On the Theoretical, Conceptual, and Philosophical Foundations for Research in Mathematics Education

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Prelude The current infatuation in the U.S. with "what works" studies seems to leave education researchers with less latitude to conduct studies to advance theoretical and model-building goals and they are expected to adopt philosophical perspectives that often run counter to their own. Three basic questions are addressed in this article: *What is the role of theory in education research? How does one's philosophical stance influence the sort of research one does?* And, *What should be the goals of mathematics education research?* Special attention is paid to the importance of having a conceptual framework to guide one's research and to the value of acknowledging one's philosophical stance in considering what counts as evidence.

Establishing a Context

The current emphasis in the United States being placed on so-called *scientific research* in education is driven in large part by political forces. Much of the public discussion has begun with an assumption that the purpose of research is to determine "what works," and the discourse has focused largely on matters of research design and data collection methods.[1] One consequence has been a renewal of attention to experimental designs and quantitative methods that had faded from prominence in education research over the past two decades or so.

Today's debate in the United States over research methods calls to mind the controversy that raged 40 years ago surrounding calls to make mathematics education research (hereafter referred to as MER) more "scientific." A concern voiced by many at that time was that MER was not answering "what works" questions precisely because it was so narrowly embedded in a research paradigm that simply was not appropriate for answering questions of real importance—specifically, the positivist, "experimental" paradigm (Lester and Lambdin 2003). Writing in 1967 about the

[1] In this article, I make no claims about the state of mathematics education research in any countries other than the United States. One can only hope that the situation is not as dire elsewhere.

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need for a journal devoted to research in mathematics education, Joe Scandura, an active researcher in the U.S. during the 1960s and 70s observed:

> [M]any thoughtful people are critical of the quality of research in mathematics education. They look at tables of statistical data and they say "So what!" They feel that vital questions go unanswered while means, standard deviations, and t-tests pile up. (Scandura 1967, p. iii)

A similar sentiment was expressed in the same year by another prominent U.S. researcher, Robert Davis:

> In a society which has modernized agriculture, medicine, industrial production, communication, transportation, and even warfare as ours has done, it is compelling to ask why we have experienced such difficulty in making more satisfactory improvements in education. (Davis 1967, p. 53)

Davis insisted then that the community of mathematics education researchers needed to abandon its reliance on experimental and quasi-experimental studies for ones situated in a more interpretive perspective. Put another way, the social and cultural conditions within which our research must take place require that we adopt perspectives and employ approaches that are very different from those used in fields such as medicine, physics, and agriculture. Today, we education researchers find ourselves in the position of having to defend our resistance to being told that the primary characteristics of educational research that is likely to receive financial support from the U.S. Department of Education are "randomized experiments" and "controlled clinical trials" (U.S. Department of Education 2002).

To a large extent, the argument against the use of experimental methods has focused on the organizational complexity of schools and the failure of experimental methods used in the past to provide useful, valid knowledge (Cook 2001). However, largely ignored in the discussions of the nature of educational research has been consideration of the conceptual, structural foundations of our work. To be more specific, the role of theory and the nature of the philosophical underpinnings of our research have been absent. This is very unfortunate because scholars in other social science disciplines (e.g., anthropology, psychology, sociology) often justify their research investigations on grounds of developing understanding by building or testing theories and models, and almost always they design their research programs around frameworks of some sort. In addition, researchers in these disciplines pay close attention to the philosophical assumptions upon which their work is based. In contrast, the current infatuation in the U.S. with "what works" studies seems to leave education researchers with less latitude to conduct studies to advance theoretical and model-building goals and they are expected to adopt philosophical perspectives that often run counter to their own. In this paper, I address three basic questions: *What is the role of theory in education research? How does one's philosophical stance influence the sort of research one does?* And, *what should be the goals of mathematics education research?*

The Role of Theory

Although MER was aptly characterized less than 15 years ago by Kilpatrick (1992) as largely *atheoretical*, a perusal of recent articles in major MER journals reveals that references to theory are commonplace. In fact, Silver and Herbst (2004) have noted that expressions such as “theory-based,” “theoretical framework,” and “theorizing” are commonly used by reviewers of manuscripts submitted for publication in the *Journal for Research in Mathematics Education* during the past four or five years. Silver and Herbst insist that manuscripts are often rejected for being atheoretical. I suspect the same is true of proposals submitted to other MER journals.

But, what does it mean for research to be theory based? In what follows, I argue that the role of theory should be determined in light of the research framework one has adopted. So, before proceeding further, let me discuss the broader notion of *research framework* and then situate the role of theory within this notion.

