r_1||r_2||\beta_{i \leftrightarrow j})$  and  $r_2$  is a number of length  $\ell$ . If the tag is in strong security mode,  $r_2$  is a fresh random number generated by the tag; otherwise,  $r_2 = 0$ .

The tag reading protocol is illustrated in Figure 2. In the weak security mode,  $R_i$  can pre-compute the response of each tag  $x_j$  and store them in  $D_i$ . On receiving a response x,  $R_i$  identifies the tag if it can find a record  $d_j = \langle \beta_{i \leftrightarrow j}, x_j, s_j, switch_j \rangle$  in  $D_i$  such that  $x_j = x$  and  $s_j = 0$ . In the strong security mode, however, the response of each tag cannot be pre-computed due to the use of fresh tag random number; the value of x is set to NULL in each tuple of  $D_i$  in this case. Given a response  $(r_2, x)$ , the reader identifies a tag by searching all of the unmarked (s = 0) tuples in  $D_i$  until it finds a tuple  $d_j$ 

Reader $R_i$ $[D_i]$		$\operatorname{Tag}_{[\beta_{i \leftrightarrow j}]} T_j$
Chooses $r_1 \in R$ .	<i>r</i> 1	Sets $r_2 = 0$ if $T_j$ is in the weak
Searches a value of $\beta$ in $D_i$ that $H(r_1    r_2    \beta) = x$ .	<i>r</i> <sub>2</sub> , <i>x</i>	security mode, else chooses $r_2 \in R$ ; computes $x = H(r_1    r_2    \beta_{i \leftrightarrow j})$ .
if found, then tag is identified; else, the identification falls.		

Fig. 2. Tag reading protocol

which satisfies  $x = H(r_1||r_2||\beta_{i \leftrightarrow j})$ . For each identified tag  $T_j$ , the reader sets its status  $s_j = 1$ . If  $p_j$  is not empty, the reader can also obtain relevant product information following the pointer  $p_j$ .

The above reading process can be performed multiple times by supply chain partner  $P_i$  if necessary.

#### 3.3 Security Mode Switching

Once the party  $P_i$  receives  $D_i$  from TA, it can change the security mode of its tags in different environments. Although the strong security mode is secure against both active attacks and passive attacks, the RFID tags can be processed more efficiently in the weak security mode in an environment where the active attacks are impossible. We design a security mode switching protocol below.

- 1.  $R_i \to T_j$ : To update the security mode of tag  $T_j$ , the reader chooses a fresh random number  $r_3$  and computes  $a = \beta_{i \leftrightarrow j} \oplus r_3$ , and  $b = H(switch_0||a||r_3)$ , where  $switch_0$  is the new value of switch. The reader then sends the triple  $(switch_0, a, b)$  to the tag.
- 2.  $T_j \to R_i$ : When tag  $T_j$  receives  $(switch_0, a, b)$ , it computes  $r_3 = \beta_{i \leftrightarrow j} \oplus a$ , and checks whether  $b = H(switch_0||a||r_3)$  holds; if so, it updates  $switch = switch_0$ . After update of switch value, the tag  $T_j$  will send a confirmation  $(r_2, x)$  back to the reader, where  $r_2$  is generated based on the switch value and  $x = H(r_2||r_3||\beta_{i \leftrightarrow j})$ .
- 3.  $R_i$ : Upon receiving  $(r_2, x)$ , the reader  $R_i$  confirms the update of *switch* by checking whether  $x = H(r_2||r_3||\beta_{i \leftrightarrow j})$ .

The protocol is also illustrated in Figure 3. Since a tag will send a confirmation to the reader after security mode update, any failure can be detected by the reader.

Reader $R_i$ $[D_i]$		$\operatorname{Tag}_{[\beta_{i \leftrightarrow j}]} T_j$
Chooses $r_3 \in R$ , computes $a = \beta_{i \leftrightarrow j} \oplus r_3$ , $b = H(switch_0   a  r_3)$	$switch_0, a, b$	Computes $r_3 = a \oplus \beta_{i \leftrightarrow j};$ if $b = H(switch_0    a    r_3),$
$\mathbf{f}_{n} = H(\theta, \  \ _{n} \ _{n})$	$r_2, x$	switch $\leftarrow$ switch <sub>0</sub> , choose $r_2 \in R$ , computes $x = H(\beta_{i \leftrightarrow j}    r_2    r_3)$ ,
If $x = H(\beta_{i \leftrightarrow j}    r_2    r_3)$ , the security mode switching secesses; else, it fails.		

Fig. 3. Security mode switching protocol

### 3.4 Ownership Handover

Ownership handover is performed between two supply chain parties  $P_i$  and  $P_{i+1}$  with RFID tags in weak security mode without TA's active involvement. Before the handover,  $P_i$  will update the temporal secrets of its tags and informs TA, who will send  $D_{i+1}$  to  $P_{i+1}$  in a secure manner. Note that the database  $D_{i+1}$  is not needed during the handover process.

In order to prevent the tagged products from being tracked by party  $P_i$  after ownership handover, the tag's temporary secret must be updated from  $\beta_{i \leftrightarrow j}$ to  $\beta_{i+1 \leftrightarrow j}$ . This updating process is performed by  $P_i$  before handover. Without being appropriately updated, a tag will not be accepted by  $P_{i+1}$  in the handover process (see below). The update of tag temporal secrets guarantees that only  $P_{i+1}$  can access the tags although the update is conducted by  $P_i$ . This is under the assumption that  $P_i$  cannot get access to tag root secrets nor the new database  $D_{i+1}$ . The temporary secret updating protocol is shown in Figure 4. During ownership handover, both parties need to agree upon a list of the tagged products and report the agreed list to TA for supply chain visibility.

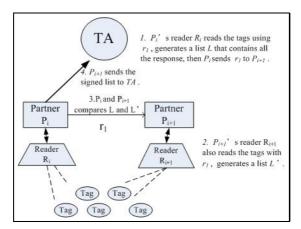
Reader $R_i$ $[D_i]$		$\operatorname{Tag} T_j \\ [\beta_{i \leftrightarrow j}]$
Chooses $r_3 \in R$ , computes $a = \beta_{i \leftrightarrow j} \oplus r_3$ , $b = H(a    r_3)$ .	<i>a</i> , <i>b</i>	Computes $r_3 = a \oplus \beta_{i \leftrightarrow j}$ ; if $b = H(a \  r_3)$ , computes $\beta_{i+1 \leftrightarrow j} = H(\alpha_j \  \beta_{i \leftrightarrow j})$ .

Fig. 4. Temporary secret updating protocol

During ownership handover, both parties need to agree upon a list of the tagged products and report the agreed list to TA for supply chain visibility.

- 1. For a batch of tagged products to be handed over to  $P_{i+1}$ ,  $P_i$  performs tag reading protocol with the same random number  $r_1$  of length  $\ell$  and records a list L of responses  $x_j$  from all tags in the batch, where  $x_j = H(r_1||r_2||\beta_{i+1\leftrightarrow j}), r_2 = 0$ . Then,  $P_i$  sends  $r_1$  to  $P_{i+1}$ .
- 2. Upon receiving  $r_1$ ,  $P_{i+1}$  performs the reading protocol with the same random number  $r_1$  and records a list L' of all responses  $x'_j$  from all tags in the batch, where  $x'_i = H(r_1 || r_2 || \beta_{i+1 \leftrightarrow j})$ .
- 3.  $P_i$  and  $P_{i+1}$  compares the two list L and L'. If the lists match, then both sign on the matched list with the current time-stamp and keep a copy of the signed list. Party  $P_i$  sends the signed list to the TA for supply chain visibility. If the two lists do not match, the two parties will settle the disagreement till they reach an agreement. After the handover process,  $P_{i+1}$  should switch the tags into the strong security mode if  $P_i$  is still around.

