Understanding a Teacher's Actions in the Classroom by Applying Schoenfeld's Theory *Teaching-In-Context*: Reflecting on Goals and Beliefs

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Introduction

Different from numerous research projects in mathematics education, at the beginning of this work, we did pursue neither a specific research plan nor a certain investigation goal, even no concrete research question guided the analysis we will report on in the following. To the contrary, in the foreground stood the development of teaching videos for a bi-national in-service teacher training organized by the University of Duisburg-Essen in Germany and the Freudenthal Institute in the Netherlands. The workshop was chosen to concentrate on the treatment of linear functions in school in the two countries, in order to work out some cultural differences or commonalities. Therefore, the researchers agreed to tape an exemplary classroom video on teaching the abovementioned subject in a German and a Netherlands classroom. The comparison between the two countries had been planned as a weekend in-service training event for more than 50 teachers. The teachers discussed both teaching examples in small groups while pursuing different focal points.

This is not the place to report on the inspiring event that took place in Germany, but we will discuss interesting teaching processes, which were observable in the video of the German lesson and attracted our attention. Ultimately, these observations developed a momentum of their own, and were thus dedicated a separate

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B. Sriraman, L. English (eds.), *Theories of Mathematics Education*,
Advances in Mathematics Education,
DOI 10.1007/978-3-642-00742-2_38,

analysis. In this context, if one assesses the videoed lesson as successful or not, is a question that we consider superfluously, and we do not aim at evaluating the lesson in this sense. However, the lesson served its purpose within the scope of the in-service training event, and moreover, provided very interesting material and incited the scientific discourse that we will elaborate on.

To cut right to the chase of the matter: At first, we are very thankful that a German teacher gave us insight into her lesson, which is surely no matter of course. At second, a very relevant issue lies in the fact that the teacher had recently attained an in-service training course on the use of open tasks in the classroom. Therefore, issues of professional development come also to the fore. That is, the teacher tried very eagerly and engaged to implement newly imparted issues into her teaching on linear functions, a topic that she has taught in rather traditional ways for several times. Although the teacher planned the lesson thoroughly, its course developed unexpected so that she shifted back to her solid and approved methods. Like beating a hasty retreat, the teacher drew on the mathematical structure and its systematic as a safety net (Törner et al. 2006).

To remain fair, one has to assume that field-testing and implementing new ideas is not automatically a successful endeavor, even though new conceptions appear reasonable on first sight and were provided adequately and correctly (Sowder 2007). Implementing *good* approaches, and that is well known in the field of professional development, is not always crowned with success (Cooney and Krainer 1996). In his famous paper, Cohen (1990) gives an impressive example for the constraints of professional development by his well-known case study of a teacher named Mrs. Oublier. Mrs. Oublier was very open for implementing new curriculum material and activities, but surprisingly, the initiated change just remained at the surface (Pehkonen and Törner 1999). Accordingly, Cohen (1990) concludes that “Mrs. O. seemed to treat new mathematical topics as though they were part of traditional school mathematics” (p. 311), and describes correspondingly her teaching style as mélange of “something old and something new” (p. 312). What is striking is that although the teacher was open for new approaches, well-established beliefs, knowledge, routines and scripts were not simply replaced, but new experiences added or assimilated. Pehkonen and Törner (1999) report on a similar observation and stress the influence of the established teaching style as follows:

> Teachers can adapt a new curriculum, for example, by interpreting their teaching in a new way, and absorbing some of the ideas of the new teaching material into their old style of teaching. (p. 260)

To sum up, it is the old style of teaching based on established knowledge and beliefs that runs counter implementing the appreciated new aspects of teaching, an issue that is approached in more detail in the following.

Although the video material of the whole lesson provides incitements for many aspects of analysis, we, however, restrict ourselves in this chapter to exploring the unexpected turning point in the videoed lesson. Thereby, we pursue the question, *To what extend was the sudden change in the teaching style inevitable or at least predictable?* In the following, we will provide some answers while taking seriously the teacher’s perspective, who planned her teaching carefully, above all to left nothing to chance with a view to the videotaping team.

Understanding a Teacher's Action in Terms of Knowledge, Goals and Beliefs

The unexpected turning point in the videoed lesson, going beyond the originally intended context to introduce linear functions, is the main subject of our analysis. The question arising for us is how the related processes can be explained or understood rationally (Cobb 1986). Of course, there are a number of approaches relevant for such an analysis. First of all, one can refer to theoretical approaches on everyday practices in mathematics lessons (Andelfinger and Voigt 1986; Krummheuer and Fetzer 2004), which provide an *interactional theory* of learning and teaching mathematics. Additionally, we like to refer to an interesting paper by Bauersfeld (1978) that discusses the relevance of communicative processes for developments in teaching. It is beyond debate that communication is a driving force in mathematics teaching; particularly, when the teacher is assigned a rather dominant role. The phenomenon we observed resembles a *hunted* teacher saving him or herself via well-known trails, like animals in a forest that prefer the route along a deer crossing (Bauersfeld 1978).