The Nature of Research Frameworks

The notion of a research framework is central to every field of inquiry, but at the same time the development and use of frameworks may be the least understood aspect of the research process. The online *Encarta World English Dictionary* defines a framework as “a set of ideas, principles, agreements, or rules that provides the basis or the outline for something that is more fully developed at a later stage.” I also like to think of a framework as being like a scaffold erected to make it possible for repairs to be made on a building. A scaffold encloses the building and enables workers to reach otherwise inaccessible portions of it. Thus, a research framework is a basic structure of the ideas (i.e., abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated. These abstractions and the (assumed) interrelationships among them represent the relevant features of the phenomenon as determined by the research perspective that has been adopted.[2] The abstractions and interrelationships are then used as the basis and justification for all aspects of the research.

Using a framework to conceptualize and guide one’s research has at least four important advantages.

1. *A framework provides a structure for conceptualizing and designing research studies*. In particular, a research framework helps determine:
   - the nature of the questions asked;
   - the manner in which questions are formulated;

[2]By “perspective” I mean the viewpoint the researcher chooses to use to conceptualize and conduct the research. There are various kinds of perspectives: discipline-based (e.g., anthropology, psychology), practice-oriented (e.g., formative vs. summative evaluation), philosophical (e.g., positivist, interpretivist, critical theorist), etc.

- the way the concepts, constructs, and processes of the research are defined; and
- the principles of discovery and justification allowed for creating new "knowledge" about the topic under study (this refers to acceptable research methods).

2. *There is no data without a framework to make sense of those data.* We have all heard the claim, "The data speak for themselves!" Dylan Wiliam and I have argued elsewhere that actually data have nothing to say. Whether or not a set of data can count as evidence of something is determined by the researcher's assumptions and beliefs as well as the context in which it was gathered (Lester and Wiliam 2000). One important aspect of a researcher's beliefs is the framework, theory-based or otherwise, he or she is using; this framework makes it possible to make sense of a set of data.
3. *A good framework allows us to transcend common sense.* Andy diSessa (1991) has argued that theory building is the linchpin in spurring practical progress. He notes that you don't need theory for many everyday problems—purely empirical approaches often are enough. But often things aren't so easy. Deep understanding that comes from concern for theory building is often essential to deal with truly important problems. I find diSessa's insistence on grounding research in theory alone too restrictive. As I discuss later in this paper, a theoretical framework is not the only, or even the best, choice for guiding our inquiry. However, building one's research program around a carefully conceptualized structure (i.e., framework) is essential.
4. *Need for deep understanding, not just "for this" understanding.* Related to the above, is the need we should have as researchers to deeply understand the phenomena we are studying—the important, big questions (e.g., What does it mean to understand a concept? What is the teacher's role in instruction?)—not simply find solutions to immediate problems and dilemmas (i.e., determine "what works"). A research framework helps us develop deep understanding by providing a structure for designing research studies, interpreting data resulting from those studies, and drawing conclusions.

Types of Frameworks

Educational anthropologist, Margaret Eisenhart (1991) has identified three types of research frameworks: theoretical, practical, and conceptual. Each category has its own characteristics, and each has a role to play in MER, but as I argue below, two of these frameworks have serious shortcomings.

Theoretical Frameworks

Another way to consider the role of theory in our research is to think of a theory as a specific kind of framework. A theoretical framework guides research activities by its reliance on a formal theory; that is, a theory that has been developed by using an established, coherent explanation of certain sorts of phenomena and relationships—Piaget's theory of intellectual development and Vygotsky's theory of socio-historical constructivism are two prominent theories used in the study of children's learning. At the stage in the research process in which specific research questions are determined, these questions would be rephrased in terms of the formal theory that has been chosen. Then, relevant data are gathered, and the findings are used to support, extend, or modify the theory. When researchers decide on a particular theory to use as a basis for a research framework they are deciding to follow the programmatic research agenda outlined by advocates of the theory. That is, the researcher is deciding to conform to the accepted conventions of argumentation and experimentation associated with the theory. This choice has the advantage of facilitating communication, encouraging systematic research programs, and demonstrating progress among like-minded scholars working on similar research problems. For example, researchers who wish to test the applicability of Piaget's theory of conservation of quantity in different settings and with different people, work together with a shared set of terms, concepts, expected relationships, and accepted procedures for testing and extending the theory.

Martyn Hammersley, a sociologist and ethnographer, has insisted that it is the duty of sociologists (and perhaps educators as well) "to attempt the production of well-established theory" because doing so "gives us the best hope of producing effective explanations for social phenomena and thereby a sound basis for policy" (Hammersley 1990, pp. 108–109). Also, Garrison (1988) has provided an interesting argument to the effect that it is impossible for research to be atheoretical and as a result it is essential that a theoretical framework be explicitly identified and articulated by the researcher. But, there are at least four problems associated with the use of a theoretical framework.