The ownership handover process is illustrated in Figure 5. Note that  $P_i$  is responsible to report to TA since it is for  $P_i$ 's interest to finalize the handover process as early as possible. TA is responsible to coordinate the handover process and manage the supply chain visibility accordingly. Our system remains secure even the tags are set to the weak security mode in the ownership handover process. The reason is that the tagged products remain static in this process; there is no point to track tags while they are not moving. After ownership handover, the tags are in  $P_{i+1}$ 's control, who will keep the tags secure by switching to appropriate security modes. If  $P_i$  has not updated some tags appropriately before ownership handover due to de-synchronization attacks or communication errors, both parties will detect the mismatch;  $P_i$  can re-update the tags to facilitate the handover. Therefore, our solution has the de-synchronization resilience property.



**Fig. 5.** Ownership handover process between  $P_i$  and  $P_{i+1}$ 

# 4 Analysis

In this section, we analyze our protocols with respect to the requirements of security, visibility, and efficiency. We summarize our analysis results in the form of statements. Proofs and explanations of the statements will be given in the full version of this paper.

**STATEMENT 4.1** (Authoritative access to RFID tags). Only a valid reader with a tag's temporary secret authorized by TA is able to conduct reading and updating on the tag successfully.

**STATEMENT 4.2** (Authenticity of tags). Given two numbers  $r_1$  and  $r_2$ , the probability for an adversary without the knowledge of  $\beta_{i \leftrightarrow j}$  to find a valid response x such that  $x = H(r_1||r_2||\beta_{i \leftrightarrow j})$  holds is  $\frac{1}{2\ell}$ , where  $\ell$  is the length of  $\beta_{i \leftrightarrow j}$ .

**STATEMENT 4.3** (Weak unlinkability). Given a response  $x_1$  from a tag prior to being processed by party  $P_i$  and a response  $x_2$  from a tag after being processed by  $P_i$ , it is computationally infeasible for a rogue reader to determine whether  $x_1$  and  $x_2$  is from the same tag.

**STATEMENT 4.4** (Strong unlinkability). A tag is strong unlinkable in the strong security mode. It is also strong unlinkable in the weak security mode in an environment of no active attacks.

**STATEMENT 4.5** (Forward and backward secrecy). If the protocol communication between a tag and a reader is compromised in certain party, it will not affect the security of the protocol communication between the tag and the reader in any other parties.

**STATEMENT 4.6** (De-synchronization resilience). The temporary secret updating protocol and the security mode switching protocol in our security solution are resilient to de-synchronization attacks.

**STATEMENT 4.7** (Visibility). While unauthorized entities are prevented from tracking the movement of material flow, authorized entities have access to the information about where and when a tag is processed.

**STATEMENT 4.8** (Efficiency). In the weak security mode, the time complexity for an authorized reader to identify a batch of n tags is  $O(n \log n)$ . In the strong security mode, the time complexity is  $O(n^2)$ .

The bottleneck of most RFID-enabled supply chain systems including ours is the process of identifying a large number of tags by each reader. According to [16,117], we roughly estimate that it takes about 210 CPU cycles of a Pentium CPU to perform a hash function (e.g., SHA-1) for digesting a 128-bit message and about 40 CPU cycles per sort operation for merge-sort or quick-sort algorithms. We assume that there are  $2^{20}$  tags in each batch, and a 1-GHz Pentium machine is used in each reader's servant computer. In the weak security mode, it requires about 800ns in database search for identifying each tag. In the strong security mode, however, the batch size is better below  $10^4$  so that a reader can identify about 500 tags per second. Since an RFID reader usually can perform about 100 times reading, the speed of searching tags in database is sufficiently fast enough as it is higher than 100 tags per second.

### 5 Related Work

A bundle of research papers addressing RFID security and privacy problems have been published (refer to [4] for a detailed literature survey) in recent years. Among them, tens of (privacy enhanced) tag/reader mutual authentication protocols have been proposed in the literature such as the HB family of RFID protocols [5,6], symmetric cipher based protocols [7,8,9], and lightweight primitive based protocols [10,11]. Unfortunately, all of these RFID protocols have focused on protecting tag to reader communication in a single domain, where no cross-domain relationship is considered among various supply chain parties. Therefore, their techniques cannot be directly used to ensure the information sharing protocols interacting multiple supply chain parties.

In this section, we compare our solution with the most related works **12.14.13** on RFID authentication protocols in supply chain environments. We realize that these works might have been attacked in some way **15**, their original ideas are still meritable and worth being reviewed. A summary of our comparison is given in Table 1.

In [12], Li and Ding proposed a de-centralized solution for secure RFID communications in supply chains. In their solution, an access key is shared between each tag and each supply chain party. The access key of a tag can be updated by the current supply chain party before the tag is handed over to the next supply chain party. Since the updated access key is shared between the current supply chain party and the next supply chain party, their solution is vulnerable to insider attacks without backward or forward secrecy. Their solution is similar to our solution in the weak security mode in a sense that only weak unlinkability is provided. The time complexity of their solution is also similar to our solution in the weak security mode, which is  $O(n \log n)$  for processing each batch of tags. Since there is no trusted authority involved in their solution, the supply chain visibility should be maintained by each party's database in a distributed manner.

Juels, Pappu, and Parno proposed an interesting solution for secure RFIDenabled supply chains **[13]**. In their work, a secret sharing method is used to break a secret key to multiple shares, with each share stored in a single tag along with the cipher of the tag id encrypted with the secret key. An authorized supply chain party, which is supposed to get access to a large number of tags can collect enough shares to recover the secret key, and thus decrypt the tags' IDs. An adversary is assumed to have limited access to the tags; thus, he or she cannot recover the secret key nor decrypt any tags' IDs. It is clear that this solution does not have any unlinkability feature. Anyone can track the movement of a tag even if it is encrypted. The advantage of this solution is that it can be directly used with the current EPC Gen2 tags **[1]** without any cryptographic extensions; therefore, the cost of tags is apparently lower than other solutions which have to incorporate hash and random number generation computations in tags.

Song proposed an RFID ownership transfer protocol recently 14. In her solution, a supply chain party and a tag share a couple of secrets (t, s), where t = h(s). The tag stores the value t, and the reader authorizes itself to the tag

	Unlinkability	Visibility	Efficiency	Cost
	(anti-tracking)			(tag)
12	Weak	Distributed	Batch process	Moderate
13	Null	Distributed	Decryption	Low
14	Strong	Distributed	Tag by tag	Moderate
Our solution	Strong	Centralized	Switch	Moderate

Table 1. Comparison of our solution with three other solutions

by proving its possession of s. In ownership handover, the current party must be online to help the next party to identify a tag; then, the current party will send the tag secret to the next party, which will use the secret information to update the tag secret to a new value. To provide strong unlinkability, each tag will generate its reply based on a fresh random number, which is similar to our solution in the strong security mode. The weakness of this solution is its low efficiency, especially in the handover process which takes  $O(n^2)$  time for processing n tags one by one.

Comparing to the above works, our proposal solution is the only solution that involves a trusted authority. Therefore the supply chain visibility can be easily maintained in a centralized manner. On the one hand, our solution provides strong unlinkability under the assumption that the weak security mode is used in a relative secure environment of no active attacks. On the other hand, it can switch to the weak security mode for higher efficiency in tag reading. It thus provides higher efficiency in certain environment without downgrading the security features. In terms of tag cost, our solution is similar to [12,14] as it involves random number generation and hash computations in tags. Note that our solution is suitable for the RFID tags of cost around US\$0.5 and RFID reader of cost around US\$1000. Such RFID readers and tags are currently available in the market and their costs are affordable in supply chain management at container, pallet, or case level (probably not at the item level).