As mentioned earlier, we have opted for Schoenfeld's (1998) theory of *teaching-in-context*, especially paying attention to the three fundamental parameters knowledge, goals, and beliefs, which we abbreviate here as KGB framework. This theory explains developments in teaching from a more multi-faceted perspective and allows the didactical analysis of focusing on understanding, explaining, and prognosticating rich and complex teaching coherences. A teacher's spontaneous decision-making is characterized in terms of available knowledge, high priority goals and beliefs. Insofar, the teaching process is understood as a continuous decision-making algorithm. Schoenfeld's (2000, 2003, 2006) fundamental assumption is that these processes are accomplished typically from an inner perspective, and are thus understandable rationally. Teaching processes depend on multitudinous influencing factors, but a theoretically based description calls for minimizing the variables, in order to identify the most significant ones. Thus, we follow Schoenfeld, who considers the three variables of knowledge, goals and beliefs as sufficient for understanding and explaining numerous teaching situations.

Available Teacher Knowledge

Within the discussion of teacher professional knowledge, Shulman's (1986) venerable paper *Those who understand: Knowledge growth in teaching* remains central, and his notions of subject matter knowledge, and particularly, pedagogical content knowledge initiated the discourse significantly, and much subsequent research has followed. By this basic work, Shulman (1986, 1987) developed both a topology as well as a typology of professional knowledge of teachers (Baumert and Kunter 2006), which was modified by several authors (Grossman 1990; Bromme 1994). Over the last two decades, essential research in mathematics teacher education has also focused on other accounts of teacher knowledge (Sherin et al. 2000), particularly maintaining the decisive role of substantial mathematical skills for teaching (Ball 2000a, 2000b, 2002; Ball and Bass 2000; Ma 1999), or of skilled teachers' mental structures in terms of routines, agendas, and curriculum scripts (Leinhardt and Greeno 1986). The approaches have in common that they extend merely theorizing about knowledge to additionally considering knowledge that is relevant in praxis, i.e., when teaching in the classroom. In the current German debate, Tenorth (2006) tries correspondingly to draw more attention to teaching practice and its associated essential routines. He points out that it is not sufficient to just focus on knowledge and derived competencies, but also necessary to consider professional schemes, which represent the practical organization of teaching for a live in-class performance (Roesken accepted). Tenorth's (2006) provocative subtitle *Theory stalled but practice succeeds* does not herald an argument against knowledge, but one against abstractly theorizing about knowledge (Rösken et al. 2008).

Of course, it is trivial that a teacher's knowledge takes a decisive role with a view to teaching, like the wool is definitely needed for knitting, in adaptation of a bon mot by Heinrich Winter (1975). Nevertheless, we follow Leinhardt and Greeno as well as Tenorth, and we give, compared to Schoenfeld, more prominence to the fact that a teacher not only retrieves encyclopedical knowledge continuously, but relies directly on scripts and routines, which are modifiable, but often represented in similar action patterns. Accordingly, for the decision-making processes that are necessary steadily throughout the course of a lesson, practical knowledge is significant that is directly relevant for teaching. Situations, where teachers are able to experiment and to vary their behavior, are easily conceivable, but rather unlikely when teachers come under stress, or feel like they are on the run like described in the metaphor used by Bauersfeld (1978). Indeed, then the teacher is more likely to draw on well-practiced experiences.

Teacher Beliefs

In the literature, beliefs have been described as a *messy construct* with different meanings and accentuations (Pajares 1992), and indeed the term belief has not yet been clearly defined. However, there is some consensus that mathematical beliefs are considered as personal philosophies or conceptions about the nature of mathematics as well as about teaching and learning mathematics (Thompson 1992). Following Schoenfeld (1998), beliefs can be interpreted as "mental constructs that represent the codification of people's experiences and understandings", and he continues to state that "people's beliefs shape what they perceive in any set of circumstances, what they consider to be possible or appropriate in those circumstances, the goals they might establish in those circumstances" (p. 19).

A teacher's beliefs about the mathematical content and the nature of mathematics as well as about its teaching and learning have an influence on what he or she does in the classroom, and what decisions he or she takes. Quite recently, Goldin et al. (2008) elaborate on exploring the psychological and epistemological consequences of metaphors or analogies used to describe beliefs in order to understand their definitions or interpretations. With respect to mathematics teaching and learning, they differentiate between *problem solving approaches, change and development approaches*, as well as *sense-making approaches*, and shed light on some essential roles of beliefs.