1. *Theoretical frameworks force the research to explain their results are given by "decree" rather than evidence.* Some researchers (e.g., Eisenhart 1991) insist that educational theorists prefer to address and explain the results of their research by "theoretical decree" rather than with solid evidence to support their claims. That is to say, there is a belief among some researchers that adherence to a theoretical framework forces researchers to make their data fit their theory. In addition, rigid adherence to a theoretical position makes it likely that the researcher will omit or ignore important information.
2. *Data have to "travel."* Sociologist and ethnographer, John Van Maanen (1988), has observed that data collected under the auspices of a theory have to "travel" in the sense that (in his view) data too often must be stripped of context and local meaning in order to serve the theory.
3. *Standards for theory-based discourse are not helpful in day-to-day practice.* Related to the previous concern, is the belief that researchers tend to use a theoretical framework to set a standard for scholarly discourse that is not functional outside the academic discipline. Conclusions produced by the logic of scholarly discourse too often are not at all helpful in day-to-day practice (cf., Lester and Wiliam 2002). "Researchers don't speak to practitioners!" and "Theory is irrelevant to the experience of practitioners." are commonly voiced complaints. More-

over, the academics who use theory to explain their results too often establish a standard for scholarly discourse that is not functional to persons not familiar with the theory.

4. *No triangulation*. Sociologist, Norman Denzin (1978) was one of the first social scientists to discuss the importance of theoretical triangulation, by which he meant the process of compiling currently relevant theoretical perspectives and practitioner explanations, assessing their strengths, weaknesses, and appropriateness, and using some subset of these perspectives and explanations as the focus of empirical investigation. By embedding one's research in a single theory, such triangulation does not happen.

Practical Frameworks

In response to what he perceived as the irrelevance of theoretical research, educational evaluator and philosopher, Michael Scriven (1986), has suggested practical frameworks as an alternative. For Scriven, a practical framework guides research by using "what works" in the experience of doing something by those directly involved in it.[3] This kind of framework is not informed by formal theory but by the accumulated practice knowledge of practitioners and administrators, the findings of previous research, and often the viewpoints offered by public opinion. Research questions are derived from this knowledge base and research results are used to support, extend, or revise the practice (see also Cobb 2007).

Although this sort of framework has at least one obvious advantage over theoretical frameworks—the problems are those of the people directly involved—it has one serious limitation: findings resulting from use of a practical framework tend to be, at best, only locally generalizable (i.e., the researcher finds out "what works" now under certain specific conditions and constraints, but learns little or nothing that goes beyond the specific context). Another drawback of practical frameworks is that they depend on the insiders' (i.e., local participants') perspectives. Although insiders know the behaviors and ideas that have meaning for people like themselves, they are unlikely to consider the structural features and causes of social practices or the norms that they unwittingly internalize and use in communication and action; these practices and norms are the taken-for-granted context of the insiders' lives. Because insiders take these constraints for granted, practical frameworks tend to ignore macro-level constraints on what and how insiders act and how they make sense of their situation. Put another way, all too often insiders can't see the forest for the trees.

[3] Although there are similarities between the "what works" mentality that is driving much of the current educational research in the U.S. and a practical framework perspective, it is not appropriate to conclude that they are the same. Indeed, political ideology seems to be driven today's research agendas; there typically is no underlying structure of ideas that describe the phenomena being studied.

Conceptual Frameworks

Eisenhart (1991) has described a conceptual framework as "a skeletal structure of *justification*, rather than a skeletal structure of *explanation*" (p. 210; italics added). Furthermore, it is "an argument including different points of view and culminating in a series of reasons for adopting some points ... and not others" (p. 210). A conceptual framework is an argument that the concepts chosen for investigation, and any anticipated relationships among them, will be appropriate and useful given the research problem under investigation. Like theoretical frameworks, conceptual frameworks are based on previous research, but conceptual frameworks are built from an array of current and possibly far-ranging sources. The framework used may be based on different theories and various aspects of practitioner knowledge, depending on what the researcher can argue will be relevant and important to address about a research problem. Eisenhart (1991) argued that

> Conceptual frameworks are not constructed of steel girders made of theoretical propositions or practical experiences; instead they are like scaffoldings of wooden planks that take the form of arguments about what is relevant to study and why ... at a particular point in time. As changes occur in the state-of-knowledge, the patterns of available empirical evidence, and the needs with regard to a research problem, used conceptual frameworks will be taken down and reassembled. (pp. 210–211)

Furthermore, conceptual frameworks accommodate both outsiders' and insiders' views and, because they only outline the kinds of things that are of interest to study for various sources, the argued-for concepts and their interrelationships must ultimately be defined and demonstrated in context in order to have any validity.

Of special importance for conceptual frameworks is the notion of *justification*. In my view, although explanation is an essential part of the research process, too often educational researchers are concerned with coming up with good "explanations" but not concerned enough with justifying why they are doing what they are doing and why their explanations and interpretations are reasonable. In my experience reviewing manuscripts for publication and advising doctoral students about their dissertations, I have found a lack of attention to clarifying and justifying why a particular question is proposed to be studied in a particular way and why certain factors (e.g., concepts, behaviors, attitudes, societal forces) are more important than others.