# 6 Conclusion

In this paper, we investigate the security, visibility, and efficiency issues for RFID-enabled supply chain systems. High efficiency is particularly desirable in RFID-enabled supply chains since a large quantity of tagged products are routinely processed and exchanged among multiple supply chain parties such as suppliers, manufacturers, distributors, and retailers. In order to enhance the efficiency of a RFID-enabled supply chain system without sacrificing its security, we distinguish the environments into two secure levels. In a relatively secure environment with no active attacks, our RFID system can be set to the weak security mode so as to provide high processing speed. While in a relatively less secure environment that is exposed to active attacks, our RFID system can be switched to the strong security mode so as to maintain strong unlinkability. In the future, we are interested in investigating the scalability of our solution and verifying its practicality in real EPCglobal network or simulated environments. Acknowledgment. This work is partly supported by A\*Star SERC Grant No. 082 101 0022 in Singapore.

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# Achieving Better Privacy Protection in Wireless Sensor Networks Using Trusted Computing

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**Abstract.** A wireless sensor network (WSN) is an ad-hoc wireless network composed of small sensor nodes deployed in large numbers. Sensor nodes are usually severely resource limited and power constrained. Security enforcement in WSNs is thus a challenging task. In this paper we propose a clustered heterogeneous architecture for WSNs, where highend cluster heads are incorporated, and they are further equipped with trusted computing technology (TC). As such, the cluster heads act as trusted parties, and are expected to help effectively address privacy issues in WSNs. As concrete examples, we discuss in details how user query privacy and source location privacy can be better protected.

# 1 Introduction

A WSN consists of a large number of sensor nodes collecting environmental data. The nodes are usually severely constrained in computation, storage, communication and power resources. When deployed in critical applications, mechanisms must be in place to secure a WSN. Security issues associated with WSNs can be categorized into two broad classes [24]: content-related security, and contextual security. *Content-related security* deals with security issues related to the content of data traversing the sensor network such as data secrecy, integrity, and key exchange. Numerous efforts have recently been dedicated to content-related security issues, such as secure routing [16]17]22[32], key management and establishment [5]8]10]25]26]34]37], and data aggregation [6]15]27]33]. In many cases, it does not suffice to just address the content-related security issues. Suppose a sensitive event triggers a packet being sent over the network; while the content of the packet is encrypted, knowing which node sends the packet reveals the location where the event occurs. *Contextual security* is thus concerned with protecting such contextual information associated with data collection and transmission.

It is commonly acknowledged that the resource-constrained nature of sensor nodes makes security enforcement in WSNs a challenging task. The majority of the above mentioned efforts attempted to solve security issues in *homogeneous* WSNs where all sensor nodes have the same capabilities. However, both theoretical and empirical studies have concluded that homogeneous WSNs are not scalable. Through a theoretical analysis it is shown in **[12]** that the throughput of each sensor node decreases rapidly as the numbers of nodes increases; in

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addition, it is demonstrated via simulations in [9] that, as the traffic becomes heavy, the overhead due to routing and control consumes a large portion of the available bandwidth.

*Our Contribution.* To improve the effectiveness of security enforcement, we present a trusted computing (TC)-enabled clustered heterogeneous WSN architecture, composed of not only resource constrained sensor nodes, but also a number of more powerful high-end devices acting as *cluster heads*. Compared to sensor nodes, a high-end cluster head has higher computation capability, larger storage, longer power supply, and longer radio transmission range, and it thus does not suffer from the resource scarceness problem as much as a sensor node does. A distinct feature of our heterogeneous architecture is that cluster heads are equipped with trusted computing (TC) technology, and in particular a TCG-compliant TPM (Trusted Platform Module) [35] is embedded into each cluster head. The TC-enabled cluster heads act as online *trusted parties*; security enforcement is thus expected to be substantially simplified and improved. We further substantiate the above assertion by demonstrating how trusted cluster heads help to provide elegant solutions for two important contextual security problems, user query privacy [7] and source location privacy [24]28[29]31].

### 2 TC-Enabled Heterogeneous Architecture for WSNs

We partition a WSN into a number of *clusters*. A high-end device is placed into each cluster, acting as the *cluster head*. In contrast to sensor nodes, high-end cluster heads have relatively higher computation capability, larger storage size, and longer radio range. They also have longer power supply, and in some circumstances they can even be line-powered. Therefore unlike sensor nodes, cluster heads do not drastically suffer from the resource scarceness problem. Depending on applications, hardware capabilities of cluster head may vary from that comparable to a bluetooth device to that of a high end PDA. The introduction of high-end cluster heads into a WSN makes the once homogeneous network *heterogeneous*. The general heterogeneous architecture is depicted in Figure []

Downlink communication (from base station to sensor nodes) and uplink communication (from senor nodes to base station) in the architecture are asymmetric. Messages broadcast by the base station can directly reach sensor nodes, whereas messages sent by a sensor node need to be forwarded by its corresponding cluster head. As a result, uplink communication follows a hierarchical manner and consists of intra-cluster and inter-cluster communications, respectively. At the level of *intra-cluster communication*, a cluster head acts as a gateway for the sensor nodes within the cluster; a sensor node can reach the cluster head directly, or through a *short* multi-hop channel. *Inter-cluster communication* is concerned with communication among cluster heads and the base station. Since inter-cluster communication does not suffer from the limits upon sensor nodes, it can utilize more advanced communication infrastructure, e.g. 802.11. Consequently, the heterogeneous architecture is expected to enormously improves the overall system performance and lifetime of the network.

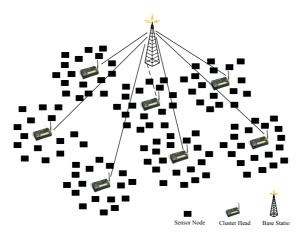


Fig. 1. Heterogeneous Wireless Sensor Network

A feature distinguishing our architecture from other similar heterogeneous ones (e.g., **18,19,36**) is that we equip each cluster head with a TPM, so as to simplify and improve security enforcement in WSNs. The rationale is that under the auspices of TPM, cluster heads can act as online *trusted parties* in enforcing security mechanisms. Relevant entities, e.g., the base station or users who query the network (see Section 4.1), can ascertain the trustfulness of cluster heads by means of attestation. We notice that the authors of **13,21** also discussed the idea of applying trusted computing technology to equipping more powerful cluster heads within WSNs, and there should be no distinction in the network architecture between ours and theirs. However, in their proposals, sensor nodes are responsible for verifying the platform state of the cluster heads through attestation, whereas our proposal is that sensor nodes never challenge their cluster heads for attestation (we believe they actually do not afford to do so), and they simply trust their respective cluster heads. Our argument relies on the fact that it is the end users (or the owner) of the network who should be concerned with the trustfulness of the cluster heads and the network.

## 3 Addressing Privacy Issues in WSNs

To show the effect of the trusted cluster heads on security enforcement, we present solutions to two important contextual security problems in WSNs: user query privacy and source location privacy. Compared to existing solutions in [7]24,28,29,31, our schemes achieve better privacy and higher efficiency.

Adversary Model. For the schemes presented below, we assume a global eavesdropper who eavesdrops on the entire network. In particular, the adversary is able to watch all the traffic traversing the sensor network. An adversary of this nature poses a great threat, especially to contextual security in WSNs. The global adversary in our consideration is much stronger than the *local eavesdrop*per assumed by other work such as [7]24,29,31]. A local eavesdropper only has knowledge on the sensor node it stays with, and it can only trace communication hop by hop. We must stress that the privacy offered by the schemes in [7]24,29[31] is immediately broken under the global adversary, because through global eavesdropping the adversary can always know which node(s) initiates the targeted data transmission.