Törner and Sriraman (2007) add a slightly different aspect to the discussion by stressing the need to develop a philosophy of mathematics compatible with the one of mathematics education. Accordingly, in his book review of Byers' *How Mathematicians Think*, Hersh (2007) points out that one commonly shared and prevalent belief, not at least among teachers, is to perceive mathematics as precise while Byers elucidates that ambiguity is always present when dealing with mathematics. Even as elaborating on the notion of ambiguity, Byers (2007) stresses how strongly held and non-reflected beliefs permeate and influence our knowledge base, and play a decisive role since they serve additionally as identification base for teachers. Correspondingly, teachers often assign a crucial role to the mathematical structure and its systematic, as was already pointed out in the introduction.

Lerman (2001) identifies two major strands of research concerning beliefs: the analysis and classification of beliefs, and monitoring changes in beliefs over time. In this regard, Cooney (2001) refers to an essential aspect when he underlines that much literature is concerned with beliefs but not with their structure. Further, he considers the structure as crucial since from information about how beliefs are formed can arguably be derived how they change. A few studies use well-established categorizations of beliefs in order to document change in a person's beliefs about the nature of mathematics and its teaching and learning (Liljedahl et al. 2007). Other research draws on the fundamental work by Green (1971) and identifies structural features in beliefs research in terms of dimensions (Pehkonen 1995; De Corte et al. 2002; Rösken et al. 2007). Correspondingly, beliefs can not be regarded in isolation; they must always be seen in coherence with other beliefs. In the literature, this phenomenon is described by using the term *belief system*. Green (1971) points out that "beliefs always occur in sets or groups. They take their place always in belief systems, never in isolation" (p. 41). Aguirre and Speer (2000) introduce the construct *belief bundle,* which "connects particular beliefs from various aspects of the teacher's entire belief system (beliefs about learning, beliefs about teaching, etc.)" (p. 333). Furthermore, they consider the activation level of certain beliefs by stating that "a bundle is a particular manifestation of certain beliefs at a particular time" (p. 333), an observation that is especially relevant for teaching.

Goals, Their Interdependencies with Beliefs, and Structural Features

Typically, a teacher begins a lesson with a specific agenda, in particular with certain goals he or she wants to accomplish. With regard to these goals, the underlying structure of a lesson can be identified, especially the teacher's choices can be modeled. Schoenfeld (2003) distinguishes three categories of goals: overarching goals, major instructional goals, and local goals, which occur at different grain sizes:

> Overarching goals [...] are consistent long-term goals the teacher has for a class, which tend to manifest themselves frequently in instruction. [...] Major instructional goals may be oriented toward content or toward building a classroom community. Such goals tend to be more short-term, reflecting major aspects of the teacher's agenda for the day or unit. Local goals are tied to specific circumstances. [...]; a goal becomes active when a student says something that the teacher believes needs to be refined in some way. (p. 20)

All these goals have different and altering priorities in any situation during a lesson. This partial re-prioritization of goals is the topic of our reflections, and guides the analysis we will report on.

Most research on beliefs and goals has focused on these constructs quite isolated from one another (Aguirre and Speer 2000). Nevertheless, there are several interdependencies between the set of beliefs and the one of goals. A teacher's goals are part of his or her action plan for a lesson. He or she enters the classroom with a specific agenda, in particular, with a certain constellation of goals, which might change in relation to the development of the lesson. Looking at these goals elucidates the teacher's actions. In this respect, Schoenfeld (2006) emphasizes that a shift in a teacher's goals provides an indication of the beliefs he or she holds. Moreover, he states that beliefs influence both the prioritization of goals when planning the lesson *and* the pursuance of goals during the lesson. Taken together, beliefs serve to re-prioritize goals when some of them are fulfilled and/or new goals emerge (Schoenfeld 2003).

Cobb (1986) has already pointed out that beliefs are allocated the link between goals and the actions arising as a consequence of them:

> The general goals established and the activity carried out in an attempt to achieve those goals can therefore be viewed as expressions of beliefs. In other words, beliefs can be thought of as assumptions about the nature of reality that underlie goal-oriented activity. (p. 4)

However, in the research literature beliefs are often given priority over goals, as indicated by Schoenfeld (2003) in his statements mentioned above. He further points out that "a teacher's beliefs and values shape the prioritization both of goals and knowledge employed to work toward those goals" (p. 8), or "they [beliefs] shape the goals teachers have for classroom interactions" (p. 248).

Whereas the interdependencies between goals and beliefs are sometimes mentioned in the literature, these ideas are not explicitly worked out (Cobb 1986; Schoenfeld 1998, 2000, 2003). Also, one finds only a few clues on a suitable internal modeling within the set of beliefs and the one of goals (Cooney et al. 1998; Törner 2002). Prioritizations, hierarchies and other dependencies seem to be relevant in this context. And again, the fundamental work by Green (1971) gives some hints for a quasi-logical relationship, at least for a set of beliefs:

> We may, therefore, identify three dimensions of belief systems. First there is the quasi-logical relation between beliefs. They are primary or derivative. Secondly, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it

> were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. (p. 47/48)

Törner (2002) pursues a comparable approach by understanding the highly individualized beliefs of a person as a *content set*. According to this, Goldin et al. (2008) point out the following:

> The content set can be modeled as akin to the mathematical notion of a *fuzzy set*, which means that the elements of the content set possess different weights that are attributed to various perceptions or assumptions. This membership function may be regarded as a measure of the level of consciousness and certitude of the belief bearer, or the degree of activation of the belief. (p. 12)

Given that goals possess an altering priority, as inherent in the abovementioned categorization by Schoenfeld, it is apparent that the set of goals can be modeled accordingly. The aim of our analysis goes beyond simply identifying the variables of knowledge, goals, and beliefs the teachers of the videoed lesson possesses, but includes elaborating on the specific interdepencies between goals and beliefs.