One prime example of a conceptual framework that has been very useful in MER is the "models and modeling perspective" developed over several years of systematic work by Dick Lesh and his colleagues (Lesh 2002; Lesh and Doerr 2003; Lesh and Kelly 2000). Lesh's "models and modeling perspective" is not intended to be a grand theory. Instead, it is a system of thinking about problems of mathematics learning that integrates ideas from a variety of theories. Other key features of the models and modeling framework are that it: (a) makes use of a variety of representational media to express the models that have been developed, (b) is directed toward solving problems (or making decisions) that lie outside the theories themselves (as a result, the criteria for success also lie outside the theories), (c) is situated (i.e., models are created for a specific purpose in a specific situation), and (d) the models are developed so that they are modifiable and adaptable (see Lesh and Sriraman 2005).

The development of theory is absolutely essential in order for significant advances to be made in the collective and individual thinking of the MER community. But, not everything we know can be collapsed into a single theory. For example, models of realistic, complex situations typically draw on a variety of theories. Furthermore, solutions to realistic, complex problems usually need to draw on ideas from more than a single mathematics topic or even a single discipline. So, a grand "theory of everything" cannot ever be developed and efforts to develop one are very likely to keep us from making progress toward the goals of our work. Instead, we should focus our efforts on using smaller, more focused theories and models of teaching, learning and development. This position is best accommodated by making use of conceptual frameworks to design and conduct our inquiry. I propose that we view the conceptual frameworks we adopt for our research as sources of ideas that we can appropriate and modify for our purposes as mathematics educators. This process is quite similar to the thinking process characterized by the French word *bricolage*, a notion borrowed by Gravemeijer (1994) from Claude Levi-Strauss to describe the process of instructional design. A *bricoleur* is a handyman who uses whatever tools are available to come up with solutions to everyday problems. In like manner, we should appropriate whatever theories and perspectives are available in our pursuit of answers to our research questions.

Why Research Frameworks Are Ignored or Misunderstood

In my mind, there are two basic problems that must be dealt with if we are to expect conceptual (or other) frameworks to play a more prominent role in our research.[4] The first has to do with the widespread misunderstanding of what it means to adopt a theoretical or conceptual stance toward one's work. The second is that some researchers, while acknowledging the importance of theory development or model building, do not feel qualified to engage in this sort of work. I attribute both of these problems in large part to the failure of: (a) our graduate programs to properly equip novice researchers with adequate preparation in theory, and (b) our research journals to insist that authors of research reports offer serious theory-based explanations of their findings (or justifications for their explanations).

Writing about the state of U.S. doctoral programs, Hiebert et al. (2001) suggested that mathematics education is a complex system and that improving the process of preparing doctoral students means improving the entire system, not merely changing individual features of it. They insist that "the absence of system-wide standards for doctoral programs [in mathematics education] is, perhaps, the most serious challenge facing systemic improvement efforts.... Indeed, participants in the system have grown accustomed to creating their own standards at each local site [univer-

[4]I am not suggesting that these are the only problems that must be dealt with regarding theoretical frameworks; external forces, such as the present-day pressure to do "what works" research is at least as serious a problem. However, I think the two problems I discuss in this article are ones that we can actually address from within our research community.

sity]" (p. 155). One consequence of the absence of commonly accepted standards is that there is a very wide range of requirements of different doctoral programs. At one end of the continuum of requirements are a few programs that focus on the preparation of researchers. At the other end are those programs that require little or no research training beyond taking a research methods course or two. In general, with few exceptions, doctoral programs are replete with courses and experiences in research methodology, but woefully lacking in courses and experiences that provide students with solid theoretical and philosophical grounding for future research. Without solid understanding of the role of theory and philosophy in conceptualizing and conducting research, there is little chance that the next generation of mathematics education researchers will have a greater appreciation for theory than is currently the case. Put another way, we must do a better job of cultivating a predilection for carefully conceptualized frameworks to guide our research.

During my term as editor of the *Journal for Research in Mathematics Education* in the early to mid 1990s, I found the failure of authors of research reports to pay serious attention to explaining and justifying the results of their studies among the most serious shortcomings of their research reports. A simple example from the expert-novice problem solver research literature may help illustrate what I mean. A report of an expert-novice study might conclude that *experts do X when they solve problems and novices do Y.* Were the researcher guided by a framework, it would be natural to ask *Why* questions (e.g., Why is it that experts perform differently from novices?). Having a framework guiding the research provides a structure within which to attempt to answer *Why* questions. Without a framework, the researcher can speculate at best or offer no explanation at all.