#### 3.1 Achieving User Query Privacy

**Problem Statement.** WSNs are often deployed to provide services to other users than the network owner [7]. Users are allowed to query a network to get sensed data from particular areas. In such a scenario, a user may wish to protect her "areas of interest" from being disclosed to other users or even the network owner. User query privacy is thus concerned with the following problem: suppose a user queries the network, intending to get the sensed data in cluster  $c_i$ , a user query privacy scheme ensures that the user ends up getting the desired data, but the adversary does not learn  $c_i$  by observing the communication.

#### Our Algorithm

Network Model. We support roaming users querying a wireless sensor network. The network follows the heterogeneous architecture proposed earlier: the whole network is partitioned into a set of n clusters,  $c_1, c_2, ..., c_n$ , where  $c_i$  is the identifier of the *i*th cluster; each cluster is grouped around a TC-enabled cluster head and we denote  $ch_i$  the cluster head in  $c_i$ . A user who desires to query the network first contacts the nearest cluster head is called *access point*. Taking Figure 2 as an example,  $ch_1$  of  $c_1$  is the access point for the user. Once the access point is determined, all the other cluster heads need to dynamically establish routing pathes to the access point. The single-path routing in [24] can achieve this objective. The single-path routing uses a greedy algorithm where each node

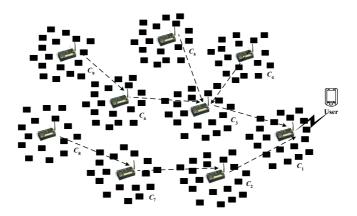


Fig. 2. User Querying Scenario

chooses one of its neighboring nodes that is nearest to the sink (i.e., the access point in our case) as its *forwarding node* in data transmission, and the node itself is a *dependent node* of the forwarding node. As a result, the routing pathes form a tree rooted at the access point. In Figure 2 the routing pathes are shown in dotted lines. After the routing pathes are formed, every cluster head knows which node is its forwarding node, and which nodes are its dependent nodes.

Assumption. To obtain services from a WSN, a user is assumed to have a certain means to authenticate to the WSN. Also, a TC-enabled cluster head can authenticate to users using the AIK of its embedded TPM/MTM. Therefore, the access point and the querying user can accomplish mutual authentication, based on which we assume the two entities share a secret key for semantic secure symmetric encryption. Further, it is also easy for each cluster head to get this secret key from the access point, since they can clearly authenticate each other with the help of their respective AIKs. We denote  $\mathcal{E}_{ch_i}(.)$  and  $\mathcal{D}_{ch_i}()$  the encryption and decryption, respectively, by  $ch_i$  using this secret key. We also assume each cluster head shares a secret cluster key (for semantic secure symmetric encryption) with all sensor nodes in its cluster. Several well studied key exchange schemes [26] can achieve this objective.  $\mathcal{CE}_{c_i}(.)$  and  $\mathcal{CD}_{c_i}(.)$  denote the encryption and decryption, respectively, using the cluster key of  $c_i$ .

Algorithm Overview. A straightforward way to achieve query privacy is that every cluster head sends encrypted data to the access point, who then forwards only the data desired by the user. This however unnecessarily wastes communication bandwidth among cluster heads. Even for this straightforward method, we should still be very cautious not to leak information about the queried cluster from the size of the data returned to the user. More specifically, clusters normally have different number of sensor nodes, so data from different clusters are likely to have different lengthes. Let us suppose the data from each sensor node forms a packet for simplicity. The total number of packets from a cluster equals the number of sensor nodes. Without privacy treatment, the number of packets eventually returned to the user by the access point would clearly indicate the cluster from which the data originates. A method to fix this problem is that regardless of which cluster is queried, the access point returns a fixed-number of packets, corresponding to the biggest cluster size. We use l to denote this number thereafter. For a cluster whose size is smaller than l, dummy packets are generated.

The basic idea of our approach is to only transmit packets from the queried cluster to the access point. However, to hide the target cluster, all the remaining clusters have to generate *fake* data transmission of the same pattern as that of the queried cluster. To better convey our idea, let us continue to use Figure 2 as an example. Suppose  $c_6$  is the target queried cluster. The data transmission starts with  $ch_9$ ,  $ch_8$ ,  $ch_5$  and  $ch_4$ , who are the tails of the respective routing pathes. Let us however only explain the transmission involving the target cluster  $c_6$ , as others follow the same manner. The cluster head  $ch_9$  generates and sends l dummy

packets  $\mathcal{E}_{ch9}^{l}(dummy\_data)$  to  $ch_{6}$ . We use  $\mathcal{E}_{ci}^{l}(data)$  to denote l packets of encryption of data by  $ch_{i}$ .  $ch_{6}$  knows that its cluster is being queried, so he discards the data from  $ch_{9}$ , and generates and passes  $\mathcal{E}_{ch_{6}}^{l}(sensed\_data)$  to  $ch_{3}$ . At this point of time,  $ch_{3}$  may have already received  $\mathcal{E}_{ch_{5}}^{l}(dummy\_data)$  and  $\mathcal{E}_{ch_{4}}^{l}(dummy\_data)$  from its dependent nodes  $ch_{5}$  and  $ch_{4}$ , respectively. If not,  $ch_{3}$  waits until it gets the data from all its dependent nodes. The reason why a node does not proceed until receives from all its dependent nodes is to avoid disclosure of information from timing patterns. Then  $ch_{3}$  decrypts  $\mathcal{E}_{ch_{6}}^{l}(sensed\_data)$  and re-encrypts sensed\\_date to yield  $\mathcal{E}_{ch_{3}}^{l}(sensed\_data)$ , it then passes the data to the access point  $ch_{1}$ . The access point  $ch_{1}$  waits until it gets data from all other routing pathes, and then decrypts  $\mathcal{E}_{ch_{3}}^{l}(sensed\_data)$  and re-encrypts sensed\\_data). Finally,  $ch_{1}$  sends  $\mathcal{E}_{ch_{1}}^{l}(sensed\_data)$  to the querying user as the query result. In our approach, every cluster head sends out l packets. Due to the semantic security of encryption, re-encryptions of the same data are not distinguishable. Therefore, the adversary watching the network cannot tell if the l packets sent out by a cluster head originate from the cluster head itself or from its dependent nodes.

Algorithm Details. A complete description of the algorithm in pseudo codes is shown in Algorithm  $\square$  where u denotes the querying user and ap denotes the access point.

To start, the user contacts the access point by sending a *hello* message, including a *nounce* that will be used in the ensuing attestation process (Step 1). The access point then informs all the other cluster heads to form routing pathes using the method described earlier (Step 2). Before sending a query, the user must have assurance of the trustfulness of the cluster heads. This is achieved by means of attestation (Step 3). Note that it is unnecessary for the user to check the status of all cluster heads, which is quite expensive; it suffices to adopt the strategy of "chained attestation" along the established routing pathes. In particular, referring to Figure  $\mathbf{2}$ , u only verifies the access point:  $ch_9$  is verified by  $ch_6$ ;  $ch_4$ ,  $ch_5$  and  $ch_6$  are verified by  $ch_3$ ;  $ch_8$  is verified by  $ch_7$  who is in return verified by  $ch_2$ ;  $ch_2$  and  $ch_3$  are verified by the access point. Once attestation is successful, the user sends to the access point the query  $e_u$ , which is the encryption of the identifier c of the target cluster using the shared secret key (Step 4). The access point broadcasts the query to all other cluster heads (Step 5), each decrypting the query and knowing which cluster the user is querying (Step 7). Each cluster head then collects sensed data (encrypted using the cluster key) from the sensor nodes of its cluster (Step 8).