Empirical Approach and Methodology

Since we report in the following on methodological issues and basic principles, we actually have to distinguish two different levels: At first, we give an overview on the available data sources, i.e., the conception of the videoed lesson and the subsequently conducted interview with the teacher. At second, we will justify and explain our approach of analyzing the data. We will start to elaborate on the former aspect:

Data Sources

Besides the topic *introduction to linear functions*, the responsible teacher was allowed to design the lesson free of any directives or restrictions. Obviously, linear relationships between objects can serve for initiating motivation for the treatment of linear functions. In the lesson, converting currencies, and measures of length and height were used to introduce the topic, while the individual tasks were embedded in the story of a European couple planning a travel to the US. The lesson started rather open and problem-oriented, using examples such as petrol consumption of vehicles or temperature changes in dependency of the height. Units like miles, gallons and Dollar were converted into kilometers, liters and Euro, while feet and degrees Fahrenheit were converted into meters and degrees Centigrade. Students worked in small groups of three or four using Excel. However, as the lesson developed and time seemed to run out, the teacher suddenly changed her teaching style in favor of a more traditional approach. That is, she switched to a monologue on definitions in a formalized structure. These observations have challenged the question whether this turn in the teaching trajectory and the discontinuities involved could be understood rationally.

A teacher with thirty years of professional experience taught the lesson in question. She has mainly been teaching in grade 11 and 12, but also in grade 5 to 10 in a German high school. Remarkably, she has attended numerous in-service teacher training courses, in particular on using computer algebra systems and open tasks in mathematics teaching. The lesson was videoed professionally, using three cameras, and was then carefully verbatim transcribed. We obtained a multi-layered transcript, involving different perspectives on the lesson that included the students' and the teacher's statements.

In order to gain a comprehensive comment of the teacher, we collected additionally data by an interview, for which the second author was responsible. The interview with the teacher allowed for gaining deep insight into her perspective. It has been our particular concern to use the words of the teacher herself to explain the turning point in the lesson, and to understand how she experienced what happened in the classroom. The interview data contributes essentially to a better understanding of the findings observed in the videoed lesson. That is, the questions used in the interview were concerned with concretizing some of the remarkable issues that became apparent in the course of the lesson. The interview was conducted mainly open, and the interviewer approached the topic rather indirect since this approach is more likely to obtain frank and open responses. The primary technique applied during the interview was trying to stimulate the teacher to deepen her descriptions and explanations. That is, the vocabulary used by the interviewee was further taken up, and used as stimulus to probe for more in-depth responses. The teacher was very sensitive to this invitation, and elaborated intensely on her statements. Thus, the interview data portrays substantially the teachers' knowledge, goals, and beliefs related to the incident in the lesson on linear functions. The interview was undertaken in the German language. Those parts selected to be subject of an intensive analysis were then translated into English. Thereby, the aim was to translate literally as far as possible, but also in an accessible way. As an additional source of information a questionnaire was developed, that was handed out to the students and enabled them to reflect the processes of the lesson. However, in this paper, the students' responses served only partly to support the teacher's expressed goals and beliefs.

Data Analysis

Content analysis was applied to the interview statements, our focus was a theory-driven approach to the data (Kvale 1996; Lamnek 2005). We applied Schoenfeld's theory as frame of reference, our data analysis is hence essentially based on the knowledge, goals and beliefs that were observable in the lesson as well as those the teacher expressed in the interview, where she reflected the lesson process in retrospect. In the following, we employ the category *knowledge* in the sense of Leinhardt and Greeno (1986). We consider it worth mentioning that available knowledge—that is, action scripts through which lessons are designed or carried out—must be viewed as representing limitations and restrictions inherent in the beliefs and goals, a teacher holds in any situation. Consequently, our analysis is concerned particularly with elaborating on the interdependencies between goals and beliefs, and documenting the duality of both constructs.

Results on Goals and Involved Beliefs

The comments of the teacher in the interview show clearly that goals and beliefs can hardly be separated. That is, when reading through the interview one can identify statements, which can be regarded either as goals or as beliefs. For instance, it appeared that goals were founded in beliefs, but also that beliefs did not always imply direct actions although they definitely influenced or induced defined goals. That is why we chose to speak of the duality of both constructs. In particular, the open interview style incited the teacher to justify her goals, partly without being explicitly asked to do so. Subjective convictions hereby become evident, which we understand as beliefs.