The Influence of One's Philosophical Stance on the Nature of One's Research

By suggesting, as I have at the beginning of this article, that the MER community in the U.S. has been preoccupied of late with methodological issues I do not mean to suggest that this community has completely ignored philosophical issues. Indeed, discussions and debates over philosophical issues associated with MER are common (e.g., Cobb 1995; Davis et al. 1990; Lesh and Doerr 2003; Orton 1995; Simon 1995; Steffe and Thompson 2000). Also, in a paper written for the forthcoming second edition of the *Handbook of Research on Mathematics Teaching and Learning*, Cobb (2007) puts "philosophy to work by drawing on the analyses of a number of thinkers who have grappled with the thorny problem of making reasoned decisions about competing theoretical perspectives" (p. 3). He uses the work of noted philosophers such as (alphabetically) John Dewey, Paul Feyerabend, Thomas Kuhn, Imre Lakatos, Stephen Pepper, Michael Polanyi, Karl Popper, Hilary Putnam, W.V. Quine, Richard Rorty, Ernst von Glasersfeld, and several others to build a convincing case for considering the various theoretical perspectives being used today "as sources of ideas to be appropriated and adapted to our purposes as mathematics

Table 1 Sources of evidence for five inquiry systems

Inquiry system	Source of evidence
Leibnizian	Reasoning
Lockean	Observation
Kantian	Representation
Hegelian	Dialectic
Singerian	Ethical values & practical consequences

educators." In this section I demonstrate the value of philosophy to MER by discussing how one's philosophical stance influences the process of making claims and drawing conclusions.

A System for Classifying Systems of Inquiry[5]

Churchman (1971) classified all systems of inquiry into five broad categories, each of which he labeled with the name of the philosopher (viz., Leibniz, Locke, Kant, Hegel, and Singer) he felt best exemplified the stance involved in adopting the system. He gave particular attention in his classification to what can be regarded as the primary or most salient form of evidence, as summarized in Table 1 (each is discussed below).

Churchman's classification is particularly useful in thinking about how to conduct research insofar as it suggests three questions that researchers should attempt to answer about their research efforts:

Are the claims we make about our research based on inferences that are warranted on the basis of the evidence we have assembled?

Are the claims we make based on convincing arguments that are more warranted than plausible rival claims? and

Are the consequences of our claims ethically and practically defensible?

The current controversy over reform versus traditional mathematics curricula has attracted a great deal of attention in the United States and elsewhere among educators, professional mathematicians, politicians, and parents and can serve to illustrate how these three questions might be used. For some, the issue of whether the traditional or reform curricula provide the most appropriate means of developing mathematical competence is an issue that can be settled on the basis of logical argument. On one side, the proponents of reform curricula might argue that a school mathematics curriculum should resemble the activities of mathematicians, with a focus on the *processes* of mathematics. On the other side, the anti-reform movement might

[5]The following section is an abridged and slightly modified version of a section of a paper by Lester and Wiliam (2002).

argue that the best preparation in mathematics is one based on skills and procedures. Despite their opposing views, both these points of view rely on rhetorical methods to establish their position, in an example of what Churchman called a *Leibnizian* inquiry system. In such a system certain fundamental assumptions are made, from which deductions are made by the use of formal reasoning rather than by using empirical data. In a Leibnizian system, reason and rationality are held to be the most important sources of evidence. Although there are occasions in educational research when such methods might be appropriate, they usually are not sufficient. In fact, typically the educational research community requires some sort of evidence from the situation under study (usually called empirical data).

The most common use of data in inquiry in both the physical and social sciences is via what Churchman calls a *Lockean* inquiry system. In such an inquiry, evidence is derived principally from observations of the physical world. Empirical data are collected, and then an attempt is made to build a theory that accounts for the data. Consider the following scenario.

A team of researchers, composed of the authors of a reform-minded mathematics curriculum and classroom teachers interested in using that curriculum, decide after considerable discussion and reflection to design a study in which grade 9 students are randomly assigned either to classrooms that will use the new curriculum or to those that will use the traditional curriculum. The research team's goal is to investigate the effectiveness (with respect to student learning) of the two curricula over the course of the entire school year. Suppose further that the research design they developed is appropriate for the sort of research they are intending to conduct.

From the data the team will gather, they hope to be able to develop a reasonable account of the effectiveness of the two curricula, relative to whatever criteria are agreed upon, and this account could lead them to draw certain conclusions (i.e., inferences). Were they to stop here and write a report, they would essentially be following a scientific rationalist approach situated in a Lockean perspective. The major difficulty with a Lockean approach is that, because observations are regarded as evidence, it is necessary for all observers to agree on what they have observed. But, because what we observe is based on the (perhaps personal) theories we have, different people will observe different things, even in the same classroom.

For less well-structured questions, or where different people are likely to disagree what precisely is the problem, a *Kantian* inquiry system is more appropriate. This involves the deliberate framing of multiple alternative perspectives, on both theory and data (thus subsuming Leibnizian and Lockean systems). One way of doing this is by building different theories on the basis of the same set of data. Alternatively, we could build two (or more) theories related to the problem, and then for each theory, generate appropriate data (different kinds of data might be collected for each theory).