Before sending out l packets of data to its forwarding node (Step 29), a cluster head must wait until it receives packets from all its dependent nodes (Step 9). Afterwards, if the cluster itself is the target cluster (Step 10-13), the cluster head simply ignores the packets from its dependent nodes, and encrypts the sensed data from its cluster. Note that every set of l packets consists of *head* and *content*, where the *head* is used to inform cluster heads enroute the origin of the lpackets while without decrypting the *content*. For a cluster that is not the target one (Step 14-27), the cluster head checks whether one of its dependent nodes

#### Algorithm 1. Achieving User Query Privacy.

1:  $u \rightarrow ap$ : hello 2:  $ap \leftrightarrow \{ch_i\}_i$ : establish routing pathes. 3:  $u \leftrightarrow ap$ : attestation 4:  $u \to ap$ :  $e_u = \mathcal{E}_u(c) / *c$  is the identifier of the target cluster\*/ 5:  $ap \rightarrow \{ch_i\}_i: e_u$ 6: for EACH  $ch_i$  do 7:  $c = \mathcal{D}_{ch_i}(e_u)$ 8:  $\{sensor\} \rightarrow ch_i: data_{c_i} = \{\mathcal{CE}_{c_i}(sensed\_data)\}$ 9:  $\{dependent\_node\} \rightarrow ch_i: \{packets\} / *ch_i \text{ waits until gets packets from all its}$ dependent nodes\*/ 10: if  $ch_i \in c$  then /\*if  $ch_i$  is cluster head of the target cluster c \*/ 11: 12: $packets = \mathcal{CD}_{c_i}(data_{c_i})$  $packets = head ||content = \mathcal{E}_c(c)||\mathcal{E}_{ch_i}^l(packets)|$ 13:14:else found queried cluster = FALSE15:16:for packets from EACH dependent\_node do 17: $c' = \mathcal{D}_{ch_i}(head)$ if c' == c then 18:19: $content' = \mathcal{D}_{ch_i}(content)$ 20: $packets = head ||content = \mathcal{E}_{ch_i}(c)||\mathcal{E}_{ch_i}^l(content')|$ 21:foundqueriedcluster = TRUE22:break 23:end if 24:end for 25:if found queried cluster == FALSE then 26: $packets = head ||content = \mathcal{E}_{ch_i}(c_i)||\mathcal{E}_{ch_i}^l(dummy\_date)$ 27:end if 28:end if 29: $ch_i \rightarrow forward\_node: packets$ 30: end for 31:  $ap \rightarrow u$ : packets

sends in the data of the target cluster. If yes, the cluster head re-encrypts the data (Step 16-22); otherwise, the cluster head generates l dummy packets (Step 25-27). Eventually, the access point passes the l packets of the target cluster to the querying user (Step 31).

**Improvement.** In the above scheme, to answer a user query all the sensor nodes are asked to send data to their respective cluster heads. This may shorten the lifetime of the network because of excess energy consumption. To mitigate this problem, We can alternatively trade off data freshness for energy efficiency, especially when queries come in at a high rate. In particular, sensor nodes *periodically* provide the sensed data to their respective cluster heads, who cache the data. The cluster heads then handle user queries using the cached data rather than collecting realtime data from the sensor nodes.

	Adversary	Sensor stor-	Sensor com-	Sensor com-	User regis-
	model	age	putation	munication	tration
Our scheme	Global eaves-	Constant	Constant	Constant	No
	dropper				
Scheme in 7	Local eaves-	$Linear \ to \ the$	Related to	Related to	yes
	dropper	total $number$	routing path	routing path	
		of users			

Table 1. Comparison Results

**Comparison.** The only earlier work we are aware of studying user query privacy is  $\boxed{7}$ . The two-server approach in  $\boxed{7}$  uses a routing scheme for data transmission similar to onion routing 14. The routing path is constructed dynamically among the sensor nodes for each query. We list the comparison results between our approach and that in  $\boxed{7}$  in Table  $\boxed{1}$  To be more specific, our scheme assumes a global eavesdropper, so achieving better privacy. A sensor node in our scheme only needs to store a secret cluster key, while each sensor in 7 is required to store secret data linear to the total number of users authorized to query the network. On computation and communication, each sensor node in our scheme needs to encrypt its sensed data and sends to its cluster head, or is required to relay data from other nodes in the same cluster; as the routing pathes must be short within a cluster, the computation and communication are thus constant. In contrast, computation and communication for a sensor node in 7 depend on the routing path from the target cluster to the querying user. Finally, in 7 every authorized user is required to register to the servers, which in turn store secret data for the user's virtual name on each sensor node; sensor nodes in our scheme on the contrary have nothing to do with user registration. To conclude, the comparison results show that our scheme outperforms 7 in all aspects.

#### 3.2 Achieving Source Location Privacy

Source location privacy is concerned with not letting the adversary know which sensor node sends data to a sink (the base station). Most existing approaches, e.g., [24,29,31], consider local eavesdroppers who can only trace transmission hop by hop. An exception is [28], which assumes a global eavesdropper. The commonly used methods for achieving source location privacy are random walk, source simulation, or a combination of the two. The random walk method generates a random routing path from the source node to the sink for each transmission, while the source simulation method generates a number of fake source nodes simulating the actual source sensor, so as to confuse the adversary.

It appears that the random walk method is not effective under the global adversary overseeing the entire network, since the adversary always knows from which node packets originated. Our heterogeneous network architecture facilitates a much more efficient implementation of source simulation. In particular, to simulate the actual source node sending a packet to its cluster head, every other cluster head randomly chooses a sensor node in its cluster to send a fake packet. Afterwards, cluster heads send their packets to the sink. Intuitively, source location privacy can be considered as a special case of user query privacy in our architecture. In order to keep the paper compact, we omit the details of the scheme, but it should not be difficult to specify, given the earlier scheme for user query privacy.

Source simulation in our architecture is more efficient than that in [28], due to the shortened routing pathes and reduced control messages resulting from the use of high-end cluster heads.

## 4 Related Work

Partitioning a WSN into clusters were proposed by several authors for achieving scalability and better performance **23.11.20**. In these schemes, one or more sensor nodes within a cluster are chosen as cluster head (i.e., homogeneous clustered WSNs). On the other hand, considering the limited capabilities of sensor nodes, studies from both the research community and the industry sector have tried to enhance network performance by incorporating a number of more powerful nodes in WSNs (i.e., heterogeneous clustered WSNs). A detailed theoretical analysis on the effect of adding powerful nodes to WSNs was given in <u>36</u>. It is concluded that only a modest number of reliable, long-range backhaul links and line-powered nodes are required to have a significant effect, and if properly deployed, heterogeneity can triple the average delivery rate and a 5-fold increase in the lifetime of a large battery-powered sensor networks. Intel has an on-going experimental effort **19** to incorporate Intel XScale<sup>®</sup> based nodes into WSNs. The experiment indicated that data traversing across a network are routed biased towards the XScale<sup>®</sup> nodes over simple sensor nodes, thereby indeed enhancing the overall system performance. **30,36** are among the work to study security issues in the homogeneous or heterogeneous clustered WSNs.

Our study of TC-enabled WSNs is motivated by the above idea of network clustering and heterogeneity, and TCG's trusted computing technology. The heterogeneous architecture we proposed differs from the existing heterogeneous networks such as [4,19,30,36] in that the high-end cluster heads in our WSNs are TC-enabled. This makes our WSNs not only have improved network performance, but also greatly facilitate security enforcement, as we have demonstrated earlier.