As mentioned in the theory section, goals and beliefs present a complex network of dependencies. The videoed lesson reveals an abrupt change in the teaching style after approximately 20 minutes, which leads us to assuming that new prioritizations and shifts did occur in the networks of goals and beliefs at this point. Our aim is to present the complexity of these networks on the one hand, and to clearly point out the characteristics of these goals and beliefs on the other hand. It would make sense to deal with goals and beliefs separately and to simply list them in chronological order as they occurred in the interview or the lesson. However, this separation would strengthen the impression that particularly goals can be understood in isolation. On the contrary, there exist reciprocal correspondences and argumentative relations to beliefs, going beyond the individual sections.

Whereas Schoenfeld (1998, 2006) distinguishes overarching goals, major instructional goals, and local goals occurring at different grain sizes, we chose a different characterization, which appears to us to be more suitable in our context. That is, in the following we will draw on the work by Shulman (1986) and adapt his categorization for the domain of knowledge to the one of beliefs, and we differentiate between formal goals (section 'Formal Goals'), pedagogical content goals and beliefs and their networking (section 'Pedagogical Content Goals and Beliefs, and Their Networking'), and subject matter goals and beliefs and their internal structure (section 'Subject Matter Goals and Beliefs and Their Internal Structure').

Formal Goals

The primary formal goal of the teacher, and this did not change during the lesson, was the production of a complete video sequence to the theme *introduction to linear functions*. For this purpose, the teacher prepared as a first approach to the topic some open tasks that dealt with several linear relationships. Obviously, the decision on how to introduce the topic was also influenced by the imagined presence of the video team. As first two goals, thus, the following statements by the teacher are identified:

> **Teacher:** More or less comprehensive video material has to be produced at the end of a 45 minutes lesson (formal goal 1). The content of the recorded lesson is an *introduction to linear functions* in grade 8 (formal goal 2).

These goals imply some restrictions on the design and reliability of the lesson, which require additionally of the teacher a certain amount of self-confidence. However, the teacher felt able to choose a new approach that drew mainly on the use of open tasks by simultaneously using the computer and the program Excel.

Pedagogical Content Goals and Beliefs, and Their Networking

According to the categorization by Shulman, we elaborate on the pedagogical content goals and beliefs that include also methodological issues. Citing the teacher statement of the interview, we sometimes only use one aspect reflected in the goals and beliefs in accordance with our initial hypothesis that beliefs and goals correspond with each other. In the following, we therefore do not always verbalize both aspects when they are attached to the same idea. When articulating, e.g., a goal, we assume that the underlying belief is undisputable and therefore does not need to be mentioned explicitly. Accordingly, we list the pedagogical content goals and beliefs and apply a consecutive numbering. In case, we mention a goal and belief related to the same idea, a similar number is assigned.

Before the lesson was videoed, the teacher had recently visited a teacher in-service training course on the use of the computer in school. Thus, the central question for the teacher on how the lesson should be designed methodologically comes as no surprise: *How shall I do it: with or without the computer?* Under the impression of the recently experienced in-service training course, her decision to employ the computer appears close at hand. This choice is rather independent of the content, as she underlines in the following statement: *You can do things in geometry with the computer.*

The following goal, emphasizing the positive aspect related to the use of the computer in the classroom, is uttered explicitly as follows:

> **Teacher:** Whenever possible, I employ the computer in mathematics lessons (pedagogical content goal 1).

This goal is complemented by beliefs attached to the aforementioned formal goal 2 (section 'Formal Goals') that is concerned with the topic of the lesson.

> **Teacher:** The theme linear functions can be mediated by the computer (pedagogical content belief 2).

The teacher views a suitable approach to this theme by employing the spreadsheet software Excel, i.e., she expresses her corresponding belief as follows:

Teacher: Excel is suitable for dealing with linear functions (pedagogical content belief 3).

The students, as their comments in the questionnaire showed, also recognize this teacher goal:

Student: I think we should try and find out whether we can solve tasks with the aid of the computer and computer programs.

An important prerequisite for the teacher is that Excel offers the required possibilities for introducing linear functions, out of which the teacher formulates a detailed, mathematics-specific goal:

Teacher: The students are to draw graphs (pedagogical content goal 4).

Again, a student reflects this goal when he or she articulates:

Student: [. . .] that we are to do these graphs correctly.

However, Excel is not primarily designed as lesson software but as an office program. That is, diagrams of linear functions are produced in standardized formats, and thus often look uniform. Differences in slope values are very quickly blurred or lost. The teacher indicated in the interview that she was aware of this fact:

Teacher: Excel fools you.