For our inquiry into the relative merits of traditional and reform curricula, our researchers might not stop with the "crucial experiment" described above, but instead, would consider as many alternative perspectives as possible (and plausible) about both their underlying assumptions and their data. They might, for example, challenge one or more of their assumptions and construct competing explanations on the basis of the same set of data. These perspectives would result in part from their engagement in serious reflection about their underlying assumptions, and in part from submitting their data to the scrutiny of other persons who might have a stake in the research, for example, teachers who taught using the traditional curriculum. An even better approach would be to consider two or more rival perspectives (or theories) while designing the study, thereby possibly leading to the generation of different sets of data. For example, a study designed with a situated cognition (or situated learning) perspective in mind might result in a very different set of data being collected than a study based on contemporary cognitive theory (see Anderson et al. 1996; Greeno 1997). These two different perspectives would also probably lead the researchers to very different explanations for the results (Boaler 2000). For example, the partisans of the situated cognition perspective might attribute results favoring the reform curriculum to certain aspects of the social interactions that took place in the small groups (an important feature of the reform curriculum), whereas cognitivists might claim that it was the increased level of individual reflection afforded by the new curriculum materials, rather than the social interaction, that caused the higher performance among students who were in the reform classrooms.

The different representations of traditional and reform classrooms developed within a Kantian inquiry system may not be reconcilable in any straightforward sense. It may not be immediately apparent where these theories overlap and where they conflict, and indeed, these questions may not be meaningful, in that the enquiries might be incommensurable (Kuhn 1962). However, by analyzing these enquiries in more detail, it may be possible to begin a process of theory building that incorporates the different representations of the situation under study.

This idea of reconciling rival theories is more fully developed in a *Hegelian* inquiry system, where antithetical and mutually inconsistent theories are developed. Not content with building plausible theories, the Hegelian inquirer takes a plausible theory, and then investigates what would have to be different about the world for the *exact opposite* of the most plausible theory itself to be plausible. The tension produced by confrontation between conflicting theories forces the assumptions of each theory to be questioned, thus possibly creating a co-ordination of the rival theories.

In our example, the researchers should attempt to answer two questions: (1) What would have to be true about the instruction that took place for the opposite of the situated learning explanation to be plausible? and (2) What would have to be true about the instruction that took place for the opposite of the cognitivist explanation to be plausible? If the answers to both these questions are "not very much" then this suggests that the available data underdetermine the interpretations that are made of them. This might then result in sufficient clarification of the issues to make possible a co-ordination, or even a synthesis, of the different perspectives, at a higher level of abstraction.

The differences among Lockean, Kantian and Hegelian inquiry systems were summed up by Churchman as follows:

> The Lockean inquirer displays the "fundamental" data that all experts agree are accurate and relevant, and then builds a consistent story out of these. The Kantian inquirer displays the same story from different points of view, emphasizing thereby that what is put into the

> story by the internal mode of representation is not given from the outside. But the Hegelian inquirer, using the same data, tells two stories, one supporting the most prominent policy on one side, the other supporting the most promising story on the other side. (1971, p. 177)

However, perhaps the most important feature of Churchman's typology is that we can inquire about inquiry systems, questioning the values and ethical assumptions that these inquiry systems embody. This inquiry of inquiry systems is itself, of course, an inquiry system, termed *Singerian*_by Churchman after the philosopher E.A. Singer (see Singer 1959). Such an approach entails a constant questioning of the assumptions of inquiry systems. Tenets, no matter how fundamental they appear to be, are themselves to be challenged in order to cast a new light on the situation under investigation. This leads directly and naturally to examination of the values and ethical considerations inherent in theory building.

In a Singerian inquiry, there is no solid foundation. Instead, everything is 'permanently tentative'; instead of asking what "is," we ask what are the implications and consequences of different assumptions about what "is taken to be":

> The "is taken to be" is a self-imposed imperative of the community. Taken in the context of the whole Singerian theory of inquiry and progress, the imperative has the status of an ethical judgment. That is, the community judges that to accept its instruction is to bring about a suitable tactic or strategy.... The acceptance may lead to social actions outside of inquiry, or to new kinds of inquiry, or whatever. Part of the community's judgement is concerned with the *appropriateness of these actions from an ethical point of view*. Hence the linguistic puzzle which bothered some empiricists—how the inquiring system can pass linguistically from "is" statements to "ought" statements—is no puzzle at all in the Singerian inquirer: the inquiring system speaks exclusively in the "ought," the "is" being only a convenient *façon de parler* when one wants to block out the uncertainty in the discourse. (Churchman 1971, p. 202; emphasis added in fourth sentence)

An important consequence of adopting a Singerian perspective is that with such an inquiry system, one can never absolve oneself from the consequences of one's research. Educational research is a process of *modeling* educational processes, and the models are never right or wrong, merely more or less appropriate for a particular purpose, and the appropriateness of the models has to be defended. It is only within a Singerian perspective that the third of our key questions (Are the consequence of our claims ethically and practically defensible?) is fully incorporated. Consider the following scenario.