We realized that we are not the first to propose the idea of equipping some more powerful cluster heads in a WSN with trusted computing technology. [13]21] also discussed similar ideas. More specifically, [21] proposed lightweight attestation techniques that can help regular sensor nodes to check the status of a TC-enabled cluster head's platform, and [13] studied key establishment and management between sensor nodes and the base station, with the help of TC-enabled cluster heads. There seems no essential difference in the network architecture between our proposal and that of [13]21]. However, the techniques proposed in [13]21] are intended for the sensor nodes to perform attestation so as to check the platform state of the cluster heads. In contrast, in our architecture the sensor nodes never attempt to check the trustfulness of the cluster heads, and they simply trust their respective cluster heads. Our justification for this is that it is the responsibility of the end users of the network to concern about the trustfulness of the cluster heads. Other difference between our work and **[13]21** is that different security issues in WSNs were addressed.

In this work, we focused on privacy protection in WSNs, and provided better solutions to two important privacy issues than existing proposals [7]:24]:28]:29]:31]. Another privacy issue in WSNs discussed in the literature is temporal privacy [23], which is concerned with the fact that an adversary, by simply monitoring the arrival of packets at the sink, can infer the temporal patterns of interested events. While we did not address the temporal privacy issue for lack of space, we believe our TC-enabled heterogeneous architecture is in a better position to solve the problem, because the *trusted* cluster heads should be able to perform timing synchronization more reliably and easily.

## 5 Conclusion and Future Work

Due to stringent resource limitations of sensor nodes, security enforcement is extremely challenging in wireless sensor networks. To solve this problem, we proposed to render a wireless sensor network heterogeneous, by incorporating TC-equipped high-end devices into clusters of the network, acting as cluster heads. We demonstrated how the TC-enabled cluster heads can effectively address privacy issues in WSNs.

This study is still in the preliminary stage. We are preparing to implement proof-of-the-concept TC-enabled WSN architecture, and further experiment with the architecture in certain real world wireless sensor network settings.

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# Trusted Privacy Domains – Challenges for Trusted Computing in Privacy-Protecting Information Sharing

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Abstract. With the growing use of the Internet, users need to reveal an increasing amount of private information when accessing online services, and, with growing integration, this information is shared among services. Although progress was achieved in acknowledging the need to design privacy-friendly systems and protocols, there are still no satisfactory technical privacy-protecting solutions that reliably enforce userdefined flexible privacy policies. Today, the users can assess and analyze privacy policies of data controllers, but they cannot control access to and usage of their private data beyond their own computing environment. In this paper, we propose a conceptual framework for user-controlled formal privacy policies and examine elements of its design and implementation. In our vision, a Trusted Personal Information Wallet manages private data according to a user-defined privacy policies. We build on Trusted Virtual Domains (TVDs), leveraging trusted computing and virtualization to construct privacy domains for enforcing the user's policy. We present protocols for establishing these domains, and describe the implementation of the building blocks of our framework. Additionally, a simple privacy policy for trusted privacy domains functioning between different organizations and entities across networks is described as an example. Finally, we identify future research challenges in this area.

### 1 Introduction

Global connectivity and easy access to distributed applications and digital services over the Internet changed the paradigm of both business and consumer use of information. The Internet offers new opportunities to individuals, e.g., e-commerce and social network services. In addition to personal computers, mobile devices, such as smart phones, allow users to access numerous services through mobile networks from any location.

Together with the new opportunities, new security threats also developed, rapidly growing in number and sophistication. Some security threats, such as identity theft, one of the fastest growing crimes on the Internet, also can cause

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privacy violations [12]. But privacy issues are much broader: individuals frequently generate and reveal a significant amount of personal and sensitive information when they use a service such as online shopping or social networking. Even if a transaction is not personalized, it always leaves a trail that can be aggregated with other information and analyzed, potentially leading to privacy leaks. Also, as devices access networks and services, information about these accesses can be recorded. The users have to trust the application provider to treat their personal data in an appropriate manner, e.g., according to best practices and regulatory requirements reflected in privacy policies. The users can read statements about privacy policies on websites, but the policies do not allow for flexibility in disclosing data necessary to access the service. There are few technical means to support this kind of enforcement. Ideally, the users should be able to grant access to their sensitive information only when the systems are trustworthy and should be allowed to revoke this permission.

Technical measures in the areas of modern IT security and cryptography provide only partial solutions. Because of the inherent vulnerabilities resulting from high complexity of systems, common computing platforms require careful and attentive system administration skills, and complete protections against execution of malicious code and tampering is impossible.

In this paper, we propose a conceptual framework for user-controlled privacy policies and examine first elements of its design and implementation. The goal is to improve the current status of data and privacy protection by supporting legal measures with novel technical solutions based on Trusted Computing (TC) as described below:

- We outline a general approach to creating privacy domains, in which a guardian agent (Trusted Personal Information Wallet) manages private data according to a user-defined privacy policy (Section 2). The agent can migrate to other platforms, but only in approved trusted domains.
- We describe a simple policy that requires trusted privacy domains between different organizations and entities. We build on the idea of Trusted Virtual Domains (TVDs), leveraging trusted computing and virtualization to automatically construct privacy domains for enforcing the user's policy. We describe protocols for establishing these domains and the implementation of the building blocks of the framework (Section 3).
- Finally, we address future research challenges, analyzing currently available policy languages that cannot yet support full solutions for the reliable enforcement of user controlled privacy (Section 4).

<sup>&</sup>lt;sup>1</sup> Revealing private information is sometimes necessary or unavoidable outside of the Internet (e.g., in supermarkets, due to surveillance, etc.). Although we do not study these methods to gain information about individuals, we note that the revealed information inside and outside the Internet can potentially be linked.

<sup>&</sup>lt;sup>2</sup> Auditing and certification are examples for at least some technology-related methods, e.g., product evaluation according to Common Criteria or certification according to ISO 27001/27002 for information security management systems in enterprises.

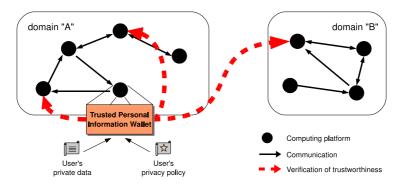


Fig. 1. Basic idea of the overall architecture

## 2 Framework for Privacy Domains

We propose to support the enforcement of privacy policies by establishing trusted domains. These policies enable the user (individual or organization) to specify fine-grained instructions for the use of private information. As the level of online activities increases and entities or organizations with complex rules interoperate, the policies may become very complex and benefit from automatic enforcement.

The proposed architecture provides mechanisms to protect sensitive and private information across IT domains and systems. The deployment of Trusted Computing technologies for privacy protection can help achieve this goal. To ensure that private information is not re-distributed to unauthorized parties, it needs to be technically bound to only those receivers that are known to comply with the policies. Communication endpoints need to attest reliably to their compliance to specified policies.

To enforce policies, we propose a "guardian agent" for the user: a *Trusted Personal Information Wallet* that is transferable between platforms and performs "verification" of the trustworthiness of a remote IT system, i.e., compliance to a specified policy. The verification helps guarantee the enforcement of the user's privacy policy when sensitive information is transmitted. Figure **1** shows an abstract illustration of the proposed concept.

In order to achieve technical enforcement of the security and privacy policies, we develop a security architecture that allows the user to share sensitive information between computing platforms while ensuring the participating platforms have technical means to comply with the policies.

Figure 2 shows a high-level view of the process of policy enforcement. A privacy policy in a machine-readable format is incorporated into the wallet. (step 1). The wallet interprets the policy and configures security and privacy services of the underlying computing platform (step 2). The security services enforce the policy by controlling communication between applications in different domains (step 3). To reliably enforce the policy, trusted security & privacy services have to run on all participating platforms, e.g., based on a security-enhanced hypervisor **3**, which allows the system owners to use legacy applications and operating

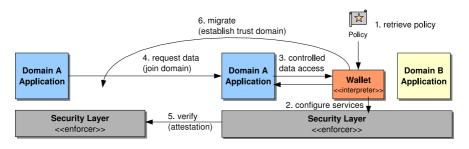


Fig. 2. Envisioned architecture for policy enforcement

systems in virtual machines, eliminating the need for new client and server side applications.