However, the teacher transforms such possibly occurring confusions positively by formulating from this circumstance a further pedagogical content goal concerning the use of the computer:

Teacher: The use of the computer has to be accompanied by a critical discussion (pedagogical content goal 5).

Simultaneously, she once more emphasizes that the computer can be an adequate mean to support learning in mathematics. The following belief is hence directly linked to the goal mentioned above:

Teacher: The computer is a modern, progressive medium (pedagogical content belief 6).

This conviction is also implicitly formulated as a goal:

Teacher: School lessons should use modern media (pedagogical content goal 6).

Complementing the assessment that the computer is a progressive medium, the teacher also sees other educational advantages in employing this medium:

Teacher: The computer supports learning through discovery (pedagogical content belief 7).

With regard to the computer, she formulates more generally:

Teacher: Mathematics lessons should offer students free space for discovery (pedagogical content goal 8).

Following this, she articulates more pointedly her belief that Excel is suitable for dealing with linear functions (pedagogical content belief 3) by linking it to the aforementioned goal that in mathematics lesson, students should be given the opportunity to discover features on their own (pedagogical content goal 8):

Teacher: You can discover a lot with Excel (pedagogical content belief 9).

This belief naturally demands circumstantial conditions concerning lesson organization by the teacher. Correspondingly, she reflects on the following thought:

> **Teacher:** Mathematics lessons have to be designed openly (pedagogical content belief 10).

Open lesson organization by the teacher and free space for students to discover are reciprocal. She completely fulfilled this requirement in her lesson planning and realized this approach consequently in the first half of the lesson. But then, the teacher realized that time was getting short and that she was running the risk of missing her formal goal 1 to provide a comprehensive video. In particular, she noticed that the lesson did not develop as desired because the students could not achieve the central terms in the context of linear functions. As a reaction, she shifted back to her approved methods and traditional teaching style, an incident that constitutes a remarkable turning point in the lesson. In the interview, the teacher reflects on her initial approach as follows:

> **Teacher:** Open questions have to be prepared (pedagogical content belief 11).

Complementary to the fundamentally positive approach, namely to design lessons open and with a discovery bias, she draws a fatal consequence when stating that open questions actually have to be prepared.

However, she is well aware of the relevance of creating a suitable motivation disposition in the students. She wishes to fulfill this by stating clearly the goal:

> **Teacher:** Mathematics lessons have to be motivating (pedagogical content goal 12).

She justifies this goal from her point of view once again with the use of the computer when she states:

> **Teacher:** The use of the computer can be motivating, in particular, in mathematically weak classes. (pedagogical content belief 13).

Even though the teacher does not establish implicitly the connection between everyday life situations in mathematics lessons and motivation, her statement in the interview gives evidence for this assumption, as the following expressed beliefs indicate:

> **Teacher:** Mathematics lessons should have a link to students' reality (pedagogical content belief 14).

> **Teacher:** Mathematics lessons have to be meaningful for the students (pedagogical content belief 15).

In spite of all these good intentions, the way the lesson turned out did, for many reasons, not meet the teacher's expectations. She deserves credit without reservation for wanting to give an innovative and successful lesson. This is also documented through her participation in several professional development activities, which she comments adequately:

> **Teacher:** Teacher training or professional development activities encourage new, progressive concepts for the improvement of lessons (pedagogical content belief 16).

Incorporating such concepts in her lessons is an intention documented by the initial lesson plan. After the lesson, the teacher therefore reflects in detail on the reasons of deviating from the original plan, and justifies her sudden modification by the following statement that is closely related to her judgment that open questions should be prepared (pedagogical content belief 11):

> **Teacher:** Students can more easily handle concrete directives than open questions (pedagogical content belief 17).

At first glance, the beliefs about the necessity and the difficulty of open lessons respectively seem to contradict each other. However, the teacher is well aware of this contradiction and reconciles it with the thought that open questions also have to be prepared:

> **Teacher:** Open questions have to be drilled. You cannot simply throw an open question at the students and then say: Okay, start!

Additionally, the teacher presents explicitly her belief on the relevance of open tasks as anchored to another individual. That is, the teacher delegates the question of responsibility to the teacher educator who conducted the in-service training course. She laughingly points out that the trainer encouraged her to try out the new approach: *The trainer is to blame with her open questions.* Abelson (1979) has presented the anchoring of beliefs to other persons as a typical characteristic.

In the following, we group together some of the pedagogical content beliefs. The beliefs identified here demonstrate that it is appropriate to speak of *belief bundles* in the terminology provided by Aguirre and Speer (2000). Thereby, we focus on mentioning explicitly beliefs that were sometimes formulated by the teacher as goals. In so doing, we draw on the duality aspect of the concepts and are able to identify five belief bundles:

Bundle (A): Beliefs about the computer

- The computer and mathematics lessons belong together (pedagogical content belief 1).
- Linear functions can be dealt with using the computer Excel is the choice tool for linear functions (pedagogical content belief 2).
- Excel is the choice tool for linear functions (pedagogical content belief 3).
- Critical reflection is necessary when using the computer (pedagogical content belief 5).
- The computer is a modern and progressive medium (pedagogical content belief 6).
- The computer is an adequate tool for learning by discovering (pedagogical content belief 7).