After studying the evidence obtained from the study, the research team has concluded that the reform curriculum is more effective for grade 9 students. Furthermore, this conclusion has resulted from a consideration of various rival perspectives. However, a sizable group of parents strongly opposes the new curriculum. Their concerns stem from beliefs that the new curriculum engenders low expectations among students, de-emphasizes basic skills, and places little attention on getting correct answers to problems. The views of this group of parents, who happen to be very active in school-related affairs, have been influenced by newspaper and news magazine reports raising questions about the new curricula, called "fuzzy math" by some pundits. To complicate matters further, although the teachers in the study were "true believers" in the new curriculum, many of the other mathematics teachers in the school district have little or no enthusiasm about changing their traditional instructional practices or using different materials, and only a few teachers have had any professional development training in the implementation of the new curriculum.

Before they begin to publicize their claims, the research team is obliged to consider both the ethical and practical issues raised by concerns and realities such as those presented above. Is it sensible to ask teachers to implement an instructional approach that will be challenged vigorously by some parents and perhaps others? Can they really claim, as the school district superintendent desires, that student performance on state mathematics tests will improve if the new curriculum is adopted? Are they confident enough in their conclusions about the merits of the new curriculum to recommend its use to inexperienced teachers? Should they encourage reluctant or resistant teachers to use this approach in their own classrooms if they may do so half-heartedly or superficially? Can these reluctant teachers be expected to implement this new curriculum in a manner consistent with reform principles? These sorts of ethical and practical questions are rarely addressed in research in mathematics education, but must be addressed if the researchers really care about moving the school district to act on their conclusions. Answers to questions such as these will necessitate prolonged dialogue with various groups, among them teachers, school administrators, parents, and students.

Implicit in the Singerian system of inquiry is consideration of the *practical* consequences of one's research, in addition to the ethical positions. Greeno (1997) suggests that educational researchers should assess the relative worth of competing (plausible) perspectives by determining which perspective will contribute most to the improvement of educational practice and we would add that this assessment must take into account the constraints of the available resources (both human and financial), the political and social contexts in which education takes place, and the likelihood of success. While the Lockean, Kantian and Hegelian inquirer can claim to be producing knowledge for its own sake, Singerian inquirers are required to defend to the community not just their methods of research, but which research they choose to undertake.

Singerian inquiry provides a framework within which we can conduct a debate about what kinds of research ought to be conducted. Should researchers work with individual teachers supporting them to undertake research primarily directed at transforming their own classrooms, or should researchers instead concentrate on producing studies that are designed from the outset to be widely generalizable? Within a Singerian framework, both are defensible, but the researchers should be prepared to defend their decisions. The fact that the results of action research are often limited to the classrooms in which the studies are conducted is often regarded as a weakness in traditional studies. Within a Singerian framework, however, radical improvements on a small-scale may be regarded as a greater benefit than a more widely distributed, but less substantial improvement.

In their discussion of Churchman's classification scheme, Lester and Wiliam (2002) demonstrated that the researcher's philosophical stance is vitally important. In particular, they showed that warrants and interpretations, and the ethical and practical bases for defending the consequences are constantly open to scrutiny and question. Unfortunately, U.S. graduate programs typically fail to provide novice researchers with adequate grounding in philosophy.

The Goals of MER and the Place of Frameworks and Philosophy

In his book, *Pasteur's Quadrant: Basic Science and Technological Innovation*, Donald Stokes (1997) presents a new way to think about scientific and technological research and their purposes. Because certain of his ideas have direct relevance for MER and the roles of theory and philosophy, let me give a very brief overview of what he proposes.

Stokes began with a detailed discussion of the history of the development of the current U.S. policy for supporting advanced scientific study (I suspect similar policies exist in other industrialized countries). He noted that from the beginning of the development of this policy shortly after World War II there has been an inherent tension between the pursuit of *fundamental understanding* and *considerations of use*. This tension is manifest in the often-radical separation between basic and applied science. He argued that prior to the latter part of the 19th Century, scientific research was conducted largely in pursuit of deep understanding of the world. But, the rise of microbiology in the late 19th Century brought with it a concern for putting scientific understanding to practical use. Stokes illustrated this concern with the work of Louis Pasteur. Of course, Pasteur working in his laboratory wanted to understand the process of disease at the most basic level, but he also wanted that understanding to be applicable to dealing with, for example, anthrax in sheep and cattle, cholera in chickens, spoilage of milk, and rabies in people. It is clear that Pasteur was concerned with both fundamental understanding and considerations of use.

Stokes proposed a way to think about scientific research that blends the two motives: the *quest for fundamental understanding* and *considerations of use*. He depicted this blending as shown in Fig. 1, where the vertical axis represents the quest for fundamental understanding and the horizontal axis considerations of use.