For data transmission, we propose new protocols based on existing attestation schemes of TC technology. When a user or application agent of another platform requests to access sensitive information (step 4), the security services of the source platform first verify the trustworthiness of the target platform using attestation mechanisms (step 5) to ensure the destination provides the required security mechanisms to enforce the policy. After successful verification, the wallet migrates to the destination platform (step 6) in order to act as policy decision module and to configure the security services of the target to enforce the defined policy. Service providers do not need to implement additional functionality on their server side (except for the underlying security layer) to interpret the policy or a clearinghouse for the policy interpretation. The wallet will interpret the policy and use the underlying security services of each platform to enforce it.

### 3 Experience with Trusted Virtual Domains

As a first step towards realizing privacy domains and policy enforcement as described before, we employ the concept of Trusted Virtual Domains (TVDs) [45]. In this section, we briefly review this concept and describe its novel application as privacy policy enforcement as well as our implementation of TVDs.

### 3.1 Concept of TVDs

A Trusted Virtual Domain (TVD) is a coalition of virtual and/or physical machines that can trust each other based on a security policy that is uniformly enforced independently of the boundaries of physical computing resources. It leverages the combination of TC and virtualization techniques in order to provide confinement boundaries for an isolated execution environment — a domain — hosted by several physical platforms.

A TVD-enforcing system supports the creation of virtual networks on physical or virtual systems. Members of a TVD can "see" and access other TVD members, but it is closed to non-members. Different instances of several TVDs can

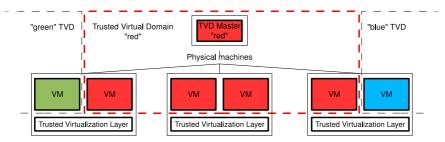


Fig. 3. Conceptual view of trusted virtual domains (TVDs)

execute on the same physical platform because the underlying virtual machine monitor isolates virtual machines of different TVDs in separate compartments and isolated virtual networks.

Figure 3 shows an example of three TVDs (identified by colors) distributed over different physical machines. The decision whether a virtual or real machine is allowed to join the TVD is enforced based on a *TVD policy*. A special node in the TVD (*TVD Master*), e.g., implemented as a central server, controls the access to the TVD by following the admission control rules specified in the TVD policy. These rules include integrity measurements of the platforms and virtual machines that are allowed to join the domain. TC technology is used to establish trust in the reported measurements, e.g., following the Trusted Computing Group (TCG) approach, hash values of the software boot stack (BIOS, bootloader, virtualization layer as well as loaded virtual machines) are stored in and signed by a Trusted Platform Module (TPM) **[6]** and reported to the TVD Master during attestation. The TVD Master can reliably verify whether the reported values comply with the TVD policy and whether it can rely on the enforcement mechanisms of the local platforms.

TVDs were first proposed by Griffin et al. [4] and Bussani et al. [5]. Recent research describes secure network virtualization [7], and discusses the management of TVDs in data centers [8]. The OpenTC project has addressed some areas of implementing TVDs in the context of enterprise rights management and managing virtual data centers. A major issue is how the domain can be managed securely: individual machines must be able to join a domain only if they fulfill the requirements for joining, and the procedures for a platform to leave a domain must be securely constructed. These aspects of TVDs have not been studied in details yet. We describe the TVD establishment and join protocols and how TC functionality is used (see Section [3.3]). The idea of applying the TVD concept to secure information sharing has been addressed by Katsuno et al. [9]. We extend this idea to privacy policy enforcement.

<sup>&</sup>lt;sup>3</sup> The definition of the required integrity measurement values in the TVD policy presupposes the knowledge about the security properties of the corresponding software. In practice, trust can be achieved via independent trusted third parties that evaluate and certify IT products according to standards like Common Criteria.

<sup>&</sup>lt;sup>4</sup> See http://www.opentc.net

#### 3.2 Realizing a Simple Privacy Policy with TVD

Let us consider a very simple privacy policy: only members of a particular TVD have access to the private information. The TVD policy expresses the requirements for virtual machines to join the TVD and to access this information. The TVD policy is used to implement the privacy policy, and the TVD infrastructure provides the policy enforcement for the wallet.

The wallet can act as TVD Master. In this case, it is directly responsible for policy enforcement. All parties that want to access the information have to join the TVD first. As they request to join, the wallet verifies the security properties of the joining parties using attestation. If the verification succeeds, the joining party becomes a member of the TVD and can then access sensitive information. The wallet can specify a set of "good" values for the platform configuration that are necessary to access the data.

Application scenarios for the case where the wallet is the TVD Master include those where the private information of one user is distributed to "homogeneous" data consumers, e.g., in an e-health scenario, the medical data and health records of patients are only accessible to computing platforms of medical personnel, but not to systems used by other departments.

In other classes of scenarios, where users belonging to a group want to exchange private data, it is unrealistic to have a virtual domain managed by a user's wallet. In these cases, a trusted party could provide a TVD Master responsible for policy enforcement for the group. The wallet of a user who wishes to exchange information within a group could attest the responsible TVD Master (e.g., using TCG attestation) before joining. If this attestation includes both the platform configuration of the TVD Master and the TVD policy, the wallet can ensure that information is only distributed within a TVD, where the master enforces a TVD policy that complies to the user's own privacy policy. The wallet can migrate to any node in the TVD (using conventional VM migration), and the required verification of the security properties of the destination is handled by the TVD establishment.

#### 3.3 Implementation

Our prototype is based on the idea that a local proxy of the corresponding TVD Master, the *TVD Proxy*, is running on each physical platform that is supposed to execute virtual machines as part of a TVD. The TVD Proxy is responsible for the local enforcement of the TVD policy and performs the admission control for joining virtual machines. Since instances of multiple TVDs should be able to run isolated on one computing platform, there can be several TVD Proxies (one for each corresponding TVD) on one platform.

The main components of the trusted virtualization layer are as follows (see also Figure  $[\underline{4}]$ ):

- TVD-Proxy-Factory: service that creates and manages TVD Proxies. During the establishment of the TVD, the TVD Master deploys the policy P and

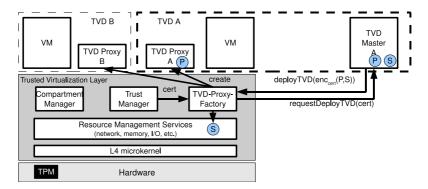


Fig. 4. TVD implementation architecture

corresponding credentials S (cryptographic keys and certificates for, e.g., network encryption) to the TVD-Proxy-Factory. To "verify" the trustworthiness of the platform and its virtualization layer, the TVD Master requests a remote attestation of the integrity measurements, using trusted computing functionality of a TPM **6**.

- CompartmentManager: service responsible for starting and terminating virtual machines (compartments) and taking integrity measurements of the virtual machines on start-up. This service also defines access rights for communication between active compartments.
- *TrustManager*: service providing an interface to the underlying TPM and used to create new binding keys, generate certificates for these keys, and unbind data encrypted with a binding key. The binding key is protected by the TPM and bound to the integrity measurements of the underlying platform and its trusted virtualization layer. The certificate includes these integrity measurements and permits a remote party to establish a *trusted channel* to the platform, i.e., a secure channel (providing confidentiality and integrity) bound to the integrity of the endpoint(s).

We have implemented this design based on an existing security kernel, Turaya, which comprises two layers: a *hypervisor layer* based on an L4 microkernel and resource management services (memory management, I/O drivers), and a *trusted software layer* providing security services, e.g., secure storage, virtualized network, compartment management, and trusted channel establishment.