Bundle (B): Beliefs about discovery-oriented lessons

- Mathematics lessons have to be discovery lessons (pedagogical content belief 8).
- The computer is an adequate tool for learning by discovering (pedagogical content belief 7).
- Excel is a useful tool for discovery lessons (pedagogical content belief 9).

Bundle (C): Beliefs about open lessons

- Mathematics lessons are to be openly designed (pedagogical content belief 10).
- Open questions have to be prepared (pedagogical content belief 11).

Bundle (D): Beliefs about motivational mechanisms

- Mathematics lessons have to be motivating (pedagogical content belief 12).
- Employing the computer enhances motivation in weak classes (pedagogical content belief 13).

Bundle (E): Beliefs about reality-related lessons

- Mathematics lessons have to be reality-related (pedagogical content belief 14).
- Mathematics lessons have to be meaningful for the students (pedagogical content belief 15).

Belief bundle (A) is characterized by a very positive assessment of the role of the computer, bundle (B) focuses on discovery-oriented lessons, whereas bundle (C) is concerned with open lessons, and (D) with motivation in lessons. Bundle (E) deals with the role of reality-related tasks for learning mathematics. Obviously, bundle (A) is central since the computer was assigned a main role in the lesson planning and relates strongly to the *corner* bundles (B), (C) and (D). For bundle (E), the teacher appears to link this bundle rather to (D) and less to (A); at least one cannot find any explicit clue pointing from (E) directly to (A). In the following figure, we display the networking of the different beliefs bundles:

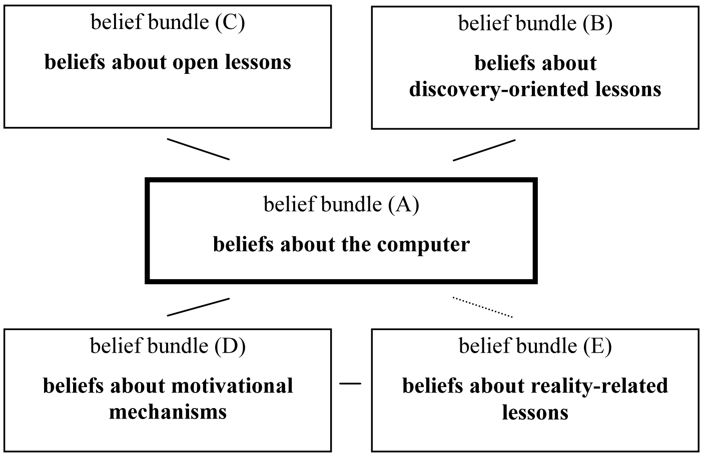

Fig. 1 Networking the belief aspects

Figure 1 sketches the relations between the individual belief bundles. We would like to point out that it is a drastic simplification, since not all subtle connections between the bundles are described. At least, the central pedagogical content beliefs are displayed and the central role of the computer is documented.

Subject Matter Goals and Beliefs and Their Internal Structure

The subject matter goals and beliefs derived from them are not only rooted strongly in mathematics, but also in beliefs about the curriculum. In the whole, they create the impression of forming a stable network, whose core is independent from the pedagogical content beliefs mentioned in the subsection before, and is consequently unaffected by the *shortcomings* in the concrete realization of the lesson plan.

> **Teacher:** The term function is a central term in mathematics (subject matter belief 1).

The teacher's diction emphatically points to the importance she gives to the theme: *I think the notion* ***function*** *is infinitely important*, whereby her stress on the word *function* is significant. This leads to a curricular goal whose manifold facets are discussed in many authoritative books on mathematical didactics:

> **Teacher:** Dealing with functions is a central issue in mathematics lessons (subject matter goal 2).

In view of that, one has to agree with the following judgment of the teacher:

> **Teacher:** Linear functions are an elementary but important subclass of functions and are suitable for grade 8 (subject matter belief 3).

This leads to the following strengthened formulation as a goal:

> **Teacher:** The treatment of linear functions is to be given more attention in grade 8 (subject matter goal 4).

The now following belief could be presented in various, slightly different perspectives, but its essential point is:

> **Teacher:** Linear functions are defined by their slopes. The slope of a linear function is its most important characteristic (subject matter belief 5).

The teacher reveals another relevant aspect by referring to the needs of future studies:

> **Teacher:** Functions are important for calculus in grade 12 (subject matter belief 6).

Extending the content of the subject matter belief 5 concerning the significance of the slope of a linear function, the teacher underlines the following belief:

> **Teacher:** The central term to be mediated in the context of linear functions is the concept of slope, which prepares students for the concept of derivative (subject matter belief 7).