So, Pasteur's research belongs in the upper right quadrant, but what of the other three quadrants of the figure? Consider first the upper left quadrant. Neils Bohr came up with a radical model of the atom, which had electrons orbiting around a nucleus. Bohr was interested solely in understanding the structure of the atom; he was not concerned about the usefulness of his work. Research in the lower right quadrant is represented by the work of Thomas Edison on electric lighting. Edison was concerned primarily with immediate applicability; his research was narrowly targeted, with little concern about deeper implications or understanding. (It may be that Edison's lack of interest in seeking fundamental understanding explains why he did not receive a Nobel Prize.) Finally, in the lower left quadrant we have research that involves explorations of phenomena without having in view either explanatory goals or uses to which the results can be put. One would hope that little, if any,

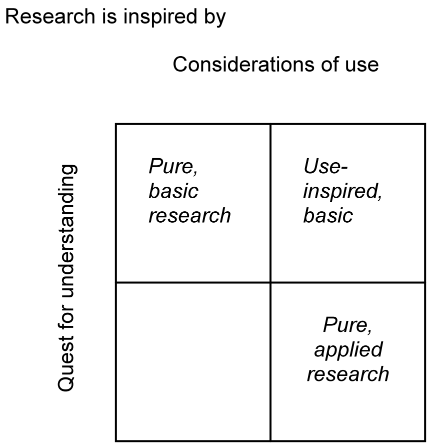


Fig. 1 Stokes's (1997) model of scientific research

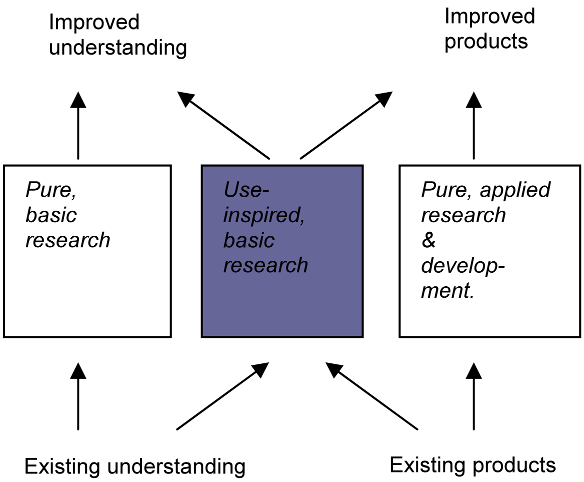
research has taken place in science, education, or any other field in this quadrant—no interest in fundamental understand or consideration of usefulness)—but I suspect that such research has been conducted.

Stokes then presented a somewhat different model (he referred to it as a "revised, dynamic model," p. 88) for thinking about scientific and technological research. In this model, the outcome of pure, basic research is still an increase in understanding and the outcome of pure, applied research is an improvement over existing technology. By melding the two types of research, we get *use-inspired, basic research* that has as its goals increased understanding and technological advancement. Adapting Stokes's dynamic model to educational research in general, and MER in particular, I have come up with a slightly different model (see Fig. 2). There are two minor, but important differences between my model and Stokes's. First, I have broadened pure, applied research to include "development" activities. Second, I have substituted "technology" with "products" (e.g., instructional materials, including curricula, professional development programs, and district educational policies).

Assuming that the case has been made for the importance of conceptual frameworks and taking account of one's philosophical stance in MER, it remains to show how researchers, especially novices, can deal with the bewildering range of theories and philosophical perspectives that are on offer. In his forthcoming chapter in the revised *Handbook of Research on Mathematics Teaching and Learning*, Cobb (2007) considers how mathematics education researchers might cope with the multiple and frequently conflicting perspectives that currently exist. He observes:

The theoretical perspectives currently on offer include radical constructivism, sociocultural theory, symbolic interactionism, distributed cognition, information-processing psychology, situated cognition, critical theory, critical race theory, and discourse theory. To add to the mix, experimental psychology has emerged with a renewed vigor in the last few years. Proponents of various perspectives frequently advocate their viewpoint with what can only be described as ideological fervor, generating more heat than light in the process. In the face of this sometimes bewildering

Fig. 2 Adaptation of Stokes's "dynamic" model to educational research



array of theoretical alternatives, the issue ... is that of how we might make and justify our decision to adopt one theoretical perspective rather than another.

Cobb goes on to question the repeated (mostly unsuccessful) attempts that have been made in mathematics education to derive instructional prescriptions directly from background theoretical perspectives. He insists that it is more productive to compare and contrast various theoretical perspectives in terms of the manner in which they orient and constrain the types of questions that are asked about the learning and teaching of mathematics, the nature of the phenomena that are investigated, and the forms of knowledge that are produced. Moreover, according to Cobb, comparing and contrasting various perspectives would have the added benefit of both enhancing our understanding of important phenomena and increasing the usefulness of our investigations.

I suggest that rather than adhering to one particular theoretical perspective, we act as *bricoleurs* by adapting ideas from a range of theoretical sources to suit our goals—goals that should aim not only to deepen our fundamental understanding of mathematics learning and teaching, but also to aid us in providing practical wisdom about problems practitioners care about. If we begin to pay serious attention to these goals, the problems of theory and philosophy are likely to be resolved.

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