The L4 microkernel ensures isolation of processes and controls inter-process communication (IPC). Compartments can be native L4 tasks or para-virtualized Linux instances (L4Linux). Communication between compartments can be allowed or denied by applying access rights to their IPC interfaces. The microkernel enforces the IPC access control.

To support wallet functionality, it is necessary to establish a TVD and attach a virtual machine to the TVD. A TVD is established in two phases:

<sup>&</sup>lt;sup>5</sup> http://www.emscb.com/content/pages/turaya.htm

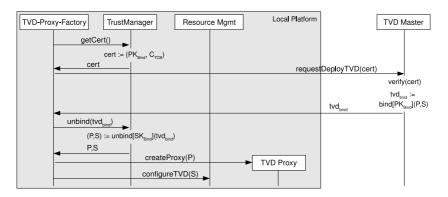


Fig. 5. TVD deployment protocol

- 1. *Deploy TVD*: First, the local TVD infrastructure must be set up, including the deployment of the TVD policy and TVD credentials from the TVD Master to the trusted virtualization layer of the local platform.
- 2. Join TVD: When policy and credentials are deployed, the local TVD Proxy enforces the policy and determines if local VMs are allowed to join the TVD.

Staged establishment of the TVD was selected to avoid a central admission control that would result in considerable performance trade-offs. In this approach, the TVD policy enforcement is partially delegated to the local platforms, but the TVD Master must verify the trustworthiness (integrity state) of the platforms to establish if they can be trusted. This is done during the deployment phase.

**Deploy TVD.** When TVD-Proxy-Factory receives a request to deploy a TVD, TrustManager generates a binding certificate *cert* :=  $(PK_{Bind}, C_{TCB})$ . The TrustManager uses the TPM to generate a new binding key pair  $(SK_{Bind}, PK_{Bind})$ , where the secret key part is protected by the TPM and bound to the integrity measurement of the trusted virtualization layer  $(C_{TCB})$ . The TVD-Proxy-Factory requests deployment from the TVD Master of the desired TVD and sends the binding certificate, including the binding key  $PK_{Bind}$ .

The TVD Master checks whether the integrity measurement of the platform matches the TVD policy. If it does, the TVD Master encrypts the TVD policy P and the corresponding TVD credentials S with the binding key  $PK_{Bind}$ , and sends the encrypted data to the local TVD-Proxy-Factory. See Figure 5.

The TVD-Proxy-Factory requests the TrustManager to unbind the data and retrieves the TVD policy and credentials (P, S). It creates a new TVD Proxy, passes the TVD policy P to it and configures the underlying resource management services (e.g., virtual network switch) with the credentials S. Now the TVD infrastructure is set up locally and ready to join virtual machines.

Join TVD. The user creates the VM using the CompartmentManager. The CompartmentManager measures the integrity of the VM image (i.e., hashing the image file), stores the measurement for future requests (during runtime),

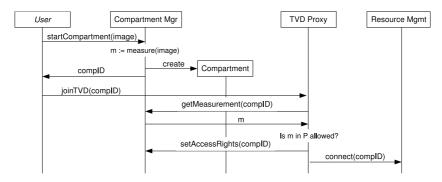


Fig. 6. TVD join protocol

starts the VM in a compartment, and returns a compartment identifier (unique during runtime of the platform). The user can request to join the compartment to the TVD by passing the compartment ID to the TVD Proxy.

The TVD Proxy obtains the integrity measurement m of the given compartment ID from the CompartmentManager. If the value m is listed in the TVD policy P as allowed to join, the TVD Proxy configures the underlying resource management to connect the compartment to the virtual resources of the TVD, e.g., "plugging" a virtual network connector to the VM.

## 4 Remaining Challenges and Related Work

Privacy policy languages are designed to translate the privacy policies for users and organizations into statements that can be interpreted by IT systems. In 10 the authors give an overview of common policy languages. W3C's Platform for Privacy Preferences (P3P) was designed to express website privacy policies in machine-readable format III, and P3P Preference Exchange Language (AP-PEL) is used to express privacy preferences of an individual and to query the P3P data 12.13. CPExchange was developed to facilitate business-to-business communication about privacy policies 14. For internal privacy policies of organizations, IBM proposed Enterprise Privacy Authorization Language (EPAL) **15**. Another language for describing both privacy and security policies in a machine readable format is the eXtensible Access Control Markup Language (XACML) 16. Other initiatives, such as DPAL 17, and XPref 18, addressed various aspects of expressing privacy requirements and related concepts. Due to the growth of services that require the transfer of context sensitive information (e.g., time and location), the Internet Engineering Task Force (IETF) initiative started work on *Geopriv*, a language that can express policies for granting access on the basis of presence and location information 17.

<sup>&</sup>lt;sup>6</sup> The details of the resource isolation and realization of TVDs on this level are out of scope for this paper. Cabuk et al. [7] show how to realize network isolation based on VLAN tagging.

In addition to the earlier work on access control policies and (privacy) languages, recent research has analyzed and developed methodologies for evaluating actual policies to compare them with the policies the users desired to use, e.g., Bauer et al. [19] conducted user study of access control policies. Cornwell et al. [20] have analyzed policy management in different applications in mobile computing and developed applications where users can define policies to control the usage of private information, e.g., location-based or contextual information. Sadeh et al. [21] analyzed user interfaces for policy definition and mechanisms for auditing the disclosure of private information.

We conclude that, while the need to ensure user control and enforcement of privacy policies was recognized, most research so far focuses on formal languages defining privacy and related policies in various contexts, user requirements for such policies, and approaches for applications to incorporate user controlled flexible policies. However, little attention was given to the mechanisms to support automatic enforcement and interpretation of these policies. In this paper, we propose an approach to policy enforcement that takes into consideration the results of earlier research, including user requirements and design of formal policy languages. The new framework offers a realistic approach to the control and enforcement of privacy policies in a variety of contexts. We think that TVDs can help construct the *privacy domains* to support privacy protection of sensitive data that need to be shared. The process to build domains where the protection of sensitive data is governed by privacy policies determined by users still needs to be defined. Policy management for privacy domains remains a major challenge as complex privacy policies need to be enforced within a domain, when a machine joins or leaves the domain, and for inter-domain communication.

The idea of the Trusted Personal Information Wallet is derived from previous work [22], which uses a password wallet as authentication agent to access web sites. It protects private data (credentials) of a user during the authentication to a remote server. This approach uses Trusted Computing technology to ensure that the wallet is executed in a trusted environment. In addition to protecting the credentials, *SpyBlock* [23] protects against the unintentional disclosure of sensitive information (like credit card numbers, name, address, etc.) as a result of malicious transactions [24].

Since the Trusted Personal Information Wallet acts as an agent for the user's private data and it can migrate to other platforms, it is comparable to mobile agents. Wilhelm et al. [25] propose to use a tamper-resistant hardware to provide a secure execution environment for mobile agent code. Balfe and Gallery [26] outline how attestation can be used to ensure that an agent only visits host platforms behaving in an expected manner and that access to the private agent data complies to the desired security policies. In [27], the main approach is the protection of an agent's private cryptographic key by binding the key to a TPM. In contrast, the wallet (agent) in the framework proposed here does not directly use the TPM, but relies on the TVD infrastructure to (automatically) deploy a trusted execution environment and enforce privacy policies.

## 5 Conclusion

In this paper, we proposed a conceptual framework for privacy policy management and enforcement to ensure security and trust for sharing of private or sensitive information. We believe that Trusted Computing technology, in particular the concept of trusted virtual domains (TVDs), can efficiently support privacy policy enforcement. We think that future research will lead to the development of trusted privacy-enhancing architectures that will be applicable to several use cases, e.g., e-commerce, enterprise rights management, e-health, and other areas. Here we outline only the first steps towards the definition of such architectures. In addition, the definition and enforcement of more complex privacy policies will be a subject of future work.

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