From this results the following specific mathematical goal, which can also be identified as a kind of output directive for the lesson:

> **Teacher:** The term slope must be mentioned in this lesson (subject matter goal 8).

It appears that the aforementioned goals describe an implication structure in the sense of a content hierarchy, finally arriving at the central subject matter goal 8. Likewise, the subject matter belief 6 that functions are important for Calculus, and the corresponding goal 4 that linear functions should be given more attention in grade 8, can be understood as important propaedeutical arguments, which in the last instance characterize unavoidably the subject matter goal 8 that the term slope needs to be accomplished.

As could be observed in the videoed lesson, it seems that the teacher never put into question this implication structure. Even in spite of the consequences that occurred during the lesson while consequently adhering to the central subject matter goal 8, she gave up her initial teaching approach and tried repeatedly to get the students to this central mathematical goal. In the interview, the teacher stated that being fixated on reaching this goal in that lesson proved to be a mistake. However, she did not question the importance of this fundamental goal, which probably could have been easily reached in the next lesson.

Interpretative Remarks on the Goals and Beliefs Structure

The previous discussion has made clear that diverse goals and beliefs cannot be simply understood as a list one can pull together according to certain overriding categories. Although it makes sense to bundle them together according to general characteristics, it should also be admitted that these are not the only relations between them. In any case, one can ascertain a deductive structure given by overriding and derived goals and beliefs that is influenced finally by mutual correlations, and reminds of Green's (1971) categorization. However, the assessments and prioritizations changed in the course of the lesson as could be shown by the analysis provided in the sections 'Formal Goals', 'Pedagogical Content Goals and Beliefs, and Their Networking', 'Subject Matter Goals and Beliefs and Their Internal Structure'.

Besides, we assign greater relevance to another mechanism, i.e., the observed uncoupling of the pedagogical content *belief bundles,* which are shown in their initial network in Fig. 1. At first, beliefs centered on the role of the computer are dominant, produce all the other connections, and are central for the conception of the lesson. As the lesson was progressing, an interesting phenomenon appeared: The use of the computer became problematic and denoted the decisive turning point in the lesson when losing its central role. The moment the teacher realized that she could not achieve her central subject matter goal that is to introduce the term slope, she let the students simply switch off the computer. As the computer lost its important role, the *belief bundles* concerning *open lessons*, *discovery lessons* and *motivation* played only marginal roles afterwards. From this point onwards, global subject matter and formal goals dominate the lesson activities to reach the one goal: *The term slope must be mentioned.* In other words, all pedagogical content goals and beliefs lost their rather positive value and stepped aside to make room for subject matter goals and beliefs.

In a deliberately provocative formulation, subject matter related goals and beliefs might be called *hard* and pedagogical content goals and beliefs *soft*. A teacher who participated in a discussion on this lesson commented aptly the situation: *When the house is on fire, who will then worry about pedagogy? Then you can rely only on the systematic nature of the content.* Obviously, pedagogy then loses out in the game pedagogy versus content (Wilson and Cooney 2002).

Conclusions

It was not the objective of this paper to conduct an analysis of the lesson with equal attention to all aspects. Thus, one could continue with further explanations for the observed phenomenon. However, we assign Schoenfeld's KGB framework convincing explanatory power, which has enabled us to illuminate central focal points. The dominance of the computer in the lesson plan is both its strength and its weakness, and thus presents a risk factor for a successful unfolding of the lesson. Switching off the computer after 20 minutes rendered its mediating function obsolete (Noss and Hoyles 1996). This regressive action decomposed and separated the well-intended pedagogical content goals and beliefs, and made room for an approach dominated by focusing on systematic subject matter content.

We have only marginally mentioned the linking of beliefs and goals to the available teacher's knowledge, e.g., to the teacher's available action scripts. Actually, here we find another reason for the turning point in the lesson: The teacher had not hitherto developed a sufficiently solid repertoire in employing Excel for the introduction of the concepts concerning linear functions, but she possesses sufficient experience for an introduction by a reliable and robust traditional approach.

A further deficit has also become apparent to the authors while dealing with this theme: There are no papers or research dealing with topological characteristics and the interweaving of these networks of beliefs and goals. Moreover, the gradation of beliefs according to their valences as could be shown in our analysis, provides illuminating insight into understanding a teacher's actions in the classroom against the background of the KGB framework.

In terms of the theoretical implications of the study and its relation to the larger scheme mathematics education, it is still unclear today whether beliefs theories should be developed independently of philosophy or classified within a framework of epistemological considerations, this are seldom recognized in most papers. The term 'epistemology' is rarely mentioned explicitly but is appropriate for any discussion of beliefs, since the teacher's knowledge, goals and actions ultimately rest on some philosophical convictions, and more specifically on their epistemology of what it means to know mathematics.

Acknowledgements The authors acknowledge gratefully the support of the Robert Bosch Foundation for the teacher workshop that generated the data for this study.

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